

## II/7A

# Composite: Curved Box Girder Load Factor Design

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### Introduction

Chapter 7 illustrates the design of a two span, rigid-frame, box-girder bridge on straight alignment. This chapter illustrates the design of the same bridge but with horizontally curved, two-span box girders and without the rigid-frame construction at the center pier.

Horizontally curved box girders are applicable for simple and continuous spans of lengths similar to those for which straight box girders are applicable, as outlined in Chapter 7. Curved box girders are used for grade-separation and elevated bridges where the structure must coincide with the curved roadway alignment. This condition occurs frequently at urban crossings and interchanges but may also be found at rural intersections where the structure must conform with the geometric requirements of the highway.

The example design of curved box girders presented in this chapter is in accordance with the 12th Edition of the "Standard Specifications for Highway Bridges" of the American Association of State Highway and Transportation Officials (AASHTO), 1977, and the 1978 "Interim Specification" (hereinafter referred to as the AASHTO Specifications), as modified by the "AASHTO Guide Specifications for Horizontally Curved Highway Bridges," 1980 (hereinafter referred to as the Guide Specifications). (See also C. P. Heins, "Box-Girder Bridge Design—State of the Art," Engineering Journal, 4th Quarter, 1978, American Institute of Steel Construction.) The design specifies ASTM A36 and A572, Grade 50, steels for the box girders.

### General Design Considerations

Curved box girders are of the same general construction as straight box girders, consisting of a bottom flange, two webs, which may be either vertical or sloped, and top flanges attached to the concrete deck with shear connectors. In negative-bending regions, where the bottom flange is in compression, it is usually stiffened by longitudinal stiffeners or both longitudinal and transverse stiffeners.

Curved box girders differ from straight box girders in that the curved boxes normally have internal diaphragms or cross frames at regular intervals along the span and lateral bracing at the top flange. The cross frames maintain the shape of the cross section and are spaced at such intervals as to keep the transverse distortional stresses and lateral bending stresses in the flanges at acceptable levels. Cross frames are discussed in more detail later.

The principles of composite construction as applied to flexure in curved box girders are assumed to be the same as for straight box girders. These have been discussed in Chapter 7 and in more detail in Chapters 3, 3A, 4 and 4A in connection with rolled beams and plate girders.

### LOADS, LOAD COMBINATIONS AND LOAD FACTORS

Loads and load factors are considered to be the same as those given in the AASHTO Specifications for straight bridges. Curvature, however, introduces additional effects, such as forces due to roadway superelevation, centrifugal forces and thermal forces. For box girders, the Guide Specifications account for centrifugal forces by means of special impact factors, which are given later. Centrifugal forces, therefore, need not be considered in any other way. Thermal forces may be neglected if the support system is designed to permit thermal movements.

The following load combinations should be considered:

**A. Construction Loads.** A partial dead load  $D_p$  and a live load due to construction vehicles  $C$  comprise the total construction load. At each construction stage, the strength of a member must be sufficient to resist the effects of the load combination  $1.3(D_p + C)$ .

**B. Service Loads.** These consist of the total dead load  $D$  plus the total design live load  $L_T$ .

$$L_T = L + I$$

where  $L$  = basic live load from vehicles that may operate on a highway legally without a specific load permit

$I$  = impact loads

The service loads are multiplied by the appropriate load factors for Maximum Design Load and Overload and then combined into group loadings in accordance with the AASHTO Specifications and as outlined in Chapters 3A, 4A, 5, and 7.

Impact is an important consideration in design of curved box girders, because of the uplift and vibrations that may occur. The Guide Specifications assign impact factors for design of components of curved box girders as given in the following table. As stated previously, these impact factors include the effects of centrifugal forces.

**Impact Factors for Curved Box Girders**

Condition to Be Determined	Impact Factor $I$
Reactions	1.00
Direct stresses in box webs and bottom plates	0.35
Direct stresses in concrete slab	0.30
Shear stresses in box web	0.50
Stresses in diaphragms	0.50
Deflections	0.30

The impact factors are valid within the following parameter ranges:

$$100 \text{ ft} \leq L \leq 300 \text{ ft}$$

$$300 \text{ ft} \leq R_c \leq 1,000 \text{ ft}$$

$$v \leq 70 \text{ mph}$$

$$\text{Number of box girders} \leq 3$$

$$\text{Number of continuous spans} \leq 2$$

$$\frac{\text{Weight of vehicles}}{\text{Weight of bridge}} \leq 0.3$$

where  $L$  = girder span, ft

$R_c$  = radius, ft, of centerline of bridge

$v$  = vehicle speed, mph

The Guide Specifications require that a dynamic analysis be made if the above ranges are exceeded.

### LATERAL DISTRIBUTION OF DEAD AND LIVE LOAD

Initial dead load and superimposed dead load are made up of the same items that constitute the dead loads for a straight bridge. Also, the lateral distribution is the same as that illustrated in Chapter 7. The live-load distribution factor for moment for a curved box girder, however, is different. This factor can be expressed as a modification of the distribution factor for straight box girders given in the AASHTO Specifications. Studies of curved box-girder bridges with radii ranging from 200 to 10,000 ft have shown that the moments are related to straight-girder moments by

$$W_{Lc} = (1,440X^2 + 4.8X + 1)W_L$$

where  $W_{Lc}$  = distribution factor for live-load moments in curved box girders

$W_L$  = distribution factor for live-load moments in straight box girders

$$X = 1/R_c$$

$R_c$  = radius of the centerline of the bridge, ft

This distribution factor is assumed in this chapter to be applicable also to shear, torque and deflection.

### STRUCTURAL ANALYSIS

Any of several different approaches may be used to analyze curved box-girder bridges of the type presented in the following design example. For instance, if the various elements of the structure were idealized as line elements, it could be treated as a planar grid and analyzed by a classical stiffness method for moment, shear and torque. The grid might be taken as the two box girders, considered totally independent of each other, or as the two girders interacting through the deck slab and diaphragms.

Idealization of the girders into line elements is appropriate when the transverse dimensions of the members are small relative to the length. If the member cross sections do not deform, the unit stresses are assumed to be obtainable by ordinary flexural theory, as illustrated in Chapters 3, 4, 5 and 7.

Finite-element methods are more general and may be used for a wide variety of structural analysis problems. Such programs require moderate- to large-size computer systems.

Other solutions for curved box-girder bridges include the use of finite-difference and folded-plate techniques.

A major disadvantage of many rigorous computer programs is that they analyze only one specific loading condition at a time. This makes it difficult to obtain maximum-stress curves for live loads, because each point on a curve represents the effects of a separate loading condition, which involves its own input and for which the load position for maximum or minimum effect is generally not known. Either trial-and-error loading, or the generation of influence lines or surfaces, is required for development of the necessary curves (see W. F. Till, W. N. Poellot, Jr., and A. W. Hedgren, Jr., "Curved Girder Workshop," textbook prepared for Federal Highway Administration, Washington, D.C., 1976).

A finite-difference program developed at the University of Maryland, however, does provide automatic generation of maximum-stress curves (see C. P. Heins and C. Yoo, "User's Manual for the Static Analysis of Curved Girder Bridges," Report No. 55, Sept., 1973, Civil Engineering Department, University of Maryland).

The *M/R* method is an appropriate calculation that makes use of the conjugate-beam analogy. It is readily understood and adaptable to any design office operation. The computations may be performed longhand or may be partly or fully programmed for a computer or electronic desk calculator. (See D. H. H. Tung and R. S. Fountain, "Approximate Torsional Analysis of Curved Box Girders by the *M/R* Method," Engineering Journal, July, 1970, American Institute of Steel Construction.)

The method loads a conjugate simple span beam with a distributed loading, which is equal to the moment in the real simple or continuous span induced by the applied load divided by the radius of curvature of the girder.

The resulting shears in the conjugate span are then numerically equal to the internal torques in the real span.

The following tables compare influence ordinates computed by the *M/R* method with those obtained by the finite-element program NASTRAN (using beam elements) and the University of Maryland finite-difference program, for the outer girder of the design example of this chapter. The comparison indicates that, for bending moment at the maximum-positive bending section (Joint 5), the maximum differences of the *M/R* calculations from NASTRAN and finite-difference moments are 1.4% and 3.9%, respectively. Average differences are 0.5% and 1.6%.

Comparison of Moments at Joint 5

Load at	<i>M/R</i>	NASTRAN	Finite-Diff.
1	0	0	0
2	71	71	71.3
3	143	143	142.8
4	218	217	217.2
5	296	295	300.8
6	231	230	240.4
7	172	170	178.9
8	119	118	121.9
9	73	72	73.6
10	33	33	33.2
11	0	0	0

For girder torques at the end support, the maximum differences of the  $M/R$  calculations from NASTRAN and finite-difference torques are 19.2% and 18.7%, respectively. Average differences are 11.2% in both cases.

Comparison of Torques at Joint 1

Load at	$M/R$	NASTRAN	Finite-Diff.
1	0	0	0
2	10.2	9.86	11.4
3	16.6	15.96	18.98
4	17.4	18.94	21.39
5	20.4	19.39	21.79
6	18.9	17.90	20.35
7	15.9	15.07	17.03
8	12.1	11.48	12.68
9	7.98	7.55	8.17
10	3.79	3.6	3.84
11	0	0	0
12	-3.5	-2.95	-3.0
13	-6.2	-5.2	-5.3
14	-8.0	-6.78	-6.9
15	-8.98	-7.58	-7.8
16	-8.8	-7.57	-7.9
17	-8.0	-6.89	-7.1
18	-6.6	-5.66	-5.7
19	-4.66	-4.03	-4.06
20	-2.4	-2.096	2.13
21	0	0	0

These comparisons provide some measure of the accuracy with which curved-box girder behavior can be predicted by currently available methods. The reasonable consistency of results should enhance the designer's confidence in selection of a method.

This chapter utilizes the  $M/R$  approach because of its ease of application and satisfactory results. The background, rationale and application of the method are fully described in the Tung and Fountain paper previously mentioned.

### TORSIONAL EFFECTS

The applied torque in a curved box girder is resisted by a combination of two kinds of internal torsion: pure, or St. Venant, torsion and warping torsion. St. Venant torsion provides most of the resistance.

The Guide Specifications state that if the box girder does not have a full-width steel top flange, the girder must be treated under initial dead load (wet-concrete stage) as an open section. This means that the St. Venant torsion constant  $K_T$  (from elementary mechanics) is given by

$$K_T = \frac{1}{3} \sum b t^3$$

where  $b$  = width of an individual plate element

$t$  = thickness of the plate element

If the section is closed, however,

$$K_T = \frac{4A^2}{\sum b/t}$$

where  $A$  = enclosed area of the section.

A closed box-girder section is usually several thousand times stiffer than an open section. For this reason, if a curved box girder does not have a permanent, solid, top-flange plate, the girder is braced by a lateral system at or near the top flange, to "quasi-close" the box during the wet-concrete stage of construction (see "Steel/Concrete Composite Box-Girder Bridge — A Construction Manual," ADUSS 88-7493-01, Dec., 1978, United States Steel Corporation).

For analysis purposes, top lateral bracing may be transformed to an equivalent thickness of plate  $t_{eq}$ , in., by

$$t_{eq} = \frac{E}{G} \frac{2A_d}{b} \cos^2 \alpha \sin \alpha$$

where  $E$  = steel modulus of elasticity, ksi

$G$  = steel shearing modulus of elasticity, ksi

$A_d$  = area of lateral-bracing diagonal, sq in.

$b$  = clear box width, in., between top flanges

$\alpha$  = angle of lateral-bracing diagonal with respect to transverse direction

To properly close the section and minimize warping stresses, the cross-sectional area of the lateral-bracing diagonal should be at least

$$A_d = 0.03b$$

The internal stresses produced by St. Venant torsion in a closed section are shearing stresses around the perimeter, as shown in the following sketch and defined by

$$\tau = \frac{T}{2At}$$

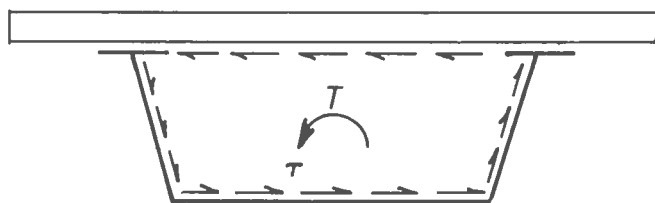
where  $\tau$  = St. Venant shear stress in any plate, ksi

$T$  = internal torque, in.-kips

$A$  = enclosed area within box girder, sq in.

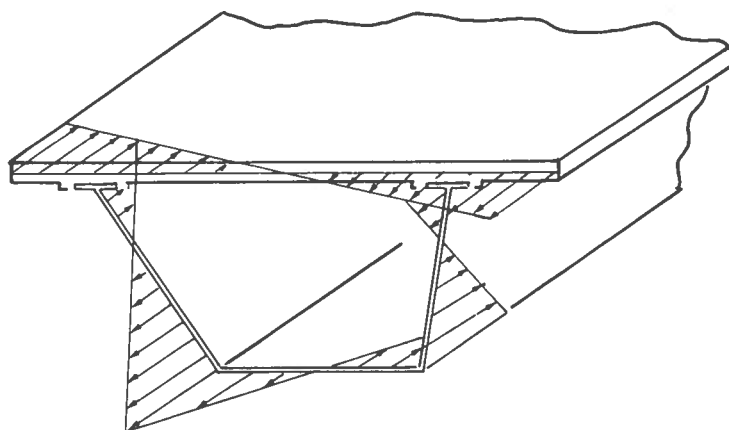
$t$  = thickness of plate, in.

These shearing stresses add to the vertical shearing stresses in one of the girder webs and subtract from the vertical shearing stresses in the other girder web.



**ST. VENANT TORSION IN A CLOSED SECTION**

Normal stresses as shown in the following sketch result from warping torsion restraint and from distortion of the cross-section. The Guide Specifications state that "the effect of normal stresses due to nonuniform torsion (warping torsion) and cross-sectional deformation shall be included in the design of curved box-girder bridges unless a rational analysis indicates that these effects are small."



### WARPING STRESSES IN A BOX GIRDER

Researchers have determined that warping stresses may be neglected for single-box, closed cross sections but may have to be taken into account for twin-box structures. Methods are available for computing warping stresses at torsionally fixed supports (see W. T. Till, W. N. Poellot, Jr., and A. W. Hedgren, Jr., "Curved Girder Workshop," Federal Highway Administration, Washington, D.C., 1976).

Warping stress due to distortion can be reduced to negligible levels through the use of internal crossframes. This is covered in detail on page 24, and has been accounted for in the design by limiting the crossframe spacing. Warping torsional stress has been neglected in the example because of its complexity and because the design of the negative bending section at the pier has otherwise been treated conservatively.

It should also be recognized that lateral bending stress in the top flange due to curvature, discussed on pages 19 and 24, is a form of local warping torsional stress and has been fully accounted for in the example.

### WEBS

The Guide Specifications require that the maximum calculated shear for design of the box-girder webs be the sum of the vertical shear  $V_v$  associated with bending moment and the shear  $V_T$  due to St. Venant torsion. If the web is inclined, the design shear associated with bending moment is

$$V_w = \frac{V_v}{\cos\theta}$$

where  $\theta$  = angle of inclination of web plate with the vertical.

No web stiffeners are required if

$$\frac{D}{t} \leq 150$$

where  $D$  = web depth, in. (depth along the slope for sloping webs)

$t$  = web thickness, in.

For an unstiffened web, the ultimate shear capacity, kips, is the smaller of the following:

$$V_{u1} = \frac{3.5Et^3}{D}$$

$$V_{u2} = 0.58F_yDt$$

where  $E$  = modulus of elasticity of web steel, ksi

$F_y$  = yield strength of web steel, ksi

When the maximum design shear exceeds  $V_{ul}$ , transverse stiffeners are required on the web. A transversely stiffened web must satisfy

$$\frac{D}{t} \leq \frac{1,154}{\sqrt{F_y}} \left[ 1 - 8.6 \frac{d_o}{R} + 34 \left( \frac{d_o}{R} \right)^2 \right]^*$$

where  $d_o$  = spacing, in., of transverse stiffeners

$R$  = radius of web curvature

The ultimate shear capacity of the stiffened web is given by

$$V_u = 0.58 F_y D t C$$

$$\text{where } C = 569.2 \frac{t}{D} \sqrt{\frac{1 + (D/d_o)^2}{F_y}} - 0.3 \leq 1.0$$

The stiffener spacing, however, is not permitted to exceed the depth of the web.

For proportioning the stiffener, the Guide Specifications limit the width to  $82.2/\sqrt{F_y}$  times its thickness. The moment of inertia of the transverse stiffener with respect to the midplane of the web must be at least

$$I = d_o t^3 J$$

$$\text{where } J = \left[ 2.5 \left( \frac{D}{d_o} \right)^2 - 2 \right] X$$

$$X = 1.0 \text{ when } d_o/D \leq 0.78$$

$$= 1 + \left( \frac{d_o/D - 0.78}{1.775} \right) Z^4 \text{ when } 0.78 \leq \frac{d_o}{D} \leq 1 \text{ and } 0 \leq Z \leq 10$$

$$Z = 0.95 d_o^2 / R t$$

$R$  = radius of curvature of the web

When the girder section is unsymmetrical, as is normally the case with composite girders, and when  $D_c$ , the clear distance between the neutral axis and the inside face of the compression flange, exceeds  $D/2$ , then  $D_c/2$  for the compressed web must not exceed the preceding limits with  $D$  taken as  $2D_c$ . In this case, the location of the neutral axis should be taken as that for the short-term composite section.

## BOTTOM FLANGES

The maximum normal tension stress  $F_b$ , ksi, including warping normal stress, is limited to

$$F_b = F_y \sqrt{1 - 3(f_v/F_y)^2}$$

where  $F_y$  = yield strength, ksi

$f_v$  = shear stress, ksi

\*The design equations presented in this section are those given for Load Factor Design in the Guide Specifications. Where applicable, the constants in the equations have been modified to give results in kips per square inch rather than pounds per square inch.



The allowable compression stress for flanges involves several parameters, which are defined as follows:

$$R_1 = \frac{97.08 \sqrt{K}}{\sqrt{\frac{1}{2} \left[ \Delta + \sqrt{\Delta^2 + 4(f_v/F_y)^2 (K/K_s)^2} \right]}}$$

$$R_2 = \frac{210.3 \sqrt{K}}{\sqrt{\frac{1}{1.2} \left[ \Delta - 0.4 + \sqrt{(\Delta - 0.4)^2 + 4(f_v/F_y)^2 (K/K_s)^2} \right]}}$$

where  $\Delta = \sqrt{1 - 3(f_v/F_y)^2}$

$K$  = buckling coefficient

= 4 when  $n = 0$

= any assumed value less than 4 when  $n > 0$

$n$  = number of equally spaced longitudinal flange stiffeners

$K_s$  = buckling coefficient

= 5.34 when  $n = 0$

=  $\frac{5.34 + 2.84(I_s/bt^3)^{1/3}}{(n+1)^2}$  when  $n > 0$

$I_s$  = moment of inertia, in.<sup>4</sup>, of a longitudinal flange stiffener about an axis parallel to the flange and at the base of stiffener

The maximum allowable compression stress  $F_b$  for flanges depends on the magnitude of the St. Venant shear stress  $f_v$  across the flange. One case is that for which  $f_v$  is less than  $0.75F_y/\sqrt{3}$ . In this case, there are several expressions for  $F_b$ , depending on the ratio  $w/t$  of width to thickness of the flange between longitudinal stiffeners:

If  $\frac{w}{t} \sqrt{F_y}$  does not exceed  $R_1$ ,

$$F_b = F_y \Delta$$

Note that, in this case,  $F_b$  is the same as the allowable tension stress for a flange.

If  $\frac{w}{t} \sqrt{F_y}$  lies between  $R_1$  and  $R_2$ ,

$$F_b = F_y \left[ \Delta - 0.4 \left( 1 - \sin \frac{\pi}{2} \frac{R_2 - \frac{w}{t} \sqrt{F_y}}{R_2 - R_1} \right) \right]$$

If  $\frac{w}{t} \sqrt{F_y}$  equals or exceeds  $R_2$ ,

$$F_b = 26,210K \left( \frac{t}{w} \right)^2 - \frac{f_v^2 K}{26,210K_s^2 (t/w)^2}$$

A second case is that for which  $f_v$  lies between  $0.75F_y/\sqrt{3}$  and  $F_y/\sqrt{3}$ . In this case,  $\frac{w}{t}\sqrt{F_y}$  is not permitted to exceed  $R_1$  nor is  $w/t$  allowed to exceed 60, except in regions of low compression stress near points of dead load contraflexure. The maximum allowable compression stress is given by

$$F_b = F_y \Delta$$

which again is the same as the allowable tension stress for a flange.

The longitudinal bottom-flange stiffeners should be equally spaced between the girder webs. These stiffeners must be proportioned so that the moment of inertia about the base of the stiffener is at least equal to

$$I_s = \phi t^3 w$$

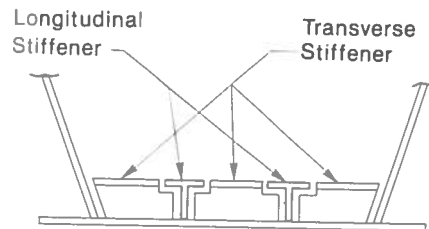
where  $\phi = 0.07K^3n^4$  for  $n > 1$

$= 0.125K^3$  for  $n = 1$

$n$  = number of stiffeners

$K$  = buckling coefficient  $\leq 4$

The Guide Specifications also state that, when longitudinal stiffeners are used, a transverse stiffener must be placed between the longitudinal stiffeners at the point of maximum compression stress and near points of dead-load contraflexure, as shown in the following sketch. The transverse stiffeners must be the same size as the longitudinal stiffeners. In addition, the Guide Specifications require that the transverse stiffeners be connected only to the bottom flange. The connection should be designed for a force equal to the calculated bending stress in the longitudinal stiffener times the stiffener area.



**TRANSVERSE BOTTOM-FLANGE STIFFENER**

## TOP FLANGES

Under total design loading, the narrow top flanges of a box girder work compositely with the concrete deck with an initial locked-in stress due to  $DL_1$ . The effective slab width for the composite section is computed in the same manner as for straight I and box girders, as shown in Chapters 3, 4, 5 and 7.

The following definitions and limits apply to all the following allowable-stress criteria for narrow top flanges:

(a) The absolute value of the ratio of the normal stress  $f_w$  due to nonuniform torsion (lateral bending) to the normal stress  $f_b$  due to flexure shall not exceed 0.5 anywhere along the length of the girder; that is  $f_w/f_b \leq 0.5$ .

(b) The unbraced length of flange  $l$  shall not exceed 25 times the width of the compression flange  $b$ .

(c) The unbraced length  $l$  shall not exceed  $0.1R$ , where  $R$  is the radius of curvature of the flange.

(d) The unbraced length of flange is the distance between cross frames or diaphragms.

(e) The ratio  $f_w/f_b$  is positive when  $f_w$  is compressive on the flange tip farthest from the center of curvature. The average flexural stress  $f_b$  shall be computed using the larger of the two bending moments at either end of the braced segment of the flange, and  $f_w$  is the corresponding value of  $f_w$  at that location.

The maximum allowable total stress for top flanges for composite construction is the same as that specified in the Guide Specifications for curved, composite I girders.

#### Compression in Top Flange—Total Design Loading

The average normal stress, ksi (exclusive of lateral bending stress) is limited to

$$F_{bu} = F_y \bar{\rho}_B \bar{\rho}_W$$

where  $F_y$  = yield strength of top-flange steel, ksi

$$f = 1 - 3 \frac{F_y}{E \pi^2} \left( \frac{l}{b} \right)^2$$

$$\bar{\rho}_B = \frac{1}{1 + \frac{l}{b} \left( 1 + \frac{l}{6b} \right) \left( \frac{l}{R} - 0.01 \right)^2}$$

$$\bar{\rho}_W = 0.95 + 18 \left( 0.1 - \frac{l}{R} \right)^2 + \frac{(f_w/f_b) \left( 0.3 - 0.1 \frac{l}{R} \frac{l}{b} \right)}{\bar{\rho}_B / f}$$

$l$  = unbraced length of compression flange, in.

$b$  = flange width, in.

$E$  = modulus of elasticity, ksi, of flange steel

$R$  = radius of curvature of flange, in.

$f_w$  = lateral bending stress due to all causes, ksi

$f_b$  = ordinary bending stress due to vertical loading, ksi

If  $\bar{\rho}_B \bar{\rho}_W$  exceeds unity,  $\bar{\rho}_B \bar{\rho}_W = 1.0$  should be used.

#### Tension in Top Flange—Total Design Loading

The average normal stress is limited to

$$F_{bs} = F_y f$$

#### Compact Flanges Under Construction Loading

Under construction loading at the wet-concrete stage, the top flanges should be considered to act as noncomposite, I-girder flanges. The allowable stress under this condition depends on whether the flange is compact or noncompact as defined by the ratio  $b/t$ . For compactness,  $b/t \leq 101.2/\sqrt{F_y}$ . When this property is checked, if the ratio  $b/t$  changes between points of bracing, the larger value of  $b/t$  should be used.

Under construction loading, if the flange is compact, the average normal stress is limited as follows:

#### Compression (Compact Flange)—Construction Loading

The allowable stress is  $F_{bu}$  as specified for composite flanges in compression under total design loading.

#### Tension (Compact Flange)—Construction Loading

$$F_{bu} = F_y \bar{\rho}_B \bar{\rho}_W$$

where  $\bar{\rho}_B$  and  $\bar{\rho}_W$  are as defined for total design loading.

