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Composite: Curved Plate Girder Load Factor Design

Introduction

This chapter discusses and illustrates design of a two-span highway bridge on horizontally curved alignment, utilizing curved I-shaped plate girders. Girders are connected to each other at regular intervals by crossframe-type diaphragms. There is no other system of lateral bracing between girders at either flange level. A reinforced concrete deck slab acts compositely with the steel girders.

Horizontally-curved plate girders are suitable for simple and continuous spans of lengths similar to those for which straight plate girders are suitable, as outlined in Chapters 4 and 4A. Curved plate girders are used for grade separation and elevated bridges, where the structure must conform to the curved roadway alignment. This condition occurs most frequently at urban crossings and interchanges but may also be found at rural intersections where the structure must follow the highway's geometric requirements.

The design example of this chapter is in accordance with the 12th Edition of the "Standard Specifications for Highway Bridges" of the American Association of State Highway and Transportation Officials (AASHTO), including the 1977 through 1982 "Interim Specifications" (all hereinafter referred to as the AASHTO Specifications), as modified by the "AASHTO Guide Specifications for Horizontally Curved Highway Bridges," 1980 (hereinafter referred to as the Guide Specifications). The design uses ASTM A36 and ASTM A572 Grade 50 steels.

General Design Considerations

Curved I-girders are of the same general construction as straight I-girders. The curvature is achieved either by cutting the flanges to a curve or by fabricating the girder straight and heat curving it. Crossframes between girders are usually spaced more closely than the 25 ft maximum allowed for straight girders. A spacing range of 12-18 ft is very common for ordinary curved grade separation structures of moderate length; the more sharply curved the structure, the more closely spaced are the crossframes. Lateral bracing is used only when it is needed to carry wind loads, just as for straight bridges. Recently adopted provisions of the AASHTO Specifications permit elimination of lateral bracing in most bridges and this is the course that should be taken for maximum economy whether the bridge is straight or curved.

LOADS AND LOAD COMBINATIONS

Composite construction is equally applicable to straight or curved I-girder bridges. Initial dead load (DL_1), superimposed dead load (DL_2), and live load plus impact ($L+I$) can be computed by the rules and principles outlined in Chapters 3, 3A, 4 and 4A. Load factors for the Group loadings are those defined by the Specifications.

Several different loadings must be considered:

A) Construction Loads

For certain structures, construction loads may govern the design of some sections. If this possibility exists, the load combination $1.3(D_p + C)$ should be examined, where D_p is a partial dead load and C is a load due to construction equipment.

B) Service Loads

Service loads consist of the dead load D , plus live load with impact ($L+I$), plus centrifugal force CF . The service load $D + (L+I) + CF$ is applied to the structure to determine stress range to be used in checking resistance to fatigue and to determine the live load deflection.

C) Maximum Design Loads

All sections of the bridge must be proportioned for sufficient strength to resist the forces due to the loading, $1.3 [D + \frac{5}{3} (L+I) + CF]$.

D) Overload

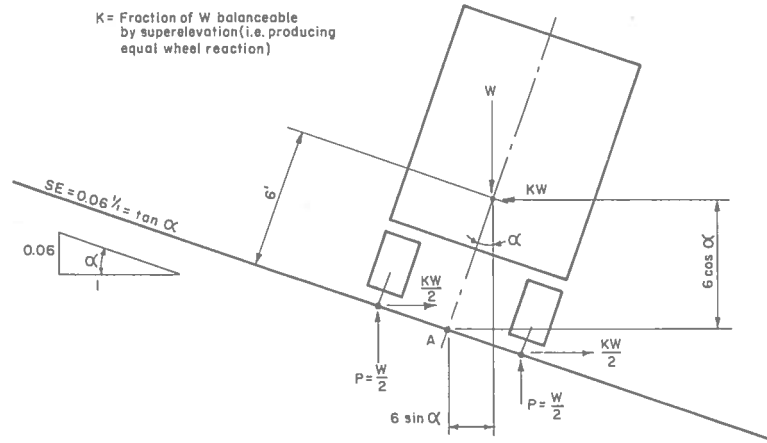
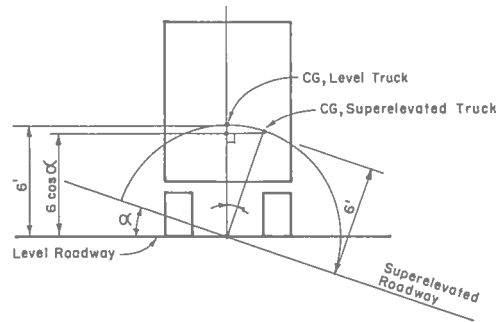
In addition to meeting the maximum strength requirement of (C), each girder section must satisfy performance criteria under the loading $D + \frac{5}{3} (L+I) + CF$.

The foregoing relationships show that centrifugal force is a primary load that must be accounted for in the design of any curved I-girder bridge. Centrifugal force is a horizontal force acting on the live load 6 ft above the deck and transmitted to the structure through the wheels of the vehicle. The entire horizontal force is assumed to be carried to the bridge bearings by way of the deck slab in transverse bending and the crossframes at each support in transverse shear. No particular account need be taken of the horizontal effect of centrifugal force on the bridge superstructure except that the support crossframe diagonals must be adequately proportioned to deliver the load to the bearings.

Because centrifugal force acts 6 ft above the deck it also causes an overturning moment. Usually, part of this overturning moment is balanced by the superelevation. The remainder is assumed to act on the steel framing treated as a pile group supporting the slab, producing a downward loading on girders outside the centerline of the bridge and an upward loading on girders inside the centerline of the bridge.

A derivation of the fraction of centrifugal force overturning moment balanceable by superelevation is given in the example below in which the superelevation is assumed to be 0.06 ft/ft.

If wheel reactions, P , are to be equal, moments about Point A must be zero. By inspection the moments due to wheel reactions cancel one another and it can be written simply:



$$N (6 \sin \alpha) - KW (6 \cos \alpha) = 0$$

$$6 \sin \alpha W = 6 \cos \alpha KW$$

$$K = \frac{6 \sin \alpha W}{6 \cos \alpha W} = \tan \alpha = SE = 0.06$$

It is seen that the fraction of W that can be balanced by superelevation is equal numerically to the superelevation rate.

DESIGN THEORY AND STRUCTURAL ANALYSIS

The Specifications require that the structure be analyzed as a system, taking into account the complete distribution of loads to various members. A number of very general computer programs are available which rigorously analyze large structural systems using variations of classical indeterminate structural analysis theory. These programs give excellent results. They do require large amounts of input, large computers, and considerable computer time, inasmuch as many load cases must be run in order to develop live load envelopes.

The United States Steel V-Load Method offers an alternative approach. Although it is an approximate solution, it does rationally and systematically treat the entire structure. The method was developed in 1963 and improved in 1965. A survey in 1973 indicated that 75 percent of curved bridges, designed in the United States up to that time, were designed by the V-Load Method.

In its original form, the method was limited to structures with radial supports. In 1982 another improvement was made, extending the applicability of the method to include structures with skewed supports. The derivation and background of the theory are presented in the USS Highway Structures Design Handbook, Chapter I/12.

The V-Load Method has a number of positive attributes:

- 1) It is rational, easily understood, and helps to improve perception of curved bridge behavior.
- 2) Its approach and order of accuracy are quite consistent with that currently practiced for straight girders.
- 3) It is readily programmed on small to medium size computers.
- 4) It analyzes the structure as a system and thus satisfies the Specification requirement that the structure be so treated.
- 5) It lends itself to automatic generation of live load envelopes, eliminating hand manipulation of loads.
- 6) Its results have shown excellent agreement with results from rigorous solutions for a variety of structures.

The V-Load Method has been implemented on the computer in a program entitled VLOAD,* which is available on a time-sharer basis from USS Engineers and Consultants, Inc. (UEC), a division of United States Steel Corporation.

The V-Load Method provides values for dead load moments, shears, reactions and deflections, as well as live load moment and shear envelopes, and live load deflections for each girder. These values all result from normal vertical bending, which consists of two parts: The first part is vertical bending due to the applied loads acting on the girders as if they were straight. This is called *primary bending moment*. The second part is additional vertical bending resulting from the shears developed at the ends of the crossframes, which act as concentrated loads on the girder. These are called *secondary moments* and are additive to the primary moments on girders outside the centerline of the bridge, and subtractive from the primary moment on girders inside the centerline of the bridge. The stress caused by vertical bending is computed by the ordinary mechanics of composite girders as described in Chapters 3, 3A, 4 and 4A. It is designated as f_b .

The table below shows a comparison of the V-Load-analysis results to MSC/NASTRAN finite-element-analysis results for peak moments in the two-span structure discussed later in this chapter. The moments tabulated are from a preliminary analysis, in which the girder sections have been estimated. The standard AASHTO distribution factors for straight girders were used to compute the L+I loads on the girders for determining the primary moments in the V-Load analysis.

*For additional information on this program contact USS Engineers and Consultants, Inc., Room 1614, 600 Grant St., Pittsburgh, PA 15230, Tel. (412) 433-7512.

MOMENT		SPAN 1			PIER			SPAN 2		
		NASTRAN	VLOAD	CUGAR	NASTRAN	VLOAD	CUGAR	NASTRAN	VLOAD	CUGAR
G1	DL ₁	1393	1383	1388	-2133	-2201	-2111	608	616	579
	DL ₂	521	527	513	-641	-669	-635	248	258	231
	L+I	1535	1606		-763	-682		1139	1247	
	TOTAL	3449	3516		-3537	-3552		1995	2121	
G2	DL ₁	1206	1198	1202	-1946	-1921	-1927	534	507	515
	DL ₂	459	445	446	-565	-558	-563	221	209	205
	L+I	1100	1518		-514	-645		882	1182	
	TOTAL	2765	3161		-3025	-3124		1637	1898	
G3	DL ₁	1007	986	998	-1687	-1647	-1666	416	395	409
	DL ₂	363	354	358	-479	-467	-472	173	161	164
	L+I	897	1271		-376	-600		670	977	
	TOTAL	2267	2611		-2542	-2714		1259	1533	
G4	DL ₁	771	760	767	-1288	-1339	-1288	320	304	324
	DL ₂	267	265	271	-351	-372	-365	130	123	127
	L+I	919	1057		-391	-520		745	752	
	TOTAL	1957	2082		-2030	-2231		1195	1179	

Note: CUGAR solution not available for L+I

Inspection of the table reveals excellent correlation of all moments for the exterior girders G1 and G4, and equally good correlation of DL₁ and DL₂ moments for interior girders G2 and G3. For L+I moments in girders G2 and G3 the V-Load results are conservative, leading to the conclusion that the AASHTO live load distribution factor for interior girders is conservative. *The accuracy of the V-Load Method for live load is totally dependent upon the accuracy of the distribution factors that are used to compute the primary moments.* Further research is needed to develop better live load distribution factors. For lack of such development at present, this example will rely solely on the AASHTO distribution factors for straight bridges. The above table indicates that the total moments on girders G2 and G3 will be approximately 10 percent conservative on the average. Stress levels will be even less conservative. *more?*

In addition to vertical bending, lateral flange bending occurs in all curved members. As shown in Chapter I/12, the lateral flange bending moment is given by

$$M_{\text{Lateral}} = \frac{M_{\text{Vertical}} d^2}{12 R h}$$

where

M_{Vertical} = total vertical bending moment at the girder section in question

d = diaphragm or crossframe spacing along the girder

R = radius of girder

h = depth of girder, center to center of flanges

The stress caused by lateral flange bending is designated as f_w and is obtained by

$$f_w = \frac{M_{\text{Lateral}}}{S_{\text{Lateral}}}$$

where

M_{Lateral} = lateral flange bending moment as computed above

S_{Lateral} = lateral section modulus of flange

The stresses f_b and f_w are additive at the tip of the flange.

These stresses, due to secondary bending moment and lateral flange bending moment, represent the effect of curvature. The more sharply curved the bridge, the greater the portion of stress attributable to curvature. The true measure of the curvature sharpness of a girder is the central angle subtended by the span length. Small central angles exist when there is little curvature. The Guide Specifications recognize the fact that for very small central angles there will be relatively little secondary bending effect. Table 1.4A of the Guide Specifications permits vertical secondary bending to be neglected under the conditions indicated.

Guide Specifications
TABLE 1.4A
Limiting Central Angle for Neglecting Curvature in
Determining Primary Bending Moments

Number of Girders	Angle for 1 span	Angle for 2 or more spans
2	2°	3°
3 or 4	3°	4°
5 or more	4°	5°

However, neglecting lateral flange bending is never permitted, because this is a function not only of central angle, but also of crossframe spacing and flange width.

DIAPHRAGMS AND CROSSFRAMES

The Guide Specifications require that crossframes or diaphragms “be provided at each support and at intermediate intervals between supports with spacings as determined by design considerations.” It has already been stated that 12-18 ft spacing is a very common range. The formula for lateral bending moments indicates that they are proportional to the square of the crossframe spacing. Thus, an increase in the crossframe spacing will require an increase in girder flange material to resist the resulting higher lateral bending moment. Conversely, if crossframe spacing is reduced, lateral bending moments will be reduced, and correspondingly less girder flange material will be needed; however, more crossframe material will be required. Selection of crossframe spacing therefore becomes a matter of judgment in choosing the most economical trade-off of girder flange material versus crossframe material.

Crossframes in curved I-girder bridges carry calculated stresses and therefore must be designed as main structural members, with each line of crossframes extending in a single plane across the width of the bridge. For crossframe members, tee sections are preferable to angles because they are symmetrical about the Y-axis and therefore carry their loads concentrically. Either “X”- or “K”-type framing may be used, depending on the geometry of the bay. The Guide Specifications require that crossframes be full depth members and that they “be framed in such a way as to transfer the horizontal and vertical forces to the flanges and web as necessary.” It is also stipulated that crossframe connection plates must be attached to both the web and the flanges of the girder.

The primary forces on curved girder crossframes are those resulting from curvature, that is, from the interaction of girders and crossframes in carrying $D + L + I + CF$ loads. This explains why they are considered main structural elements. It also means that they must resist fatigue. Members and their end connections must be examined for strength and also for assurance that stress ranges are within allowable values.

The crossframes also carry wind and CF loads. The following assumptions are made for design of crossframes in an I-girder bridge:

- 1) The crossframes carry the *curvature* effect of all vertical loads (DL_1 , DL_2 , $L + I$ and $CF_{Vert.}$) in accordance with V-Load theory.
- 2) The slab carries wind on the upper half of the structure, wind on live load, and centrifugal force ($CF_{Horiz.}$) back to the support crossframes, which in turn carry these forces down to the bridge bearings.

3) The wind on the lower half of the structure is carried through the intermediate crossframe diagonals up to the slab, and then to the bridge bearings through the support crossframes as described in 2) above. The support crossframes therefore transmit *all* wind load down to the bridge bearings.

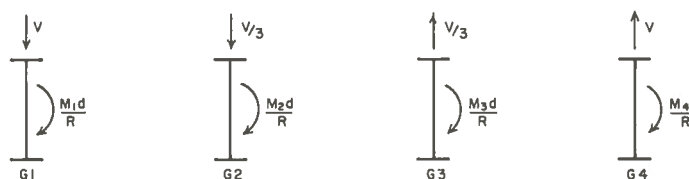
Three distinct type of crossframes may be proportioned for the forces applicable to each type and the Group loadings that govern, as indicated below:

Crossframe Type	Effects to be Considered	Group Loadings to be Checked
End Support	Wind, Centrifugal Force	Group I: $1.3 [DR + \frac{5}{3} (TR) + CF_H]$
	Dead Load & Truck Wheel Loading	Group II: $1.3 [DR + W_T + W_B]$ Group III: $1.3 [DR + TR + CF_H + 0.3 (W_T + W_B) + WL]$
Intermediate	Curvature	Group I: $1.3 [D + \frac{5}{3} (L + I) + CF_V]$ Group II: $1.3 [D + W_{Bp}]$
	Wind	Group III: $1.3 [D + (L + I) + CF_V + 0.3 W_{Bp}]$
Interior Support	Curvature	Group I: $1.3 [D + \frac{5}{3} (L + I) + CF_V + CF_H]$ Group II: $1.3 [D + W_T + W_B]$
	Wind, Centrifugal Force	Group III: $1.3 [D + (L + I) + CF_V + CF_H + 0.3 (W_T + W_B) + WL]$

where

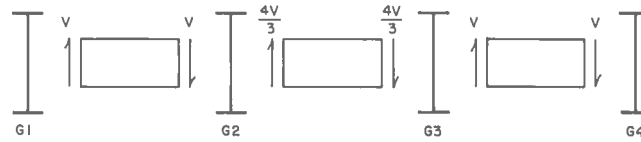
- D = dead load curvature
- L + I = live load plus impact, curvature
- W_T = wind on upper half of structure
- W_B = wind on lower half of structure
- W_L = wind on live load
- CF_V = vertical centrifugal force
- CF_H = horizontal centrifugal force
- DR = dead load on top strut
- TR = truck wheel loading on top strut
- W_{Bp} = panel wind load, lower half of structure

The crossframe forces due to curvature are easily computed from V-Load theory.* Looking at any crossframe line in the positive moment region of a four-girder system, the assumed distribution of V-Loads and torque loads to the girders is as shown below.