### Noncompact Flanges under Construction Loading

If  $b/t$  lies between  $101.2/\sqrt{F_y}$  and  $139.1/\sqrt{F_y}$ , the flange is noncompact. For noncompact flanges under construction loading, the average normal stress is limited to

$$F_{by} = F_{bs} \bar{\rho}_B \bar{\rho}_w$$

where  $F_{bs} = F_y$  for tension flanges

$$= F_y \left[ 1 - 3 \frac{F_y}{E \pi^2} \left( \frac{l}{b} \right)^2 \right] \text{ for compression flanges}$$

$$\rho_B = \frac{1}{1 + (l/R) (l/b)}$$

$\rho_w = \rho_{w1}$  or  $\rho_{w2}$ , whichever is smaller, if  $f_w/f_b$  is positive

$= \rho_{w1}$  if  $f_w/f_b$  is positive

$= \rho_{w1}$  if  $f_w/f_b$  is negative

$$\rho_{w1} = \frac{1}{1 - (f_w/f_b) (1 - l/75b)}$$

$$\rho_{w2} = \frac{0.95 + \frac{l/b}{30 + 8,000 (0.1 - l/R)^2}}{1 + 0.6 (f_w/f_b)}$$

Furthermore, for noncompact flanges, the tip stress  $f_b + f_w$  is not permitted to exceed  $F_y$ .

### SHEAR CONNECTORS

Design of shear connectors for fatigue is the same as that for straight girders as given in the AASHTO Specifications. For ultimate strength, the Guide Specifications require that the number of shear connectors between points of maximum positive moment and the end supports or dead-load inflection points be sufficient to satisfy.

$$P_c \leq \phi S_u$$

where  $\phi$  = reduction factor = 0.85

$S_u$  = ultimate strength, kips, of the shear connector as given in the AASHTO Specifications for straight girders

$P_c$  = force, kips, on the connector

$$= \sqrt{P^2 + F^2 + 2PF \sin \frac{\Theta}{2}}$$

$$P = \frac{P}{N}$$

$P = 0.85 f'_c b c$  or  $A_s F_y$ , whichever is smaller, at points of maximum positive moment

$= A_s F_y$  at points of maximum negative moment as defined by the AASHTO Specifications for straight girders

$N$  = number of connectors between points of maximum positive moment and adjacent end supports or dead-load inflection points, or between points of maximum negative moment and adjacent dead-load inflection points

$$F = \frac{P(1 - \cos \Theta)}{4KN_s \sin \Theta/2}$$

$\Theta$  = angle extended between point of maximum moment (positive or negative) and adjacent point of contraflexure or support

$f'_c$  = 28-day compressive strength of concrete slab, ksi

$b$  = effective width, in., of slab

$c$  = thickness, in., of slab

$A_s$  = total area, sq. in., of steel section, including cover plates

$A_s$  = total area, sq. in., of longitudinal reinforcing steel at the interior support within the effective width of flange

$F_y$  = yield strength, ksi, of the reinforcing steel

$$K = 0.166 \left( \frac{N}{N_s} - 1 \right) + 0.375$$

$N_s$  = number of connectors at a section

### INTERNAL DIAPHRAGMS

Curved box girders require internal diaphragms at the supports to resist transverse rotation, displacement and distortion and to transmit the girder torque to the substructure. In addition, intermediate diaphragms or cross frames should be provided unless a rational analysis indicates that they are not needed. Diaphragms or cross frames serve to limit the normal and transverse bending stresses due to distortion and the lateral bending stresses in the narrow top flanges during the wet-concrete stage of construction. Formulas for the spacing of intermediate cross frames and for the required cross-sectional area of cross-frame diagonals have been derived based on limitation of the distortion stress to 10% of the stress due to ordinary bending. For this limitation (according to Heins, page 2), cross-frame spacing should not exceed

$$S = L \sqrt{\frac{R}{200L - 7,500}} \leq 25 \text{ ft}$$

and the cross-sectional area, sq. in., of the diagonal should be at least

$$A_b = 750 \frac{Sb}{d^2} \frac{t^3}{d+b}$$

where  $S$  = diaphragm spacing, in.

$d$  = depth of box, in.

$b$  = width of box, in.

$t$  = thickness, in., of thickest component of box-girder cross section

$R$  = radius of girder, in.

$L$  = span of girder, in.

### LATERAL BENDING STRESSES UNDER $DL_1$

Two kinds of lateral bending occur in the top flanges under the initial dead-load condition, during which the flanges are not supported by the concrete deck. The first kind is lateral bending due to the horizontal component of the shear force in the sloping webs (see Chapter 7). The second kind of lateral bending is that due to curvature. The equations used for calculating lateral-bending effects are as follows:

$$M_{LC} = \frac{M_1 d^2}{10Rh}$$

where  $M_{LC}$  = lateral bending moment, kip-in., due to curvature

$M_1$  =  $DL_1$  moment, kip-in.

$d$  = distance, in., between diaphragms

$R$  = radius of curvature of the girder, in.

$h$  = depth of girder, in.

The corresponding stress at the flange tips is

$$f_{wc} = \frac{6M_{LC}}{2b^2t}$$

where  $f_{wc}$  = lateral bending stress, ksi, due to curvature

$b$  = flange width, in.

$t$  = flange thickness, in.

Lateral bending stresses due to the effects of both curvature and the sloping webs are assumed to be proportional to the square of the unbraced length of flange. Thus, lateral flange bending stresses, as well as distortional stresses, may influence selection of cross-frame spacing.

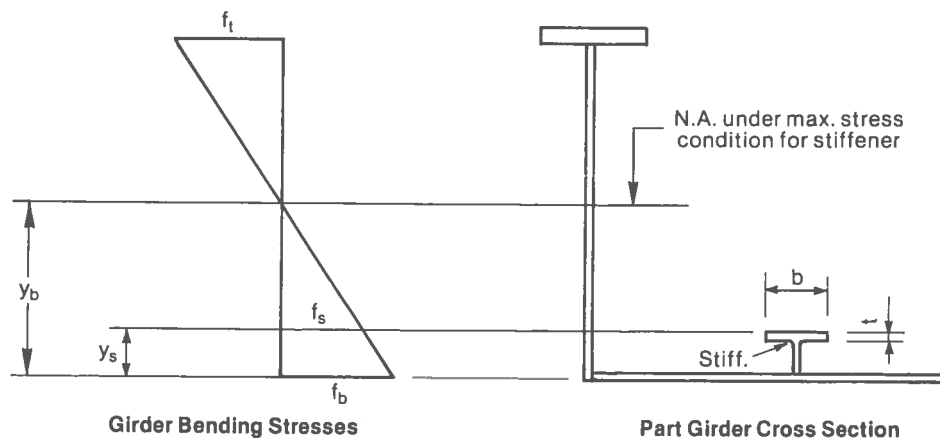
Lateral bending stress due to curvature also occurs in the flanges of the longitudinal stiffeners attached to the bottom flange. These stiffener flanges participate with the girder flanges in resisting bending moments and carry a stress  $f_s$ , ksi, as shown in the following sketch and given by

$$f_s = \frac{y_b - y_s}{y_b} f_b$$

where  $f_b$  = maximum bending stress, ksi, in the girder bottom flange

$y_b$  = distance, in., from neutral axis to bottom of girder

$y_s$  = distance, in., from neutral axis to top of stiffener flange



### BENDING STRESSES IN LONGITUDINAL STIFFENER

Since the stiffener is curved, its flange is subjected to a lateral bending moment

$$M_{LC} = \frac{f_s b t d^2}{10R}$$

where  $d$  = unbraced length, in., of stiffener flange

$t$  = thickness, in., of stiffener flange

$b$  = width, in., of stiffener flange

$R$  = radius of curvature, in., of stiffener

The corresponding lateral bending stress is

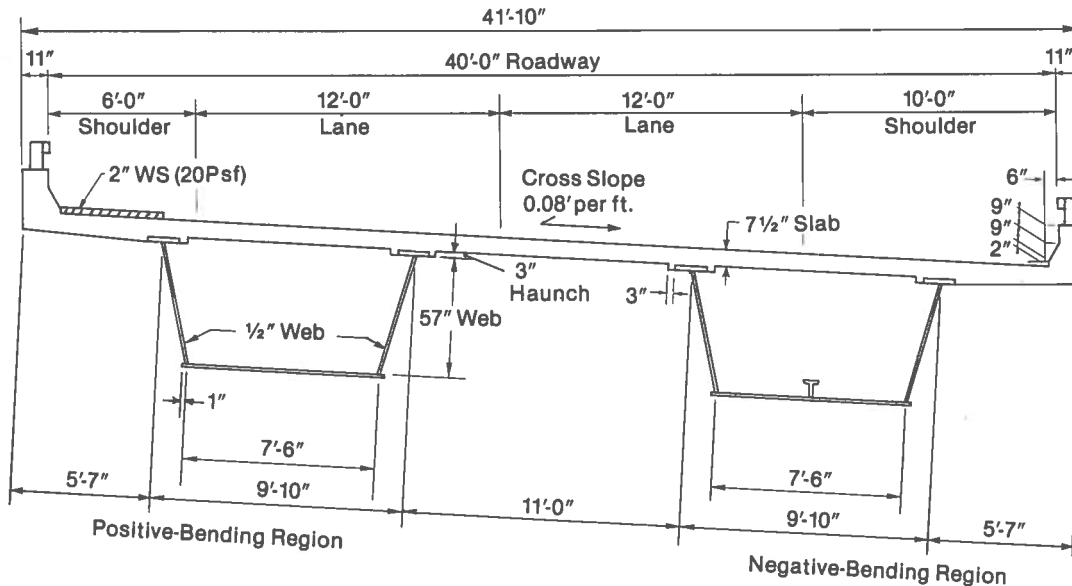
$$f_{wc} = \frac{6f_s d^2}{10Rb}$$

With the direct stress and the lateral bending stress in the stiffener flange known,  $f_s$  may be checked against the allowable stresses for noncomposite I-girder flanges given previously for top flanges of the girder under construction loading.

## Design Example—Two-span Curved Box Girder (120-120 Ft) Composite for Positive and Negative Bending

The following data apply to this design:

Roadway Section: See typical bridge cross section.



TYPICAL CROSS SECTION OF EXAMPLE BRIDGE

**Specifications:** 1977 AASHTO Standard Specifications for Highway Bridges and 1978 Interim Specifications.

**Loading:** HS20-44.

**Structural Steel:** ASTM A36 and A572, Grade 50.

**Concrete:**  $f'_c = 4,000$  psi, modular ratio  $n = 8$ .

**Slab Reinforcing Steel:** ASTM A615, Grade 40, with  $F_y = 40$  ksi.

**Loading Conditions:**

Case 1—Weight of girder and slab ( $DL_1$ ) supported by the steel girder alone.

Case 2—Superimposed dead load ( $DL_2$ ) (parapets and railings) supported by the composite section with the modular ratio  $n = 8$ . (Used in design of web-to-flange fillet welds.)

Case 3—Superimposed dead load ( $DL_2$ ) (parapets and railings) supported by the composite section with the increased modular ratio  $3n = 3 \times 8 = 24$ .

Case 4—Live load plus impact ( $L + I$ ) supported by the composite section with the modular ratio  $n = 8$ .

Fatigue—500,000 cycles of truck load  
100,000 cycles of lane loading } Redundant load-path structure.

**Loading Combinations:**

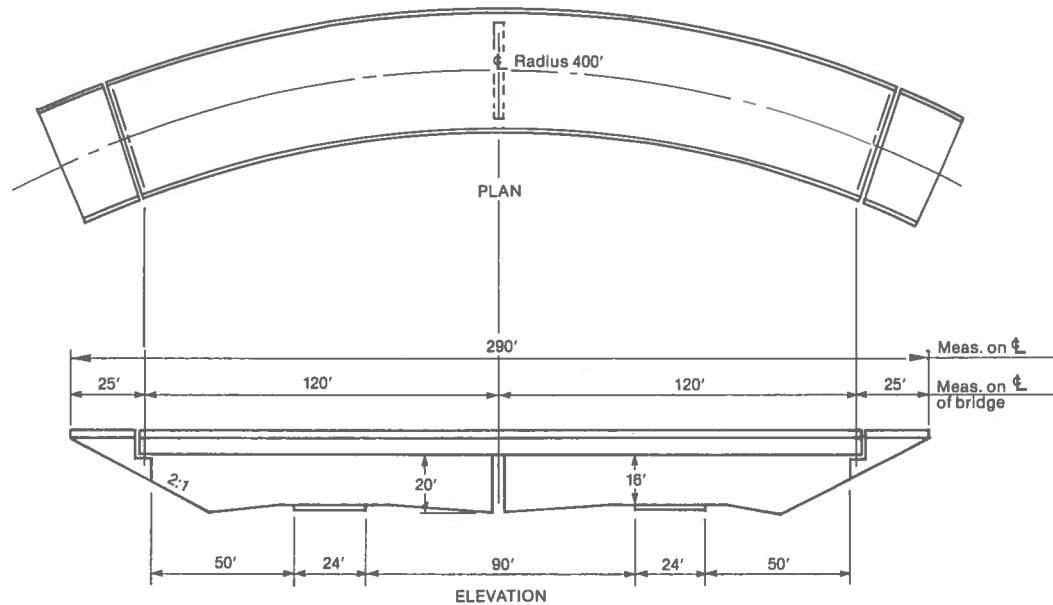
Combination A = Case 1 + 3 + 4

Combination B = Case 2 + 4

Combination C = Case 1 + 2 + 4

## GEOMETRY OF BRIDGE

The following plan and elevation views show the geometric layout for the example structure of this chapter.



### TWO-LANE CURVED OVERPASS STRUCTURE FOR 4-LANE DIVIDED HIGHWAY—90-FOOT MEDIAN

## LOADS, SHEARS, MOMENTS AND TORQUES FOR BOX GIRDER

The initial dead load and superimposed dead load are the same as those of the design in Chapter 7. Initial dead load consists of the weight of the girder, concrete slab and haunches. The superimposed dead load consists of the weight of the parapet, wearing surface and railing.

### Dead Load on Steel Box Girder

$$\text{Slab} = 0.63 \times 20.9 \times 0.150 = 1.976$$

$$0.12 \times 4.83 \times 0.150 = 0.087$$

$$\text{Haunches} = 0.19 \times 1.67 \times 0.150 \times 2 = 0.095$$

$$\text{Girder (assumed weight)} = 0.475$$

$$DL_1 \text{ per girder} = 2.633 \text{ k/ft}$$

### Dead Load Carried by Composite Section

$$\text{Parapet} = 1.50 \times 0.92 \times 0.150 = 0.207$$

$$0.37 \times 0.50 \times 0.150 = 0.028$$

$$0.17 \times 1.42 \times 0.150 = 0.036$$

$$\text{Wearing surface} = 0.020 \times 19.50 = 0.390$$

$$\text{Railing} = 0.020$$

$$DL_2 \text{ per girder} = 0.681 \text{ k/ft}$$



### Live Load on Box Girder

The live-load distribution factor for the curved box girders is the factor for straight girders, from Chapter 7, modified by the curvature factor  $1,440X^2 + 4.8X + 1$ , where  $X$  is the reciprocal of the centerline radius of the bridge. For a roadway width  $W_c = 40$  ft,

$$N_w = \frac{W_c}{12} = 3.33$$

This is reduced to the integer 3. Because there are two box girders, the factor  $R$  used in computation of live-load distribution is

$$R = \frac{N_w}{2} = \frac{3}{2}$$

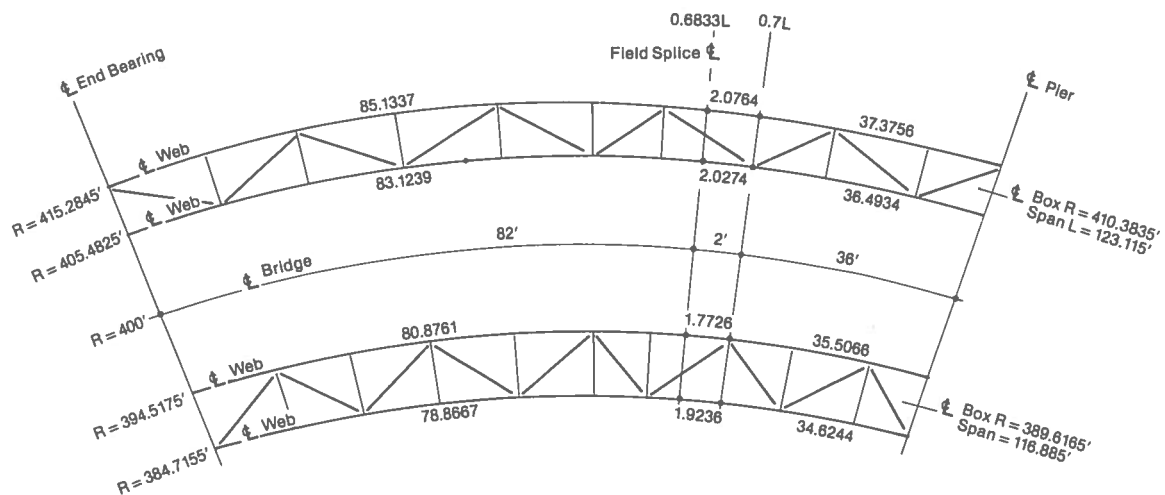
The distribution factor for moment in a straight box girder is

$$W_L = 0.1 + 1.7R + \frac{0.85}{N_w} = 0.1 + 1.7\left(\frac{3}{2}\right) + \frac{0.85}{3} = 2.933$$

Hence, the distribution factor for moment in the curved box girders, with  $X = 1/400 = 0.0025$ , is

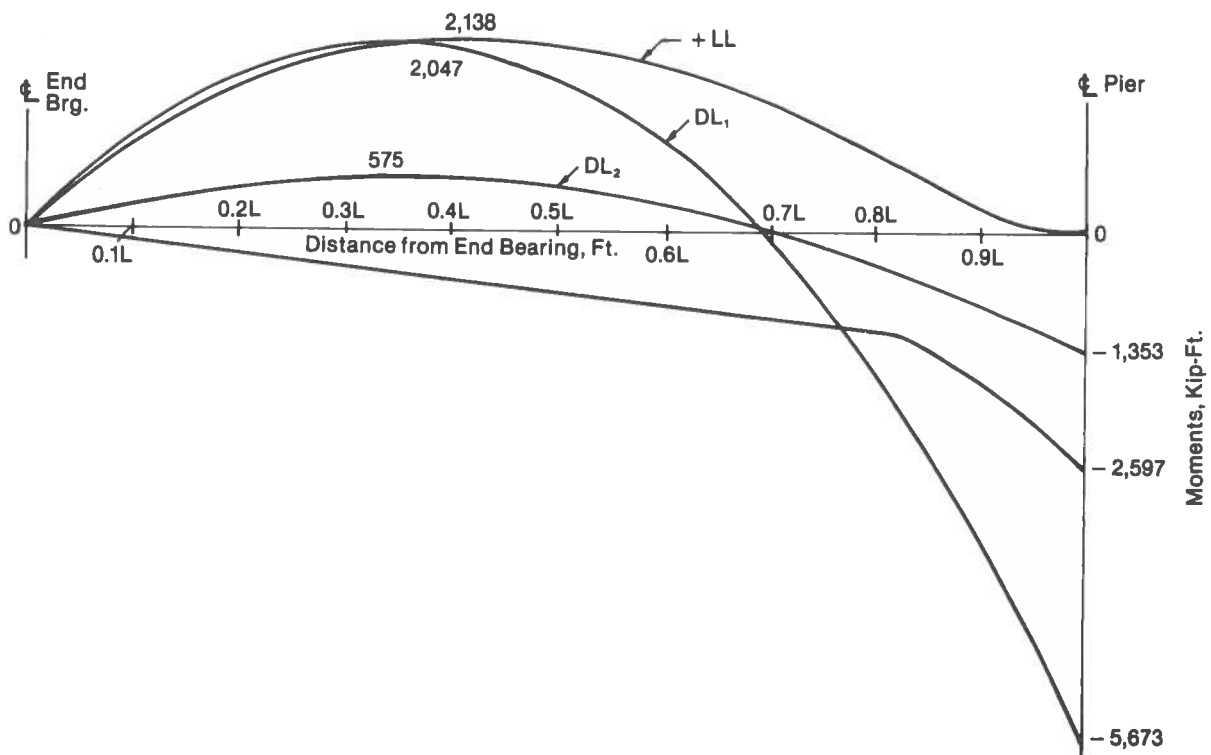
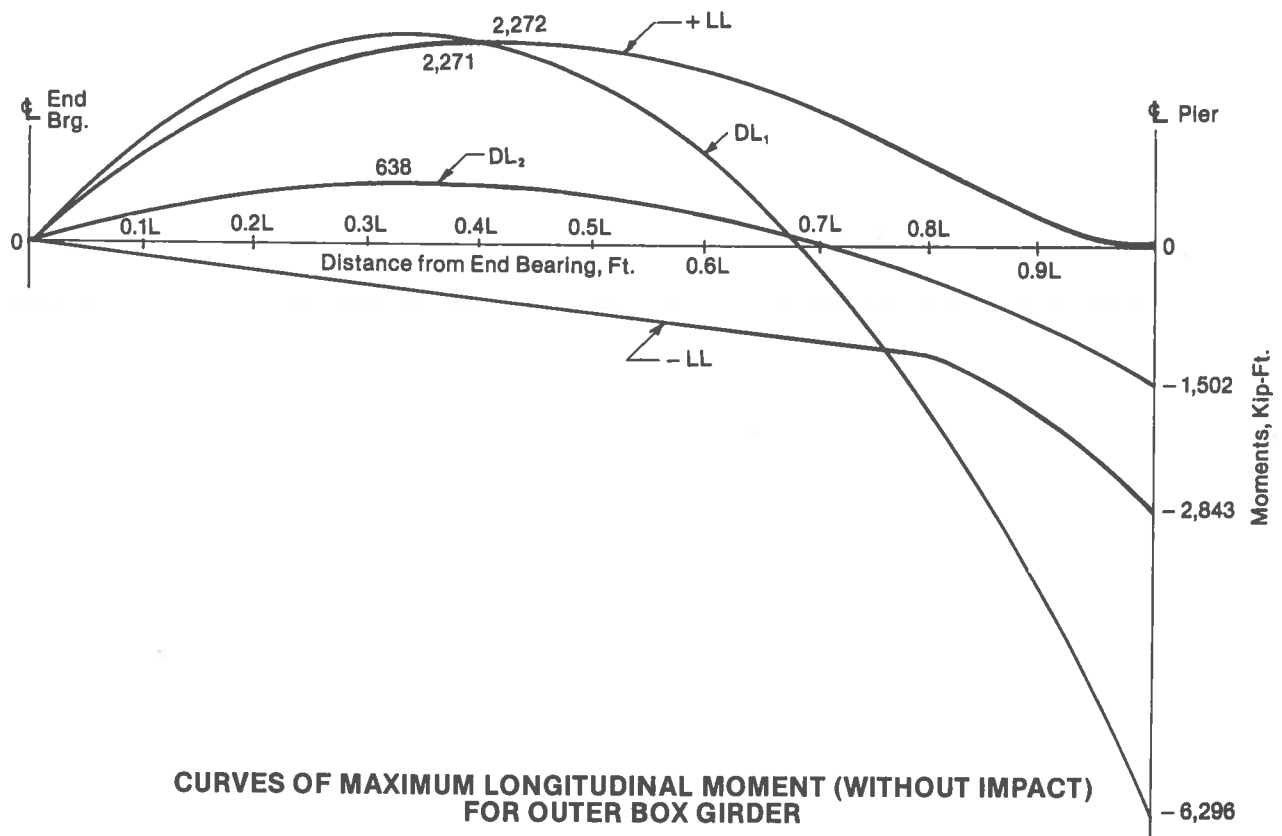
$$\begin{aligned} W_{Lc} &= W_L(1,440X^2 + 4.8X + 1) \\ &= 2.933[1,440(0.0025)^2 + 4.8 \times 0.0025 + 1] = 2.995 \text{ wheels} = 1.497 \text{ lanes} \end{aligned}$$