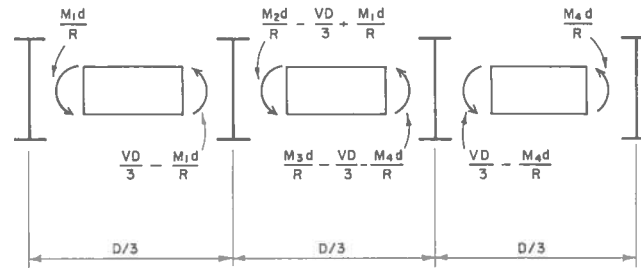


*See USS Highway Structures Design Handbook, Vol. I, Chap. 12, V-Load Analysis.

The corresponding distribution of end shears on the crossframes follows.



For equilibrium, the distribution of end moments on the crossframes can then be worked out.



In the above figures

$$V = \frac{M_{1p} + M_{2p} + M_{3p} + M_{4p}}{C(RD/d)}$$

$M_{1p}, M_{2p}, M_{3p}, M_{4p}$ = primary moments in G1, G2, G3, G4 at crossframe line under investigation

M_1, M_2, M_3, M_4 = total moments in G1, G2, G3, G4 at crossframe line under investigation

D = width, G1 to G4

d = crossframe spacing on G1

R = radius of G1

C = a constant ($10/9$ for a four-girder system)

Forces from curvature, wind, centrifugal force and direct wheel loading are added together for design, in accordance with the Group loadings.

GIRDER SECTION DESIGN

The Guide Specifications give allowable normal stresses that depend on whether the compression flange qualifies as compact or non-compact. Compact compression flanges may be stressed to full plasticity without local buckling; non-compact flanges are more slender and therefore subject to buckling. The expressions for allowable stress involve the geometric parameters of the girder such as unbraced length of compression flange, flange width and radius (l , b and R , respectively). They also include the ratio of the warping (lateral bending) normal stress to the vertical bending stress, $\frac{f_w}{f_b}$. Thus, the actual stresses must be known in order to compute the allowable stress.

There are two limitations on the unbraced compression flange length

$$l \leq 25(b)$$

$$\text{and } l \leq 0.1R$$

There are also several provisions regarding $\frac{f_w}{f_b}$. This parameter, and the allowable stress expressions that follow below, apply to an entire unbraced length of flange. If the section is

prismatic over the unbraced length, the stress f_b is computed using the larger of the two bending moments at either end of the segment, and f_w is due to the corresponding lateral bending moment at that location. If the section changes within the unbraced length, f_b and f_w are computed for the smaller section at the location of the change in section, using the moment at that point. The ratio $\frac{f_w}{f_b}$ is a signed quantity defined as positive for compression flanges and negative for tension flanges. In addition, its absolute value $\frac{f_w}{f_b}$ must not exceed 0.5, except under low stress conditions not governing the design of the section.

Compactness is defined by the $\frac{b}{t}$ ratio of the compression flange. When $\frac{b}{t} \leq \frac{3,200}{\sqrt{F_y}}$ the flange is compact. In this case the vertical bending stress in the compression flange, f_b , is limited to

$$F_{bu} = F_{bs} \bar{\rho}_B \bar{\rho}_w$$

where

$$\bar{\rho}_B = \frac{1}{1 + \frac{1}{b} \left(1 + \frac{1}{6b} \right) \left(\frac{1}{R} - 0.01 \right)^2}$$

$$\bar{\rho}_w = 0.95 + 18 \left[0.1 - \frac{1}{R} \right]^2 + \frac{f_w}{f_b} \left[0.3 - 0.1 \frac{1}{R} \frac{1}{b} \right]$$

$$\rho_B / F_{bs} / F_y$$

$$\bar{\rho}_B \bar{\rho}_w \leq 1$$

$$F_{bs} = F_y (1 - 3\lambda^2), \text{ with } \lambda = \frac{1}{\pi} \left(\frac{1}{b} \right) \sqrt{\frac{F_y}{E}}$$

For the tension flange the allowable stress is given by the same expressions except that

$$F_{bs} = F_y.$$

A non-compact compression flange falls within the range

$$\sqrt{\frac{3200}{F_y}} \leq \frac{b}{t} \leq \sqrt{\frac{4,400}{F_y}}. \text{ (In no case is } \frac{b}{t} \text{ allowed to exceed } \sqrt{\frac{4,400}{F_y}}.)$$

For non-compact compression flanges, f_b is limited to

$$F_{by} = F_{bs} \rho_B \rho_w$$

where

$$\rho_B = \frac{1}{1 + \frac{1}{R} \frac{1}{b}}$$

$$\rho_{w1} = \frac{1}{1 - \frac{f_w}{f_b} \left[1 - \frac{1}{75b} \right]}$$

$$\rho_{w2} = \frac{0.95 + \frac{1/b}{30 + 8000(0.1 - 1/R)^2}}{1 + 0.6(f_w/f_b)}$$

$$\rho_w = \text{smaller of } \rho_{w1} \text{ and } \rho_{w2} \text{ if } \frac{f_w}{f_b} \text{ is positive}$$

or

$$\rho_w = \rho_{w1} \text{ if } \frac{f_w}{f_b} \text{ is negative}$$

$$F_{bs} = F_y (1 - 3\lambda^2), \text{ with } \lambda = \frac{1}{\pi} \left(\frac{1}{b} \right) \sqrt{\frac{F_y}{E}}$$

Again, for the tension flange the allowable stress is given by the same expressions except that

$$F_{bs} = F_y.$$

In addition to these limitations on f_b , both tension and compression flanges for the non-compact case are subject to the following limit on flange tip stress:

$$f_b + f_w \leq F_y$$

This limit on flange tip stress does not apply to the case in which the compression flange is compact. The reason is that the allowable stress expressions for this case are based on an ultimate strength taken as the fully plastic strength. Since all fibers are stressed to the yield point under fully plastic bending, lateral bending stress is not a meaningful parameter.

The Guide Specifications do not recognize any transition curve between the allowable stresses for compact and non-compact flanges. This means that there is an abrupt drop in allowable stress when the compression flange goes from compact to non-compact. Some experimentation by the designer may be necessary to determine what type of compression flange is most economical for a particular section. A thicker compression flange is needed for compactness than for non-compactness, but the tension flange may be considerably thinner for the former case than for the latter, because of the significant increase in allowable stress. Experience generally indicates that sections with compact compression flanges are more economical than sections with non-compact compression flanges.

WEBS

Provisions in the Guide Specifications for webs do not consider the post-buckling strength resulting from tension field action. Thus, there are no special rules for spacing of stiffeners adjacent to an end reaction point, and generally the criteria for web design are simple.

Transverse web stiffeners are not required if $\frac{D}{t} \leq 150$, and the ultimate shear capacity of unstiffened webs is the smaller of

$$V_u = \frac{3.5Et^3}{D}$$

or

$$V_u = 0.58 F_y D t.$$

If the maximum shear exceeds $\frac{3.5Et^3}{D}$, transverse stiffeners are required for the web. The ultimate shear capacity of a stiffened web is

$$V_u = 0.58 F_y D t C$$

where

$$C = \left[18,088 \left(\frac{t}{D} \right) \sqrt{\frac{1 + (D/d_o)^2}{F_y}} \right] - 0.3 \leq 1.0$$

Both stiffened and unstiffened webs are subject to the interaction equation

$$\frac{M}{M_u} = 1.375 - 0.625 \frac{V}{V_u} \text{ when } V > 0.6 V_u.$$

When transverse web stiffeners are required they must be spaced at a distance, d_o , not exceeding D , the depth of the girder. Their proportions are subject to the following rules:

- 1) $\frac{b}{t} \leq \frac{2,600}{F_y}$ (AASHTO/Art.10.48.5.5)
 - 2) $\frac{b}{t} \leq 16$
 - 3) $b \geq 2 + \frac{D}{30}$
- } (AASHTO/Art.10.34.5.1)

where

b = stiffener width
t = stiffener thickness

The stiffeners must satisfy the rigidity requirement given below for moment of inertia about the mid-plane of the web

$$I = d_o t^3 J$$

where

$$J = [2.5 (D/d_o)^2 - 2] X, \text{ but not less than } 0.5$$

$$X = 1 \text{ when } d_o/D \leq 0.78$$

$$X = 1 + \left[\frac{d_o/D - 0.78}{1.775} \right] Z^4 \text{ when } 0.78 \leq d_o/D \leq 1.0$$

$$Z = \frac{0.95 d_o^2}{Rt}$$

R = radius of web

t = web thickness

Longitudinal stiffeners are required if

$$\frac{D}{t} > \frac{36,500}{\sqrt{F_y}} \left[1 - 8.6 \left(\frac{d_o}{R} \right) + 34 \left(\frac{d_o}{R} \right)^2 \right]$$

When one longitudinal stiffener is placed a distance $\frac{D}{5}$ from the compression flange, the web proportions must satisfy

$$\frac{D}{t} \leq \frac{73,000}{\sqrt{F_y}} \left[1 - 2.9 \sqrt{\frac{d_o}{R}} + 2.2 \left(\frac{d_o}{R} \right) \right]$$

but when another equal sized stiffener is located a distance $\frac{D}{5}$ from the tension flange, the web need only satisfy the requirement

$$\frac{D}{t} \leq \frac{73,000}{\sqrt{F_y}}$$

The longitudinal stiffener proportions are governed by the same AASHTO requirements as for straight girders.

SHEAR CONNECTORS

Design of shear connectors for fatigue is the same as that for straight girders as given in the AASHTO Specifications. For ultimate strength, the Guide Specifications require that the number of shear connectors between points of maximum positive moment and the end supports or dead-load inflection points be sufficient to satisfy:

$$P_c \leq \phi S_u$$

where ϕ = reduction factor = 0.85

S_u = ultimate strength, kips, of the shear connector as given in the AASHTO Specifications for straight girders

P_c = force, kips, on the connector

$$= \sqrt{P^2 + F^2 + 2PF \sin \frac{\Theta}{2}}$$

$$P = \frac{P}{N}$$

P = $0.85f'_c b c$ or $A_s F_y$, whichever is smaller, at points of maximum positive moment
= $A_r F_y$ at points of maximum negative moment as defined by the AASHTO Specifications for straight girders

N = number of connectors between points of maximum positive moment and adjacent end supports or dead-load inflection points, or between points of maximum negative moment and adjacent dead-load inflection points

$$F = \frac{P(1 - \cos \Theta)}{4KN, \sin \Theta/2}$$

Θ = angle extended between point of maximum moment (positive or negative) and adjacent point of contraflexure or support

f'_c = 28-day compressive strength of concrete slab, ksi

b = effective width of slab, inches

c = thickness of slab, inches

A_s = total area of steel section, including cover plates, sq. inches

A_{r_s} = total area of longitudinal reinforcing steel at the interior support within the effective width of flange, sq. inches

F_r = yield strength of the reinforcing steel, ksi

$$K = 0.166 \left(\frac{N}{N_s} - 1 \right) + 0.375$$

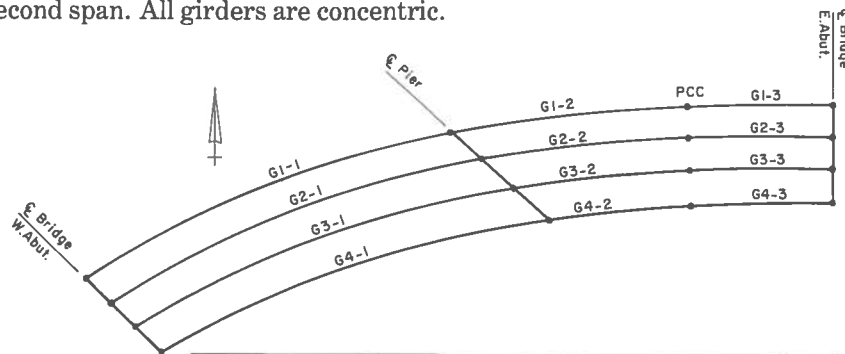
N_s = number of connectors at a section

Design Example

Two-Span Continuous I-Girder Bridge

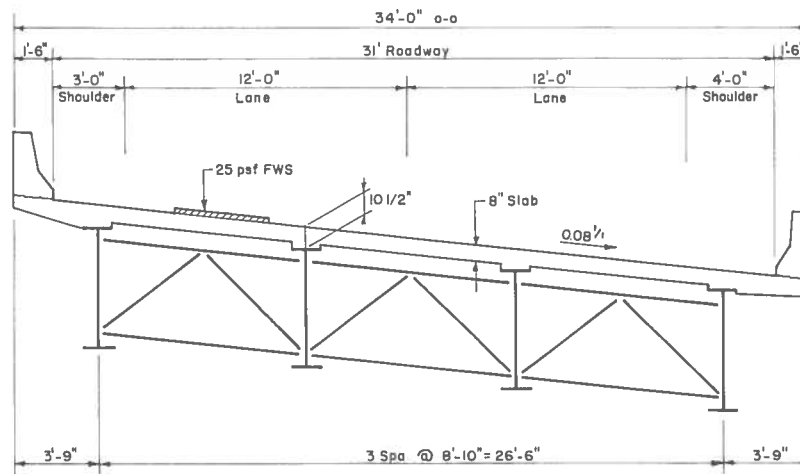
Composite for Positive and Negative Moment

A curved bridge with two spans and four girders is selected as the example structure for this chapter. A plan view appears below. The centerlines of bearing at the west abutment and center pier are parallel to an underpassing roadway and skewed to the centerline of bridge. The centerline of bearing at the east abutment is radial to the centerline of bridge. The radius of Girder G1 changes from 300 ft to 600 ft at a point of compound curvature in the second span. All girders are concentric.



Girder	Radius	Arc Length	Central Angle Δ°
G1-1	300.0000	110.0000	21.00844
G1-2	300.0000	60.0000	11.45917
G1-3	600.0000	40.0000	3.81972
G2-1	291.1667	110.4751	21.73931
G2-2	291.1667	53.1651	10.46183
G2-3	591.1667	39.4111	3.81972
G3-1	282.3333	111.0056	22.52711
G3-2	282.3333	46.2739	9.39067
G3-3	582.3333	38.8222	3.81972
G4-1	273.5000	111.6008	23.37936
G4-2	273.5000	39.3167	8.23650
G4-3	573.5000	38.2333	3.81972

A cross section of the bridge appears below:



Typical Bridge Cross Section

The following data apply to this design:

Specifications: 1977 AASHTO Standard Specifications for Highway Bridges, 1977 through 1982 Interim Specifications and 1980 Guide Specifications for Horizontally Curved Highway Bridges.

Loading: HS20-44.

Structural Steel: ASTM A36 and A572, Grade 50.

Concrete: $f'_c = 4,000$ psi, modular ratio $n = 8$.

Slab Reinforcing Steel: ASTM A615, Grade 40, with $F_y = 40$ ksi.

Loading Conditions:

Case 1—Weight of girder and slab (DL_1) supported by the steel girder alone.

Case 2—Superimposed dead load (DL_2) (parapets and railings) supported by the composite section with the modular ratio $n=8$. (Used in design of web-to-flange fillet welds.)

Case 3—Superimposed dead load (DL_2) (parapets and railings) supported by the composite section with the increased modular ratio $3_n = 3 \times 8 = 24$.

Case 4—Live load plus impact ($L+I$) supported by the composite section with the modular ratio $n = 8$.

Fatigue—500,000 cycles of truck load
100,000 cycles of lane loading } Redundant load-path structure.

Loading Combinations:

Combination A = Case 1+3+4

Combination B = Case 2+4

Combination C = Case 1+2+4

LOADS, SHEARS AND MOMENTS ON GIRDERS

The loads on the exterior and interior girders are computed in the same manner as illustrated in Chapters 3, 3A, 4 and 4A.

Exterior Girders

Dead Load Carried by Steel Section

$$\text{Slab} \quad \frac{8}{12} \times (3.75 + 4.42) \times 0.150 = 0.817$$

$$\text{Haunch} \quad 0.17 \times 1.33 \times 0.15 = 0.034$$

$$\frac{1}{2} \times 0.30 \times 3.08 \times 0.15 = 0.069$$

$$\text{Girder, diaph's (assumed weight)} = 0.290$$

$$\text{DL}_1 \text{ for Exterior Girders} = 1.210 \text{ k/ft}$$

Dead Load Carried by Composite Section

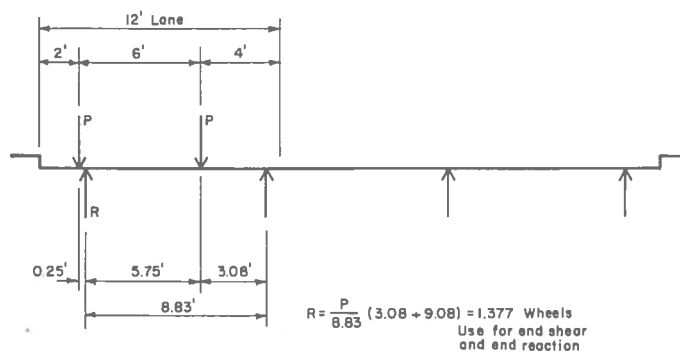
$$\text{Parapet} \quad 0.434 \times 2 \times \frac{1}{4} = 0.217$$

$$\text{Future Wearing Surface} \quad 0.025 \times 31.0 \times \frac{1}{4} = 0.194$$

$$\text{DL}_2 \text{ for Exterior Girders} = 0.411 \text{ k/ft}$$

Live Load

$$\text{Live Load Distribution} = \frac{S}{4 + 0.25S} = \frac{8.83}{4 + 0.25(8.83)} = 1.423 \text{ wheels}$$



Interior Girders

Dead Load Carried by Steel Section

$$\text{Slab} \quad \frac{8}{12} \times 8.83 \times 0.150 = 0.883$$

$$\text{Haunch} \quad 0.17 \times 1.33 \times 0.15 = 0.034$$

$$\text{Girder, diaph's (assumed weight)} = \underline{0.310}$$

$$DL_1 \text{ for Interior Girders} = 1.227 \text{ k/ft}$$

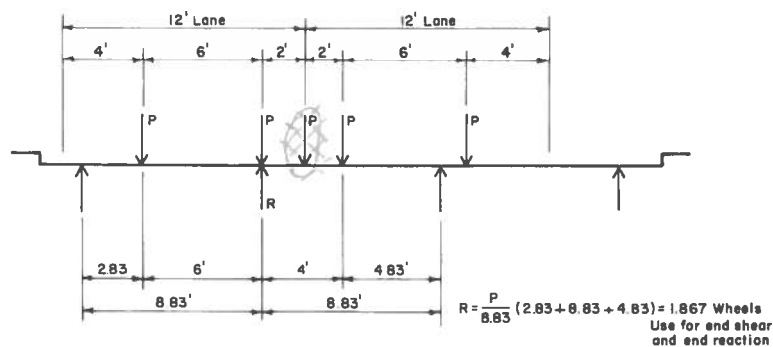
Dead Load Carried by Composite Section

Same as Exterior Girders

$$DL_2 \text{ for Interior Girders} = 0.411 \text{ k/ft}$$

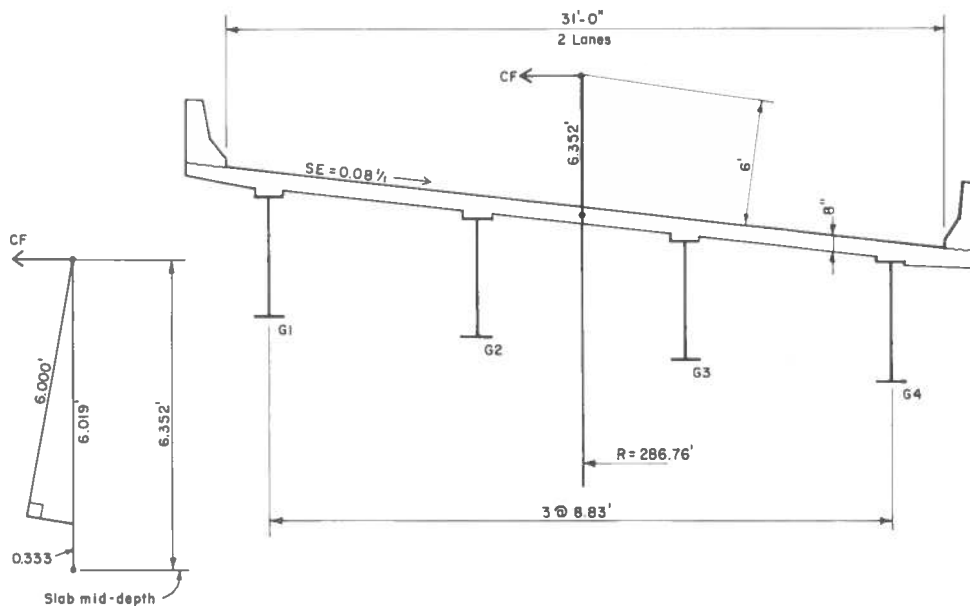
Live Load

$$\begin{aligned} \text{Live Load Distribution} &= \frac{S}{5.5} = \frac{8.83}{5.5} \\ &= 1.606 \text{ wheels} \end{aligned}$$



Centrifugal force is defined by the Specifications as a percentage of the live load, without impact. It is a function of the design speed—taken as 30 miles per hour for this two-lane ramp on a centerline radius of slightly less than 287 ft. The superelevation rate is 0.08 ft per ft, or 8.0 percent. The AASHTO formula (Article 1.2.21) gives a centrifugal force value of 21.0 percent, but 8.0 percent is balanceable by the superelevation,* leaving 13.0 percent as producing an overturning moment about the mid-depth of the slab. With two lanes of live load on the bridge, the CF force in terms of lanes is 0.260 lanes.

*See earlier section "Loads and Load Combinations."



$$S = 30 \text{ mph}$$

$$R = 286.76'$$

$$C = \frac{6.68 S^2}{R} = \frac{(6.68) (30)^2}{286.76} = 21.0\%$$

$$\text{Balanceable by superelevation} = (100) (0.08) = \frac{8.0}{13.0\%}$$

$$CF = (0.130) (2) = 0.260 \text{ lanes}$$

"Pile group distribution" is used to determine the vertical CF loads on the girders due to the overturning moment. The "moment of inertia" of the four girders treated as piles is 389.8 ft². Loads of 0.056 and 0.019 lanes are computed for G1 and G2. The CF vertical loads on G3 and G4 are assumed to be zero since such loads would act upward and be subtractive from the other vertical loads. Using these zero loads represents another loading case, the one corresponding to the situation in which live load is on the structure but not moving.

$$I_{\text{Pile Group}} = 2 \left[\left(\frac{8.83}{2} \right)^2 + (1.5 \times 8.83)^2 \right] = 389.8 \text{ ft}^2$$

$$CF_{\text{load on G1}} = \frac{(0.260)(6.352)(1.5)(8.83)}{389.8} = 0.056 \text{ lanes}$$

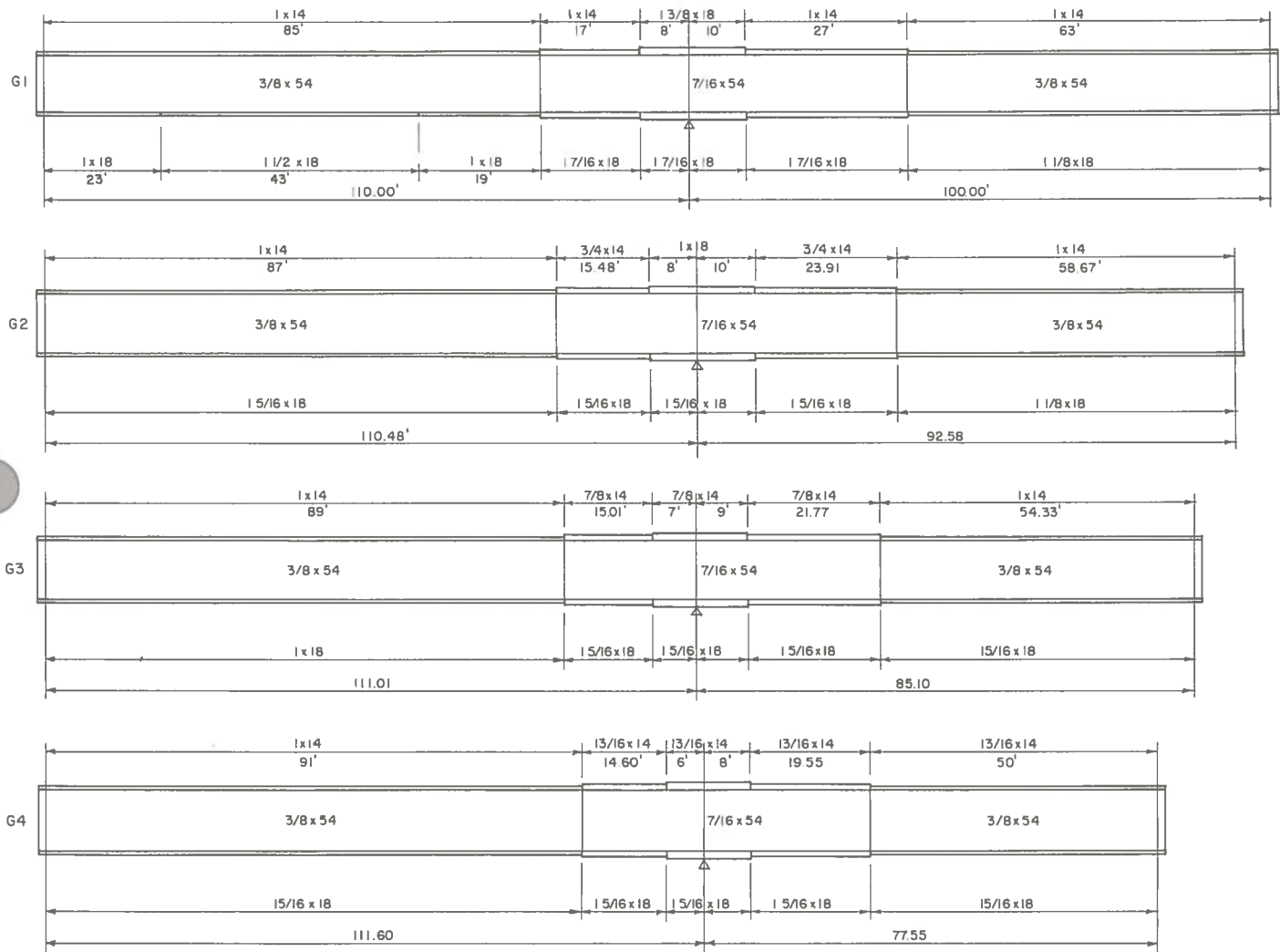
$$CF_{\text{load on G2}} = \frac{(0.260)(6.352)(0.5)(8.83)}{389.8} = 0.019 \text{ lanes}$$

$$CF_{\text{load on G3 \& G4}} = 0 \text{ lanes, by inspection, assuming } S = 0 \text{ mph}$$

We now have all the loads needed for an analysis of the curved bridge. At this point we check the central angle criterion to determine whether the secondary bending due to curvature need be considered. If it does not, there is no need to perform a V-Load analysis of the structure; all girders could be analyzed as if they were straight, with lateral flange bending stress computed and added in.

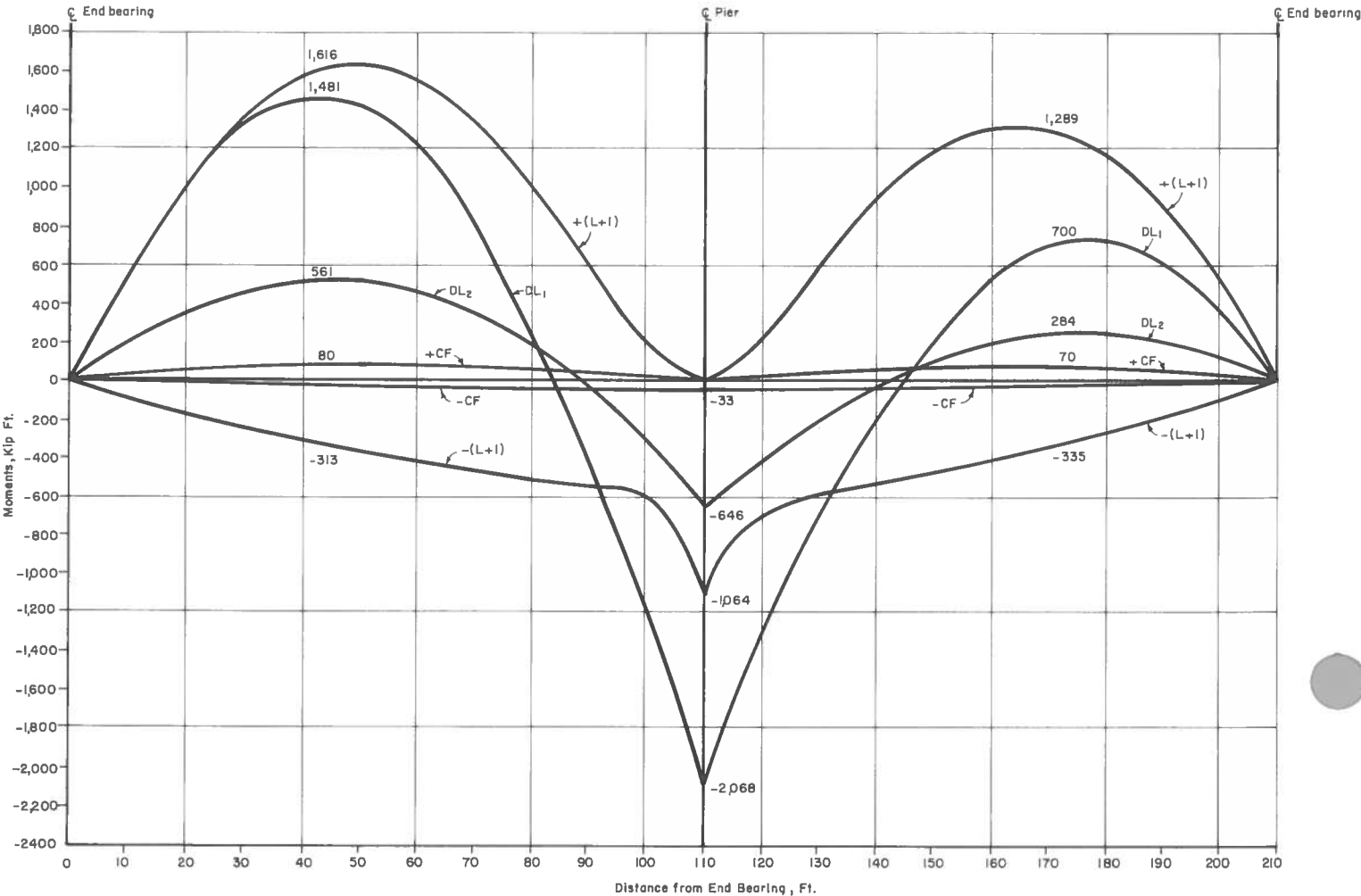
Turning back to the geometry table on page 12, we see that the central angles, Δ , are well beyond the limits of Table 1.4A in the Guide Specifications. Thus, curvature must be considered in all its aspects.

The Computer Program VLOAD is used to perform an analysis for DL_1 , DL_2 and $L+I$, on the basis of the girder sections shown below.