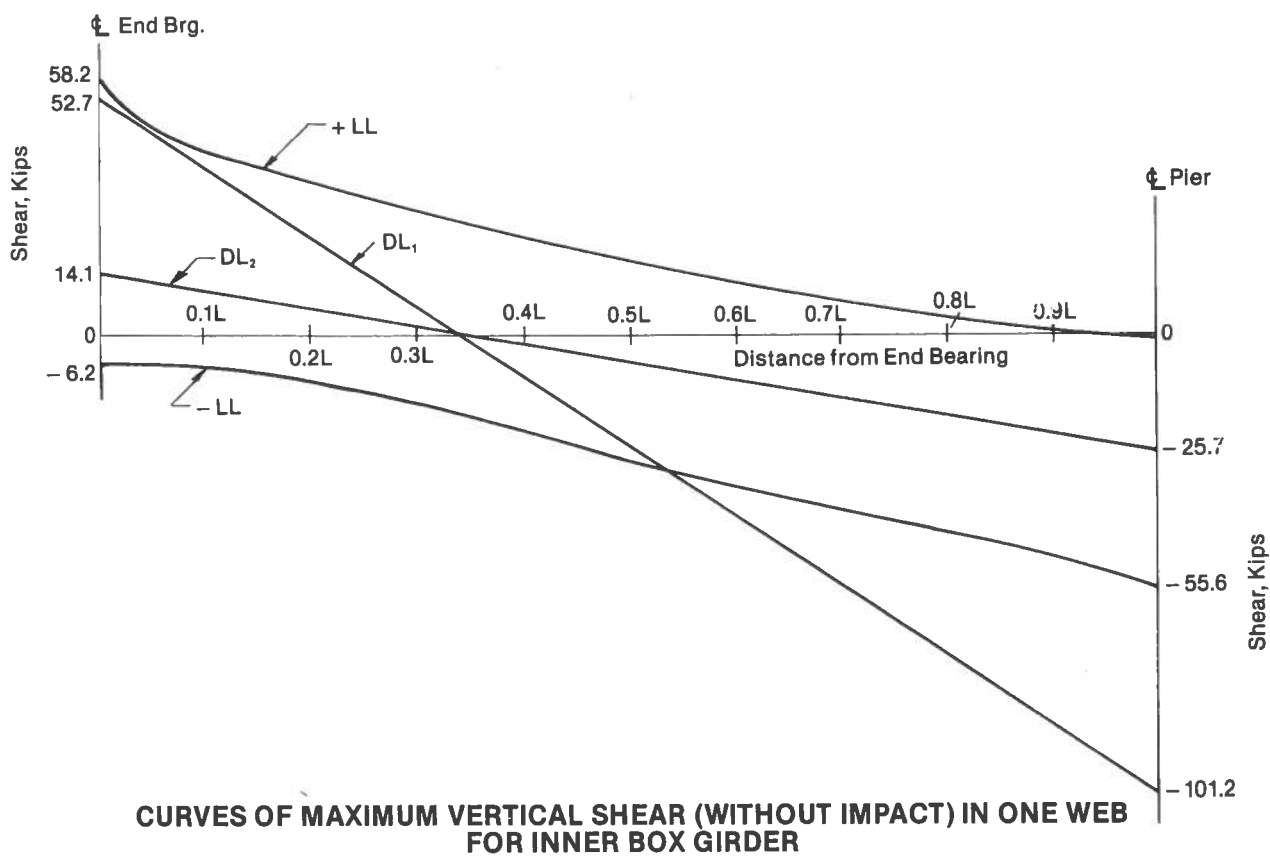
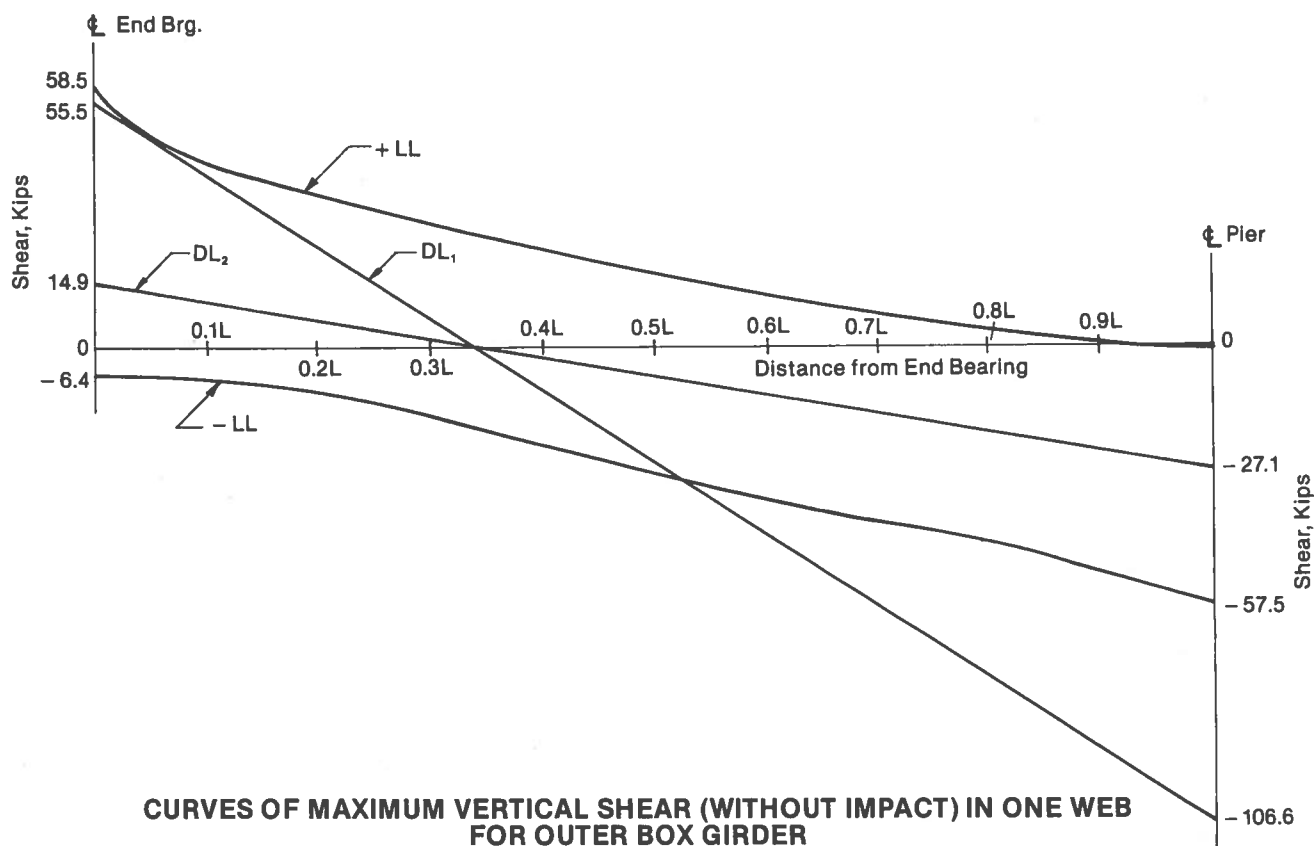
The bridge is analyzed by the  $M/R$  method outlined previously in General Design Considerations. For computation of moments and shears, the girders are treated as straight but their developed lengths are used. From the bridge centerline radius of 400 ft, centerline span of 120 ft and the geometry of the bridge cross section, the span of the outer girder is calculated to be 123.115 ft, and the span of the inner girder, 116.885 ft (see drawing showing plan geometry).



PLAN GEOMETRY AT TOP OF WEBS

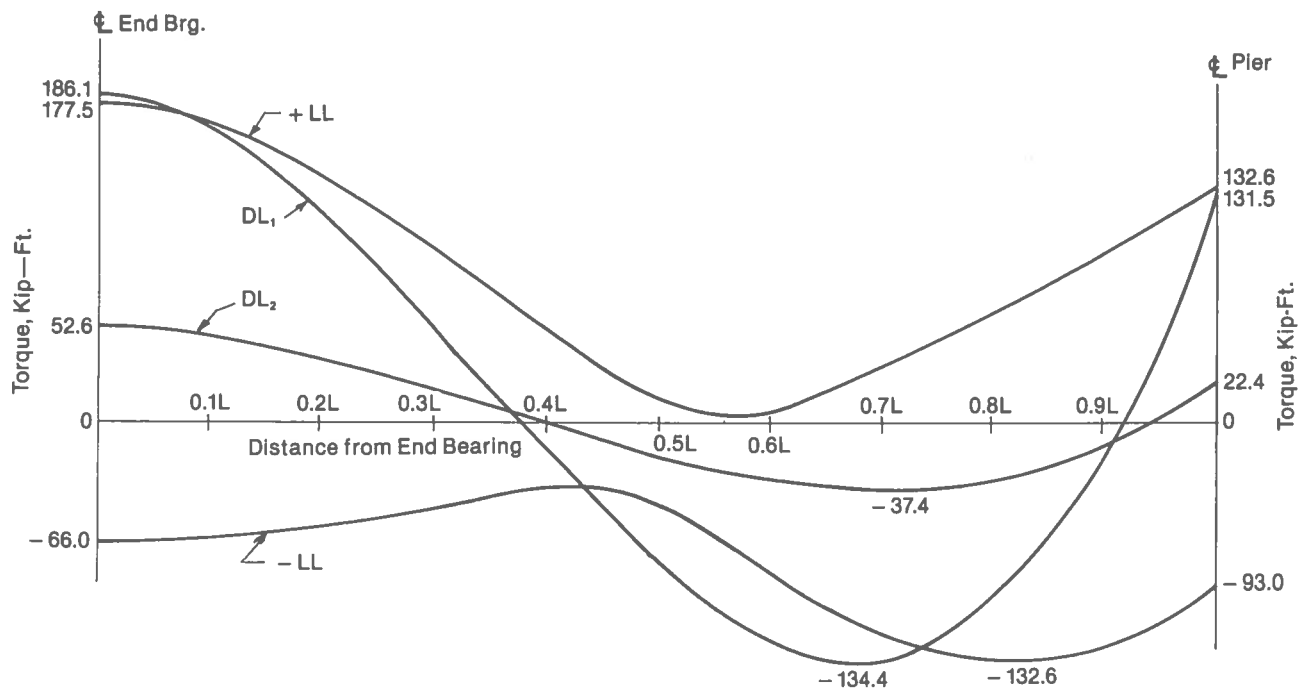
The following curves of maximum longitudinal moments and vertical shears were computed on this basis. Because the Guide Specifications impose a variety of impact factors, as tabulated previously in General Design Considerations, these curves do not include impact. It is taken into account at later points in the design.



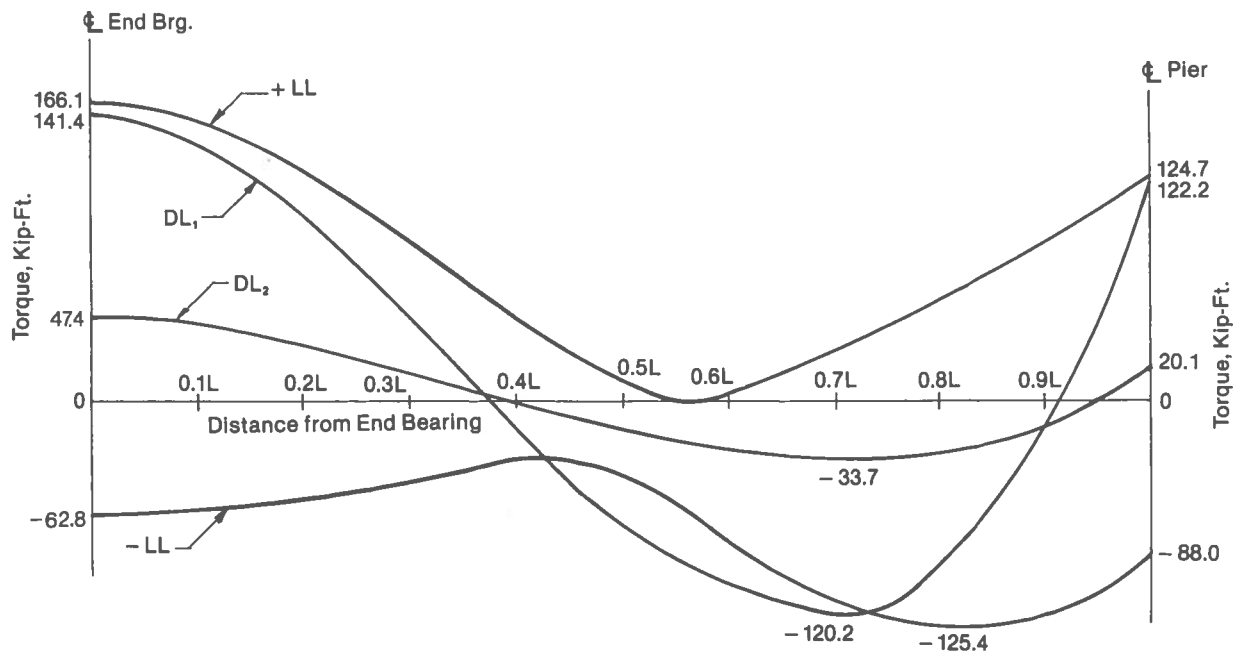


The curves of maximum torque are calculated by the  $M/R$  method outlined previously in General Design Considerations. For example, to obtain the  $DL_1$  torque diagram for the outer girder, the  $DL_1$  ordinates of the curves of maximum longitudinal moment for the outer girder are divided by 410.3835, the centerline radius of that girder. The girder is then assumed loaded with the resulting  $M/R$  diagram and the shears are computed. These shears are equivalent to the  $DL_1$  torques.

To determine live-load torques in the outer girder, a unit load is placed at the first panel point in the span and the resulting longitudinal bending moments are calculated. These moments are divided by the girder radius to obtain the  $M/R$  diagram. The span is then loaded with this diagram. Each of the resulting shears in the girder represent one influence ordinate for each of a series of torque influence lines for the girder. Placement of the unit load at the next panel point and repetition of the procedure yields another influence ordinate for each of the series of influence lines. A full set of torque influence lines may be obtained in this fashion. AASHTO truck or lane loading is then applied to the influence diagrams to obtain maximum values for live-load torque at each panel point for plotting the curves of maximum live-load torque (see following graphs). The computational steps are well suited to execution on programmable desk calculators or full-size computers in design offices.



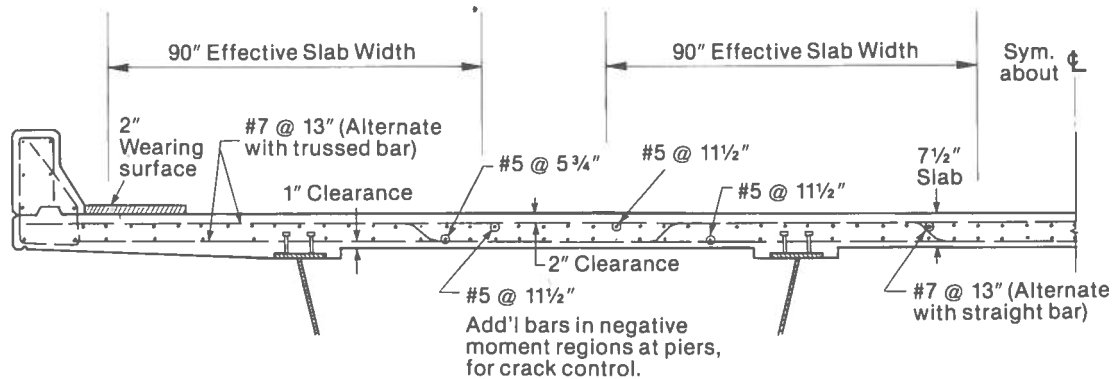
**CURVES OF MAXIMUM TORQUE FOR OUTER BOX GIRDER**



**CURVES OF MAXIMUM TORQUE FOR INNER BOX GIRDER**

### DESIGN OF GIRDER SECTIONS

The effective slab width to be used for the composite section in the positive-moment region and the area of steel reinforcement to be used for the composite, negative-moment section are the same as for the straight bridge of Chapter 7.



**SLAB HALF SECTION**

### Effective Slab Width

1. One-fourth the span:  $\frac{1}{4} \times \frac{3}{4} \times 120 \times 12 \times 2 = 540$  in.
2. Center to center of girders:  $12[9.83 + \frac{1}{2}(9.83 + 11)] = 243$  in.
3.  $12 \times$  slab thickness:  $12 \times 7.5 \times 2 = 180$  in. (governs)

### Area of Slab Reinforcement for Negative-Moment Section

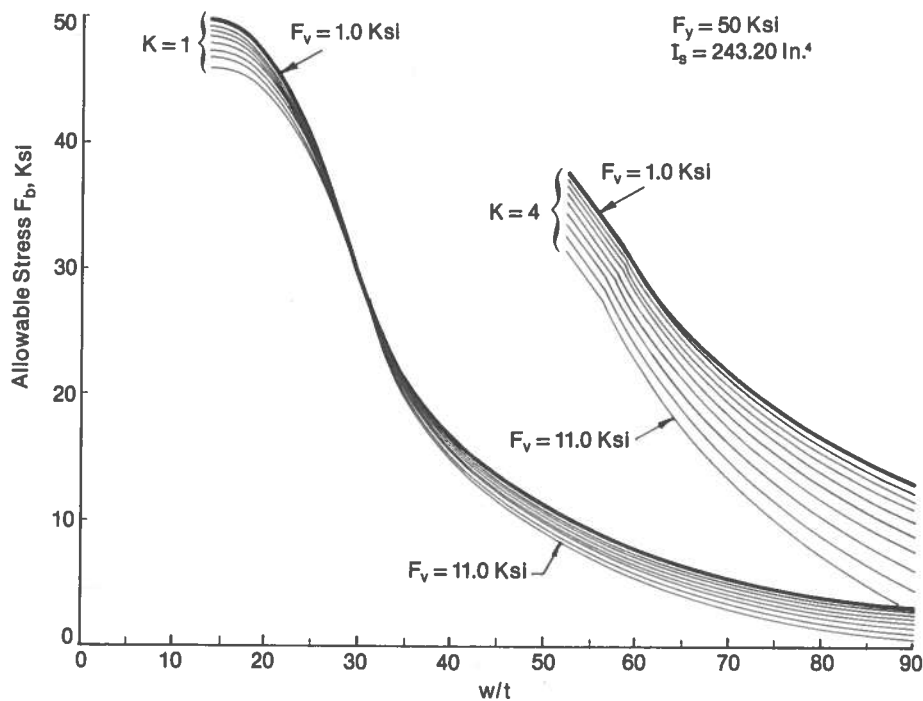
Bar Location	No. of Bars	Area per Bar	Total Area	$d$	$Ad$
Top row	31	0.31	9.61	4.313	41.45
Bottom row	18	0.31	5.58	2.188	12.21
			15.19 in. <sup>2</sup>		53.66 in. <sup>3</sup>

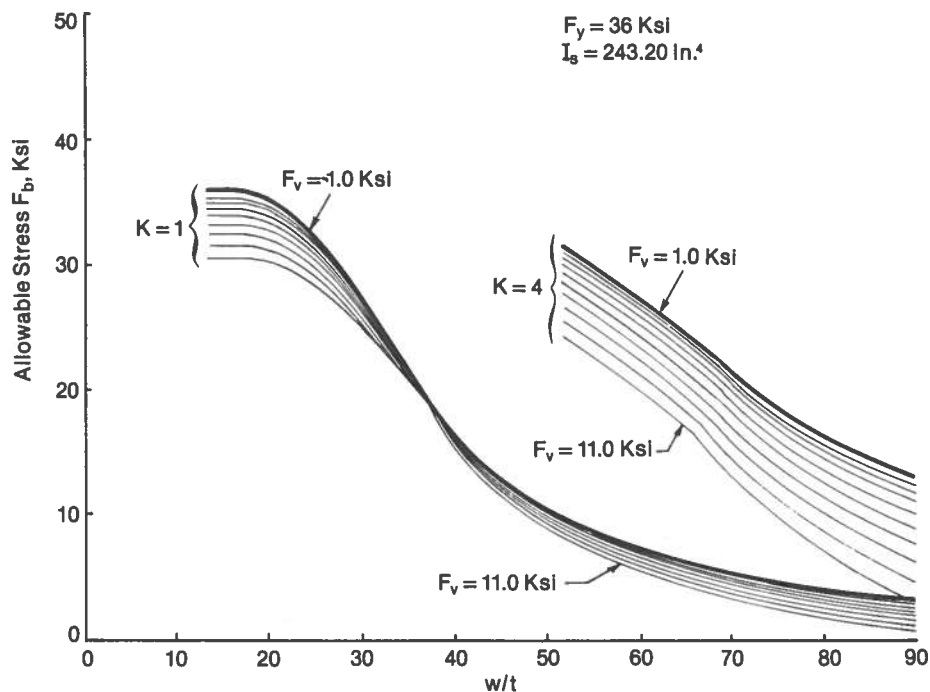
$$d_{Reinf.} = \frac{53.66}{15.19} = 3.63 \text{ in.}$$

The maximum allowable effective width of the bottom flange is one-fifth the span, or about 24 ft. The 7-ft-6-in. width of the flange is considerably less than this. Hence, the entire width of the bottom flange is considered effective.

### Allowable Bottom-Flange Compression Stress

The allowable bottom-flange compression stress  $F_b$  is defined by complex expressions and parameters. For a better understanding of  $F_b$ , it is convenient to show its variation graphically with the flange width-thickness ratio  $w/t$ . Families of curves may be plotted for specific values of  $F_v$ , the St. Venant shearing stress, for  $K=1$  and  $K=4$  (see following graphs).





#### ALLOWABLE STRESS FOR BOTTOM FLANGE OF LONGITUDINALLY STIFFENED, CURVED BOX GIRDER

A single longitudinal stiffener, as used in the bridge of Chapter 7, is efficient for flanges 7 ft 6 in. wide. The curves are therefore shown for a flange with one ST 7.5 × 25 longitudinal stiffener, with moment of inertia  $I_s = 243.2$  in.<sup>4</sup>. One graph is based on  $F_y = 36$  ksi and the second graph, on  $F_y = 50$  ksi. The curves are terminated at points where  $I_s \geq 0.125K^3t^3b$  is no longer satisfied.

To make practical use of the allowable-stress equation in design, some of the parameters may be eliminated. For example, the graphs reveal that the effect of  $f_v$  is negligible at values lower than 1 ksi. (The allowable stress converges to a limit at this low range of  $f_v$ .) Trial also shows that, in fact, this is the range of  $f_v$  to be expected in box girders of this size and type. For preliminary design, therefore, allowable stresses may be based on an arbitrary value of  $f_v = 0.5$  ksi. This assumption eliminates  $f_v$  as a variable in determination of  $F_b$ .

Another parameter is the buckling coefficient  $K$ . If one longitudinal stiffener is attached to the bottom flange, the minimum permissible moment of inertia of the stiffener about its base  $I_s$  is related to  $K$  by

$$I_s \geq 0.125K^3t^3b$$

where  $t$  = flange thickness, in.

$b$  = flange width, in.

Transposition yields the maximum  $K$  value for a given size of longitudinal stiffener with a moment of inertia  $I_s$ , as

$$K_{max} = \sqrt[3]{\frac{I_s}{0.125t^3b}}$$

A lower value of  $K$  may be used, but at the sacrifice of efficiency, because of lower allowable stress would result. Use of a larger value of  $K$  would require a larger stiffener.

For convenience in design, the allowable stresses, based on  $K_{max}$ , may be tabulated for possible combinations of bottom-flange thickness and ST stiffeners. The following ST shapes are chosen as possible longitudinal stiffeners, and the moment of inertia  $I_s$  about the base of the stem of each stiffener is calculated.

Moments of Inertia of Longitudinal ST Stiffeners	
ST Shape	Moment of Inertia, In. <sup>4</sup>
7.5×25	$40.6 + 7.35(5.25)^2 = 243.2$
6 ×25	$25.2 + 7.35(4.16)^2 = 152.4$
6 ×20.4	$18.9 + 6.00(4.42)^2 = 136.1$
5 ×17.5	$12.5 + 5.15(3.44)^2 = 73.4$
4 ×11.5	$5.03 + 3.38(2.85)^2 = 32.5$
3.5×10	$3.36 + 2.94(2.46)^2 = 21.2$

For a single longitudinal stiffener on the 7.5-ft—wide bottom flange, the stiffener spacing is  $w = 7.5 \times 11/2 = 45$  in. For this spacing, the value of the buckling coefficient  $K_{max}$  furnished by each size of stiffener is calculated for several thicknesses of bottom flange. Allowable stresses are then computed from the governing equations given previously in General Design Considerations, with a low value of  $f_v$ , such as 0.5 ksi, and tabulated. The resulting table can be used at later points of the design as an aid in selection of bottom-flange thicknesses.

Allowable Compression Stress  $F_b$  for Bottom Flange  
with One Longitudinal Stiffener ( $f_v = 0.5$  Ksi)

$t$	$w/t$	ST Stiffener	$I_s$	$K_{min}$	$K_s$	$F_b$ , Ksi	
						$F_y = 36$	$F_y = 50$
$13/16$	55.4	6 ×25	152.4	3.70	2.65	28.7	31.8
		6 ×20.4	136.1	3.56	2.60	28.1	30.7
		5 ×17.5	73.4	2.90	2.36	24.5	24.7
		4 ×11.5	32.5	2.20	2.12	18.7	18.7
$7/8$	51.4	7.5×25	243.2	4.00	2.76	31.7	37.6
		6 ×25	152.4	3.43	2.55	29.7	33.8
		6 ×20.4	136.1	3.30	2.51	29.2	32.7
		5 ×17.5	73.4	2.69	2.29	25.9	26.5
$15/16$	48.0	7.5×25	243.2	3.74	2.66	32.4	39.2
		6 ×25	152.4	3.20	2.47	30.6	35.5
		6 ×20.4	136.1	3.08	2.43	30.2	34.5
		5 ×17.5	73.4	2.51	2.23	27.1	28.4
1	45.0	7.5×25	243.2	3.50	2.58	33.0	40.5
		6 ×25	152.4	3.00	2.40	31.4	37.1
		6 ×20.4	136.1	2.27	2.36	27.5	29.2
		5 ×17.5	73.4	2.35	2.17	28.1	30.6

#### Selection of Structural Steel for Girders

The girders are designed for A36 steel in the positive-bending region and for steel with a yield strength of 50 ksi in the negative-bending region. The first step is selection of the web depth and preliminary thickness.



## GIRDER DEPTH AND WEB DESIGN

The girder depth is assumed the same as that for the straight bridge of Chapter 7. A web thickness of  $\frac{1}{2}$  in. is investigated for the outer girder.