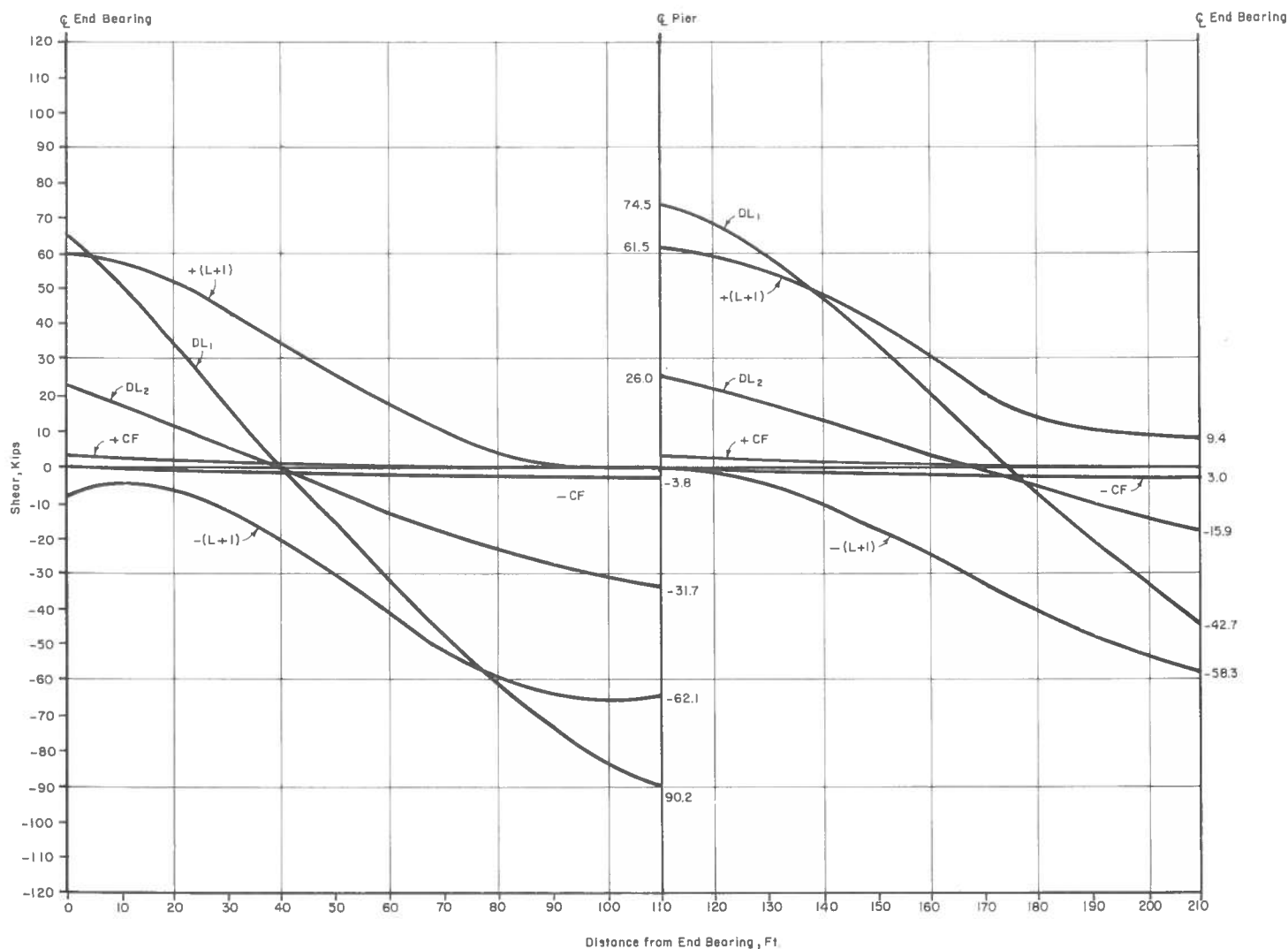


Program VLOAD could also have been used to analyze the CF loading on G1 and G2 but there is a simpler approach. It is recalled that the V-Loads are a function of the summation of primary moment transversely across the structure at each crossframe line. Since the CF loads act downward on G1 and G2, and upward on G3 and G4, the summation of moments is essentially zero and therefore the V-Loads are zero. Thus for centrifugal force loading all that need be analyzed is the primary load effect, which can be done on an isolated straight girder basis.

In order to design Girder G1 the moment and shear curves are plotted as below:



Maximum Moment Curves for Girder G1



Maximum Shear Curves for Girder G1

Although crossframe spacing has been discussed in a general way, it should be noted that the entire analysis of vertical bending in the structure has been carried out *without having actually set the crossframe spacing*. One of the advantages of the V-Load method is that the moments, shears, reactions and deflections are independent of the crossframe locations. We must now set the crossframe spacing so that the detailed operations of design can begin.

Recalling that the most important function of the crossframes is to control lateral flange bending stresses, we could make a preliminary estimate of required crossframe spacing by assuming a reasonable ratio of $\frac{f_w}{f_b}$, say 0.15. However, crossframe steel, with its high labor intensity, is expensive compared to flange steel. Current economic considerations suggest using higher ratios of $\frac{f_w}{f_b}$ for structures as sharply curved as this one. If we assume that f_w is $0.30 f_b$, and we are designing for $f_w + f_b = F_y = 50$ ksi, then $1.30 f_b = 50$ ksi and $f_b = 38$ ksi.

An expression for lateral bending stress can then be derived as follows:

Assume f_b is equal to the flange force divided by the flange area = $f_b = \frac{M_{\text{vertical}}}{h} \div bt$

where

M_{vertical} = vertical bending moment
 h = depth of girder, center to center of flanges
 b = flange width
 t = flange thickness

Transposing, $M_{\text{vertical}} = f_b h b t$.

From page 5 the lateral bending moment,

$$M_{\text{lateral}} = \frac{M_{\text{vertical}} d^2}{12R h}$$

Dividing by the lateral section modulus of the flange, we have for lateral bending stress

$$f_w = \frac{6M_{\text{vertical}} d^2}{12R h b^2}$$

If $M_{\text{vertical}} = f_b h b t$ is substituted, then

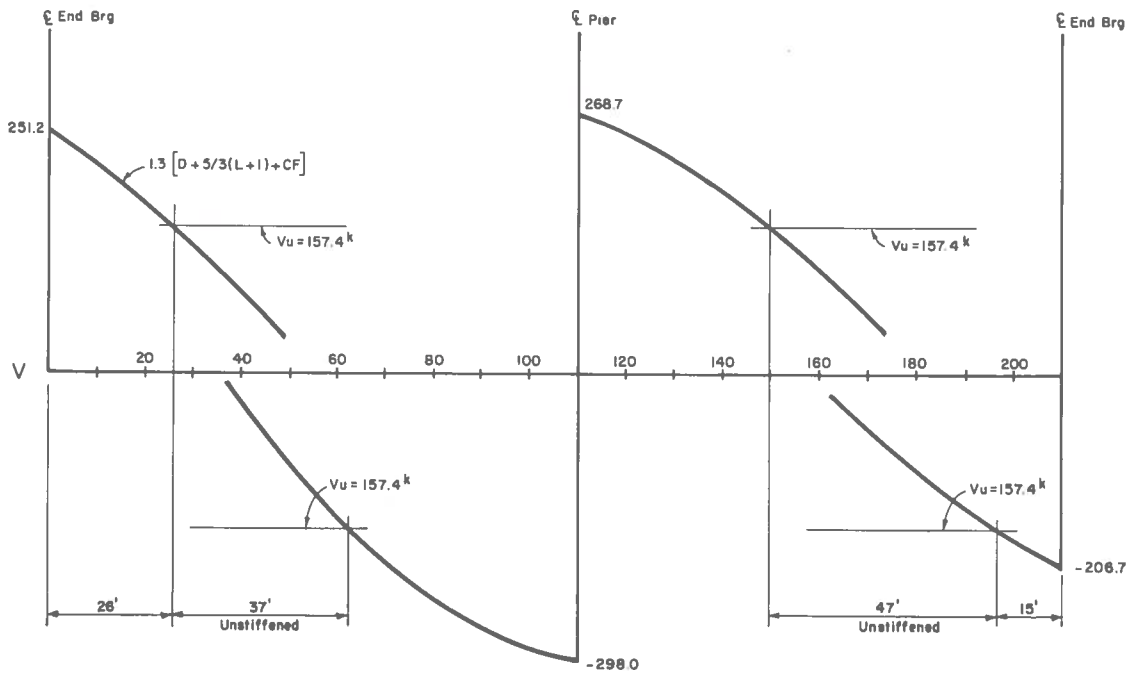
$$f_w = \frac{6f_b h b t d^2}{12R h b^2} = \frac{f_b d^2}{2R b}$$

With $f_b = 38$ ksi and $R = 300$ ft the following table giving lateral bending stresses as a function of flange width and diaphragm spacing can be generated.

Approximate Lateral Bending Stresses, f_w , ksi

d, Diaphragm Spacing (ft)	b, Flange Width (inches)				
	9	12	16	20	24
10	8.4	6.3	4.8	3.8	3.2
12	12.2	9.1	6.8	5.5	4.6
15	19.0	14.3	10.7	8.6	7.1
18	27.4	20.5	15.4	12.3	10.3
21	37.2	27.9	20.9	16.8	14.0
25	52.8	39.6	29.7	23.8	19.8

The table shows that the diaphragms must be spaced much more closely in this curved bridge than the nominal 25 ft spacing that would be required for a straight bridge, if the lateral bending stress is to be kept within reasonable limits. We are looking for an f_w in the range of 11-12 ksi to satisfy $f_w = 0.30 f_b$ and $f_b + f_w = 50$ ksi. If it is further assumed that an 18-inch bottom flange will be used, we see by scanning the table that the crossframe spacing should be about 16 ft. The framing plan below would appear to satisfy these requirements.



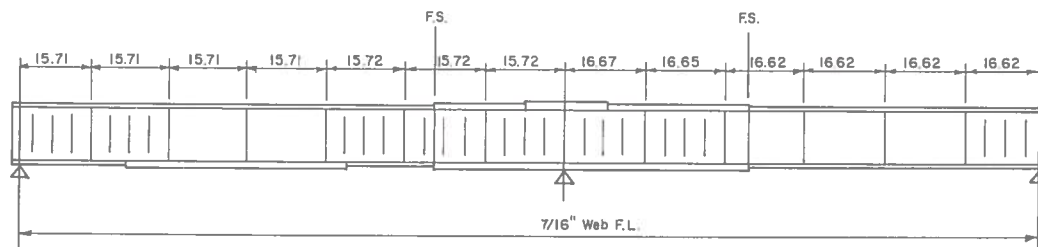
The unstiffened shear capacity is 157.4 kips, obtained as follows:

Assume $\min t_w = \frac{7}{16}$ inches

$$V_u = \frac{(3.5)(29000)(0.4375)^3}{54} = 157.4$$

The diagram shows that 37 ft of Span 1 and 47 ft of Span 2 may be left unstiffened. A $\frac{3}{8}$ -inch web in the positive moment regions could have been used but would have had an unstiffened shear capacity of only 99.1 kips, which was judged to provide unsatisfactorily short unstiffened regions.

The proposed stiffener arrangement is shown below. The dimensioned heavy lines represent the crossframes and the stiffeners are equally spaced within the crossframe panels.



The most critical region would appear to be the 16.87 ft crossframe panel adjacent to the interior support. With 5 equal stiffener spaces within the panel, the stiffener spacing, d_o , is 40.49 inches, and the resulting shear capacity is 539.3 kips. The applied shear is less than 0.6 of the shear capacity and therefore the moment/shear interaction relationship discussed on p.14 does not have to be considered.

Capacity of $\frac{7}{16}$ " web with $d_o = 40.49$ ":

$$C = 18,000 \left(\frac{t_w}{D} \right) \sqrt{\frac{1 + (D/d_o)^2}{F_y}} - 0.3 = 18,000 \left(\frac{0.4375}{54} \right) \sqrt{\frac{1 + (54/40.49)^2}{50,000}} - 0.3$$

$$= 0.7871$$

$$V_u = 0.58 F_y D t_w C = (0.58)(50)(54)(0.4375)(0.7871) = 539.3^k$$

$$0.6V_u = (0.6)(539.3) = 323.6^k > 298.0$$

Only the panels immediately adjacent to the interior support carry high shear and moment. In all other panels in which stiffeners are required, they are equally spaced to satisfy $d_o \leq D$, and this spacing provides sufficient shear capacity to resist the applied shear. More than enough capacity is available as seen by a check at the left end bearing:

Capacity of $\frac{7}{16}$ " web with $d_o = 47.13$ ":

$$C = 18,000 \left(\frac{0.4375}{54} \right) \sqrt{\frac{1 + (54/47.13)^2}{50,000}} - 0.3$$

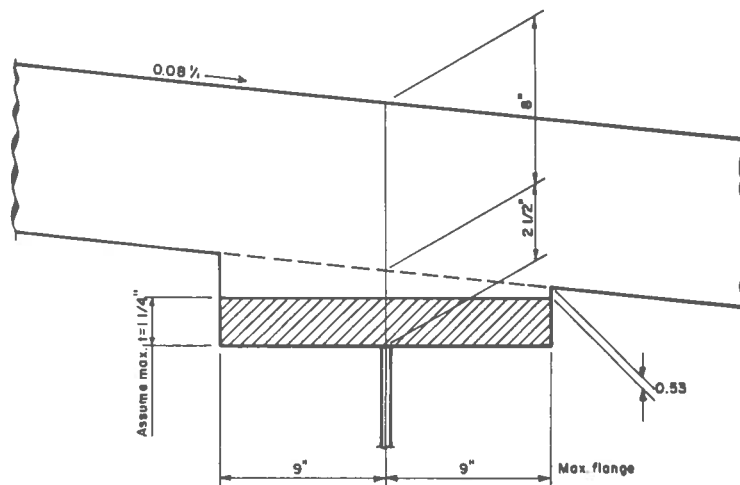
$$= 0.6918$$

$$V_u = (0.58)(50)(54)(0.4375)(0.6918)$$

$$= 474.0^k > 251.2$$

DESIGN OF SECTIONS FOR GIRDER G1

Now, we can start designing sections for Girder G1. The first section to be considered is the maximum positive moment section in Span 1. The effective width of the $7\frac{1}{2}$ -inch thick structural slab is 90 inches. A $2\frac{1}{2}$ -inch haunch is provided as illustrated below.

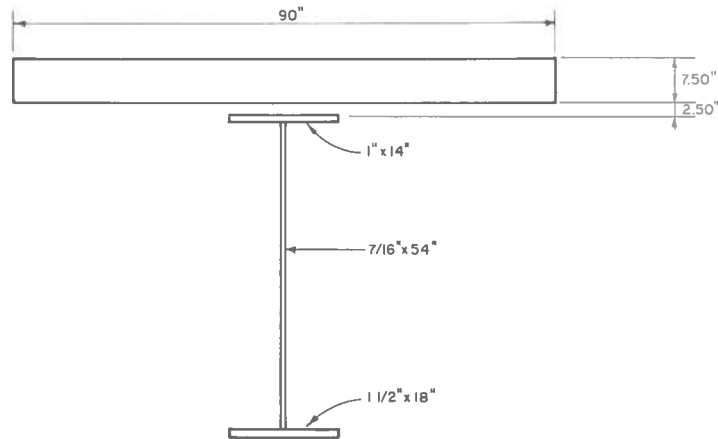


Haunch Detail

This haunch furnishes adequate clearance at the tip of the anticipated wider flange plate over the pier, and also accommodates any excess camber within the spans.

THE MAXIMUM POSITIVE SECTION—SPAN 1

The trial section has a 1" x 14" top flange, 7/16" x 54" web and 1 1/2" x 18" bottom flange. Section properties are computed for the steel and composite sections using the same procedures illustrated in Chapters 3, 3A, 4 and 4A.