### Unstiffened Web in Positive-Moment Region—Outer Girder

For shear stresses, the impact factor is 0.50. At the end bearing, the maximum design shear along the sloped web is

$$V_u = \frac{58.69}{57} \times 1.3[55.5 + 14.9 + \frac{5}{3}(58.5 \times 1.50)] = 290.0 \text{ kips}$$

Maximum capacity for buckling of the unstiffened web is

$$V_{u1} = \frac{3.5Et^3}{D} = \frac{3.5 \times 29,000(\frac{1}{2})^3}{58.69} = 216.2 < 290.0 \text{ kips}$$

Therefore stiffeners are required near the end bearing. The maximum shear strength of the web is

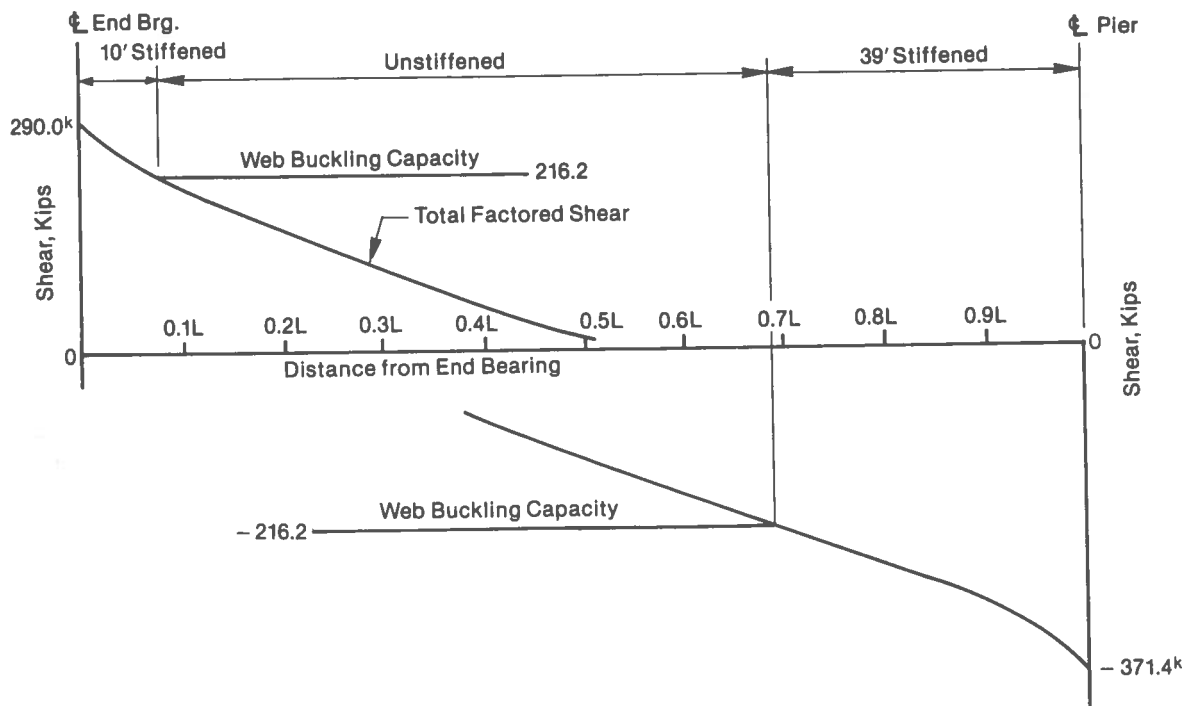
$$V_{u2} = 0.58F_yDt = 0.58 \times 36 \times 58.69 \times \frac{1}{2} = 612.7 > 290.0 \text{ kips}$$

Hence, a  $\frac{1}{2}$ -in. A36 web plate is satisfactory for ultimate strength. It also meets the requirement  $D/t \leq 150$  for unstiffened webs.

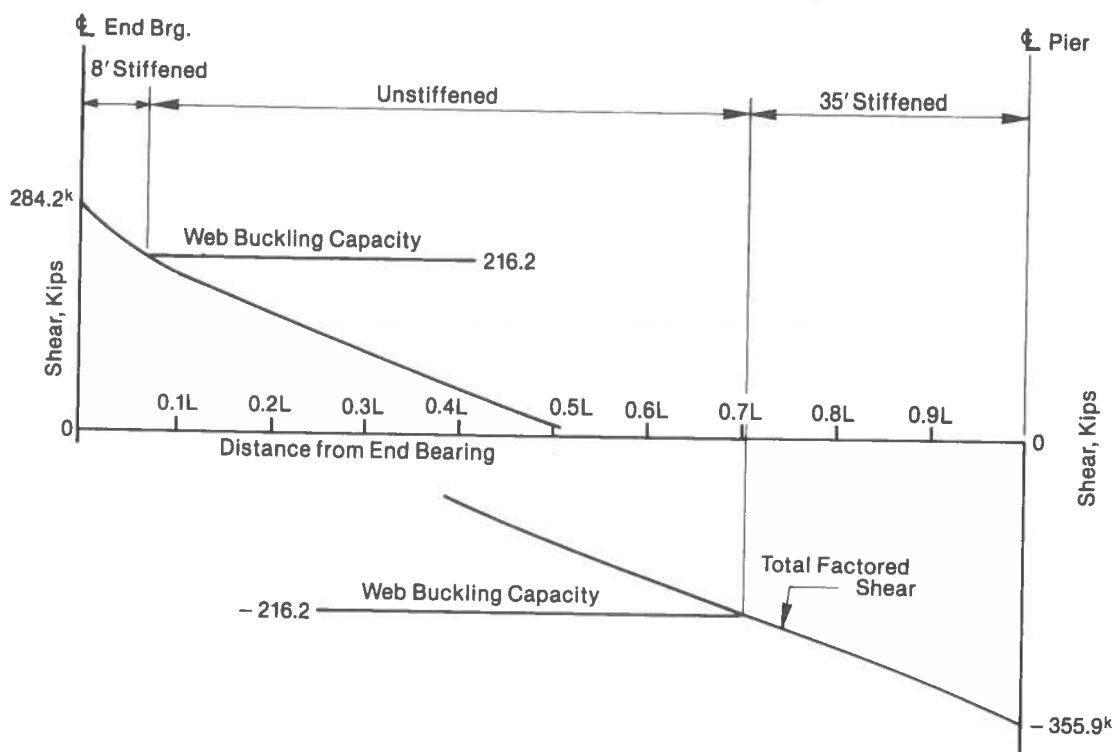
The inner box girder, with slightly smaller shear forces and a  $\frac{1}{2}$ -in. web, also satisfies these conditions.

### Stiffened Web—Outer Girder

The shear forces in the negative-bending region and also adjacent to the end bearing exceed the buckling capacity of the unstiffened  $\frac{1}{2}$ -in. web for both girders. For the outer girder, the webs will be stiffened for a distance of at least 39 ft from the interior support and at least 10 ft from the end bearing. Corresponding distances for the inner girder are 35 and 8 ft. (See the following shear curves.) More detailed final design calculations for the web are given later.



COMPARISON OF DESIGN SHEAR AND SHEAR CAPACITY  
FOR UNSTIFFENED WEB OF OUTER BOX GIRDER



**COMPARISON OF DESIGN SHEAR AND SHEAR CAPACITY  
FOR UNSTIFFENED WEB OF INNER BOX GIRDER**

Next, the maximum permissible depth-to-thickness ratio of the transversely stiffened web is checked, for an assumed maximum stiffener spacing of 58.69 in.

$$\frac{D}{t} = \frac{1,154}{\sqrt{F_y}} \left[ 1 - 8.6 \frac{d_o}{R} + 34 \left( \frac{d_o}{R} \right)^2 \right]$$

$$= \frac{1,154}{\sqrt{36}} \left[ 1 - 8.6 \times \frac{58.69/12}{384.67} + 34 \left( \frac{58.69/12}{384.67} \right)^2 \right] = 172.4$$

The  $D/t$  of the web is  $58.69/(1/2) = 117.4$  and therefore is satisfactory.

Because the preceding calculations were made only to determine whether the  $1/2$ -in. web is a possible solution, the relatively small torsional effects have been ignored. They are considered later, however, when detailed stiffener spacing is calculated. Design computations for the stiffeners are also given later.

### CROSS-FRAME SPACING

As discussed previously in General Design Considerations, internal cross frames are spaced at regular intervals within the box girders to minimize normal stresses due to distortion and to control top-flange lateral bending stresses. A tentative cross-frame spacing  $S$  is calculated as follows:

$$S = L \left( \frac{R}{200L - 7,500} \right)^{1/2} = 120 \left( \frac{400}{200 \times 120 - 7,500} \right)^{1/2} = 18.7 < 25 \text{ ft}$$

Additional studies of unsupported flange lengths versus lateral bending moments and required flange widths, however, establish a smaller cross-frame spacing as the best trade-off between girder-flange material and cross-frame material: Use 10 cross-frame spaces. Hence, measured along the bridge centerline,

$$S = \frac{120}{10} = 12 \text{ ft}$$

Measured along the centerline of the outer girder, the spacing is

$$S = \frac{123.125}{120} \times 12 = 12.31 \text{ ft}$$

and measured along the centerline of the inner girder, the spacing is

$$S = \frac{116.875}{120} \times 12 = 11.69 \text{ ft}$$

The smaller spacing is more than adequate to retain the shape of the cross section and to limit transverse distortional stresses.

It will be seen in subsequent calculations that the girder top flanges as designed are fully stressed. A larger crossframe spacing, with its corresponding larger lateral bending moments, would require larger flanges. Although crossframe material would be saved, flange material would be added. In general, reducing the unbraced flange length is a more efficient way to control lateral bending stresses than adding flange material.

### LATERAL FLANGE BENDING

As discussed previously, lateral bending moments are produced by the radial component of axial force in curved flanges and by the horizontal component of shear forces in the sloping webs. The equations used to calculate lateral bending effects due to curvature were given previously in General Design Considerations.

The horizontal forces from the sloping webs act as a uniformly distributed load along both the top and bottom flanges. This load equals the change in vertical and torsional shear per foot along the girder due to  $DL_1$ . It is applied as a uniform load on a continuous beam that is considered supported at the cross frames.

The change in vertical and torsional shears  $\Delta V_v$ , kips per ft, due to  $DL$  along the top flange of the outer girder is

$$\Delta V_v = \frac{V_{0.0} - V_{1.0}}{L}$$

where  $L$  = span, ft, of girder

$V_{0.0}$ ,  $V_{1.0}$  = vertical and torsional shear force, kips, at points  $k$  indicated by the subscript  $k$  is taken at the tenth points of the span.

The uniform load applied to the top flange  $\Delta V_H$ , kips per ft, is

$$\Delta V_H = \frac{1}{2} \frac{h}{D} \Delta V_v$$

where  $h$  = horizontal projection of web, in.

$D$  = depth, in., of box girder

The lateral bending moment  $M_{Ls}$ , kip, ft, due to the sloped web is

$$M_{Ls} = \frac{\Delta V_H S^2}{12}$$

where  $S$  = cross-frame spacing, ft

At the end bearing, the vertical shear due to  $DL_1$  is  $V_{0.0} = 52.7$  kips per web. At the interior support,  $V_{1.0} = -101.2$  kips per web.

The transverse, St. Venant shear force  $V_T$ , kips, along the sloped web due to  $DL_1$  is

$$V_T = \frac{Tl}{2A}$$

where  $T$  =

torsional moment, kip-in., obtained from the curves of maximum torque

$l$  = sloped length of web, in.

$A$  = enclosed trapezoidal area, sq in., between girder bottom flange and top of plate diaphragm

At the end bearing, the transverse torsional shear is

$$V_{T0.0} = \frac{181.1 \times 12 \times 58.69}{2[(1/2)55(90+118)]} \times \frac{57}{58.69} = 10.8 \text{ kips}$$

At a distance of 0.7 of the span from the end bearing,

$$V_{T0.7} = \frac{-134.4}{181.1} \times 10.8 = -8.0 \text{ kips}$$

At the interior support,

$$V_{T1.0} = \frac{131.5}{181.1} \times 10.8 = 7.8 \text{ kips}$$

The change in vertical and torsional shear along the girder between the end bearing (point 0.0) and the 0.7 point then is

$$\Delta V = \frac{52.7 - (-101.2)}{123.12} + \frac{10.8 - (-8)}{7 \times 12.31} = 1.250 + 0.218 = 1.468 \text{ kips per ft}$$

Between the 0.7 point and the interior support, the shear change is

$$\Delta V = 1.20 + \frac{-8.0 - 7.8}{3 \times 12.31} = 0.822 \text{ kips per ft}$$

The uniformly applied load at the top flange between the end bearing and the 0.7 point then is

$$\Delta V_H = \frac{14}{57} \times 1.468/2 = 0.180 \text{ kips per ft}$$

The uniform load between the 0.7 point and the interior support (0.988 point) is

$$\Delta V_H = \frac{14}{57} \times 0.822/2 = 0.101 \text{ kips per ft}$$

On the assumption that these loads are applied to the flange as a continuous beam, the lateral bending moments at the support—the cross-frame locations—are calculated. Between the end bearing and the 0.7 point,

$$M_{Ls} = \frac{0.180(12.31)^2}{12} = 2.27 \text{ kip-ft}$$

Between the 0.7 point and the interior support,

$$M_{Ls} = \frac{0.101(12.31)^2}{12} = 1.28 \text{ kip-ft}$$

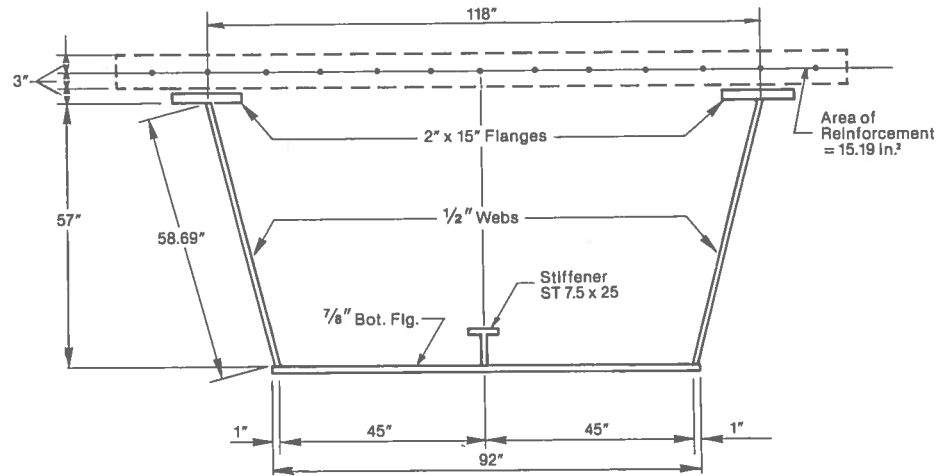
The nature of the allowable-stress equations is such that lateral flange bending may or may not have an effect on the design. This will be seen at a later point in the design example.

## NEGATIVE-MOMENT SECTION 2 FT FROM INTERIOR SUPPORT

Adjacent to the center pier, the section chosen is about the same as that used for the straight bridge of Chapter 7, except that the section is composed entirely of steel with a yield strength of 50 ksi. As shown previously, in the drawing of the slab half section, the area of steel reinforcement to be used for the composite section also is the same as that for the design example of Chapter 7.

A single, longitudinal structural tee of 50-ksi steel is used to stiffen the bottom flange in the negative-moment region. Selection of this stiffener follows very closely the procedure employed for straight box girders.

The section used for maximum negative moment extends from the center pier to a transition point to be determined later. The section is investigated for negative moment occurring 2 ft from the pier. (The section directly over the pier is treated in conjunction with the pier diaphragm.) Properties are calculated for the steel section alone and for the section plus reinforcement.



**SECTION 2 FT. FROM INTERIOR SUPPORT**

**Steel Section at Transition 2 Ft from Center of Interior Support**

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
2 T. Flg. Pl. 2×15	60.00	29.50	1,770	52,215	20	52,235
2 Web Pl. 1/2×58.69	58.69				15,891	15,891
Bot. Flg. Pl. 7/8×92	80.50	-28.94	-2,330	67,421		67,421
Stiff. ST 7.5×25	7.35	-23.25	-171	3,973	41	4,014

$$d_s = \frac{-731}{206.54} = -3.54 \text{ in.}$$

$$I_{NA} = 136,973 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 30.50 + 3.54 = 34.04 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.38 - 3.54 = 25.84 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{136,973}{34.04} = 4,024 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{136,973}{25.84} = 5,301 \text{ in.}^3$$

$$d_{\text{Top of Stiff.}} = 21 - 3.54 = 17.46 \text{ in.}$$

$$S_{\text{Top of Stiff.}} = \frac{136,973}{17.46} = 7,845 \text{ in.}^3$$

**Steel Section, with Reinforcing Steel, 2 Ft from Interior Support**

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Steel Section	206.54		-731			139,561
Reinforcement	15.19	35.13	534	18,746		18,746

$$d_c = \frac{-197}{221.73} = -0.89 \text{ in.}$$

$$I_{NA} = 158,132 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 30.50 + 0.89 = 31.39 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.38 - 0.89 = 28.49 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{158,132}{31.39} = 5,038 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{158,132}{28.49} = 5,550 \text{ in.}^3$$

$$d_{\text{Top of stiff.}} = 21 - 0.89 = 20.11 \text{ in.}$$

$$d_{\text{Reinf.}} = 35.13 \times 0.89 = 36.02 \text{ in.}$$

$$S_{\text{Top of stiff.}} = \frac{158,132}{20.11} = 7,863 \text{ in.}^3$$

$$S_{\text{Reinf.}} = \frac{158,132}{36.02} = 4,390 \text{ in.}^3$$

### Bottom-Flange Stresses 2 Ft from Interior Support

The factored moment 2 ft from the interior support is computed from

$$M = 1.3 \left( D + \frac{5}{3} L_T \right)$$

From the curves of maximum moment, the moment due to  $DL_1$  is 5,820 kip-ft, the moment due to  $DL_2$  is 1,370 kip-ft and the live-load moment is 2,670 kip-ft. The impact factor is 0.35.

$$L_T = L(1 + I) = 2,670 \times 1.35 = 3,604 \text{ kip-ft}$$

The longitudinal bending stress in the bottom flange then is

$$f_b = 1.3 \left( \frac{5,820 \times 12}{5,301} + \frac{1,370 \times 12}{5,550} + \frac{5}{3} \times \frac{3,604 \times 12}{5,550} \right) = 37.86 \text{ ksi}$$

For computation of the allowable bending stress for the bottom flange, the St. Venant shear stress at the section must first be calculated from

$$f_v = \frac{T}{2At}$$

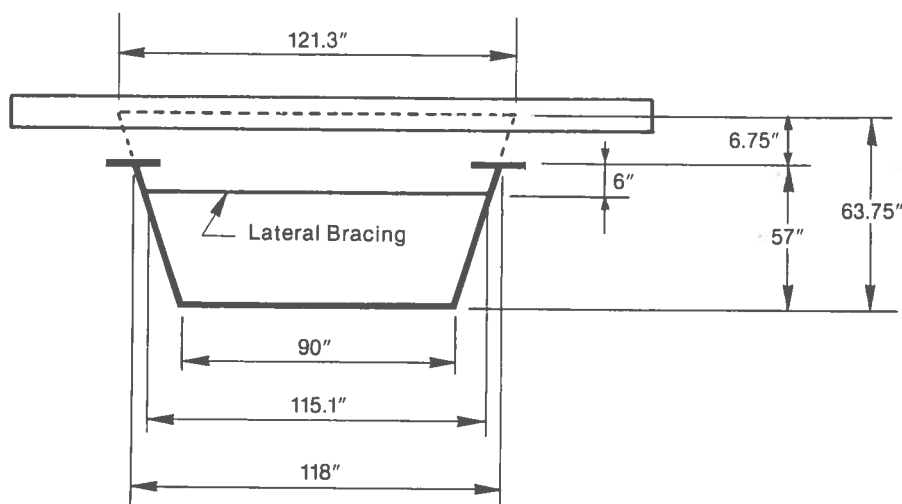
with the thickness of flange  $t = 7/8$  in. The factored torque is computed from

$$T = 1.3 \left( D + \frac{5}{3} L_T \right)$$

From the curves of maximum torque, the torque due to  $DL_1$  is 106.9 kip-ft, the torque due to  $DL_2$  is 16.6 kip-ft and the live-load torque is 120.5 kip-ft. The impact factor is 0.50.

$$L_T = L(1 + I) = 120.5 \times 1.50 = 180.8 \text{ ft-kips}$$

The stress  $f_v$  is calculated in two parts—that due to  $DL_1$  torque acting on the non-composite section and that due to  $DL_2$  and  $L_T$  acting on the composite section. These sections are defined in the following figure.



SECTION FOR CALCULATION OF  $f_v$  2 FT. FROM INTERIOR SUPPORT

The enclosed area  $A_1$  to be used in computing  $f_v$  due to  $DL_1$  is the area within the box bounded at the top by the lateral bracing, which is located 6 in. below the top flange:

$$A_1 = \frac{1}{2}(57-6)(90+115.1) = 5,230 \text{ in.}^2$$

The enclosed area  $A_2$  to be used in computing  $f_v$  due to  $DL_2$  and  $L_T$  is the area within the box bounded at the top by the mid-depth of the slab, which is 6.75 in. above the top flange:

$$A_2 = \frac{1}{2}(57+6.75)(90+121.3) = 6,735 \text{ in.}^2$$

The shear stress 2 ft from the interior support then is

$$f_v = \frac{1.3 \times 12}{2 \times 7/8} \left( \frac{106.9}{5,230} + \frac{16.6}{6,735} + \frac{5}{3} \times \frac{180.8}{6,735} \right) = 0.60 \text{ ksi}$$

Because the shear stress due to torque is much less than  $0.75F_y/\sqrt{3} = 21.65$  ksi, the allowable stress  $F_b$  for the bottom flange is determined by the parameters  $R_1$ ,  $R_2$  and  $\Delta$  as follows:

$$\Delta = \sqrt{1 - 3 \left( \frac{f_v}{F_y} \right)^2} = \sqrt{1 - 3 \left( \frac{0.60}{50} \right)^2} = 0.9998$$

For computation of  $R_1$  and  $R_2$ ,  $n=1$ ,  $b=90/2=45$  in.,  $t=7/8$  in. and  $I_s = I_o + Ad^2 = 40.6 + 7.35(7.5 - 2.25)^2 = 243.2 \text{ in.}^4$

$$K = \sqrt[3]{\frac{I_s}{0.125t^3b}} = \sqrt[3]{\frac{243.2}{0.125(0.875)^3 45}} = 4.01 > 4 \text{ Use } K = 4.$$

$$K_s = \frac{5.34 + 2.48(I_s/bt^3)^{1/3}}{(n+1)^2} = 2.76 < 5.34$$

With the use of the preceding results,

$$R_1 = \frac{97.08\sqrt{K}}{\sqrt{\frac{1}{2} \left[ \Delta + \sqrt{\Delta^2 + 4(f_v/F_y)^2 (K/K_s)^2} \right]}} = 194.2$$

$$R_2 = \frac{210.3\sqrt{K}}{\sqrt{\frac{1}{1.2} \left[ \Delta - 0.4 + \sqrt{(\Delta - 0.4)^2 + 4(f_v/F_y)^2 (K/K_s)^2} \right]}} = 420.4$$

Because  $w\sqrt{F_y}/t = 45\sqrt{50}/(7/8) = 363.7$  falls between  $R_1$  and  $R_2$ , the allowable compression stress in the bottom flange is given by

$$\begin{aligned} F_b &= F_y \left[ \Delta - 0.4 \left( 1 - \sin \frac{\pi}{2} \frac{R_2 - w\sqrt{F_y}t}{R_2 - R_1} \right) \right] \\ &= 50 \left[ 0.9998 - 0.4 \left( 1 - \sin \frac{\pi}{2} \frac{420.4 - 363.7}{420.4 - 194.2} \right) \right] = 37.68 \text{ ksi} \end{aligned}$$

This represents only a 1/2% overstress. The 7/8-in.-thick bottom-flange plate is considered adequate.

#### Lateral Bending in the Longitudinal Tee Stiffener

The direct stress in the flange of the longitudinal stiffener due to participation with the box-girder bottom flange in resisting bending is computed 2 ft from the interior support from

$$f_s = \frac{y_b - y_s}{y_b} f_b$$

with  $y_b = d_{\text{Bot of steel}} = 28.49$  in.

$y_s =$  distance from top of stiffener to underside of girder bottom flange =  
 $7.50 + 0.88 = 8.38$  in.

$$f_b = 37.86 \text{ ksi}$$

Hence, the direct stress in the longitudinal stiffener flange is

$$f_s = \frac{28.49 - 8.38}{28.49} \times 37.86 = 26.72 \text{ ksi}$$

The lateral bending stress in the flange of the stiffener is computed from

$$f_{wc} = \frac{6f_s d^2}{10Rb}$$

where  $d = l = 123.115/10 = 12.31$  ft

$$R = 410.38 \text{ ft}$$

$$b = 5.64 \text{ in.}$$

Substitution of these values in the equation for  $f_{wc}$  yields

$$f_{wc} = \frac{6 \times 26.72 (12.31)^2}{10 \times 410.38 (5.64/12)} = 12.60 \text{ ksi}$$

The allowable average compression stress in the stiffener flange is given by the same equations that apply to curved I-girder flanges, outlined in General Design Considerations. For calculations of  $F_{bu}$ , the following parameters are determined:

$$f = 1 - 3 \left( \frac{F_y}{E\pi^2} \right) \left( \frac{l}{b} \right)^2 = 1 - 3 \left( \frac{50}{29,000\pi^2} \right) \left( \frac{12.31}{5.64/12} \right)^2 = 0.640$$

$$\begin{aligned} \bar{\rho}_B &= \frac{1}{1 + \frac{l}{b} \left( 1 + \frac{l}{6b} \right) \left( \frac{l}{R} - 0.01 \right)^2} \\ &= \frac{1}{1 + \frac{12.31}{5.64/12} \left( 1 + \frac{12.31}{6 \times 5.64/12} \right) \left( \frac{12.31}{410.38} - 0.01 \right)^2} = 0.9468 \end{aligned}$$

$$\bar{\rho}_B/f = 0.9468/0.64 = 1.48$$

$$\bar{\rho}_w = 0.95 + 18 \left( 0.1 - \frac{l}{R} \right)^2 + \frac{(f_w/f_b)[0.3 - 0.1(l/R)(l/b)]}{\bar{\rho}_B/f}$$

with  $l/R = 12.31/410.38 = 0.030$

$$f_w/f_b = 12.60/26.72 = 0.472$$

$$l/b = 12.31/(5.64 \times 12) = 26.19$$

$$\bar{\rho}_w = 0.95 + 18(0.1 - 0.030)^2 \frac{0.472(0.3 - 0.1 \times 0.030 \times 26.19)}{1.48} = 1.1088$$

$$\bar{\rho}_B \bar{\rho}_w = 0.9468 \times 1.1088 = 1.0499 > 1 \text{ Use } 1.$$

With the use of the preceding results, the allowable compression stress is

$$F_{bu} = F_y \bar{\rho}_B \bar{\rho}_w = 50 \times 0.64 \times 1 = 32.02 > 26.72 \text{ ksi}$$

Braced at each cross frame, therefore, the stiffener has adequate strength for lateral bending due to curvature.

#### Top-flange Stress 2 Ft from Interior Support

The top-flange bending stress (tension) in the negative-moment section 2 ft from the interior support is

$$F_{bs} = 1.3 \times 12 \left( \frac{5,820}{4,024} + \frac{1,370}{5,038} + \frac{5}{3} \times \frac{3,604}{5,038} \right) = 45.4 \text{ ksi}$$



The top-flange stress at this section may not exceed

$$F_{bs} = F_y \left[ 1 - 3 \left( \frac{F_y}{E \pi^2} \right) \left( \frac{l}{b} \right)^2 \right]$$

With a cross-frame spacing of  $l = 12.31$  ft and flange width of 15 in.,

$$F_{bs} = 50 \left[ 1 - 3 \left( \frac{50}{29,000 \pi^2} \right) \left( \frac{12.31 \times 12}{15} \right)^2 \right] = 47.5 > 45.4 \text{ ksi}$$

Hence, the top flange is adequate.

#### Reinforcing Steel Stress 2 Ft from Interior Support

The allowable tension stress in the slab reinforcing steel is 40 ksi. At the negative-moment section 2 ft from the interior support, the stress in the reinforcing steel is

$$f_r = 1.3 \times 12 \times \frac{1,370 + (5/3)3,604}{4,390} = 26.2 < 40 \text{ ksi}$$

Therefore, the reinforcing steel is not overstressed.

#### INVESTIGATION OF WEBS AT INTERIOR SUPPORT

The ratio  $D/t$  for the webs in the negative-moment region is  $56.89/(1/2) = 113.78 < 150$ . Therefore, no web stiffeners are required in this region, if the buckling capacity of the webs is not exceeded.

The design shear per web at the center pier is a combination of direct and torsional shears. The direct shear is

$$V_v = 1.3(106.6 + 27.1 + \frac{5}{3} \times 1.5 \times 57.5) \frac{58.69}{57} = 371 \text{ kips}$$

The torsional shear is

$$V_t = 1.3 \times 12 \left( \frac{131.5}{5,230} + \frac{22.4}{6,735} + \frac{5}{3} \times 1.5 \times \frac{132.6}{6,735} \right) \frac{1}{2} \times 58.69 = 36 \text{ kips}$$

The total shear then is  $V = 371 + 36 = 407$  kips.

The ultimate shear capacity is the smaller of the following:

$$V_{u1} = \frac{3.5 E t_w^3}{D} = \frac{3.5 \times 29,000 (1/2)^3}{58.69} = 216.2 < 407 \text{ kips}$$

$$V_{u2} = 0.58 F_y D t_w = 0.58 \times 50 \times 58.69 \times \frac{1}{2} = 851 > 407 \text{ kips}$$

Because the design shear exceeds  $V_{u1}$ , the buckling capacity of the web, transverse stiffeners are required on the web. With stiffeners, the allowable shear is given by

$$V_u = 0.58 F_y D t_w C = 0.58 \times 50 \times 58.69 \times \frac{1}{2} C = 851 C$$

$$C = 569.2 \frac{t_w}{2} \sqrt{\frac{1 + (D/d_o)^2}{F_y}} - 0.3 \leq 1.0$$

$$= 569.2 \times \frac{1/2}{58.69} \sqrt{\frac{1 + (58.69/d_o)^2}{50}} - 0.3 = 0.6858 \sqrt{1 + \left( \frac{58.69}{d_o} \right)^2} - 0.3$$

When the preceding expression for the allowable shear is equated to the required shear capacity of 407 kips, the required stiffener spacing is found to be 109 in. With this result,  $d_o/D = 109/58.69 = 1.9$ . The ratio  $d_o/D$ , however, may not exceed 1. Therefore, the web stiffeners are placed 49 in. on centers, one-third the cross-frame spacing.

The design shear decreases to below the shear capacity of the unstiffened web approximately 39 ft from the interior support. Hence, all panels should be stiffened for at least this distance from the center pier. For practical reasons, transverse stiffeners are placed on the webs up to the field splice, a distance from the center pier that exceeds the 39 ft required. (The distance is measured along the outer web.)

Stiffeners also are required over a 10-ft length adjacent to the end bearing. For the purpose, two stiffeners are equally spaced in the first panel.

### WEB STIFFENERS

Transverse web stiffeners consist of  $\frac{3}{8} \times 5$ -in. plates. These are welded to the webs. The required moment of inertia of the web stiffeners, about the midplane of the web, is determined from

$$I = d_o t^3 J$$

$$\text{where } J = \left[ 2.5 \left( \frac{D}{d_o} \right)^2 - 2 \right] X$$

Because  $d_o/D = 49/58.69 = 0.835$  lies between 0.78 and 1,  $X$  should be determined from

$$X = 1 + \frac{d_o/D - 0.78}{1.775} Z^4$$

$$\text{where } Z = 0.95 d_o^2 / R t = 0.95 (49)^2 / (415.3 \times 12 \times 1/2) = 0.914.$$

$$X = 1 + \frac{0.835 - 0.78}{1.775} (0.914)^4 = 1.00$$

$$J = \left[ 2.5 \left( \frac{58.69}{49} \right)^2 - 2 \right] 1.00 = 1.59$$

Hence, the required stiffener moment of inertia is

$$I = d_o t^3 J = 49 \left( \frac{1}{2} \right)^3 1.59 = 9.74 \text{ in.}^4$$

A  $\frac{3}{8} \times 5$ -in. plate provides a moment of inertia about the web midplane of

$$I = \frac{0.375(5)^3}{12} + 0.375 \times 5 (2.5 + 0.25)^2 = 18.08 > 9.74 \text{ in.}^4$$

Hence, this plate is adequate. Also, the width-thickness ratio of  $5/(3/8) = 13.3$  is below the maximum allowed of  $82.2/\sqrt{F_y} = 82.2/\sqrt{36} = 13.7$ .

The ratio  $D/t$  for the webs is  $58.69/(1/2) = 117.4$ . The ratio below which longitudinal web stiffeners are not required is

$$\frac{D}{t} = \frac{1,154}{\sqrt{F_y}} \left[ 1 - 8.6 \frac{d_o}{R} + 34 \left( \frac{d_o}{R} \right)^2 \right] = 150 > 117.4$$

Therefore, longitudinal web stiffeners are not required.

### SHEAR-BENDING INTERACTION

The negative-moment section has been checked for both shear and bending as independent actions. AASHTO Specifications Art. 1.7.59E(4), however, limits the permissible bending moment when the design shear exceeds 60% of the critical shear so that

$$\frac{M}{M_u} \leq 1.375 - 0.625 \frac{V}{V_u}$$

where  $M$  = bending moment in girder

$M_u$  = critical bending moment

$V$  = shear in girder

$V_u$  = critical shear

From page 48, the total shear is  $V=371+36=407$  kips.

For calculation of the shear capacity with stiffeners spaced at 49 in.,

$$C=569.2 \times \frac{1}{2} \sqrt{\frac{1+(58.69/49)^2}{50}} - 0.3 = 0.770$$

The shear capacity then is

$$V_u = 0.58 \times 50 \times 58.69 \times \frac{1}{2} \times 0.770 = 655 \text{ kips per web}$$

and 60% of the shear capacity equals 393 kips, which is less than the 407-kip design shear. A reduction in permissible bending moment, therefore, is required. Because the section is highly stressed, an additional stiffener is needed to prevent a reduction in permissible bending stress. Hence, an extra stiffener is placed 24.5 in. from the pier. With this stiffener, 60% of the shear capacity will exceed the design shear at the pier.

Next, the design shear 49 in. from the pier is calculated and found to be less than 60% of the shear capacity of the section:

The direct shear is

$$V_u = 1.3(100.1 + 25.4 + \frac{5}{3} \times 1.5 \times 54.8) \frac{58.69}{57} = 351 \text{ kips}$$

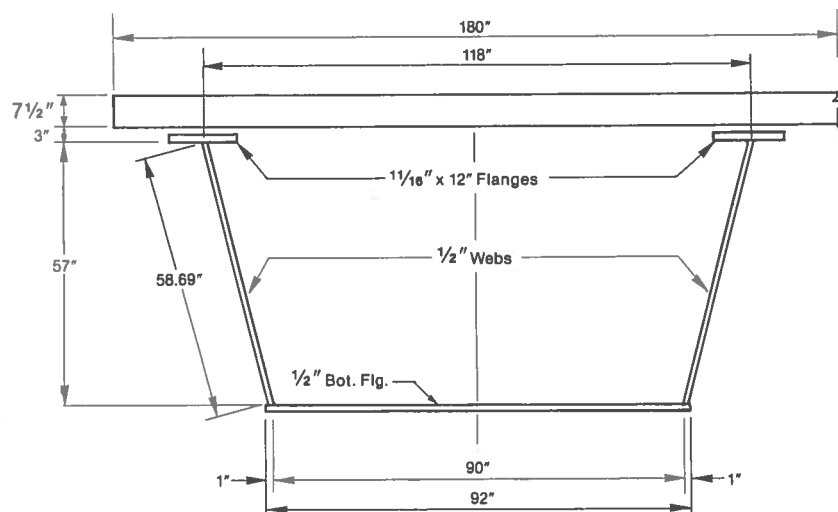
The torsional shear is

$$V_T = 1.3 \times 12 \left( \frac{70.5}{5,230} + \frac{8.2}{6,735} + \frac{5}{3} \times 1.5 \times \frac{66.5}{6,735} \right) \frac{1}{2} \times 58.69 = 18 \text{ kips}$$

The total shear then is  $351+18=369 < 393$  kips. Therefore, no bending reduction is required for the negative-moment section.

#### MAXIMUM—POSITIVE MOMENT SECTION

The section chosen for maximum positive moment is shown in the following drawing.



SECTION FOR MAXIMUM POSITIVE MOMENT

The section is composed entirely of A36 steel and is considered to act compositely with the concrete slab. A bottom-flange longitudinal stiffener is not required, because the bottom flange is in tension. The stiffener used in the negative-moment region will be terminated near the field splice.

In determination of the effective width of the concrete slab for the composite section, each half of the box girder is considered equivalent to a plate girder and the usual AASHTO criteria for effective slab width are applied. Hence, the effective slab

width for the box girder equals the sum of the effective flange widths for each flange. As shown previously in the Slab Half Section in Design of Girder Sections, the effective width of the slab for each box girder is 180 in.

#### Steel Section for Maximum Positive Moment

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
2 T. Flg. Pl. 1 <sup>1</sup> / <sub>16</sub> × 12	16.50	28.84	476	13,278		13,728
2 Web Pl. 1/2 × 58.69	58.69				15,891	15,891
Bot. Flg. Pl. 1/2 × 92	46.00	-28.75	-1,323	38,022		38,022

$$d_s = \frac{-847}{121.19} = -6.99 \text{ in.}$$

$$I_{NA} = 61,720 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.19 + 6.99 = 36.18 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.00 - 6.99 = 22.01 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{61,720}{36.18} = 1,706 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{61,720}{22.01} = 2,804 \text{ in.}^3$$

#### Composite Section, 3n=24, for Maximum Positive Moment

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Steel Section	121.19		-847			67,641
Conc. 180 × 7.5/24	56.25	35.25	1,983	69,894	267	70,161

$$d_{24} = \frac{1,136}{177.44} = 6.40 \text{ in.}$$

$$I_{NA} = 130,529 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.19 \times 6.40 = 22.79 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.00 + 6.40 = 35.40 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{130,529}{22.79} = 5,727 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{130,529}{35.40} = 3,687 \text{ in.}^3$$

#### Composite Section, n=8, for Maximum Positive Moment

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Steel Section	121.19		-847			67,641
Conc. 180 × 7.5/8	168.75	35.25	5,948	209,682	791	210,473

$$d_8 = \frac{5,101}{289.94} = 17.59 \text{ in.}$$

$$I_{NA} = 188,387 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.19 - 17.59 = 11.60 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.00 + 17.59 = 46.59 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{188,387}{11.60} = 16,240 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{188,387}{46.59} = 4,044 \text{ in.}^3$$

#### Stresses in Bottom Flange at 0.4 Point—Maximum Design Loads

Maximum positive moment occurs at a distance from the end bearing of about 0.4 the span. The bottom-flange stresses resulting from the maximum design moment for a section located at the 0.4 point are computed as follows:

$$F_b = 1.3 \times 12 \left( \frac{2,271}{2,804} + \frac{638}{3,687} + \frac{5}{3} \times 1.35 + \frac{2,272}{4,044} \right) = 35.05 \text{ ksi}$$

The St. Venant shear stress is

$$f_v = \frac{1.3 \times 12}{2 \times 0.5} \left( \frac{-159}{5,230} - \frac{1.1}{6,735} + \frac{5}{3} \times 1.5 \times \frac{50}{6,735} \right) = -0.19 \text{ ksi}$$

The allowable bending stress for a bottom flange in tension is

$$F_b = F_y \sqrt{1 - 3(f_v/F_y)^2} = 36 \sqrt{1 - 3(0.19/36)^2} = 36.00 > 35.05 \text{ ksi}$$

Hence, the bottom flange is adequate.

#### Stresses in Top Flanges at 0.4 Point—Maximum Design Loads

The top flanges in the positive-moment region are in compression and therefore the allowable stress is limited to

$$F_{bu} = F_y f \bar{\rho}_B \bar{\rho}_w$$

$$\text{where } f = 1 - 3 \left( \frac{F_y}{E \pi^2} \right) \left( \frac{l}{b} \right)^2$$

The unbraced length  $l$  will be taken as zero, corresponding to a composite top flange that is continuously restrained by the slab. For  $l=0$ ,  $f=1$ ,  $\bar{\rho}_B=1$ ,

$$\bar{\rho}_w = 0.95 + 18(0.1)^2 + \frac{f_w}{f_b} \times 0.3 = 1.13 + \frac{f_w}{f_b} \times 0.3 > 1$$

Because  $F_b = F_y(1)\bar{\rho}_w$  and  $\bar{\rho}_w > 1$ ,  $F_b$  must be equal to the yield strength. This is always the case for top flanges of composite box girders. (A check should be made, however, that, before the concrete of the deck hardens, the steel section is not overstressed under  $DL_1$ . For this check,  $l$  should be taken equal to the distance between cross frames or diaphragms or other points of top-flange lateral support. This calculation is performed at a later point in the example.)

The bending stress under Maximum Design Load is

$$f_b = 1.3 \times 12 \left( \frac{2,271}{1,706} + \frac{638}{5,727} + \frac{5}{3} \times 1.35 \times \frac{2,272}{16,240} \right) = 27.4 < 36 \text{ ksi}$$

The stress under full design load is substantially less than the allowable stress. A reduction in the thickness of the  $1\frac{1}{16}$ -in. flange plates, however, is not desirable, because a thickness of  $\frac{9}{8}$ -in. is considered the minimum desirable thickness to the top flange.

As discussed in General Design Considerations previously in this chapter, the webs will not require stiffeners if the ratio  $D/t < 150$  and the shear is small. The  $\frac{1}{2}$ -in. web plates therefore need not be stiffened in the region of maximum positive bending.

The section designed for maximum positive moment extends from the field splice to a point to be determined later for transition to a lighter section in the positive bending region near the abutment.

#### Stresses in Top Flanges at 0.4 Point—Construction Loads

As mentioned previously, the top flanges should be checked at the 0.4 point for adequacy under  $DL_1$  and construction loads. With the deck not in place,  $b/t$  for the  $1\frac{1}{16} \times 12$ -in. flanges equals 17.45. The flanges are noncompact because 17.45 exceeds  $101.2/\sqrt{F_y} = 16.87$  and is less than  $139.1/\sqrt{F_y} = 23.19$ . Hence, the average normal stress is limited to

$$F_{by} = F_{bs} \rho_B \rho_w$$

With  $l$  = unsupported length of flange = 12.31 ft = 148 in.,  $R$  = radius of curvature of the flange = 410 ft,  $b$  = flange width = 12 in.,  $l/b = 12.33$  and  $l/R = 12.31/410 = 0.030$ ,

$$\rho_B = \frac{1}{1 + (l/R)(l/b)} = \frac{1}{1 + 0.030 \times 12.33} = 0.73$$

For determination of  $\rho_w$ , the bending stress  $f_b$  due to vertical loading and the lateral bending stress  $f_w$  must first be computed.

The vertical-bending stresses arise from a moment of 2,271 kip-ft due to  $DL_1$  plus a moment from a concentrated load. This load is taken as 4 kips, simulating a concrete screeding or finishing machine, and is placed at the 0.4 point. It produces a moment of 96 kip-ft. The total moment for construction loads then is  $2,271 + 96 = 2,367$  kip-ft. The resulting maximum bending stress in the top flanges is

$$f_b = 1.3 \times 12 \times \frac{2,367}{1,706} = 21.6 \text{ ksi}$$

Lateral bending is caused by curvature and web inclination. The bending moment due to curvature is

$$M_{Lc} = \frac{M_1 l^2}{10 R h} = \frac{2,367 \times 12 (148)^2}{10 \times 410 \times 12 \times 57} = 221.9 \text{ in.-kips}$$

The lateral bending stress due to curvature then is

$$f_w = \frac{6 M_{Lc}}{2 b^2 t} = \frac{6 \times 221.9}{2 (12)^2 (11/16)} = 6.7 \text{ ksi}$$

Web inclination causes a lateral bending moment of

$$M_{Ls} = 2.27 \times 12 = 27.2 \text{ in.-kips}$$

as calculated previously in LATERAL FLANGE BENDING. The lateral bending stress due to web inclination then is

$$f_w = \frac{27.2}{(12)^2 (11/16)/6} = 1.7 \text{ ksi}$$

Hence, the total lateral bending stress is

$$f_w = 6.7 + 1.7 = 8.4 \text{ ksi}$$

and  $f_w/f_b = 8.4/21.6 = 0.389$ .

Since the ratio  $f_w/f_b$  is, by definition, positive for the top flanges at the cross frames in positive-bending regions of the girder,  $\rho_w$  is taken as the smaller of  $\rho_{w1}$  and  $\rho_{w2}$ . With  $l/b = 148/12 = 12.3$ ,

$$\begin{aligned} \rho_{w1} &= \frac{1}{1 - (f_w/f_b)(1 - l/75b)} = \frac{1}{1 - 0.389(1 - 12.3/75)} = 1.48 \\ \rho_{w2} &= \frac{0.95 + \frac{l/b}{30 + 8,000(0.1 - l/r)^2}}{1 + 0.6 f_w/f_b} \\ &= \frac{0.95 + \frac{12.3}{30 + 8,000(0.1 - 12.31/410)^2}}{1 + 0.6 \times 0.389} = 0.91 \text{ Governs} \end{aligned}$$

and

$$F_{bs} = F_y \left[ 1 - 3 \left( \frac{F_y}{E \pi^2} \right) \left( \frac{l}{b} \right)^2 \right] = 36 \left[ 1 - 3 \frac{36}{29,000 \pi^2} (12.3)^2 \right] = 33.9 \text{ ksi}$$

The maximum allowable stress is

$$F_{by} = F_{bs} \rho_B \rho_w = 33.9 \times 0.73 \times 0.91 = 22.5 > 21.6 \text{ ksi O.K.}$$

The combined bending stress due to vertical and lateral bending at the flange tip is

$$f_b + f_w = 21.6 + 8.2 = 30.0 < 36 \text{ ksi}$$

The top flange is, therefore, adequate for  $DL_1$  and construction loads.

## FATIGUE INVESTIGATIONS

In the check of fatigue resistance, because the twin box girders have four webs interacting with the concrete deck and are continuous over two spans, the bridge is considered to have sufficient redundancy to be treated as a redundant load-path structure.

### Fatigue of Stiffener Welds to Webs

The welds of the transverse stiffeners to the web near the bottom flange in the positive-moment region constitute a Category C detail, for which the allowable stress range is 19 ksi. For computation of the bending stress 1½ in. above the bottom flange (near the toe of the stiffeners) section moduli are computed for the steel section alone and for the short- and long-term composite section that are used for positive bending moment.

Section Moduli, In.<sup>3</sup>

Steel Section	Composite, $n=8$	Composite, $n=8$
$\frac{61,720}{27.00-6.99}=3,084$	$\frac{188,387}{27.00+17.59}=4,225$	$\frac{130,529}{27.00+6.40}=3,907$

At the point of maximum positive moment, the bending stress for positive live-load moment, with an impact factor of 0.35, is

$$f_b = 1.35 \times 12 \times \frac{2,272}{4,225} = 8.7 \text{ ksi}$$

The bending stress for negative live-load moment is

$$f_b = 1.35 \times 12 \times \frac{-579}{3,907} = -2.4 \text{ ksi}$$

Thus, the maximum stress range is  $8.7 - (-2.4) = 11.1 < 19$  ksi. Hence, the stiffener welds are satisfactory.

### Fatigue of Shear-Connector Welds

Studies of the box girder indicate that, beginning at a distance from the end bearing of about 0.6 of the span, the section experiences sufficient negative live-load bending to put the top flange into tension. Hence, the shear connectors on the top flange at the 0.6 point should be checked for fatigue as a Category C detail, for which the stress range permitted is 19 ksi. Properties needed for computation of stress range at the 0.6 point are those computed previously for the maximum-positive-moment section. In addition, properties of the section to be used for negative bending at the 0.6 point must be calculated. This section consists of the steel section plus the reinforcing steel in the concrete slab. The reinforcement area is less than that used at the interior support, because half of the top layers of bars is terminated near the point of contraflexure. The area is obtained from the size and number of bars shown in the SLAB HALF SECTION in DESIGN OF GIRDER SECTIONS and the location of the center of gravity of the reinforcement relative to the bottom of the concrete slab is obtained as follows:

Reinforcement at 0.6 Point

Bar Location	No. of Bars	Area per Bar	Total Area	$d$	$Ad$
Top row	13	0.31	4.03	4.313	17.38
Bottom row	18	0.31	5.58	2.188	12.21

9.61 in.<sup>2</sup>

29.59 in.<sup>3</sup>

$$d_{\text{Reinf.}} = \frac{29.59}{9.61} = 3.08 \text{ in.}$$

### Steel Section, with Reinforcing Steel, at 0.6 Point

Material	$A$	$d$	$Ad$	$Ad^2$	$I_o$	$I$
Steel Section	121.19		-847			67,641
Reinforcement	9.61	34.58	332	11,491	...	11,491

$$130.80 \text{ in.}^2$$

$$-515 \text{ in.}^3$$

$$79,132$$

$$d_s = \frac{-515}{130.80} = -3.94 \text{ in.}$$

$$-3.94 \times 515 = -2,028$$

$$I_{NA} = 77,104 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.19 \times 3.94 = 33.13 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.00 - 3.94 = 25.06 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{77,104}{33.13} = 2,237 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{77,104}{25.06} = 3,077 \text{ in.}^3$$

The positive live-load moment, with an impact factor of 0.35, at the point is

$$M = 1.35 \times 1,948.3 = 2,630 \text{ kip-ft}$$

The negative live-load moment is

$$M = 1.35(-867.9) = -1,172 \text{ kip-ft}$$

The bending stress due to positive live-load moment is

$$f_b = \frac{2,630 \times 12}{16,240} = 1.9 \text{ ksi}$$

The bending stress due to negative live-load moment is

$$f_b = \frac{-1,172 \times 12}{2,327} = -6.0 \text{ ksi}$$

Thus, the stress range is  $1.9 - (-6.0) = 7.9 < 19$  ksi. Therefore, the shear-connector welds are satisfactory.

### Fatigue Weld at End of Longitudinal Stiffener

Another fatigue consideration in the positive-bending region is the weld at the termination of the bottom-flange longitudinal stiffener. A square termination of the stiffener is a Category E detail, for which the allowable stress range is 12.5 ksi. The stiffener can be terminated at a point at which the bottom-flange compression is within the allowable stress for an unstiffened flange. The 0.6 point of the span is tried.

First, the normal stress in the bottom flange is checked, requiring computation of the shear stress due to torsion. Then, the fatigue stress range at the stiffener termination is checked.

### Moments and Torques at 0.6 Point

	$DL_1$	$DL_2$	$+(L+I)$	$-(L+I)$
$M$ , kip-ft	1,013	338	2,630	-1,172
$T$ , kip-ft	-121.6	-31.9	$1.5 \times 5.2 = 7.8$	$1.5(-84.6) = -126.9$

At the 0.6 point, the shear flow at the bottom flange due to torsion under  $DL_1$  is

$$\tau = \frac{-121.6 \times 12}{2 \times 51(90 + 118)/2} = -0.14 \text{ kips per in.}$$

The shear flow due to torsion under  $DL_2$  is

$$\tau = \frac{-31.9 \times 12}{2 \times 63.75(90 + 121.3)/2} = -0.03 \text{ kips per in.}$$



For live-load, the negative shear flow is

$$\tau = \frac{-126.9 \times 12}{2 \times 6,735} = -0.11 \text{ kips per in.}$$

The total shear flow then is  $0.14 + 0.03 + 0.11 = 0.28$  kips per in.

The shear stress in the bottom flange at the 0.6 point is therefore

$$f_v = \frac{0.28}{1/2} = 0.56 \text{ ksi}$$

The vertical-bending stress in the bottom flange at this point is

$$f_b = 1.3 \times 12 \left( \frac{1,013}{2,804} + \frac{338}{3,687} + \frac{5}{3} \times \frac{-1,172}{3,077} \right) = -2.84 \text{ ksi}$$

For computation of the critical compression stress for the bottom flange, assume for  $R_2$  its maximum value:

$$R_2 = \frac{210.3}{1} \sqrt{4} = 420.6$$

$$\frac{w}{t} \sqrt{F_y} = \frac{90}{1/2} \sqrt{36} = 1,080 > (R_2 = 420.6)$$

Hence, the maximum allowable normal stress in the bottom flange is

$$\begin{aligned} F_b &= 26,210K \left( \frac{t}{w} \right)^2 - \frac{f_v^2 K}{26,210K_s^2 (t/w)^2} \\ &= 26,210 \times 4 \left( \frac{1/2}{90} \right)^2 - \frac{(0.56)^2 4}{26,210(5.34)^2 (0.5/90)^2} = 3.18 > 2.84 \text{ ksi} \end{aligned}$$

Since the critical compression stress is larger than the design bending stress, the bottom flange is adequate without a stiffener, and the stiffener may be terminated at the 0.6 point.