Steel Section

Steel Section						
Material	A	d	Ad	Ad ²	I _o	I
Top flg. 1 x 14	14.00	27.50	385	10,588		10,588
Web 7/16 x 54	23.63				5,741	5,741
Bot. Flg. 1 1/2 x 18	27.00	-27.75	-749	20,792		20,792

$$64.63 \text{ in}^2$$

$$-364 \text{ in}^3$$

$$37,121$$

$$d_s = \frac{-364}{64.63} = 5.63 \text{ in}$$

$$-5.63 \times 304 = -2,049$$

$$I_{NA} = 35,072 \text{ in}^4$$

$$d_{\text{Top of Steel}} = 28.00 + 5.63 = 33.63 \text{ in}$$

$$d_{\text{Bot. of Steel}} = 28.50 - 5.63 = 22.87 \text{ in}$$

$$S_{\text{Top of Steel}} = \frac{35,072}{33.63} = 1,043 \text{ in}^3$$

$$S_{\text{Bot. of Steel}} = \frac{35,072}{22.87} = 1,534 \text{ in}^3$$

Composite Section, 3n = 24						
Material	A	d	Ad	Ad ²	I _o	I
Steel Section	64.63		-364			37,121
Conc. 7.5/24 x 90	28.13	33.25	935	31,099	132	31,231

$$92.76 \text{ in}^2$$

$$571 \text{ in}^3$$

$$68,352$$

$$d_s = \frac{571}{92.76} = 6.16 \text{ in.}$$

$$-6.16 \times 571 = -3,517$$

$$64,835 \text{ in}^4$$

$$d_{\text{Top of Steel}} = 28.00 - 6.16 = 21.84 \text{ in}$$

$$d_{\text{Bot. of Steel}} = 28.50 + 6.16 = 34.66 \text{ in}$$

$$S_{\text{Top of Steel}} = \frac{64,835}{21.84} = 2,969 \text{ in}^3$$

$$S_{\text{Bot. of Steel}} = \frac{64,835}{34.66} = 1,871 \text{ in}^3$$

Composite Section, n = 8						
Material	A	d	Ad	Ad ²	I _o	I
Steel Section	64.63		-364			37,121
Conc. $\frac{7.5}{8} \times 90$	84.38	33.25	2,806	93,287	396	93,683

$$149.01 \text{ in}^2 \quad 2,442 \text{ in}^3 \quad 130,804$$

$$d_s = \frac{2,442}{149.01} = 16.39 \text{ in} \quad -16.39 \times 2,442 = \frac{-40,024}{90,780 \text{ in}^4}$$

$$d_{\text{Top of Steel}} = 28.00 - 16.39 = 11.61 \text{ in} \quad d_{\text{Bot. of Steel}} = 28.50 + 16.39 = 44.89 \text{ in}$$

$$S_{\text{Top of Steel}} = \frac{90,780}{11.61} = 7,819 \text{ in}^3 \quad S_{\text{Bot. of Steel}} = \frac{90,780}{44.89} = 2,022 \text{ in}^3$$

$$S_{\text{Top Flg, Lat.}} = \frac{(1)(14)^2}{6} = 32.7 \text{ in}^3 \quad S_{\text{Bot. Flg, Lat.}} = \frac{(1.5)(18)^2}{6} = 81.0 \text{ in}^3$$

Moments are tabulated at the 0.4 point of the span, 44 ft from the end support. Lateral bending moments are computed using the formula from p. 5 , and vertical and lateral flange bending stresses are calculated.

Bending Moments 44 ft from End Support				
	DL ₁	DL ₂	L + I	CF
M, kip-ft	1,481	561	1,616	80

$$M_{\text{Top Flg, Lat}_{DL_1}} = \frac{M_{DL_1} d^2}{12Rh} = \frac{(1,481)(15.71)^2}{(12)(300) \left(\frac{55.25}{12} \right)} = 22.1 \text{ k-ft}$$

$$M_{\text{Bot. Flg, Lat}_{DL_1}} = 22.1 \text{ k-ft}$$

$$M_{\text{Bot. Flg, Lat}_{DL_2}} = \frac{(561)(15.71)^2}{(12)(300) \left(\frac{55.25}{12} \right)} = 8.4 \text{ k-ft}$$

$$M_{\text{Bot. Flg, Lat}_{L+I}} = \frac{(1,616)(15.71)^2}{(12)(300) \left(\frac{55.25}{12} \right)} = 24.1 \text{ k-ft}$$

$$M_{\text{Bot. Flg, Lat}_{CF}} = \frac{(80)(15.71)^2}{(12)(300) \left(\frac{55.25}{12} \right)} = 1.2 \text{ k-ft}$$

Steel Stresses Due to Maximum Design Loads			
Top of Steel (Compression)		Bottom of Steel (Tension)	
Vertical Bending:			
For DL ₁ : f _b = $\frac{1,481 \times 12}{1,043} \times 1.30$	= 22.2	f _b = $\frac{1,481 \times 12}{1,534} \times 1.30$	= 15.1
For DL ₂ : f _b = $\frac{561 \times 12}{2,969} \times 1.30$	= 2.9	f _b = $\frac{561 \times 12}{1,871} \times 1.30$	= 4.7
For L+I: f _b = $\frac{1,616 \times 12}{7,819} \times 1.30 \times \frac{5}{3}$	= 5.5	f _b = $\frac{1,616 \times 12}{2,022} \times 1.30 \times \frac{5}{3}$	= 21.4
For CF: f _b = $\frac{80 \times 12}{7,816} \times 1.30$	= $\frac{0.2}{30.8 \text{ ksi}}$	f _b = $\frac{80 \times 12}{2,022} \times 1.30$	= $\frac{0.6}{41.8 \text{ ksi}}$
Lateral Bending:			
For DL ₁ : f _w = $\frac{22.1 \times 12}{32.7} \times 1.30$	= 10.5	f _w = $\frac{22.1 \times 12}{81.0} \times 1.30$	= 4.3
For DL ₂ : f _w	= 0.0	f _w = $\frac{8.4 \times 12}{81.0} \times 1.30$	= 1.6
For L+I: f _w	= 0.0	f _w = $\frac{24.1 \times 12}{81.0} \times 1.30 \times \frac{5}{3}$	= 7.7
For CF: f _w	= $\frac{0.0}{10.5 \text{ ksi}}$	f _w = $\frac{1.2 \times 12}{81.0} \times 1.30$	= $\frac{0.2}{13.8 \text{ ksi}}$

To determine the allowable stresses, the compression flange geometry is checked and found to satisfy the requirements for compactness. The appropriate equations from the Guide Specifications yield an allowable compression stress of 45.2 ksi, considerably higher than the 30.8 ksi actual average stress. This will be discussed below.

Allowable Steel Stresses	
Top Flange (Compression):	
$\frac{b}{t} = \frac{14}{1} = 14.0$	
$\frac{b}{t}_{\text{Max for compact flg}} = \frac{3,200}{\sqrt{F_y}} = \frac{3,200}{\sqrt{50,000}} = 14.31 > 14.0$	
	∴ Flange is compact
$\bar{\rho}_B = \frac{1}{1 + \frac{1}{b} \left(1 + \frac{1}{6b} \right) \left(\frac{1}{R} - 0.01 \right)^2}$	
$= \frac{1}{1 + \frac{15.71}{14/12} \left[1 + \frac{15.71}{(6)(14/12)} \right] \left(\frac{15.71}{3.00} - 0.01 \right)^2} = 0.92729$	
$\lambda = \frac{1}{\pi} \left(\frac{1}{b} \right) \sqrt{\frac{F_y}{E}} = \frac{1}{\pi} \left(\frac{15.71}{14/12} \right) \sqrt{\frac{50}{29,000}} = 0.17798$	
$F_{bs} = F_y (1 - 3 \lambda^2) = 50 [1 - (3)(0.17798)^2] = 45.248$	
$\bar{\rho}_w = 0.95 + 18 \left[0.1 - \frac{1}{R} \right]^2 + \frac{f_w}{f_b} \frac{\left[0.3 - 0.1 \frac{1}{R} \frac{1}{b} \right]}{\bar{\rho}_B F_{bs}/F_y}$	
$= 0.95 + 18 \left[0.1 - \frac{15.71}{300} \right]^2 + \frac{10.5}{30.8} \frac{\left[0.3 - 0.1 \left(\frac{15.71}{300} \right) \left(\frac{15.71}{14/12} \right) \right]}{(0.92729) 45.248/50}$	
	= 1.08407

$$\bar{\rho}_B \bar{\rho}_W = (0.92729)(1.08407) = 1.00525 > 1.0, \text{ use } 1.0$$

$$F_{bu} = F_{bs} \bar{\rho}_B \bar{\rho}_W = (45.248)(1.0) = 45.2 \text{ ksi} > 30.8$$

The allowable tensile stress in the bottom flange is computed in similar manner as 43.1 ksi. It slightly exceeds the actual average stress of 41.8 ksi.

$$\bar{\rho}_B = \frac{1}{1 + \frac{15.71}{18/12} \left[1 + \frac{15.71}{(6)(18/12)} \right] \left(\frac{15.71}{300} - 0.1 \right)^2} = 0.95092$$

$$F_{bs} = F_y = 50$$

$$\bar{\rho}_W = 0.95 + 18 \left[0.1 - \frac{15.71}{300} \right]^2 + \frac{13.8}{41.8} \left[0.3 - 0.1 \left(\frac{15.71}{300} \right) \left(\frac{15.71}{18/12} \right) \right] \\ = 0.90573$$

$$\bar{\rho}_B \bar{\rho}_W = (0.95092)(0.90573) = 0.86128 < 1.0$$

$$F_{bu} = (50)(0.86128) = 43.1 \text{ ksi} > 41.8$$

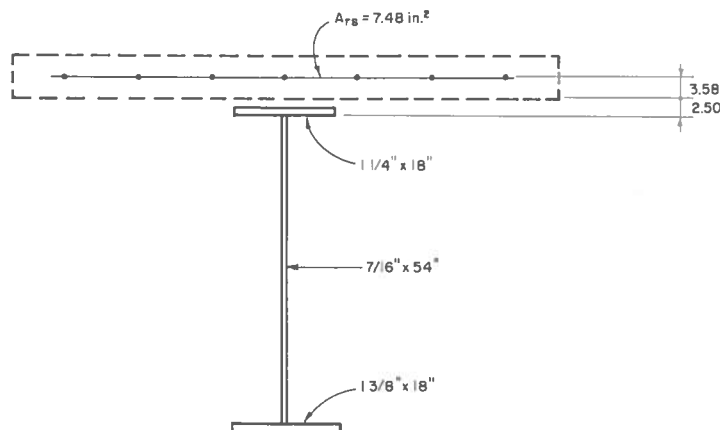
While there appears to be inefficiency of the top flange (30.8 ksi actual stress vs 45.2 ksi allowable stress), we should consider what would happen if the top flange were made thinner and thereby become a noncompact compression flange. The following material and stresses would result:

	Material	Bottom Flange Stress	Top Flange Stress
Top Flange	1 ⁵ / ₁₆ x 16	Actual 24.6 ksi	Actual 28.2 ksi
Web	7 ¹ / ₁₆ x 54	Allowable 24.9 ksi	Allowable 28.2 ksi
Bottom Flange	2 ⁷ / ₈ x 18		

The design with a non-compact compression flange has all its elements stressed to the maximum and has the appearance of being very efficient. However, the allowable stresses are much lower than for the section with a compact compression flange. With the compact compression flange, the total cross-sectional area is 64.63 in² and with the non-compact compression flange the cross-sectional area is 90.38 in². Thus, the seemingly inefficient section with the compact compression flange requires only 72 percent as much steel as the section with the non-compact compression flange. As is noted in the introduction, compact compression flange sections are usually (but not always) more economical.

THE MAXIMUM NEGATIVE MOMENT SECTION

We next design the maximum negative moment section over the interior support. The area of longitudinal reinforcing steel within the effective slab width is 7.48 inches². A steel section with a 1¹/₄" x 18" top flange, 7¹/₁₆" x 54" web and 1³/₈" x 18" bottom flange is investigated.



Properties for the steel section alone and the steel section plus reinforcement are computed.

Steel Section at Interior Support						
Material	A	d	Ad	Ad ²	I _o	I
Top Flg. 1¼ x 18	22.50	27.63	622	17,177		17,177
Web 7/16 x 54	23.63				5,741	5,741
Bot. Flg. 1¾ x 18	24.75	-27.69	-685	18,977		18,977

$$\begin{aligned}
 & 70.88 \text{ in}^2 \qquad - 63 \text{ in}^3 \qquad 41,895 \\
 d_s = & \frac{-63}{70.88} = 0.89 \text{ in} \qquad - 0.89 \times 63 = -56 \\
 & \qquad \qquad \qquad I_{NA} = 41,839 \text{ in}^4 \\
 d_{\text{Top of Steel}} = & 28.25 + 0.89 = 29.14 \text{ in} \qquad d_{\text{Bot. of Steel}} = 28.38 - 0.89 = 27.49 \text{ in} \\
 S_{\text{Top of Steel}} = & \frac{41,839}{29.14} = 1,436 \text{ in}^3 \qquad S_{\text{Bot. of Steel}} = \frac{41,839}{27.49} = 1,522 \text{ in}^3
 \end{aligned}$$

Steel Section with Reinforcing Steel at Interior Support						
Material	A	d	Ad	Ad ²	I _o	I
Steel Section	70.88		-63			41,895
Reinforcement	7.48	33.08	247	8,185		8,185
	78.36 in ²		184 in ³			50,080

$$\begin{aligned}
 d_s = & \frac{184}{78.36} = 2.35 \text{ in} \qquad - 2.35 \times 184 = -432 \\
 & \qquad \qquad \qquad 49,648 \text{ in}^4 \\
 d_{\text{Top of Steel}} = & 28.25 - 2.35 = 25.90 \text{ in} \qquad d_{\text{Bot. of Steel}} = 28.38 + 2.35 = 30.73 \text{ in} \\
 S_{\text{Top of Steel}} = & \frac{49,648}{25.90} = 1,917 \text{ in}^3 \qquad S_{\text{Bot. of Steel}} = \frac{49,648}{30.73} = 1,616 \text{ in}^3 \\
 d_{\text{Reinf.}} = & 33.08 - 2.35 = 30.73 \text{ in} \qquad S_{\text{Reinf.}} = \frac{49,648}{30.73} = 1,616 \text{ in}^3 \\
 S_{\text{Top Flg. Lat.}} = & \frac{(1.25)(18)^2}{6} = 67.5 \text{ in}^3 \qquad S_{\text{Bot. Flg. Lat.}} = \frac{(1.38)(18)^2}{6} = 74.5 \text{ in}^3
 \end{aligned}$$

The tabulation and computations below give all the vertical and lateral bending moments needed to compute stresses in the section. For lateral bending stresses at an interior support, a fraction-of-moment (FOM) parameter is used to modify the lateral bending effect. This is because the derivation of the V-Load theory is based on a smoothly curved moment diagram. The sharply peaked moment that occurs at an interior support, used as is, would unduly and irrationally penalize the design. A moment is used in lieu of the actual peak moment, which is the average of the peak moment and the moments at the crossframes immediately adjacent on either side of the support in question. The FOM value is the ratio of the average moment to the peak moment.

Bending Moments at Interior Support						
	DL ₁	DL ₂	+ (L + I)	+ CF	– (L + I)	– CF
M, kip-ft	–2,068	–646	0	0	–1,064	–33

$$M_{\text{Top Flg, Lat}_{DL_1}} = \frac{(2,068)(16.87)^2}{(12)(300) \left(\frac{55.31}{12} \right)} = 35.5 \text{ k-ft}$$

$$M_{\text{Bot Flg, Lat}_{CF}} = \frac{(33)(16.87)^2}{(12)(300) \left(\frac{55.31}{12} \right)} = 0.6 \text{ k-ft}$$

$$M_{\text{Bot Flg, Lat}_{DL_1}} = \quad = 35.5 \text{ k-ft}$$

$$M_{\text{Bot Flg, Lat}_{DL_2}} = \frac{(696)(16.87)^2}{(12)(300) \left(\frac{55.31}{12} \right)} = 11.1 \text{ k-ft}$$

$$M_{\text{Bot Flg, Lat}_{L+I}} = \frac{(1,064)(16.87)^2}{(12)(300) \left(\frac{55.31}{12} \right)} = 18.2 \text{ k-ft}$$

Fraction of Moment (FOM) Calculation			
Crossframe Location	Span 1 0.8571	Pier	Span 2 0.1687
M _{DL₁}	750 k-ft	2,068 k-ft	914 k-ft
M _{DL₂}	186	646	247
M _{L+I}	536	1,064	588
M _{CF}	28	33	28
M _{TOTAL}	1,500 k-ft	3,811 k-ft	1,777 k-ft

$$\text{Avg. moment} = \frac{1,500 + 3,811 + 1,777}{3} = 2,363 \text{ k-ft}$$

$$\text{FOM} = \frac{2,363}{3,811} = 0.620$$

Stresses at top and bottom of steel are computed below.