The bending stress at the bottom of steel for positive live-load moment is

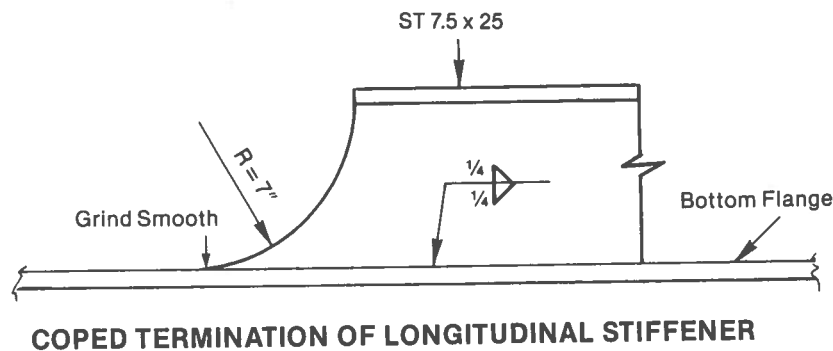
$$f_b = \frac{2,630 \times 12}{4,044} = 7.8 \text{ ksi}$$

and for negative live-moment is

$$f_b = \frac{-1,172 \times 12}{3,077} = -4.6 \text{ ksi}$$

Thus, the stress range is  $7.8 - (-4.6) = 12.4 < 12.5$  ksi. By a narrow margin, a square termination of the longitudinal stiffener can be made at the 0.6 point.

Fatigue characteristics of the termination, however, may be improved considerably by introducing a radius at the end of the stiffener, as shown in the following drawing. The 7-in. radius coping as shown is considered to upgrade the detail to Category D, with an allowable stress range of 16 ksi. Inasmuch as there are only four of these details in the structure, the coped termination is judged to be worth the small extra cost for providing a more fatigue-resistant design.



The longitudinal stiffener is terminated 11 ft from the field splice, so that it ends near the 0.6 point.

### POSITIVE-MOMENT TRANSITION 20 FT FROM END SUPPORT

At a distance of 20 ft from the end bearing, the section used for maximum positive moment may be reduced. The top-flange thickness is decreased from  $1\frac{1}{16}$  to  $\frac{9}{16}$  in. and the bottom flange is reduced from  $\frac{1}{2}$  to  $\frac{5}{16}$  in. The section is made of A36 steel.

#### Steel Section Adjacent to end Support

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
2 T. Flg. Pl. $1\frac{1}{16} \times 12$	13.50	28.78	389	11,182	15,891	11,182
2 Web Pl. $\frac{1}{2} \times 58.69$	58.69					15,891
Bot. Flg. Pl. $\frac{5}{16} \times 92$	28.75	-28.66	-824	23,615		23,615

$$d_s = \frac{-435}{100.94} = -4.31 \text{ in.}$$

$$100.94 \text{ in.}^2 \quad -435 \text{ in.}^3 \quad 50,688$$

$$-4.31 \times 435 = -1,875$$

$$I_{NA} = 48,813 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.06 + 4.31 = 33.37 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 28.81 - 4.31 = 24.50 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{48,813}{33.37} = 1,463 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{48,813}{24.50} = 1,992 \text{ in.}^3$$

#### Composite Section, 3n=24, Adjacent to End Support

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Steel Section	100.94		-435		267	50,688
Conc. 180×7.5/24	56.25	35.25	1,983	69,894		70,161

$$d_{24} = \frac{1,548}{157.19} = 9.85 \text{ in.}$$

$$157.19 \text{ in.}^2 \quad 1,548 \text{ in.}^3 \quad 120,849$$

$$-9.85 \times 1,548 = -15,248$$

$$I_{NA} = 105,601 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.06 + 9.85 = 19.21 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 28.81 + 9.85 = 38.66 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{105,601}{19.21} = 5,497 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{105,601}{38.66} = 2,732 \text{ in.}^3$$

#### Composite Section, n=8, Adjacent to End Support

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Steel Section	100.94		-435		791	50,688
Conc. 180×7.5/8	168.75	35.25	5,948	209,682		210,473

$$d_s = \frac{5,513}{269.69} = 20.44 \text{ in.}$$

$$269.69 \text{ in.}^2 \quad 5,513 \text{ in.}^3 \quad 261,161$$

$$-20.44 \times 5,513 = -112,686$$

$$I_{NA} = 148,475 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.06 + 20.44 = 8.62 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 28.81 + 20.44 = 49.25 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{148,475}{8.62} = 17,224 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{148,475}{49.25} = 3,015 \text{ in.}^3$$

$$d_{\text{Top of conc.}} = 39.00 - 20.44 = 18.56 \text{ in.}$$

$$S_{\text{Top of conc.}} = \frac{148,475}{18.56} = 8,000 \text{ in.}^3$$

From the curves of maximum moment, for the section 20 ft from the end bearing, the moment due to  $DL_1$  is 1,670 kip-ft, due to  $DL_2$  460 kip-ft and due to live-load 1,480 kip-ft. The impact factor to be applied to live load is 0.35. The bottom-flange bending stress due to design moment then is

$$F_b = 1.3 \times 12 \left( \frac{1,670}{1,992} + \frac{460}{2,732} + \frac{5}{3} \times 1.35 \times \frac{1,480}{3,015} \right) = 32.9 \text{ ksi}$$

From the curves of maximum torque, the torque due to  $DL_1$  is 137 kip-ft, due to  $DL_2$  41 kip-ft and due to live load 150 kip-ft. The impact factor is 0.50. The St. Venant shear stress in the bottom flange due to torque then is

$$f_v = \frac{1.3 \times 12}{2 \times 0.31} \left( \frac{137}{5,230} + \frac{41}{6,735} + \frac{5}{3} \times 1.50 \times \frac{150}{6,735} \right) = 2.20 \text{ ksi}$$

Accordingly, the bottom-flange allowable bending stress is

$$F_b = F_y \sqrt{1 - 3(f_v/F_y)^2} = 36 \sqrt{1 - 3(2.20/36)^2} = 35.8 > 32.9 \text{ ksi}$$

Hence, the bottom flange is adequate.

As indicated in the previous discussion of top-flange stresses at the 0.4 point, the allowable stress for the composite top flange under service conditions is the yield stress, 36 ksi. The design bending stress in the flange is

$$F_b = 1.3 \times 12 \left( \frac{1,670}{1,463} + \frac{460}{5,497} + \frac{5}{3} \times 1.35 \times \frac{1,480}{17,224} \right) = 22.1 < \text{ksi}$$

Hence, the  $\frac{1}{16} \times 12$ -in. flange is more than adequate.

A check of this flange under construction loads by the procedure used for bottom-flange stresses at the 0.4 point also indicates that the section has adequate strength and stability.

Inasmuch as the torsional stresses are low, the manhole detail for the bottom flange near the end bearing, as used in the example design of Chapter 7, may also be used for this example.

### NEGATIVE-MOMENT TRANSITION 13 FT FROM INTERIOR SUPPORT

Inspection of the curves of maximum moment indicates that a smaller section than that used for maximum negative moment may be introduced at some distance from the pier. A reduced section made of A572, Grade 50, steel is chosen. Since the portion of the girder near the pier is also made of this material, the same steel is used throughout the negative-bending region, from field splice to field splice.

The reduced section is investigated at a distance from the interior support of 13 ft, where a transition is made to the heavier negative-moment section.

Steel Section 13 Ft from Interior Support

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
2 T. Flg. Pl. $1\frac{1}{8} \times 15$	33.75	29.06	981	28,508	15,891	28,508
2 Web Pl. $\frac{1}{2} \times 58.69$	58.69					15,891
Bot. Flg. Pl. $\frac{3}{4} \times 92$	69.00	-28.88	-1,993	57,558		57,558
Stiff. ST $7.5 \times 25$	7.35	-23.25	-171	3,973	41	4,014

$$d_s = \frac{-1,183}{168.79} = -7.01 \text{ in.} \quad \begin{array}{l} 168.79 \text{ in.}^2 \quad -1,183 \text{ in.}^3 \quad 105,971 \\ -7.01 \times 1,183 = -8,293 \end{array}$$

$$I_{NA} = 97,678 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.62 + 7.01 = 36.63 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.25 - 7.01 = 22.24 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{97,678}{36.63} = 2,667 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{96,678}{22.24} = 4,392 \text{ in.}^3$$

**Steel Section, with Reinforcing Steel, 13 Ft from Interior Support**

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Steel Section	168.79		-1,183			105,971
Reinforcement	15.19	35.13	534	18,746		18,746

$$d_c = \frac{-649}{183.98} = 3.53 \text{ in.}$$

$$I_{NA} = 122,428 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.63 + 3.53 = 33.16 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.25 - 3.53 = 25.72 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{122,428}{33.16} = 3,693 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{122,428}{25.72} = 4,760 \text{ in.}^3$$

$$d_{\text{Reinf.}} = 35.13 + 3.53 = 38.66 \text{ in.}$$

$$S_{\text{Reinf.}} = \frac{122,428}{38.66} = 3,167 \text{ in.}^3$$

**Stresses in Bottom Flange**

At 13 ft from the interior support, the moment due to  $DL_1$  is 3,705 kip-ft, due to  $DL_2$  845 kip-ft and due to live load 1,768 kip-ft. The impact factor is 0.35. The bending stress in the bottom flange then is

$$F_b = 1.3 \times 12 \left( \frac{3,705}{4,392} + \frac{845}{4,760} + \frac{5}{3} \times 1.35 \times \frac{1,768}{4,760} \right) = 29.0 \text{ ksi}$$

The torque due to  $DL_1$  is -30 kip-ft, due to  $DL_2$  -15 kip-ft and due to live load -127 kip-ft. The impact factor is 0.50. The shear stress in the bottom flange due to torque then is

$$f_v = \frac{1.3 \times 12}{2 \times 3/4} \left( \frac{30}{5,230} + \frac{15}{6,735} + \frac{5}{3} \times 1.5 \times \frac{127}{6,735} \right) = 0.57 \text{ ksi}$$

Because the shear stress is much less than  $0.75F_y/\sqrt{3} = 15.6 \text{ ksi}$ , the allowable stress  $F_b$  for the bottom flange is determined by the parameters  $R_1$ ,  $R_2$  and  $\Delta$  as follows:

$$\Delta = \sqrt{1 - 3 \left( \frac{f_v}{F_y} \right)^2} = \sqrt{1 - 3 \left( \frac{0.57}{50} \right)^2} = 1.000$$

For computation of  $R_1$  and  $R_2$ ,  $n=1$ ,  $b=90/2=45 \text{ in.}$ ,  $t=3/4 \text{ in.}$  and  $I_s=243.2 \text{ in.}^4$

$$K = \sqrt[3]{\frac{I_s}{0.125t^3b}} = \sqrt[3]{\frac{243.2}{0.125(0.75)^3 45}} = 4.68 > 4 \text{ Use } K=4.$$

$$K_s = \frac{5.34 + 2.84(I_s/bt^3)^{1/3}}{(n+1)^2} = 3.00 < 5.34$$

With the use of the preceding results,

$$R_1 = \frac{97.08\sqrt{4}}{\sqrt{\frac{1}{2} \left[ 1 + \sqrt{(1)^2 + 4 \left( \frac{0.57}{50} \right)^2 \left( \frac{4}{3} \right)^2} \right]}} = 194.2$$

$$R_2 = \frac{210.3\sqrt{4}}{\sqrt{\frac{1}{1.2} \left[ 1 - 0.4 + \sqrt{(1-0.4)^2 + 4 \left( \frac{0.57}{50} \right)^2 \left( \frac{4}{3} \right)^2} \right]}} = 420.4$$

Because  $w\sqrt{F_y}/t = 45\sqrt{50}/0.75 = 424.2 > (R_2 = 420.4)$ , the allowable compression stress in the bottom flange is given by

$$f_b = 26,210K \left( \frac{t}{w} \right)^2 - \frac{f_y^2 K}{26,210K_s^2 (t/w)^2}$$

$$= 26,210 \times 4 \left( \frac{0.75}{45} \right)^2 - \frac{(0.57)^2 \times 4}{26,210(3)^2 (0.75/45)^2} = 29.1 > 29.0 \text{ ksi}$$

The bottom flange is satisfactory

#### Stress in Top Flange

The bending stress in the top flange 13 ft from the interior support is

$$F_{bs} = 1.3 \times 12 \left( \frac{3,705}{2,667} + \frac{845}{3,693} + \frac{5}{3} \times 1.35 \times \frac{1,768}{3,693} \right) = 42.0 \text{ ksi}$$

The allowable stress in the top flange is computed as follows:

$$F_{bs} = F_y \left[ 1 - 3 \left( \frac{F_y}{E\pi^2} \right) \left( \frac{l}{b} \right)^2 \right] = 50 \left[ 1 - 3 \left( \frac{50}{29,000\pi^2} \right) \left( \frac{148}{15} \right)^2 \right] = 47.5 > 42.0 \text{ ksi}$$

Hence, the  $1\frac{1}{8} \times 15$ -in. top flange is adequate.

#### Fatigue Check—13 Ft from Interior Support

Fatigue is checked at the butt-welded top-flange transition. The weld is in category B, with an allowable stress range of 27.5 ksi. The positive live-load moment is 372 kip-ft at 13 ft from the interior support, and the negative live-load moment is -1,768 kip-ft. Hence, the moment range is  $372 - (-1,768) = 2,140$  kip-ft, and the stress range is

$$f_b = 1.35 \times 12 \times \frac{2,140}{3,693} = 9.39 < 27.5 \text{ ksi}$$

The flange weld is satisfactory for fatigue.

#### FLANGE-TO-WEB WELDS

Size of the flange-to-web welds for the straight box girder of Chapter 7 are governed by material-thickness requirements, rather than by horizontal shear flow, by a substantial margin. Torsional effects for the curved box girders do not add to the stresses in the flanges-to-web welds sufficiently to change this condition. Consequently, welds in this example are sized by material thickness.

The requirement that the web be fully developed by the flange-to-web weld, to insure adequate fatigue resistance with respect to transverse distortional stresses, should be checked, however. By AASHTO 1.749(E),

$$\text{Weld size required} = \frac{\text{web thickness}}{0.707 \times 2} \frac{\frac{1}{2}}{1.414} = \frac{3}{8} \text{ in. Governs}$$

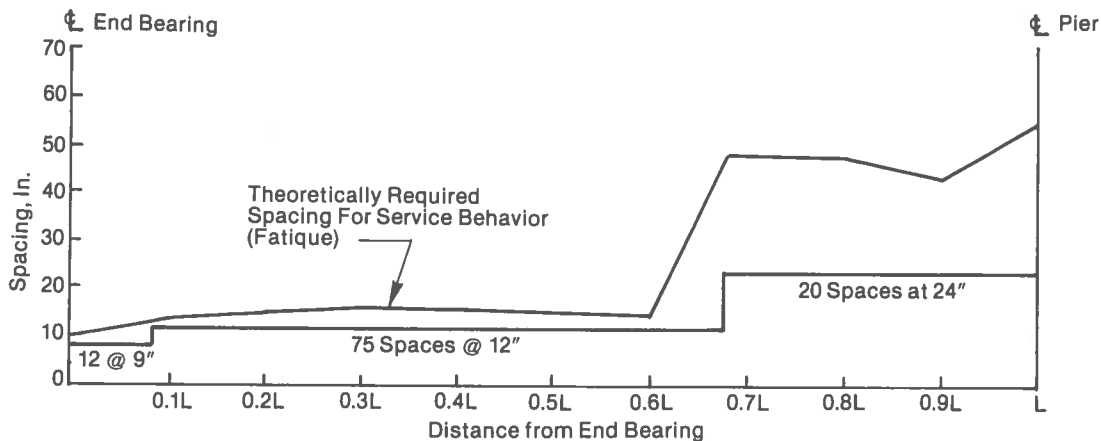
#### SHEAR CONNECTORS

Two  $\frac{7}{8}$ -in.-dia, 5-in.-high, welded stud shear connectors are welded per row to each flange. The 5-in. height satisfies the 2-in. minimum concrete cover over the connectors as well as the requirement for 2-in. minimum penetration into the concrete slab. The spacing of the shear connectors to meet fatigue criteria is determined at tenth points along the span. Subsequently, the connectors at the spacing that results are checked for ultimate strength.

### Computation of Shear-Connector Spacing

Distance from End Bearing	Positive Live-Load Shear, Kips	Negative Live-Load Shear, Kips	Shear Range $V_r$ , Including 50% Impact, Kips	$Q$ In. <sup>3</sup>	$I$ In. <sup>4</sup>	$S_r = \frac{V_r Q}{I}$ kips per in.	Spacing, In. $= \frac{4Z_r}{S_r} = \frac{4 \times 8.11}{S_r}$
0	117.0	— 12.8	194.7	2,500	148,475	3.28	9.9
0.1L	83.3	— 14.3	146.4	2,500	148,475	2.47	13.1
0.2L	69.9	— 21.6	137.3	2,980	188,387	2.17	15.0
0.3L	57.1	— 30.9	132.0	2,980	188,387	2.09	15.5
0.4L	45.0	— 44.1	133.7	2,980	188,387	2.11	15.4
0.5L	34.0	— 56.6	135.9	2,980	188,387	2.15	15.1
0.6L	24.1	— 68.2	138.5	2,980	188,387	2.19	14.8
0.7L	15.4	— 78.7	141.5	587	122,428	0.68	47.7
0.8L	7.9	— 88.0	143.9	587	122,428	0.69	47.1
0.9L	2.5	— 101.3	155.7	587	122,428	0.75	43.3
L	0	— 114.9	172.4	547	158,132	0.60	54.1

Try the connector spacing shown in the following graph.



### SHEAR-CONNECTOR SPACING

#### Shear Connectors—Strength Requirements

For ultimate strength, the number of shear connectors between critical points must be such that the design load  $P_c$ , kips per shear connector, does not exceed the ultimate strength, kips, of a shear connector:

$$P_c = \phi S_u$$

where  $\phi = 0.85$

$$S_u = 0.4d^2 \sqrt{f'_c E_c} = \frac{0.4(7/8)^2}{1,000} \sqrt{4,000(150)^{3/2} 33 \sqrt{4,000}} = 37.93 \text{ kips}$$

In a straight girder, the design load  $\bar{P}$ , kips per shear connector, is given by

$$\bar{P} = \frac{P}{N}$$

where  $N$ =number of shear connectors between point of maximum positive moment and the end bearing or dead-load inflection points, or between points of maximum negative moment and adjacent dead-load inflection points

For positive bending moment,  $P$  is the smaller of the following:

$$P_1 = 0.85f'_c b c$$

$$P_2 = A_s F_y$$

where  $b$ =effective width of concrete slab=180 in.

$c$ =slab thickness=7.5 in.

$A_s$ =area of steel section=121.19 in.<sup>2</sup>

$F_y$ =yield strength of the steel=36 ksi

$$P_1 = 0.85 \times 4 \times 180 \times 7.5 = 4,590 \text{ kips}$$

$$P_2 = 121.19 \times 36 = 4,363 < 4,590 \text{ kips Governs.}$$

For negative moment, with the area of longitudinal reinforcing steel in the slab  $A'_s$  =15.19 in.<sup>2</sup> and yield strength of this steel  $F'_y$ =40 ksi,

$$P = A'_s F'_y = 15.19 \times 40 = 608 \text{ kips}$$

For curved box girders, the design load per shear connector is

$$P_c = \sqrt{\bar{P}^2 + F^2 + 2\bar{P}F \sin \theta/2}$$

where  $\theta$ =angle subtended between the point of maximum positive moment and the end bearing (6.87°) or the point of contraflexure (4.87°) or between the point of maximum negative moment and the point of contraflexure (5.45°)

$$F = \frac{P(1 - \cos \theta)}{4KN_s \sin \theta/2}$$

$N_s$ =number of shear connectors on the two flanges at a section=4

$$K = 0.166(N/N_s - 1) + 0.375$$

Between the point of maximum positive moment and the end bearing,  $N=212$ . Hence, with  $P=4,363$  kips,

$$K = 0.166 \left( \frac{212}{4} - 1 \right) + 0.375 = 9.007$$

$$\bar{P} = \frac{4,363}{212} = 20.58 \text{ kips}$$

$$F = \frac{4,363(1 - \cos 6.87^\circ)}{4 \times 9.007 \times 4 \sin 6.87/2} = 3.63 \text{ kips}$$

The design load per shear connector for the curved girder then is

$$P_c = \sqrt{(20.58)^2 + (3.63)^2 + 2 \times 20.58 \times 3.63 \sin 6.87/2} = 21.11 \text{ kips}$$

The ultimate strength of a shear connector is

$$P_c = 0.85 \times 37.93 = 32.2 > 21.11 \text{ kips}$$

Hence, the number of shear connectors between the point of maximum moment and the end bearing is satisfactory.

Between the point of maximum positive moment and the dead-load inflection point,  $N=140$ .

$$K = 0.166 \left( \frac{140}{4} - 1 \right) + 0.375 = 6.019$$

$$\bar{P} = \frac{4,363}{140} = 31.16 \text{ kips}$$

$$F = \frac{4,363(1 - \cos 4.77^\circ)}{4 \times 6.019 \times 4 \sin 4.77/2} = 3.77 \text{ kips}$$

The design load per shear connector for the curved girder then is

$$P_c = \sqrt{(31.16)^2 + (3.77)^2 + 2 \times 31.16 \times 3.77 \sin 4.77/2} = 31.54 < 32.2 \text{ kips}$$

Hence, the number of shear connectors between the point of maximum positive moment and the dead-load inflection point is satisfactory.

Between the point of maximum negative moment and the dead-load inflection point,  $N=80$ .

$$K = 0.166 \left( \frac{80}{4} - 1 \right) + 0.375 = 3.529$$

$$\bar{P} = \frac{608}{80} = 7.6 \text{ kips}$$

$$F = \frac{608(1 - \cos 5.45^\circ)}{4 \times 3.529 \times 4 \sin 5.45/2} = 1.02 \text{ kips}$$

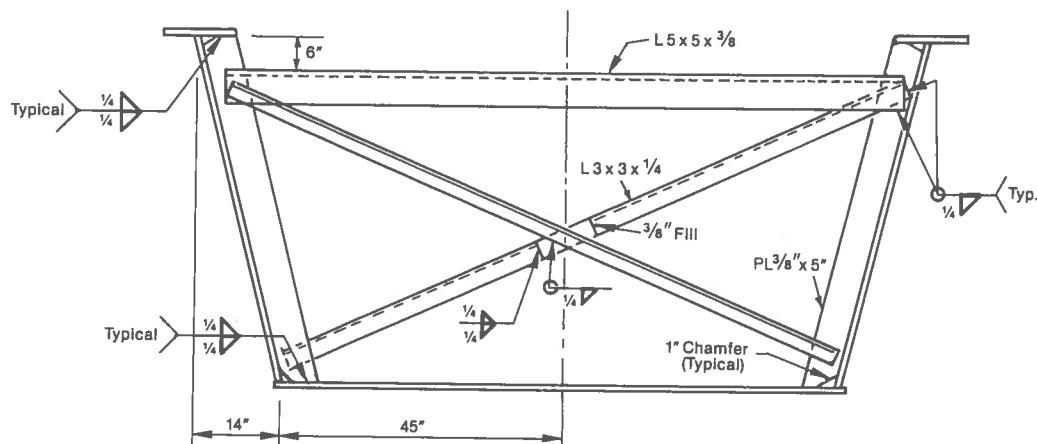
The design load per shear connector for the curved girder then is

$$P_c = \sqrt{(7.6)^2 + (1.02)^2 + 2 \times 7.6 \times 1.02 \sin 5.45/2} = 7.72 < 32.2 \text{ kips}$$

Hence, the number of shear connectors between the point of maximum negative moment and the dead-load inflection point is satisfactory. Thus, the spacing selected to meet fatigue requirements also satisfies ultimate-strength requirements.

### CROSS FRAMES

Three different designs for intermediate cross frames of A36 steel are employed for the girders in this example. Two of these are used for regions of the box girders where a longitudinal stiffener is attached to the bottom flange. The third design is used for regions of the girder without this stiffener. The cross frame shown in the following drawing is the third type.



CROSS FRAME IN SECTIONS WITHOUT LONGITUDINAL STIFFENER



### Design of Top Strut of Cross Frames

For simplicity, cross-frame members are designed with working-stress criteria. For the top strut, an angle  $5 \times 5 \times \frac{3}{8}$  in., with an area of  $3.61 \text{ in.}^2$ , is investigated for overall buckling ( $L/r < 120$ ), local buckling ( $b/t < 12$ ) and for capacity as a compression member. The computations show that the lateral reaction of the strut on the curved flange is 10.6 kips, considerably less than the strut capacity of 34.2 kips.

The unbraced length of the strut is 118 in. For overall buckling of the  $5 \times 5 \times \frac{3}{8}$ -in. angle,

$$\frac{L}{r} = \frac{118}{0.99} = 119 < 120$$

For local buckling,

$$\frac{b}{t} = \frac{5 - 0.38}{0.38} = 12.2$$

This is close enough to the limiting value of 12 for main compression members to be acceptable.