Steel Stresses Due to Maximum Design Loads					
Top of Steel (Tension)			Bottom of Steel (Compression)		
<i>Vertical Bending:</i>					
For DL ₁ : f _b =	$\frac{2,068 \times 12}{1,436} \times 1.30$	= 22.5	f _b =	$\frac{2,068 \times 12}{1,522} \times 1.30$	= 21.2
For DL ₂ : f _b =	$\frac{646 \times 12}{1,917} \times 1.30$	= 5.3	f _b =	$\frac{646 \times 12}{1,616} \times 1.30$	= 6.2
For L+I: f _b =	$\frac{1,064 \times 12}{1,917} \times 1.30 \times \frac{5}{3}$	= 14.4	f _b =	$\frac{1,064 \times 12}{1,616} \times 1.30 \times \frac{5}{3}$	= 17.1
For CF: f _b =	$\frac{33 \times 12}{1,917} \times 1.30$	$= \frac{0.3}{42.5 \text{ ksi}}$	f _b =	$\frac{33 \times 12}{1,616} \times 1.30$	$= \frac{0.3}{44.8 \text{ ksi}}$
<i>Lateral Bending:</i>					
For DL ₁ : f _w =	$\frac{35.5 \times 12 \times 0.620}{67.5} \times 1.30$	= 5.1	f _w =	$\frac{35.5 \times 12 \times 0.620}{74.5} \times 1.30$	= 4.6
For DL ₂ : f _w =		= 0.0	f _w =	$\frac{11.1 \times 12 \times 0.620}{74.5} \times 1.30$	= 1.4
For L+I: f _w =		= 0.0	f _w =	$\frac{18.2 \times 12 \times 0.620}{74.5} \times 1.30 \times \frac{5}{3}$	= 3.9
For CF: f _w =		$= \frac{0.0}{5.1 \text{ ksi}}$	f _w =	$\frac{0.6 \times 12 \times 0.62}{74.5} \times 1.30$	$= \frac{0.1}{10.0 \text{ ksi}}$

Allowable stresses are computed from the Guide Specification formulas and found to be satisfactory. The compression flange is again compact.

Allowable Steel Stresses

Bottom Flange (Compression):

$$\frac{b}{t} = \frac{18}{1.375} = 13.1 < 14.31 \therefore \text{Flange is compact}$$

$$\begin{aligned}\bar{\rho}_B &= \frac{1}{1 + \frac{1}{b} \left(1 + \frac{1}{6b} \right) \left(\frac{1}{R} - 0.01 \right)^2} \\ &= \frac{1}{1 + \frac{16.87}{18/12} \left[1 + \frac{16.87}{(6)(18/12)} \right] \left(\frac{16.87}{300} - 0.01 \right)^2} = 0.93536\end{aligned}$$

$$\lambda = \frac{1}{\pi} \left(\frac{1}{b} \right) \sqrt{\frac{F_y}{E}} = \frac{1}{\pi} \left(\frac{16.87}{18/12} \right) \sqrt{\frac{50}{29,000}} = 0.14865$$

$$F_{bs} = F_y (1 - 3 \lambda^2) = 50 [1 - (3)(0.14865)^2] = 46.685$$

$$\begin{aligned}\bar{\rho}_w &= 0.95 + 18 \left[0.1 - \frac{1}{R} \right]^2 + \frac{\frac{f_w}{f_b} \left[0.3 - 0.1 \frac{1}{R} \frac{1}{b} \right]}{\frac{\bar{\rho}_B F_{bs}}{F_y}} \\ &= 0.95 + 18 \left[0.1 - \frac{16.87}{300} \right]^2 + \frac{\frac{10.0}{44.8} \left[0.3 - 0.1 \left(\frac{16.87}{300} \right) \left(\frac{16.87}{18/12} \right) \right]}{(0.93536)(46.685/50)} = 1.04499\end{aligned}$$

$$\bar{\rho}_B \bar{\rho}_w = (0.93536)(1.04499) = 0.97744 < 1.0$$

$$F_{bu} = F_{bs} \bar{\rho}_B \bar{\rho}_w = (46.685)(0.97744) = 45.6 \text{ ksi} > 44.8$$

Top Flange (Tension)

$$\bar{\rho}_B = 0.93536$$

$$F_{bs} = F_y = 50$$

$$\begin{aligned}\bar{\rho}_w &= 0.95 + 18 \left[0.1 - \frac{16.87}{300} \right]^2 + \frac{\left(\frac{5.1}{42.5} \right) \left[0.3 - 0.1 \left(\frac{16.87}{300} \right) \left(\frac{16.87}{18/12} \right) \right]}{\frac{(0.93536)(50)}{50}} \\ &= 0.95411\end{aligned}$$

$$\bar{\rho}_B \bar{\rho}_w = (0.93536)(0.95411) = 0.89244 < 1.0$$

$$F_{bu} = (50)(0.89244) = 44.6 \text{ ksi} > 42.5$$

Finally the Stress range in the reinforcement is computed and seen to be much lower than the 20,000 psi allowable range.

Stress Range in Reinforcement Due to Service Loads

$$\text{For } -(L+I): \frac{1,064 \times 12}{1,616} = 7.9$$

$$\text{For } -(CF): \frac{33 \times 12}{1,616} = \frac{0.2}{8.1} \text{ ksi} < 20.00$$

Though not illustrated, all sections of all four girders were designed two ways: 1) with compact compression flange; 2) with non-compact compression flange. The design used was the one with the least cross-sectional area and therefore least steel. The section with

compact compression flange was the most economical in nearly all cases. This is the reason the flange material of Girders G2, G3 and G4 may not appear to follow typical patterns associated with straight girders.

In addition to the computations for maximum strength, stress ranges under service loading must be checked against allowable fatigue stress ranges for the details at hand. For any detail located on the girder flange at some distance away from the web (such as a stiffener-to-flange weld), the computed stress range must include lateral bending stress. Fatigue checks are not illustrated here because they are similar in principle to those shown in other chapters.

The calculations for Overload are not shown here either. As with straight girders, Overload does not govern the design of curved plate girders of normal proportions.

The computation of allowable stresses for curved girders by the Guide Specification formulas is a tedious and time-consuming procedure. It is virtually imperative that such operations be carried out either with a programmable calculator or computer assistance.

TRANSVERSE WEB STIFFENER DESIGN

Transverse web stiffeners are designed by the rules outlined in the introduction. The rigidity requirement is a function of the stiffener spacing, web thickness, radius of curvature and web depth. It is checked first at the largest stiffener spacing, $d_o = 49.95$ in.

At $d_o = 49.95$ ":

$$\frac{d_o}{D} = \frac{49.95}{54} = 0.925 \quad 0.78 < 0.925 < 1.0$$

$$Z = \frac{(0.75)(49.95)^2}{(300)(12)(0.4375)} = 1.505$$

$$X = 1 + \left[\frac{(49.95/54) - 0.78}{1.775} \right] (1.505)^4 = 1.00042$$

$$J = \left[2.5 \left(\frac{54}{49.95} \right)^2 - 2 \right] 1.00042 = 0.92222 > 0.5$$

$$I = (49.95) (0.4375)^3 (0.92222) = 3.86 \text{ in.}^4$$

Since we want to use the same size stiffeners throughout the structure, we must consider the other three girders G2, G3 and G4. We now check the smallest stiffener space, 30.36 inches, located on Girder G2.

At $d_o = 30.36$ ":

$$\frac{d_o}{D} = \frac{30.36}{54} = 0.562 < 0.78$$

$$\therefore X = 1$$

$$J = \left[2.5 \left(\frac{54}{30.36} \right)^2 - 2 \right] (1) = 5.909$$

$$I = (30.36) (0.4375)^3 (5.909) = 15.02 \text{ in.}^4$$

Because the structure is to be painted, A36 steel is chosen for the stiffeners. A $\frac{3}{8}$ " x 5" stiffener is satisfactory. For the crossframe connection plates the size chosen is $\frac{9}{16}$ " x 7" to accommodate the crossframe connection.

Use A36 steel for stiffeners

Try $\frac{3}{8}$ x 5" for intermediate stiffeners:

$$\frac{b}{t} = \frac{5}{0.375} = 13.33$$

$$\frac{b}{t}_{\text{Allow}} = \frac{2,600}{\sqrt{F_y}} = \frac{2,600}{\sqrt{36,000}} = 13.70 > 13.33$$

$$b_{\text{Min}} = \frac{1}{4} \text{ max. flg. width} = (\frac{1}{4})(18) = 4.5" < 5$$

$$t_{\text{Min}} = \frac{b}{16} = \frac{5}{16} < \frac{3}{8}$$

$$I_{\text{Furn'd}} = \frac{1}{3} tb^3 = \left(\frac{1}{3}\right)\left(\frac{3}{8}\right)(5)^3 = 15.63 \text{ in.}^4 > 15.02$$

Use $\frac{3}{8}$ x 5" stiffeners

Try $\frac{9}{16}$ x 7" for crossframe connection plates:

$$\frac{b}{t} = \frac{7}{0.5625} = 12.44 < 13.70$$

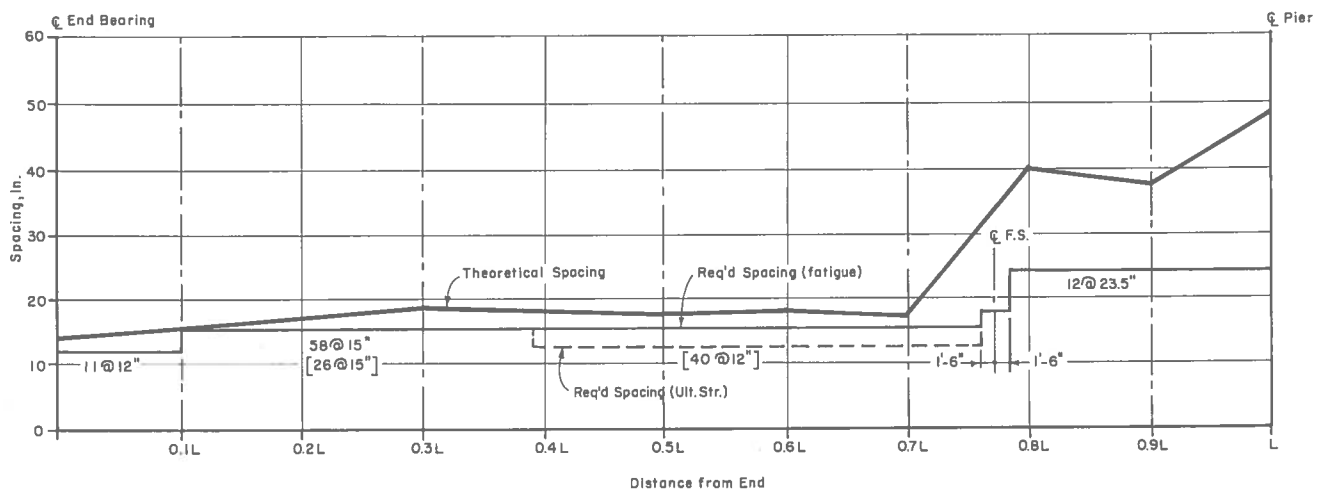
Use $\frac{9}{16}$ x 7" connection plates

SHEAR CONNECTOR SPACING

Shear connector spacing is first computed for fatigue using exactly the same procedure that would be used for a straight bridge. Two $\frac{7}{8}$ in. diameter by 4 in. long studs per row are chosen. The fatigue strength, Z_r , per connector is equal to $(10.6)(0.875)^2 = 8.12$ kips.

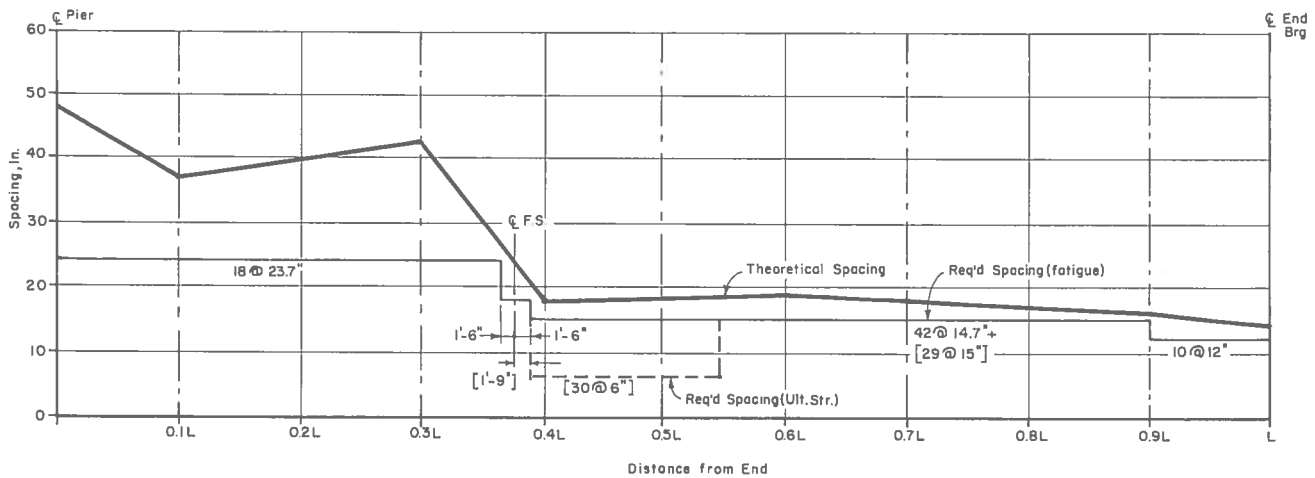
Shear Connector Spacing—Girder G1—Span 1

Dist. from End Brg.	Positive Live Load Shear (kips)	Negative Live Load Shear (kips)	Shear Range (kips)	Q (in ³)	I (in ⁴)	$S_r = \frac{VQ}{I}$ (kpi)	$\text{Spceg} = \frac{2Z_r}{S_r}$ (inches)
0	60.1+3.1 = 63.2	-7.2-0.3 = -7.5	70.7	1,215	74,137	1.16	14.0
0.1L	56.5+2.7 = 59.2	-5.3-0.4 = -5.7	64.9	1,215	74,137	1.06	15.3
0.2L	49.6+2.2 = 51.8	-7.4-0.7 = -8.1	59.9	1,215	74,137	0.98	16.6
0.3L	40.7+1.8 = 42.5	-13.4-0.9 = -14.3	56.8	1,508	90,820	0.94	17.3
0.4L	30.7+1.4 = 32.1	-23.6-1.3 = -24.9	57.0	1,508	90,820	0.95	17.1
0.5L	20.9+1.1 = 22.0	-34.7-1.7 = -36.4	58.4	1,508	90,820	0.97	16.7
0.6L	11.9+0.7 = 12.6	-44.5-2.1 = -46.6	59.2	1,508	90,820	0.98	16.6
0.7L	5.2+0.5 = 5.7	-52.6-2.5 = -55.1	60.8	1,215	74,137	1.00	16.2
0.8L	1.2+0.2 = 1.4	-58.6-3.0 = -61.6	63.0	260	88,998	0.18	90.1
0.9L	-0.1+0.1 = 0.0	-61.5-3.4 = -64.9	64.9	260	88,998	0.19	85.4
L	0.0+0.0 = 0.0	-62.1-3.8 = -65.9	65.9	237	89,833	0.17	95.4



Shear Connector Spacing—Girder G1—Span 2

Dist. from End Brg.	Positive Live Load Shear (kips)	Negative Live Load Shear (kips)	Shear Range (kips)	Q (in ³)	I (in ⁴)	$S_r = \frac{VQ}{I}$ (kpi)	$Spcg = \frac{2Z_r}{S_r}$ (inches)
0	61.5+3.6 = 65.1	-0.0-0.0 = -0.0	65.1	237	89,833	0.17	95.4
0.1L	59.4+3.2 = 62.6	-0.6-0.1 = -0.7	63.3	260	88,998	0.18	90.1
0.2L	55.2+2.8 = 58.0	-2.8-0.2 = -3.0	61.0	260	88,998	0.18	90.1
0.3L	48.9+2.4 = 51.3	-7.3-0.4 = -7.7	59.0	260	88,998	0.17	95.4
0.4L	40.9+2.0 = 42.9	-13.9-0.6 = -14.5	57.4	1,279	79,221	0.93	17.4
0.5L	31.9+1.7 = 33.6	-22.3-1.0 = -23.3	56.9	1,279	79,221	0.92	17.6
0.6L	22.4+1.3 = 23.7	-30.8-1.3 = -32.1	55.8	1,279	79,221	0.90	18.0
0.7L	15.5+1.0 = 16.5	-38.8-1.7 = -40.5	57.0	1,279	79,221	0.92	17.6
0.8L	11.6+0.6 = 12.2	-46.1-2.1 = -48.2	60.4	1,279	79,221	0.98	16.6
0.9L	8.8+0.4 = 9.2	-52.7-2.5 = -55.2	64.4	1,279	79,221	1.04	15.6
L	9.4+0.0 = 9.4	-8.3-3.0 = -61.3	70.7	1,279	79,221	1.14	14.2



The shear connector spacing as determined for fatigue is now checked for ultimate strength. The ultimate strength of an individual connector is 37.93 kips and the governing load on the connector group (in the positive moment region of Span 1) is 2,295 kips.