The force acting on the strut is given by

$$R = 1.1wl$$

where  $w$  = load imposed during the wet-concrete condition, kips per ft

$l$  = cross-frame spacing = 12.31 ft

For the change in vertical and torsional shear between the end bearing and the 0.7 point, as computed for lateral flange bending,

$$w = \frac{14}{57} \times 1.468 \times \frac{1}{2} = 0.18 \text{ kips per ft}$$

From maximum positive moment due to  $DL_1$ ,

$$w = \frac{2,271 \times 12}{2 \times 55 \times 4,925} = 0.05 \text{ kips per in.} = 0.60 \text{ kips per ft}$$

Hence, the force on the strut is

$$R = 1.1(0.18 + 0.60)12.31 = 10.6 \text{ kips}$$

The allowable force on the strut is

$$R = F_c A = [16,980 - 0.53(119)^2]13.61 = 34.2 > 10.6 \text{ kips}$$

The strut is satisfactory.

### Design of Cross-Frame Diagonals

An angle  $3 \times 3 \times \frac{1}{4}$  in. checked for the diagonal for overall and local buckling. The diagonal has an area of  $1.44 \text{ in.}^2$  and makes an angle with the horizontal of  $\alpha = \tan^{-1}(51/104) = 26.12^\circ$ . The minimum area permissible for the member is

$$A_b = 75 \frac{Sb}{\alpha^2} \frac{t_w^3}{d+b} = 75 \times \frac{12.31 \times 12 \times 90}{(26.12)^2} \times \frac{(\frac{1}{2})^3}{57+90} = 0.017 < 1.44 \text{ in.}^2$$

The diagonal is checked for local buckling as a secondary member.

$$\frac{b}{t} = \frac{3 - 0.25}{0.25} = 11 < 16$$

The  $3 \times 3 \times \frac{1}{4}$ -in. angle, therefore is satisfactory.

### Design of Bottom Strut Placed Above the Longitudinal Stiffener

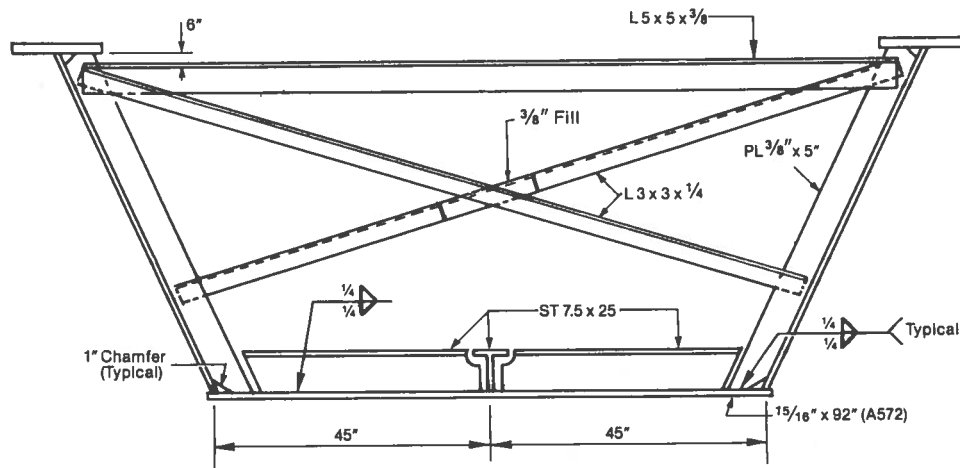
The cross frame used in conjunction with the longitudinal stiffener on the bottom flange of the girder has a bottom strut (see the following sketch). This strut serves as a transverse bottom-flange stiffener for the girder and also as a transverse lateral


$$f_b = 12 \left( \frac{3,870}{5,301} + \frac{887}{5,550} + \frac{5}{3} \times 1.35 \times \frac{1,819}{5,550} \right) = 19.5 \text{ ksi}$$
$$A_f = 92 \times \frac{7}{8} + 7.35 = 87.85 \text{ in.}^2$$
$$I_t = 0.1(n+1)^3 w^3 \frac{f_b}{E} \frac{A_f}{l}$$

$$= 0.1(1+1)^3 (45)^3 \frac{19.5}{29,000} \times \frac{87.85}{12.31 \times 12} = 29.2 < 40.6 \text{ in}^4$$
$$\frac{b}{t} = \frac{7.5 - 0.62}{0.55} = 12.5$$
$$\frac{b}{t} = \frac{82.2}{\sqrt{F_y}} = \frac{82.2}{\sqrt{36}} = 13.7 > 12.5$$

### Design of Bottom Strut at Same Level as Longitudinal Stiffener

the girder, at the same level as the longitudinal stiffener, as shown in the following drawing.



**CROSS FRAME AT POINTS OF DEAD-LOAD CONTRAFLEXURE AND MAXIMUM FLANGE STRESS**

Sizes of all members of this cross frame are the same as those of the other cross frames. Because a solid plate diaphragm is placed over the interior support (point of maximum flange stress), only one cross frame of this type is needed. It is placed near the field splice.

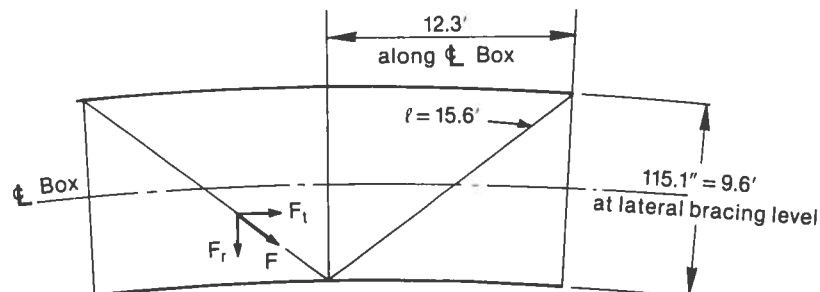
#### Cross-Frame Connections

All cross-frame connections are made with 1/4-in. fillet welds, which provide more than adequate strength.

#### LATERAL BRACING

The lateral bracing, which will be placed about 6 in. below the top flanges of the box girders, is designed to carry the St. Venant shear that exists across the top of the box due to torsion under initial dead load. For computation of this shear, a solid plate is assumed as a substitute for the open bracing actually used.

The curves of maximum torque indicate that maximum  $DL_1$  torque occurs at the end bearing. The shear is obtained by multiplying the shear flow produced by the torque by the width of the box at the level of the lateral bracing (115.1 in.). This shear force is the lateral component  $F_r$  of the force  $F$  in the 15.6-ft-long bracing diagonal (see the following drawing).



**PLAN VIEW OF GIRDER LATERAL BRACING**

The  $DL_1$  torque at the end bearing is 181.1 kip-ft. For an enclosed area of the box of  $A_1 = 5,230 \text{ in.}^2$ , the shear flow is

$$S = \frac{T}{2A_1} = \frac{1.3 \times 181.1 \times 12}{2 \times 5,230} = 0.270 \text{ kips per in.}$$

The resulting transverse force is

$$F_r = 0.270 \times 115.1 = 31.1 \text{ kips}$$

The force is the diagonal bracing therefore is

$$F = 31.1 \times \frac{15.6}{9.6} = 50.5 \text{ kips}$$

and the longitudinal component of the force is

$$F_t = 31.1 \times \frac{12.3}{9.6} = 39.8 \text{ kips}$$

Because the lateral-bracing diagonals are considered main members, for which the slenderness ratio  $L/r$  must be equal to or less than 120, the radius of gyration of the diagonal should be at least

$$r = \frac{15.6 \times 12}{120} = 1.56 \text{ in.}$$

For the diagonals, try a WT7  $\times$  26.5. It has a radius of gyration about the Y-Y axis  $r_y = 1.92$ , area  $A_s = 7.81 \text{ in.}^2$  and section modulus  $S_x = 4.94 \text{ in.}^3$ . The slenderness ratio for the diagonal for the Y-Y axis is

$$\frac{KL_c}{r_y} = \frac{0.75 \times 15.6 \times 12}{1.92} = 73.1$$

For computation of the critical buckling stress in the diagonal,

$$\sqrt{\frac{2\pi^2 E}{F_y}} = 107 > 73.1$$

Hence, the critical buckling stress for the Y-Y axis is

$$F_{cr} = F_y \left[ 1 - \frac{F_y}{4\pi^2 E} \left( \frac{KL_c}{r_y} \right)^2 \right] = 36 \left[ 1 - \frac{36}{4\pi^2 \times 29,000} (73.1)^2 \right] = 29.95 \text{ ksi}$$

The bending strength of the diagonal as an unbraced beam is

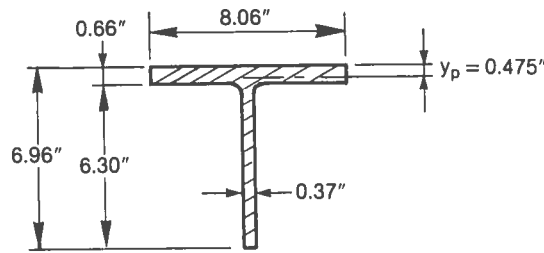
$$\begin{aligned} M_u &= F_y S \left[ 1 - \frac{3F_y}{4\pi^2 E} \left( \frac{L_b}{0.9b'} \right)^2 \right] \\ &= 36 \times 4.94 \left[ 1 - \frac{3 \times 36}{4\pi^2 \times 29,000} \left( \frac{15.6 \times 12}{0.9 \times 3.845} \right)^2 \right] = 128.7 \text{ kip-in.} \end{aligned}$$

The slenderness ratio for the X-X axis is

$$\frac{KL_c}{r_x} = \frac{0.75 \times 15.6 \times 12}{1.88} = 74.7$$

The Euler buckling stress for the X-X axis is

$$F_c = \frac{\pi^2 E}{(KL_c/r_x)^2} = \frac{\pi^2 \times 29,000}{(74.7)^2} = 51.29 \text{ ksi}$$



**WT 7 X 26.5**

The neutral axis of the WT for bending under Maximum Design Load is located at a distance  $y_p$  below the outer surface of the flange. The area of the section above the axis equals the area below.

$$8.06y_p = 6.30 \times 0.37 + 8.06(0.66 - y_p)$$

Solution of the equation yields  $y_p = 0.475$ -in. and  $0.66 - y_p = 0.185$ -in.

The plastic section modulus then is

$$Z = \frac{8.06(0.475)^2}{2} + \frac{8.06(0.185)^2}{2} + 6.30 \times 0.37 \times 3.335 = 8.82 \text{ in.}^3$$

The capacity of the WT as a compact beam therefore is

$$M_p = F_y Z = 36 \times 8.82 = 317.5 \text{ kip-in.}$$

The maximum bending moment in the diagonal is

$$M_{ecc} = 50.5 \times 1.38 = 69.7$$

$$M_{DL} = 1.30 \times 0.0265(15.6)^2 \times 12 = 12.6$$

$$M = 82.3 \text{ kip-in.}$$

Substitution of the preceding results in the interaction equation for combined compression and bending yields

$$\frac{50.5}{0.85 \times 7.81 \times 29.95} + \frac{82.3}{128.7 \left(1 - \frac{50.5}{7.81 \times 51.29}\right)} = 0.986 < 1.0$$

$$\frac{50.5}{0.85 \times 7.81 \times 36} + \frac{82.3}{317.5} = 0.471 < 1.0$$

The WT7X26.5 is satisfactory.

### **Bolted Gusset-Plate Connection**

The connections of the lateral bracing to the girder webs are made with bolts, because of their excellent fatigue characteristics. This type of connection corresponds to an AASHTO Category B detail, with an allowable girder stress ranges of 45.0 ksi (lane loading and 27.5 ksi (truck loading).

The bolted connections of the lateral bracing at a girder are made to a  $\frac{3}{4}$ -in. gusset plate, which is, in turn, welded to another  $\frac{3}{4}$ -in. plate that is bolted to the girder web (see following drawing). Bolts are  $\frac{7}{8}$  in. in diameter and have a capacity of 12.63 kips each. Maximum permissible length of the unsupported edge of the gusset is

$$L = \frac{347.8t}{\sqrt{F_y}} = \frac{347.8 \times 3/4}{\sqrt{36}} = 43.5 \text{ in.}$$

The unsupported edge of the gusset is about 12 in. < 43.5 in.


$$0.45F_y = 0.45 \times 70 = 31.5 \text{ ksi}$$
$$f_w = \frac{1.3 \times 79.6}{30 \times 2 \times 0.707 \times \frac{1}{4}} = 9.75 < 31.5 \text{ ksi}$$

The diagonals are connected to the gusset plate with  $\frac{7}{8}$ -in.-dia, A325 bolts. Each bolt has a capacity of 12.63 kips. Hence, the number of bolts required for a diagonal is

$$\frac{1.3 \times 50.5}{12.63} = 5.2 \text{ bolts}$$

## The plate-to-girder web attachment requires

$$\frac{1.3 \times 79.6}{12.63} = 8.2 \text{ bolts}$$

The bolts in the connection to the girder web are subject to combined tension and shear. The tensile forces are caused by a direct pull and prying action. The maximum direct tensile force is 31.1 kips. Divided among the six bolts, the tension is

$$T = \frac{31.1}{6} = 5.2 \text{ kips per bolt}$$

Prying action results from both the tension on the bolts and distortion of the connected parts. The lever arms involved are the distance  $a=1\frac{1}{2}$  in. from the center of the bolts to the edge of the  $\frac{3}{4}\times 9$ -in. connection plate and the distance  $b=1\frac{7}{8}$  in. from the toe of the fillet weld between the two connection plates and the center of the bolts. The thickness of the girder web may be assumed to be 0.5 in.  $< 0.75$  in. The prying force on the bolts then is

$$Q = \left( \frac{3b}{8a} - \frac{t^3}{20} \right) T = \left[ \frac{3 \times 2.375}{8 \times 1.5} - \frac{(\frac{3}{4})^3}{20} \right] 5.2 = 3.0 \text{ kips per bolt}$$

The total tension on the bolts  $= 5.2 + 3.0 = 8.2$  kips per bolt. The tensile stress in each bolt therefore is

$$f_t = \frac{8.2}{0.601} = 13.6 \text{ ksi}$$

The shear stress in each bolt is

$$f_v = \frac{39.8}{6 \times 0.601} = 11.0 \text{ ksi}$$

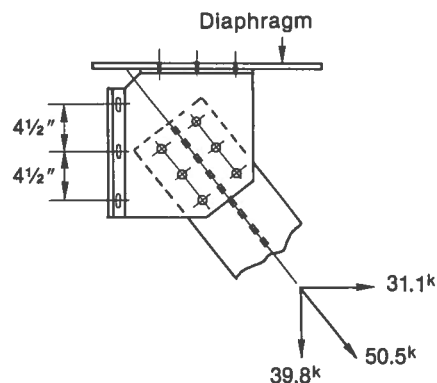
The allowable shear stress is

$$f_v = 1.33F_v \left( 1 - \frac{f_t}{159} \right) = 1.33 \times 16.0 \left( 1 - \frac{13.6}{159} \right) = 19.5 > 11.0 \text{ ksi}$$

The connection, therefore, is satisfactory.

A similar design is made for the lateral bracing connection at the ends of the girders, where the longitudinal shear is 39.8 kips (see the following drawing). The gusset plate is about 12 in. long along the connection to the girder web. For two  $\frac{1}{4}$ -in. fillet welds, the stress in a weld is

$$f_w = \frac{1.3 \times 39.8}{12 \times 2 \times 0.707 \times \frac{1}{4}} = 12.20 < 31.5 \text{ ksi}$$

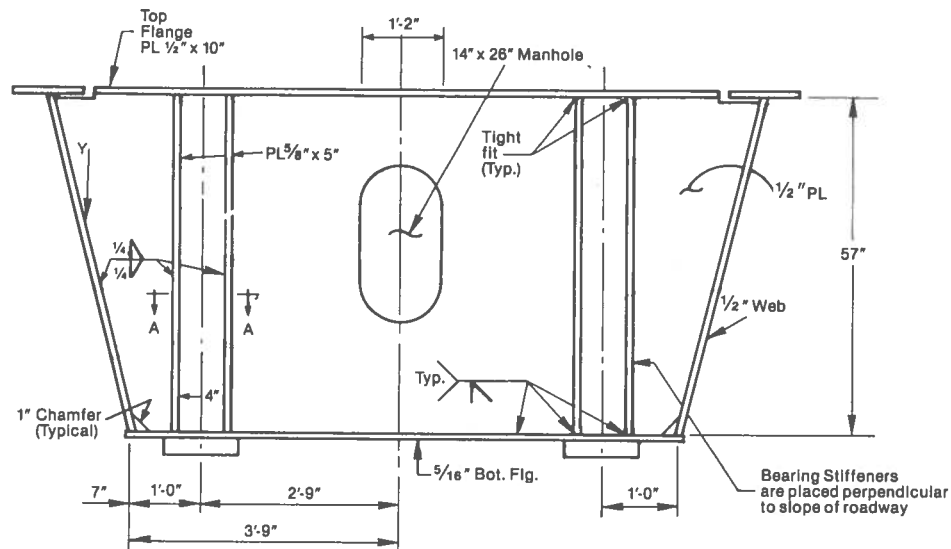


### BRACING CONNECTION WITH BOLTED GUSSET AT END OF GIRDER

By inspection, the connection shown is adequate.

### END DIAPHRAGM

The box girders are supported at the end bearings on two shoes, 5.5 ft. apart. These shoes need not be designed for uplift inasmuch as this condition cannot occur. A  $\frac{1}{2}$ -in.-thick plate diaphragm with a  $\frac{1}{2}\times 10$ -in. top flange is used to transfer the girder-web shear and the girder torque into the shoes. Investigation of the diagram begins with a tabulation of these shears and torques and a computation of St. Venant shear flow. As noted previously, an impact factor of 1.0 is used for live-load reactions.



**SECTION AT DIAPHRAGM AT END BEARING**

**Vertical Shear and Torque at End Bearing**

	$DL_1$	$DL_2$	$L+I$	Total
$V$ , kips	55.5	14.9	117.0	187.4
$T$ , kip-ft	181.1	52.6	355.0	588.7

The enclosed area to be used for the box girder in computing the shear due to  $DL_1$  torque is

$$A_1 = \frac{1}{2}(90 + 118)57 = 5,928 \text{ in.}^2$$

The enclosed area of the box girder to be used in computing the shear due to  $DL_2$  and  $L+I$  torque is

$$A_2 = \frac{1}{2}(90 + 118)63.75 = 6,630 \text{ in.}^2$$

**Shear Flow Due to Torque**

$$\text{For } DL_1: S = \frac{181.1 \times 12}{2 \times 5,928} = 0.183$$

$$\text{For } DL_2: S = \frac{52.6 \times 12}{2 \times 6,630} = 0.048$$

$$\text{For } L+I: S = \frac{355 \times 12}{2 \times 6,630} = 0.321$$

0.552 kips per in.

The shear force among the web, therefore, is

$$V = 0.552 \times 58.69 = 32.4 \text{ kips}$$

The shoe reactions are calculated by superimposing the torque reactions on the reactions due to vertical loads.

$$\text{Torque Reaction} = \frac{588.7}{5.5} = 107.0 \text{ kips}$$

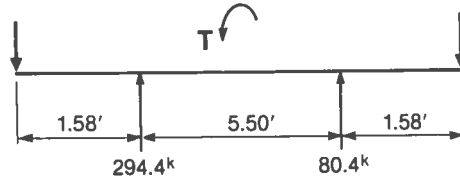


One shoe reaction then is

$$R_L = 187.4 + 107.0 = 294.4 \text{ kips Governs.}$$

and the other reaction is

$$R_R = 187.4 - 107.0 = 80.4 \text{ kips}$$



### SHOE REACTIONS AT END BEARING

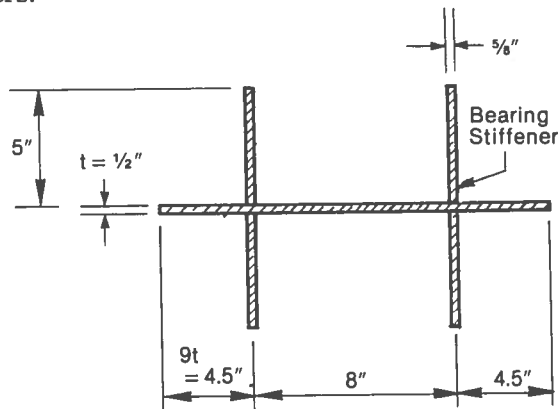
#### Design of Bearing Stiffeners at End Support

Bearing stiffeners are designed on the basis of the maximum reaction of 294.4 kips, with working-stress principles. The stiffeners are then checked as columns under ultimate-strength loads.

Try two 5-in.-wide stiffeners, welded on each side of the diaphragm web, over each shoe. The required stiffener thickness is computed with 29 ksi as the allowable stress under service load. Since there are four bearing stiffeners,

$$t = \frac{294.4}{29(5 - 0.375)4} = 0.549$$

Try  $5 \times \frac{5}{8}$ -in. stiffeners.



SECTION A-A  
BEARING STIFFENERS

#### End Reactions, kips

	$DL_1$	$DL_2$	$L+I$
Direct	55.5	14.9	117.0
Torque	32.9	9.6	64.5
Total	88.4	24.5	181.5

The equivalent column area of the diaphragm web and stiffeners is

$$A = \frac{1}{2}(2 \times 4.5 + 8) + 4 \times \frac{5}{8} \times 5 = 21.0 \text{ in.}^2$$

The moment of inertia of the section is

$$I = \frac{bd^3}{12} = \frac{0.625(10.5)^3}{12} = 120.6 \text{ in.}^4$$

The radius of gyration of the section is

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{120.6}{21.0}} = 2.40 \text{ in.}$$

and the slenderness ratio is

$$\frac{KL}{r} = \frac{D}{r} = \frac{57}{2.40} = 23.75$$

The maximum permissible slenderness ratio is

$$\frac{KL}{r} = \sqrt{\frac{2\pi^2 E}{F_y}} = \sqrt{\frac{2\pi^2 \times 29,000}{36}} = 126.1 > 23.75 \text{ O.K.}$$

Hence, the column critical stress is

$$F_{cr} = F_y \left[ 1 - \frac{F_y}{4\pi^2 E} \left( \frac{D}{r} \right)^2 \right] = 36 \left[ 1 - \frac{36}{4\pi^2 \times 29,000} (23.75)^2 \right] = 35.4 \text{ ksi}$$

The column load capacity then is

$$P_u = 0.85 \times 21.0 \times 35.4 = 631.9 \text{ kips}$$

The maximum design load is

$$V_u = 1.3 (88.4 + 24.5 + \frac{5}{3} \times 181.5) = 540 < 631.9 \text{ kips O.K.}$$

Use PL  $\frac{5}{8} \times 5$  in. as bearing stiffeners.

#### Check of End Diaphragm in Bending

Next, the end diaphragm is checked in bending, beginning with computation of section properties. A 10-in.-wide strip of the bottom flange of the girder is taken as the bottom flange of the diaphragm.