Ultimate Strength Check for Shear Connectors—Girder G1—Span 1

$$S_u = 0.4d^2 \sqrt{F_c E_c} = \frac{0.4(7/8)^2}{1,000} \sqrt{4,000(150)^{3/2} 33} \sqrt{4,000} = 37.93 \text{ k}$$

$$P_1 = 0.85f_c b c = (0.85)(4.0)(90)(7.50) = 2,295 \text{ k Governs}$$

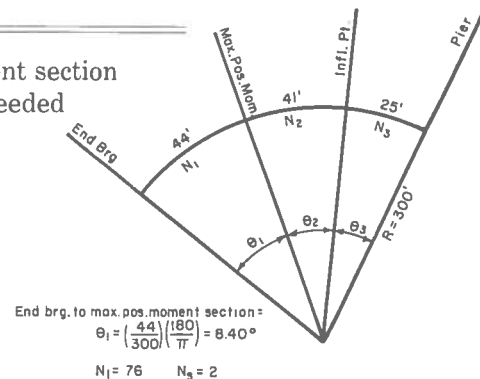
$$A_s = (1 \times 14) + (7/16 \times 54) + (1 1/2 \times 18) = 64.625 \text{ in}^2$$

$$P_2 = A_s F_y = (64.625)(50) = 3,231 \text{ k}$$

$$P_r = A_s^r F_y^r = (7.48)(40) = 299 \text{ k}$$

The subtended angle from the maximum positive moment section to the end bearing is 8.40° and the number of connectors needed for fatigue is 76.

The resulting connector force is just within the ultimate strength of the connector and the 76 studs are adequate.



$$K = 0.166 \left(\frac{N}{N_s} - 1 \right) + 0.375 = 0.166 \left(\frac{76}{2} - 1 \right) + 0.375 = 6.517$$

$$F = \frac{P(1-\cos \Theta)}{4KN_s \sin \Theta/2} = \frac{(2,295)(1-\cos 8.40^\circ)}{(4)(6.517)(2)(\sin 8.40^\circ/2)} = 6.448$$

$$P = \frac{P}{N} = \frac{2,295}{76} = 30.20$$

$$\begin{aligned} P_c &= \sqrt{P^2 + F^2 + 2P \cdot F \cdot \sin \frac{\Theta}{2}} \\ &= \sqrt{(30.20)^2 + (6.448)^2 + (2)(30.20)(6.448) \left(\sin \frac{8.40^\circ}{2} \right)} \\ &= 31.34^k \end{aligned}$$

$$P_{cult.} = (0.85)(37.93) = 32.24^k > 31.34$$

For the 41 ft from the maximum moment section to the inflection point the subtended angle is 7.83° and the number of connectors needed for fatigue is 64. In this case, the connector force exceeds the connector ultimate strength, and more studs must be added.

$$K = 0.166 \left(\frac{64}{2} - 1 \right) + 0.375 = 5.521$$

$$F = \frac{(2,295)(1-\cos 7.83^\circ)}{(4)(5.521)(2) \left(\sin \frac{7.83^\circ}{2} \right)} = 7.095$$

$$P = \frac{2,295}{64} = 35.86$$

$$P_c = \sqrt{35.86^2 + (7.095)^2 + (2)(35.86)(7.095) \left(\sin \frac{7.83^\circ}{2} \right)} = 37.03^k$$

$$P_{cult} = 32.24^k < 37.03$$

The number of studs furnished is increased to 76 and the connector force for ultimate strength re-calculated. The results are satisfactory.

$$\therefore \text{ Let } N_2 = \frac{37.03}{32.24} (64) = 73.5 \quad \text{Say } 76$$

$$K = 6.517$$

$$F = (7.095) \left(\frac{5.521}{6.517} \right) = 6.011$$

$$\bar{P} = 30.20$$

$$P_c = \sqrt{(30.20)^2 + (6.011)^2 + (2)(30.20) \left(\sin \frac{7.83^\circ}{2} \right)} = 30.86^k < 32.24$$

Calculations for the remainder of the girder are similar and are not shown to avoid repetition. The ultimate strength requirement dictates an increase in the number of studs beyond that needed for fatigue in several regions. These are indicated by dashed lines and bracketed spacings on the diagrams on pages 32 and 33.

BEARING STIFFENERS

Bearing stiffeners are designed by the same procedures as illustrated in Chapters 4A and 5. Calculations are not shown here.

FIELD SPLICES

The high-strength bolted field splice is designed in the same manner as splices in earlier chapters, on the basis that only lateral flange bending would make a curved girder splice differ from a straight girder splice. With splices located at inflection points where the vertical bending moment is low, lateral bending moments would be correspondingly low. It is assumed that lateral flange bending can be accounted for simply by conservative proportioning of fasteners and splice plates, and does not justify the added complication of a detailed analysis in this case.

CROSSFRAME DESIGN

An intermediate crossframe at the 0.429 point of Span 1 is selected to illustrate analysis and design of this element of the curved bridge. The analysis is broken into two parts—the curvature effect and the wind effect. For the curvature effect it is convenient to tabulate the calculations. The theory is outlined in the Introduction.

Design of Intermediate Crossframe at 0.429 Pt., Span 1			
Girder/Description	Moments	Girder/Description	Moments
G1		G1	
$M_{DL_1 + DL_2}$	1,481+561	$1.3[D + \frac{5}{3}(L+I) + CF_v]$	6,260
M_{L+I}	1,616	$1.3[D]$	2,655
M_{CF_v}	80	$1.3[D+(L+I)+CF_v]$	3,738
G2		G2	
$M_{DL_1 + DL_2}$	1,288+473	$1.3[D + \frac{5}{3}(L+I) + CF_v]$	5,656
M_{L+I}	1,537	$1.3[D]$	2,289
M_{CF_v}	28	$1.3[D+(L+I)+CF_v]$	3,326
G3		G3	
$M_{DL_1 + DL_2}$	1,028+368	$1.3[D + \frac{5}{3}(L+I)]$	4,569
M_{L+I}	1,271	$1.3[D]$	1,815
M_{CF_v}	—	$1.3[D+(L+I)]$	3,467
G4		G4	
$M_{DL_1 + DL_2}$	755+266	$1.3[D + \frac{5}{3}(L+I)]$	3,598
M_{L+I}	1,048	$1.3[D]$	1,327
M_{CF_v}	—	$1.3[D+(L+I)]$	2,690
$\Sigma 1.3[D + \frac{5}{3}(L+I) + CF_v]$	20,082		
$\Sigma 1.3[D]$	8,086		
$\Sigma 1.3[D+(L+I)+CF_v]$	15,340		

$$d = 15.71 \text{ ft.}$$

$$R = 300 \text{ ft.}$$

$$D = 26.5 \text{ ft.}$$

$$C = \frac{10}{9} \text{ (See Highway Structures Design Handbook, Vol. I, Chap. 12)}$$

Calculation of member forces for Group I loading is as follows:

Group I Loading: $1.3 [D + \frac{5}{3}(L+I)+CF_v]$

$$V = \frac{\Sigma M}{CRD/d} = \frac{20,082}{(\frac{10}{9})(300)(26.5)/15.71} = 35.7^k$$

$$\frac{V}{3} = \frac{35.7}{3} = 11.9^k$$

$$M_1 = 6,260 \text{ k-ft. } \frac{M_1 d}{R} = \frac{(6,260)(15.71)}{300} = 327.8 \text{ k-ft.}$$

$$M_2 = 5,656 \text{ k-ft. } \frac{M_2 d}{R} = \frac{(5,656)(15.71)}{300} = 296.2 \text{ k-ft.}$$

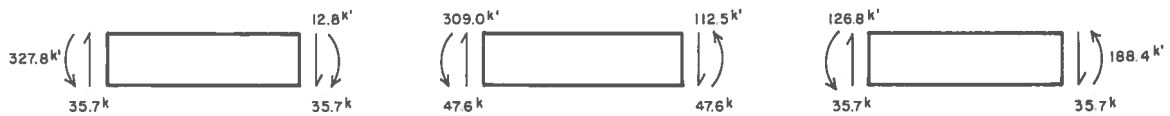
$$M_3 = 4,569 \text{ k-ft. } \frac{M_3 d}{R} = \frac{(4,569)(15.71)}{300} = 239.3 \text{ k-ft.}$$

$$M_4 = 3,598 \text{ k-ft. } \frac{M_4 d}{R} = \frac{(3,598)(15.71)}{300} = 188.4 \text{ k-ft.}$$

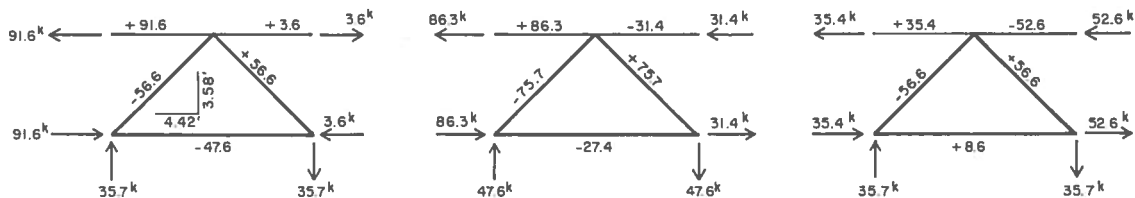
Forces on Girders:



Forces on Crossframes:



Internal Forces in Crossframes:



$$\frac{327.8}{3.58} = 91.6^k \quad \frac{309.0}{3.58} = 86.3^k \quad \frac{126.8}{3.58} = 35.4^k$$

$$\frac{12.8}{3.58} = 3.6^k \quad \frac{112.5}{3.58} = 31.4^k \quad \frac{188.4}{3.58} = 52.6^k$$

Next, member forces for Group II loading are computed.

Group II Loading: 1.3 [D]

$$V = \frac{8,086}{(10/9)(300)(26.5)/15.71} = 14.4^k$$

$$\frac{V}{3} = \frac{14.4}{3} = 4.8^k$$

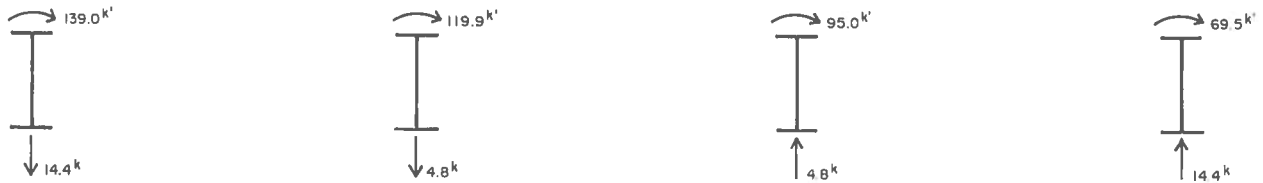
$$M_1 = 2,655 \text{ k-ft.} \quad \frac{M_1 d}{R} = \frac{(2,655)(15.71)}{300} = 139.0 \text{ k-ft.}$$

$$M_2 = 2,289 \text{ k-ft.} \quad \frac{M_2 d}{R} = \frac{(2,289)(15.71)}{300} = 119.9 \text{ k-ft.}$$

$$M_3 = 1,815 \text{ k-ft.} \quad \frac{M_3 d}{R} = \frac{(1,815)(15.71)}{300} = 95.0 \text{ k-ft.}$$

$$M_4 = 1,327 \text{ k-ft.} \quad \frac{M_4 d}{R} = \frac{(1,327)(15.71)}{300} = 69.5 \text{ k-ft.}$$

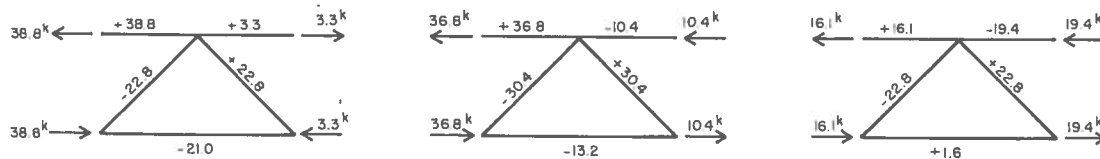
Forces on Girders:



Forces on Crossframes:



Internal Forces on Crossframes:



$$\frac{139.0}{3.58} = 38.8^k \quad \frac{131.7}{3.58} = 36.8^k \quad \frac{57.7}{3.58} = 16.1^k$$

$$\frac{11.8}{3.58} = 3.3^k \quad \frac{37.3}{3.58} = 10.4^k \quad \frac{69.5}{3.58} = 19.4^k$$

Finally, forces are analyzed under Group III loading.

Group III Loading: 1.3 [D + (L+I) + CF_v]

$$V = \frac{15,340}{(10/9)(300)(26.5/15.71)} = 27.3^k$$

$$\frac{V}{3} = \frac{27.3}{3} = 9.1^k$$

$$M_1 = 4,859 \text{ k-ft.} \quad \frac{M_1 d}{R} = \frac{(4,859)(15.71)}{300} = 254.4 \text{ k-ft.}$$

$$M_2 = 4,324 \text{ k-ft.} \quad \frac{M_2 d}{R} = \frac{(4,324)(15.71)}{300} = 226.4 \text{ k-ft.}$$

$$M_3 = 3,467 \text{ k-ft.} \quad \frac{M_3 d}{R} = \frac{(3,467)(15.71)}{300} = 181.6 \text{ k-ft.}$$

$$M_4 = 2,690 \text{ k-ft.} \quad \frac{M_4 d}{R} = \frac{(2,690)(15.71)}{300} = 140.9 \text{ k-ft.}$$

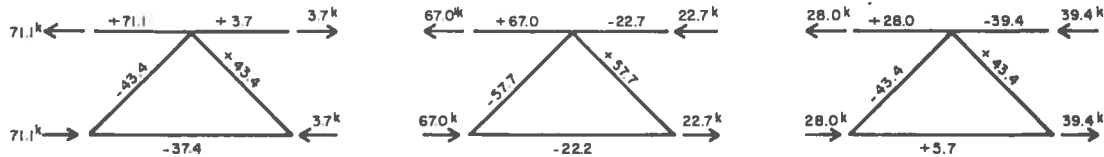
Forces on Girders:



Forces on Crossframes:



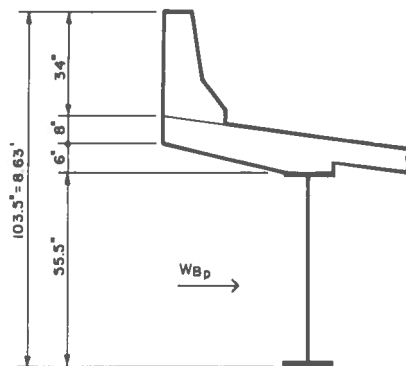
Internal Forces in Crossframes:



$$\frac{254.4}{3.58} = 71.1 \text{ k} \quad \frac{239.7}{3.58} = 67.0 \text{ k} \quad \frac{100.2}{3.58} = 28.0 \text{ k}$$

$$\frac{13.3}{3.58} = 3.7 \text{ k} \quad \frac{81.4}{3.58} = 22.7 \text{ k} \quad \frac{140.7}{3.58} = 39.4 \text{ k}$$

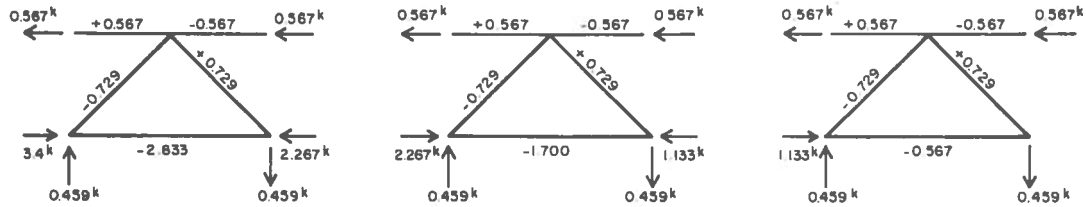
In addition to the forces from curvature, the intermediate crossframes must transfer the panel wind force from the lower half of the structure up to the slab. The resulting member forces are relatively small but nonetheless serve to complete the Group II and III loading cases. For computational purposes they are shown to three decimal places, then rounded to one decimal place for consistency with the forces due to curvature.



Wind Loading, W_{Bp}

$$W_{Bp} = \left(\frac{8.63}{2} \right) (15.71) (0.050) = 3.4 \text{ k}$$

Internal Forces in Crossframes:



Crossframe member design loads are summarized in the table below.

Summary of Maximum Design Loads for Intermediate Crossframes				
Member		Group I $1.3[D + \frac{5}{3}(L + I) + CF_v]$	Group II $1.3[D + W_{Bp}]$	Group III $1.3[D + (L + I) + CF_v] + 0.3W_{Bp}]$
Top Strut	T	91.6 ^k	$38.8 + (1.3)(0.567) = 39.5^k$	$71.1 + (1.3)(0.3)(0.567) = 71.3^k$
	C	-52.6 ^k	$-19.4 + (1.3)(-0.567) = -20.1^k$	$-39.4 + (1.3)(0.3)(-0.567) = -39.6^k$
Diagonal	T	75.7 ^k	$30.4 + (1.3)(0.729) = 31.3^k$	$57.7 + (1.3)(0.3)(0.729) = 58.0^k$
	C	-75.7 ^k	$-30.4 + (1.3)(-0.729) = -31.3^k$	$-57.7 + (1.3)(0.3)(-0.729) = -58.0^k$
Bot. Strut	T	8.6 ^k	$1.6 + (1.3)(-0.567) = 0.9^k$	$57.7 + (1.3)(0.3)(0.567) = 5.5^k$
	C	-47.6 ^k	$-21.0 + (1.3)(-2.833) = -24.7^k$	$-37.4 + (1.3)(0.3)(-2.833) = -38.5^k$

Inspection of the table reveals that the Group I loading combination governs the design of all members. The top strut must be proportioned for a compressive force of 52.6 kips and a tensile force of 91.6 kips. A WT5x15 section is checked first as a compression member. Gross and net areas are computed; the net area being the area of the section with half of the flange coped at the end connection. The critical $\frac{Kl}{r}$ is that about the y-axis of the member, for which Kl is taken as the full length. The capacity of the member is calculated as the smaller value of

$$P_u = 0.85 A_{sgross} F_{cr}$$

or

$$P_u = 0.85 A_{snet} F_y$$

The critical buckling stress, F_{cr} , is that defined in Article 1.7.69(A) of the Specifications. To guard against local buckling, the width to thickness ratio of the relatively slender stem of the WT section is checked against

$$\frac{b'}{t} \leq \frac{2,200}{\sqrt{F_y}} \sqrt{\frac{P_u}{P}}$$

Top Strut Design

Try WT5x15 ASTM A572 Grade 50

As compression member:

$$p = 52.6^k$$

$$A_{s_{gross}} = 4.42 \text{ in}^2$$

$$A_{s_{net}} = 4.42 - \left(\frac{5.81 - 0.30}{2} \right) (0.51) = 3.01 \text{ in}^2$$

$$\frac{Kl}{r_x} = \frac{(4.42)(12)}{1.45} = 36.6 < 120$$

$$\frac{Kl}{r_y} = \frac{(8.83)(12)}{1.37} = 77.3 < 120$$

$$\sqrt{\frac{2\pi^2 E}{r_y}} = \sqrt{\frac{(2)(\pi)^2(29,000)}{50}} = 107.0 > 77.3$$

$$\therefore F_{cr} = F_y \left[1 - \frac{F_y}{4\pi^2 E} \left(\frac{Kl}{r} \right)^2 \right] = 50 \left[1 - \frac{50}{4\pi^2(29,000)} (77.3)^2 \right] = 37.0 \text{ ksi}$$

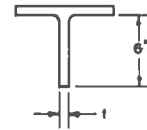
$$P_u = 0.85 A_{s_{gross}} F_{cr} = (0.85) (4.42) (37.0) = 139.0^k$$

or

$$P_u = 0.85 A_{s_{net}} F_y = (0.85) (3.01) (50) = 127.9^k > 52.6 \text{ Governs}$$

$$\frac{b'}{t} = \frac{5.235 - 0.51}{0.300} = 15.8$$

$$\frac{b'}{t_{allow}} = \frac{2,200}{\sqrt{F_y}} \sqrt{\frac{P_u}{P}} = \frac{2,200}{\sqrt{50,000}} \sqrt{\frac{139.0}{52.6}} = 16.0 > 15.8$$



As a tension member, the strut must have a capacity of 91.6 kips. The capacity is assumed to be

$$P_u = A_{s_{net}} F_y$$

where $A_{s_{net}}$ is the net area with half the flange coped for the end connection and half of the remaining flange discounted as specified in Article 1.7.8 of the Specifications.

As tension member:

$$P = 91.6^k$$

$$A_{s_{net}} = 4.42 - (1.5) \left(\frac{5.81 - 0.30}{2} \right) (0.51) = 2.31 \text{ in}^2$$

$$P_u = A_{s_{net}} F_y = (2.31) (50) = 115.5^k > 91.6$$

The WT5x15 section has adequate strength for the top strut.

The diagonal member of the crossframe is designed in similar fashion. Maximum strength requirements are satisfied by an ST4x11.5 section.

Diagonal Design

Try ST4 x 11.5 ASTM A572 Grade 50

As compression member:

$$P = 75.7^k$$

$$A_{s_{gross}} = 3.38 \text{ in}^2$$
$$A_{s_{net}} = 3.38 - \left(\frac{4.171 - 0.441}{2} \right) (0.425) = 2.59 \text{ in}^2$$

$$\frac{Kl}{r_x} = \frac{(5.69)(12)}{1.22} = 56.0 < 120$$

$$\frac{Kl}{r_y} = \frac{(5.69)(12)}{0.798} = 85.6 < 120$$

$$\sqrt{\frac{2\pi^2 E}{F_y}} = 107.0 > 85.6$$

$$\therefore F_{cr} = 50 \left[1 - \frac{50}{4\pi^2(29,000)} (85.6)^2 \right] = 34.0 \text{ ksi}$$

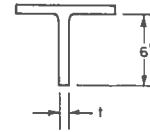
$$P_u = 0.85 A_{s_{gross}} F_{cr} = (0.85) (3.38) (34.0) = 97.7^k$$

or

$$P_u = 0.85 A_{s_{net}} F_y = (0.85) (2.59) (50) = 110.1^k$$

$$\frac{b'}{t} = \frac{4.000 - 0.425}{0.441} = 8.1$$

$$\frac{b'}{t_{allow}} = \frac{2,200}{\sqrt{F_y}} \sqrt{\frac{P_u}{P}} = \frac{2,200}{\sqrt{50,000}} \sqrt{\frac{97.7}{75.7}} = 11.2 > 8.1$$



As tension member:

$$P = 75.7^k$$

$$A_{s_{net}} = 3.38 - 1.5 \left(\frac{4.171 - 0.441}{2} \right) (0.425) = 2.19 \text{ in}^2$$

$$P_u = A_{s_{net}} F_y = (2.19) (50) = 109.5^k > 75.7$$

For the bottom strut the same WT5x15 as used for the top strut proves to be satisfactory.

Bottom Strut Design

Try WT 5x15 ASTM A572 Grade 50

As compression member:

$$P = 47.6^k$$

$$A_{s_{gross}} = 4.42 \text{ in}^2$$
$$A_{s_{net}} = 3.01 \text{ in}^2$$

$$\frac{Kl}{r_x} = 36.6$$

$$\frac{Kl}{r_y} = 77.3$$

$$F_{cr} = 37.0 \text{ ksi}$$

$$P_{u_{gross \text{ sect}}} = 139.0^k$$

$$P_{u_{net \text{ sect}}} = 127.9^k > 47.6 \text{ Governs}$$

$$\frac{b'}{t} = 15.8$$

$$\frac{b'}{t_{allow}} = \frac{2,200}{\sqrt{50,000}} \sqrt{\frac{139.0}{47.6}} = 16.8 > 15.8$$

As tension member:

$$P = 8.6^k$$

$$A_{snet} = 2.31 \text{ in}^2$$

$$P_{u \text{ net sect.}} = 115.5^k > 8.6$$

Strength design of the crossframe is completed by design of the end connections. Welds and connection material are proportioned for a design load, P_D , defined as follows:

$$P_D = 0.75 P_u$$

or

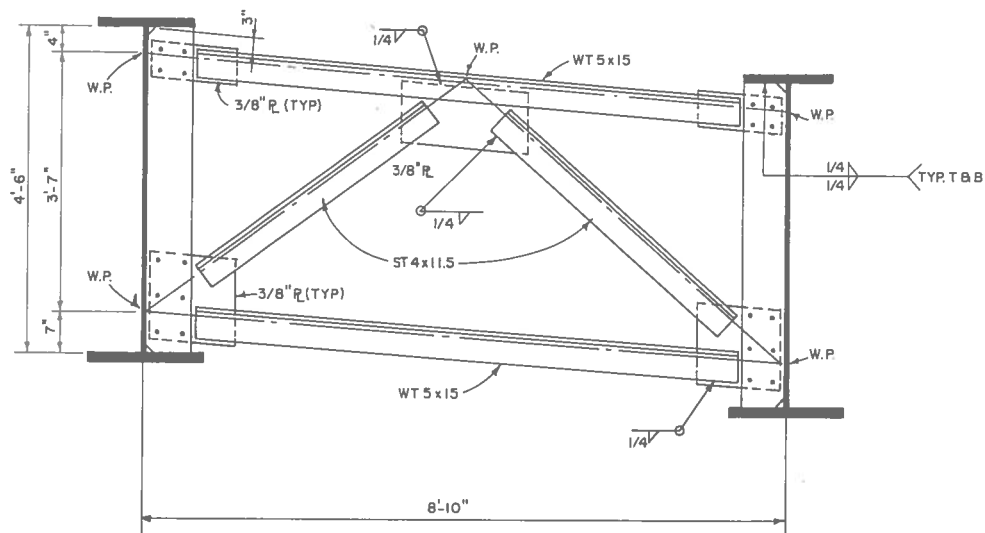
$$P_D = \frac{P + P_u}{2}, \text{ whichever is greater,}$$

and where P_u = capacity of section

P = applied load in member

The fasteners in high-strength bolted friction joints are designed for the above load P_D , divided by 1.3. For the sake of brevity, design calculations for the end connections are not illustrated.

Details of the intermediate crossframe are illustrated below.



Typical Intermediate Crossframe

However, work on this crossframe is still not complete until fatigue has been accounted for. Stress ranges in all members with welded end connections should be checked against allowable stress ranges for Category E. Additionally, stress ranges in the weld metal should be compared to allowable stress ranges for Category F. For fully bolted connections, only Category B need be considered.

Load ranges in the crossframe members are calculated by the same procedures used for the maximum design loads. All loads are service loads. Pp.43–45 show tabulated computations and a summary for fatigue load ranges. However, stress range calculations are not shown.

This completes the design example for a curved I-girder bridge. The total weight of fabricated structural steel is 191,041 lb, giving a weight per square foot of deck area of 28.2 lb. Of the total weight, 171,850 lb is girder material and 9,191 lb is crossframe material, a breakdown of approximately 90 percent–10 percent. Plans for this structure are shown on pages 46, 47 and 48.

Fatigue Check for Intermediate Crossframe at 0.429 Pt., Span 1	
Girder/Description	Moments
G1	
$M_{(L+I+CF_v)}$	$1,616 + 80 = 1,696$
$M_{-(L+I+CF_v)}$	-313
G2	
$M_{(L+I+CF_v)}$	$1,537 + 28 = 1,565$
$M_{-(L+I+CF_v)}$	-238
G3	
$M_{(L+I+CF_v)}$	$1,271$
$M_{-(L+I+CF_v)}$	-140^*
G4	
$M_{(L+I+CF_v)}$	$1,048$
$M_{-(L+I+CF_v)}$	-83^*
$\Sigma M_{(L+I+CF_v)}$	$5,580$
$\Sigma M_{-(L+I+CF_v)}$	-774

*Negative CF_v neglected

Positive $L + I + CF_v$ Moments:

$$V = \frac{\Sigma M}{GRD/d} = \frac{5,580}{(10/9)(300)(26.5)/15.71} = 9.92^k$$

$$\frac{V}{3} = \frac{9.92}{3} = 3.31^k$$

$$M_1 = 1,696 \text{ k-ft} \quad \frac{M_1 d}{R} = \frac{(1,696)(15.71)}{300} = 88.81 \text{ k-ft}$$

$$M_2 = 1,565 \text{ k-ft} \quad \frac{M_2 d}{R} = \frac{(1,565)(15.71)}{300} = 81.95 \text{ k-ft}$$

$$M_3 = 1,271 \text{ k-ft} \quad \frac{M_3 d}{R} = \frac{(1,271)(15.71)}{300} = 66.56 \text{ k-ft}$$

$$M_4 = 1,048 \text{ k-ft} \quad \frac{M_4 d}{R} = \frac{(1,048)(15.71)}{300} = 54.88 \text{ k-ft}$$

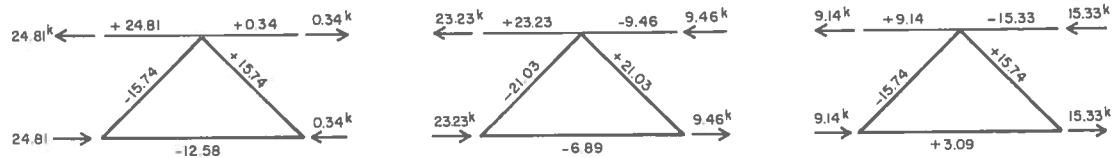
Forces on Girders:



Forces on Crossframes:



Internal Forces in Crossframes:



$$\frac{88.81}{3.58} = 24.81^k \quad \frac{83.17}{3.58} = 23.23^k \quad \frac{32.71}{3.58} = 9.14^k$$

$$\frac{1.22}{3.58} = 0.34^k \quad \frac{33.85}{3.58} = 9.46^k \quad \frac{54.88}{3.58} = 15.33^k$$

Negative L + I + CF_v Moments:

$$V = \frac{774}{\left(\frac{10}{9}\right)(300)(26.5)/15.71} = 1.38^k$$

$$\frac{V}{3} = 0.46^k$$

$$M_1 = 313 \text{ k-ft} \quad \frac{M_1 d}{R} = \frac{(313)(15.71)}{300} = 16.39 \text{ k-ft}$$

$$M_2 = 238 \text{ k-ft} \quad \frac{M_2 d}{R} = \frac{(238)(15.71)}{300} = 12.46 \text{ k-ft}$$

$$M_3 = 140 \text{ k-ft} \quad \frac{M_3 d}{R} = \frac{(140)(15.71)}{300} = 7.33 \text{ k-ft}$$

$$M_4 = 83 \text{ k-ft} \quad \frac{M_4 d}{R} = \frac{(83)(15.71)}{300} = 4.35 \text{ k-ft}$$

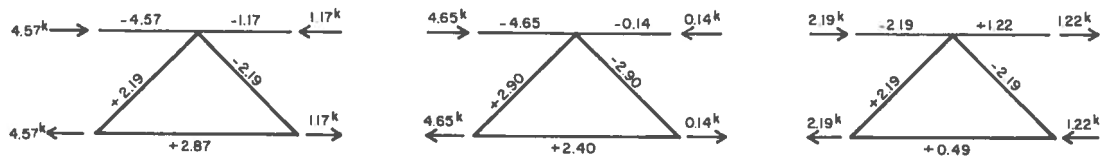
Forces on Girders:



Forces on Crossframes:



Internal Forces in Crossframes:



$$\frac{16.39}{3.58} = 4.57^k \quad \frac{16.66}{3.58} = 4.65^k \quad \frac{7.83}{3.58} = 2.19^k$$

$$\frac{4.20}{3.58} = 1.17^k \quad \frac{0.50}{3.58} = 0.14^k \quad \frac{4.35}{3.58} = 1.22^k$$

Summary of Fatigue Design Loads for Intermediate Crossframes

Member	L + I + CF _v	
Top Strut	T	24.81
	C	4.57
	Range	29.38 ^k
Diagonal	T	21.03
	C	2.90
	Range	23.93 ^k
Bottom Strut	T	2.87
	C	12.58
	Range	15.45 ^k