Section Properties of End Diaphragm

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Top Flg. $\frac{1}{2} \times 10$	5.00	28.75	143.8	4,133	7,716	4,133
Web $\frac{1}{2} \times 57$	28.50					7,716
Bot. Flg. $\frac{5}{16} \times 10$	3.12	28.66	-89.4	2,563		2,563

$$y = \frac{54.4}{36.62} = 1.49 \text{ in.}$$

$$I_{NA} = 14,331 \text{ in.}^4$$

$$d_{\text{Top}} = 57/2 + \frac{1}{2} - 1.49 = 27.51 \text{ in.}$$

$$d_{\text{Bot.}} = 57/2 + \frac{5}{16} - 1.49 = 30.30 \text{ in.}$$

$$S_{\text{Top}} = \frac{14,331}{27.51} = 521 \text{ in.}^3$$

$$S_{\text{Bot.}} = \frac{14,331}{30.30} = 473 \text{ in.}^3$$

Bending moments in the end diaphragm are calculated next, beginning with those due to torque and continuing with those due to vertical loads. As tabulated previously, the shear flow due to torque under  $DL_1$  is 0.183 kips per in. The shear along the top flange then is

$$V = 0.183 \times 118 = 21.6 \text{ kips}$$

The vertical component of the shear along one web is

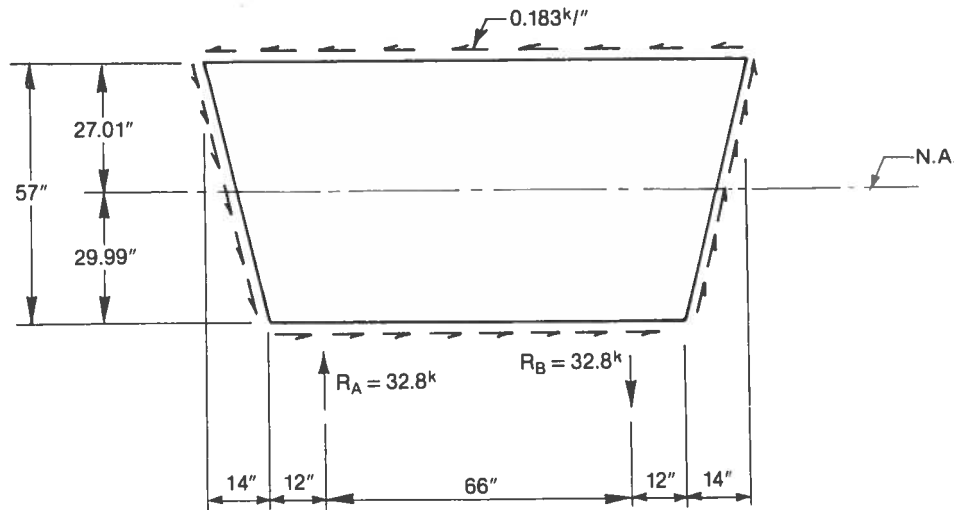
$$V = 0.183 \times 58.69 \times \frac{57}{58.69} = 10.4 \text{ kips}$$

and the horizontal component is

$$V = 0.183 \times 58.69 \times \frac{14}{58.69} = 2.6 \text{ kips}$$

The reactions at each shoe are computed by equating to zero the sum of the moments of the shears about each shoe. From moments about  $R_B$  (see following drawing):

$$R_A = \frac{21.6 \times 57 - 10.4 \times 104 - 2 \times 2.6 \times 28.50}{66} = 32.8 \text{ kips}$$



### SHEAR FLOW IN DIAPHRAGM AT END BEARING

The  $DL_1$  bending moment taken about a point on the neutral axis above the reactions, due to the shear on the projecting portions of the diaphragm is

$$M_A = -M_B = 0.183 \times 26 \times 27.01 + 10.4 \times 19 + 2.6 \times 1.49 + 0.183 \times 29.99 = 396 \text{ kip-in.}$$

Bending moments due to torque under  $DL_2$  and  $L+I$  are computed on the assumption, for simplicity, that shear flows due to  $DL_2$  and  $L+I$  act on the same perimeter as does the shear flow due to  $DL_1$ . The bending moments due to torque thus are proportional to the shear flows.

$$\text{For } DL_2: M_A = -M_B = 396 \times \frac{0.048}{0.183} = 104 \text{ kip-in.}$$

$$\text{For } L+I: M_A = -M_B = 396 \times \frac{0.321}{0.183} = 695 \text{ kip-in.}$$

### Bending in Diaphragm Due to Vertical Loads (Unfactored)

$$\text{For } DL_1: M_A = M_B = 55.5 \left( 19 + \frac{14}{57} \times 1.49 \right) = 55.5 \times 19.37 = 1,075 \text{ kip-in.}$$

$$\text{For } DL_2: M_A = M_B = 14.9 \times 19.37 = 288 \text{ kip-in.}$$

$$\text{For } L+I: M_A = M_B = 117.0 \times 19.37 = 2,266 \text{ kip-in.}$$

### Factored Bending Moments

$$\text{For } DL_1: M = 1.3(1,075 + 396) = 1,912$$

$$\text{For } DL_2: M = 1.3(288 + 104) = 510$$

$$\text{For } L + I: M = 1.3 \times \frac{5}{3} (2,266 + 695) = \frac{6,416}{8,838 \text{ kip-in.}}$$

### Factored Shears

$$\text{For } DL_1: V = 1.3 \left( 55.5 + 0.183 \times 58.69 \times \frac{57}{58.69} \right) = 86$$

$$\text{For } DL_2: V = 1.3 \left( 14.9 + 0.048 \times 58.69 \times \frac{57}{58.69} \right) = 23$$

$$\text{For } L + I: V = 1.3 \left( 117.0 + 0.321 \times 58.69 \times \frac{57}{58.69} \right) = \frac{293}{402 \text{ kips}}$$

The maximum allowable shear  $V_u$  kips, without stiffeners on the diaphragm, is

$$V_u = 101,500 \frac{t_w^3}{D} = 101,500 \frac{(\frac{1}{2})^3}{57} = 222.6 < 402 \text{ kips}$$

Because the design shear exceeds the allowable shear, stiffeners are required on the diaphragm. For the purpose, the effect of the bearing stiffeners at the shoes may be taken into account. Hence, the shear capacity of the diaphragm is calculated, with the distance from mid-depth of the girder web to the exterior bearing stiffener taken as the stiffener spacing  $d_o = 15$  in.

$$V_u = V_p \left[ C + \frac{0.87(1-C)}{\sqrt{1+(d_o/D)^2}} \right]$$

$$\text{where } V_p = 0.58 F_y D t_w = 0.58 \times 36 \times 57 \times \frac{1}{2} = 595 \text{ kips}$$

$$\begin{aligned} C &= 569.2 \frac{t_w}{D} \sqrt{\frac{1+(D/d_o)^2}{F_y}} - 0.3 \leq 1.0 \\ &= 569.2 \times \frac{1/2}{57} \sqrt{\frac{1+(57/15)^2}{36}} - 0.3 = 2.97 > 1.0 \end{aligned}$$

Use  $C = 1$  for calculating the ultimate shear strength:

$$V_u = 595(1.0 + 0) = 595 > 402 \text{ kips}$$

Hence, the section is adequate for shear capacity. A reduction in calculated bending strength is required, however, because the design shear exceeds  $0.6V_u$ .

$$0.6V_u = 0.6 \times 595 = 357 < 402 \text{ kips}$$

The thickness of the diaphragm web meets the requirement  $D/t_w \leq 150$ .

$$\frac{D}{t_w} = \frac{57}{1/2} = 114 < 150$$

Since the requirement for web thickness is satisfied and the compression flange of the diaphragm can be considered supported over its full length, the diaphragm meets requirements for a braced, noncompact section. Its bending strength independent of shear, therefore, is

$$M_u = F_y S_{Bot.} = 36 \times 473 = 17,030 > 8,838 \text{ kip-in.}$$

The moment reduction required is computed from

$$\frac{M}{M_u} = 1.375 - 0.625 \frac{V}{V_u}$$

Hence, the allowable moment is

$$M = 17,030 \left( 1.375 - 0.625 \times \frac{402}{595} \right) = 16,220 > 8,838 \text{ kip-in.}$$

Therefore, the 1/2-in. diaphragm meets bending requirements.

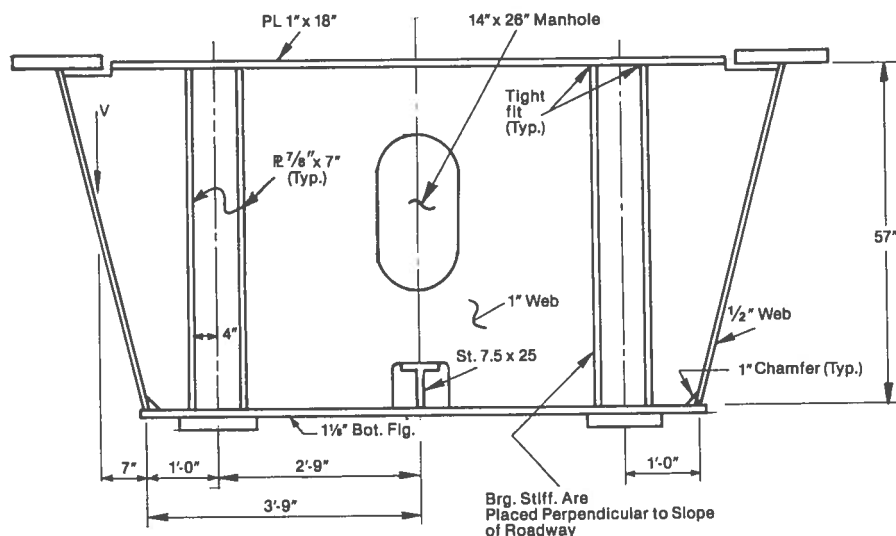
#### Expansion Dam at End Support

The support brackets and beam developed for the straight girder in the example of Chapter 7 are adequate for the expansion dams at the end bearings.

#### DIAPHRAGM AT INTERIOR SUPPORT

The diaphragm over the pier is similar to the end diaphragm and is shown in the following drawing. Computations for the bearing stiffeners and treatment of the manhole for the pier diaphragm are not given inasmuch as they are similar to those for the diaphragm at the end bearing.

The following computations indicate that a thicker bottom flange than that used for the adjoining portion of the box girder is desirable. This change is necessary because of the combination of transverse and longitudinal bending in the diaphragm.



#### SECTION AT DIAPHRAGM AT PIER

The following table lists design loads at the interior support with an impact factor of 1. Torque reactions are calculated by dividing the torque by 5.5 ft, the distance between shoes.

Design Loads at Interior Support

	Shear per Web, Kips	Torque on Section, Kip-Ft	Torque Reaction, Kips
$DL_1$	213.2	265.2	48.2
$DL_2$	54.2	44.8	8.1
$L+I$	230.0	526.0	95.6
Total	497.4	836.0	151.9

As for the end diaphragm, the enclosed area to be used in computing the shear due to  $DL_1$  torque is 5,928 in.<sup>2</sup> and the enclosed area for  $DL_2$  and  $L+I$  is 6,630 in.<sup>2</sup>

#### Shear Flow Due to Torque

$$\text{For } DL_1: S = \frac{265.2 \times 12}{2 \times 5,928} = 0.268$$

$$\text{For } DL_2: S = \frac{44.8 \times 12}{2 \times 6,630} = 0.041$$

$$\text{For } L+I: S = \frac{526 \times 12}{2 \times 6,630} = 0.476$$

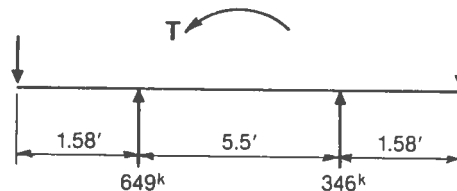
0.785 kips per in.

The shoe reactions equal the sum of the torque reaction and the reaction due to vertical loads. One shoe reaction then is

$$R_L = 497.4 + 151.9 = 649 \text{ kips}$$

and the other reaction is (see following drawing)

$$R_R = 497.4 - 151.9 = 346 \text{ kips}$$



#### SHOE REACTIONS AT PIER

#### Check of Pier Diaphragm

Next, the pier diaphragm is checked for bending in its plane, beginning with computation of section properties. The diaphragm section has flanges of A572, Grade 50, steel, and a web of A36 steel. Allowable stresses must be reduced because this section is hybrid.

#### Section Properties of Diaphragm at Pier

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Top Flg. 1×18	18.00	29.00	522	15,138	15,433	15,138
Web 1×57	57.00					15,433
Bot. Flg. 1½×26	29.25	-29.06	-850	24,703		24,703

$$\bar{y} = \frac{-328}{104.25} = -3.146 \text{ in.}$$

$$I_{NA} = 54,242 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.5 + 3.146 = 32.65 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.625 - 3.146 = 26.479 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{54,242}{32.65} = 1,662 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{54,242}{26.479} = 2,048 \text{ in.}^3$$

Then, bending moments in the pier diaphragm are calculated, beginning with those due to torque and continuing with those due to vertical loads. As tabulated previously, the shear flow due to torque under  $DL_1$  is 0.268 kips per in. The shear along the top flange then is

$$V = 0.268 \times 118 = 31.6 \text{ kips}$$

The vertical component of the shear along one web is

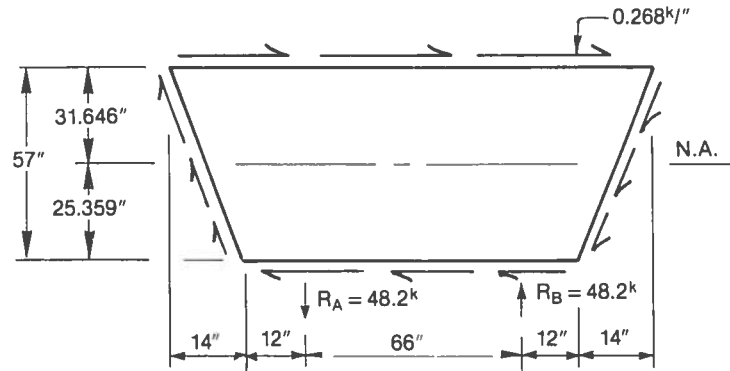
$$V = 0.268 \times 58.69 \times \frac{57}{58.69} = 15.3 \text{ kips}$$

and the horizontal component is

$$V = 0.268 \times 58.69 \times \frac{14}{58.69} = 3.75 \text{ kips}$$

The reactions at each shoe are computed by equating to zero the sum of the moments of the shear about each shoe. From moments about  $R_A$  (see following drawing):

$$R_B = \frac{31.6 \times 57 + 15.3 \times 104 - 2 \times 3.75 \times 28.50}{66} = 48.2 \text{ kips}$$



**SHEAR FLOW IN DIAPHRAGM AT PIER**

The  $DL_1$  bending moment in the plane of the diaphragm, taken about a point on the neutral axis above the reactions, due to the shears on the projecting portions of the diaphragm is  $M_B = -M_A = 0.268 \times 26 \times 31.646 + 15.3 \times 19 - 3.75 \times 3.146 + 0.268 \times 12 \times 25.354 = 581 \text{ kip-in.}$  As for the end diaphragm, moments due to torque for  $DL_2$  and  $L+I$  are taken proportional to shear flows.

$$\text{For } DL_2: M_B = -M_A = 581 \times \frac{0.041}{0.268} = 89 \text{ kip-in.}$$

$$\text{For } L+I: M_B = -M_A = 581 \times \frac{0.476}{0.268} = 1,032 \text{ kip-in.}$$

#### **Bending in Pier Diaphragm Due to Vertical Loads (Unfactored)**

$$\text{For } DL_1: M_B = M_A = 213.2 \times 19 = 4,051 \text{ kip-in.}$$

$$\text{For } DL_2: M_B = M_A = 54.2 \times 19 = 1,030 \text{ kip-in.}$$

$$\text{For } L+I: M_B = M_A = 230.0 \times 19 = 4,370 \text{ kip-in.}$$

#### **Factored Bending Moments**

$$\text{For } DL_1: M = 1.3 (4,051 + 581) = 6,606$$

$$\text{For } DL_2: M = 1.3 (1,030 + 89) = 1,455$$

$$\text{For } L+I: M = 1.3 \times \frac{5}{3} (4,370 + 1,032) = \frac{11,704}{19,765} \text{ kip-in.}$$

### Factored Shears

$$\text{For } DL_1: V = 1.3 \left( 213.2 + 0.268 \times 58.69 \times \frac{57}{58.69} \right) = 297.0$$

$$\text{For } DL_2: V = 1.3 \left( 54.2 + 0.041 \times 58.69 \times \frac{57}{58.59} \right) = 73.5$$

$$\text{For } L+I: V = 1.3 \times \frac{2}{3} \left( 230.0 + 0.476 \times 58.69 \times \frac{57}{58.69} \right) = \frac{557.1}{927.6} \text{ kips}$$

The maximum shear capacity  $V_u$ , kips, without stiffeners on the diaphragm web, is the smaller of the following:

$$V_u = 101,500 \frac{t_w^3}{D} = 101,500 \frac{(1)^3}{57} = 1,781 > 927.6 \text{ kips}$$

$$V_u = 0.58 F_y D t_w = 0.58 \times 36 \times 57 \times 1 = 1,190 > 927.6 \text{ kips. Governs.}$$

Inasmuch as the shear capacity exceeds the design shear, stiffeners are not required. Bearing stiffeners, however, are placed over the shoes, as in the case of the end diaphragm.

A reduction in the computed bending strength is required because the design shear exceeds  $0.6 V_u$ .

$$0.6 V_u = 0.6 \times 1,190 = 714 < 927.6 \text{ kips}$$

The moment reduction required is computed from

$$\frac{M}{M_u} = 1.375 - 0.625 \frac{V}{V_u}$$

where  $M_u$  = computed bending strength, kip-independent of shear. Hence the allowable moment is

$$M = \left( 1.375 - 0.625 \times \frac{927.6}{1,190} \right) M_u = 0.888 M_u$$

and the reduced allowable bending stress is given by

$$F_b = 0.888 F_y R$$

where  $R$  is the reduction factor for a hybrid section. For computation of  $R$ , the following parameters are computed:

$$\rho = \frac{F_w}{F_f} = \frac{36}{50} = 0.72$$

$$\beta = \frac{A_w}{A_f} = \frac{57}{57} = 3.167$$

$$\psi = \frac{d_{\text{top}}}{D} = \frac{32.65}{57} = 0.573$$

Substitution of these parameters yields

$$\begin{aligned} R &= 1 - \frac{\beta \psi (1 - \rho)^2 (3 - \psi + \rho)}{6 + \beta \psi (3 - \psi)} \\ &= 1 - \frac{3.167 \times 0.573 (1 - 0.72)^2 (3 - 0.573 + 0.72 \times 0.573)}{6 + 3.167 \times 0.573 (3 - 0.573)} = 0.961 \end{aligned}$$

For tension and compression, the allowable stress for bending in the plane of the diaphragm then is

$$F_b = 0.888 \times 50 \times 0.961 = 42.7 \text{ ksi}$$



### Bending Stresses in Plane of Diaphragm

For the factored bending moment of 19,765 kip-in., the stress in the tension flange is

$$f_t = \frac{19,765}{1,662} = 11.9 < 42.7 \quad \text{O.K.}$$

and in the compression flange,

$$f_c = \frac{19,765}{2,048} = 9.7 < 42.7 \text{ ksi O.K.}$$

### Bending Normal to Plane of Diaphragm

The preceding stresses must be combined with those, at the interior support, from longitudinal bending of the box girder. Section properties of the girder are computed for a 1-in.-thick, A572, Grade 50, bottom flange, which is thicker than the  $\frac{7}{8}$ -in. plate used for the adjoining section of box girder. The thicker flange extends for a distance of 2 ft on both sides of the interior support.

Box-Girder Steel Section at Pier

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
2 T. Flg. Pl. 2×15	60.00	29.50	1,770	52,215		52,215
2 Web Pl. $\frac{1}{2}$ ×58.69	58.69				15,891	15,891
Bot. Flg. Pl. $1\frac{1}{8}$ ×92	103.50	-29.06	-3,008	87,404	11	87,415
Stiff. ST 7.5×25	7.35	-23.25	-171	3,973	41	4,014

$$d_s = \frac{-1,409}{229.54} = -6.14 \text{ in.} \quad \begin{array}{l} 229.54 \text{ in.}^2 \\ -1,409 \text{ in.}^3 \\ 159,535 \\ -6.14 \times 1,409 = -8,651 \\ I_{NA} = 150,884 \text{ in.}^4 \end{array}$$

$$d_{\text{Top of steel}} = 30.50 + 6.14 = 36.64 \text{ in.} \quad d_{\text{Bot. of steel}} = 29.62 - 6.14 = 23.48 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{150,884}{36.64} = 4,118 \text{ in.}^3 \quad S_{\text{Bot. of steel}} = \frac{150,884}{23.48} = 6,426 \text{ in.}^3$$

Steel Section, with Reinforcing Steel, at Pier

Material	A	d	Ad	Ad <sup>2</sup>	I <sub>o</sub>	I
Steel Section	229.54		-1,409			159,535
Reinforcement	15.19	35.13	534	18,746		18,746

$$d_c = \frac{-875}{244.73} = -3.58 \text{ in.} \quad \begin{array}{l} 244.73 \text{ in.}^2 \\ -875 \text{ in.}^3 \\ 178,281 \\ -3.58 \times 875 = -3,128 \\ I_{NA} = 175,153 \text{ in.}^4 \end{array}$$

$$d_{\text{Top of steel}} = 30.50 + 3.58 = 34.08 \text{ in.} \quad d_{\text{Bot. of steel}} = 29.62 - 3.58 = 26.04 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{175,153}{34.08} = 5,139 \text{ in.}^3 \quad S_{\text{Bot. of steel}} = \frac{175,153}{26.04} = 6,724 \text{ in.}^3$$

$$d_{\text{Reinf.}} = 35.13 + 3.58 = 38.71 \text{ in.}$$

$$S_{\text{Reinf.}} = \frac{175,153}{38.71} = 4,525 \text{ in.}^3$$

Bending stresses are computed for full design load, with an impact factor of 0.35, with moments obtained from the curves of maximum moment.

### Factored Moments at Pier

	$DL_1$	$DL_2$	$-(L+I)$
$M$ , kip-ft	-6,296	-1,502	-3,838

### Stresses Due to bending Normal to Plane of Diaphragm

Top of Steel (Tension)	Bottom of Steel (Compression)
For $DL_1$ : $F_b = \frac{6,296 \times 12}{4,118} \times 1.30 = 23.8$	$F_b = \frac{6,296 \times 12}{6,426} \times 1.30 = 15.3$
For $DL_2$ : $F_b = \frac{1,502 \times 12}{5,139} \times 1.30 = 4.6$	$F_b = \frac{1,502 \times 12}{6,724} \times 1.30 = 3.5$
For $L+I$ : $F_b = \frac{3,838 \times 12}{5,139} \times 1.30 \times \frac{5}{3} = 19.4$ 47.8 ksi	$F_b = \frac{3,838 \times 12}{6,724} \times 1.30 \times \frac{5}{3} = 14.8$ 33.6 ksi

### Reinforcing Steel Stress (Tension) at Pier

$$f_r = \frac{1.3 \times 12 \left( 1,502 + \frac{5}{3} \times 3,838 \right)}{4,525} = 27.3 < 40 \text{ ksi O.K.}$$

The tension stress in the top flange of the box girder at the pier may not exceed

$$F_{bs} = F_y \left[ 1 - 3 \left( \frac{F_y}{E \pi^2} \right) \left( \frac{l}{b} \right)^2 \right]$$

$$= 50 \left[ 1 - 3 \times \frac{50}{29,000 \pi^2} \left( \frac{12.31 \times 12}{15} \right)^2 \right] = 47.5 \text{ ksi}$$

The design stress of 47.8 ksi in the top flange is close enough to the allowable stress that the flange is considered adequate. Stresses in the top flange for bending in the plane of the diaphragm and bending normal to that plane, in the longitudinal direction of the box girder, need not be combined, because these stresses occur in different plates.

For computation of the allowable bending stress, normal to the plane of the diaphragm, in the bottom flange, the St. Venant shear stress at the pier section must first be calculated. From the shear flows tabulated previously, the shear stress in the 1 1/8-in. flange due to torque is

$$f_v = \frac{1.3}{1.125} \left( 0.268 + 0.041 + \frac{5}{3} \times 0.476 \right) = 1.27 \text{ ksi}$$

The shear stress is so small that, for calculation of the allowable compression stress in the flange, the parameter  $\Delta$  may be taken as unity. The other parameters are computed as follows: With  $I_s$  for the ST7.5×25 stiffener equal to 243.2 in.<sup>4</sup>,

$$K = \sqrt[3]{\frac{I_s}{0.125 t^3 b}} = \sqrt[3]{\frac{243.2}{0.125 (1.125)^3 45}} = 3.12$$

$$K_s = \frac{5.34 + 2.84 (I_s / b t^3)^{1/3}}{(n+1)^2} = 2.443$$

With the use of the preceding results,

$$R_1 = \frac{97.08 \sqrt{K}}{\sqrt{\frac{1}{2} \left[ \Delta + \sqrt{\Delta^2 + 4 (f_v / F_y)^2 (K / K_s)^2} \right]}} = 171.4$$

$$R_2 = \frac{210.3\sqrt{K}}{\sqrt{\frac{1}{1.2} \left[ \Delta - 0.4 + \sqrt{(\Delta - 0.4)^2 + 4(f_v/F_y)^2 (K/K_s)^2} \right]}} = 370.9$$

Because  $w\sqrt{F_y}/t = 45\sqrt{50}/1.125 = 282.8$  falls between  $R_1$  and  $R_2$ , the allowable compression stress in the bottom flange, in the direction normal to the plane of the diaphragm, is

$$\begin{aligned} F_b &= F_y \left[ \Delta - 0.4 \left( 1 - \sin \frac{\pi}{2} \frac{R_2 - w\sqrt{F_y}t}{R_2 - R_1} \right) \right] \\ &= 50 \left[ 1 - 0.4 \left( 1 - \sin \frac{\pi}{2} \frac{370.9 - 282.8}{370.9 - 171.4} \right) \right] = 42.8 > 33.6 \text{ ksi} \end{aligned}$$

Hence, the bottom flange is satisfactory for bending in the direction normal to the plane of the web as well as for bending in the plane of the web. The flange, however, must also be investigated for the combined bending stresses.

#### Combined Bending Stresses

An interaction equation  $f_{bx}/F_{bx} + f_{by}/F_{by} \leq 1$  is used to determine the adequacy of the bottom flange for the combined bending stresses parallel and normal to the plane of the diaphragm.

$$\frac{33.6}{42.8} + \frac{9.7}{42.7} = 0.785 + 0.227 = 1.01$$

This is close enough to unity that the 1 $\frac{1}{8}$ -in. flange plate is considered satisfactory.

#### BOLTED FIELD SPLICE

For Load-Factor design of a bolted field splice, AASHTO specifications require that the splice material be proportioned for the Maximum Design Load and resistance to fatigue under Service Loads. Because friction connections must resist slip Overload, fastener size must be selected for an allowable stress of  $1.33F_v$  under the overload of  $D = \frac{5}{3}(L+I)$ , where  $F_v$  is the allowable shear stress as given in AASHTO Table 1.7.41C1.

Anticipating that curvature will require a heavier splice than that used for the straight bridge of Chapter 7 and attempting to maintain reasonably compact bolt patterns and plate sizes, we select  $\frac{7}{8}$ -in.-dia, A490 fasteners. The allowable load in double shear is

$$P = 2 \times 0.6013 \times 1.33 \times 20 = 32.0 \text{ kips per bolt}$$

For design of the splice material for the Maximum Design Load, the design moment is chosen as the greater of:

Average of the calculated moment on the section and maximum capacity of the section.

75% of the maximum capacity of the section.

The calculated moment is that induced by the Maximum Design Load  $1.3[D + \frac{5}{3}(L+I)]$ . Splice material should have a capacity equal at least to the design moment. The section capacity is based on the gross section minus any loss in flange area due to bolt holes with area exceeding 15% of each flange area.

**Bending Moments 38 Ft from Interior Support, Kip-Ft**

	For Service Loads	Factor	For Overload	Factor	Maximum Design Loads
$DL_1$	10	1	10	1.30	13
$DL_2$	100	1	100	1.30	130
$+LL$	1,595	$\frac{5}{3}$	2,658	1.30	3,455
$-LL$	-1,000				

**Shears 38 Ft from Interior Support**

	For Service Loads	Factor	For Overload	Factor	Maximum Design Loads
$DL_1$	-54.9	1	-54.9	1.30	-71.4
$DL_2$	-14.0	1	-14.0	1.30	-18.2
$LL$	-38.6	$\frac{5}{3}$	-64.3	1.30	-83.6

**Torques 38 Ft from Interior Support, Kip-Ft**

	For Service Loads	Factor	For Overload	Factor	Maximum Design Loads
$DL_1$	-134.9	1	-134.9	1.30	-175.4
$DL_2$	- 37.0	1	- 37.0	1.30	- 48.1
$LL$	-113.4	$\frac{5}{3}$	-189.0	1.30	-245.7

The section at the splice is subject to the following moments:

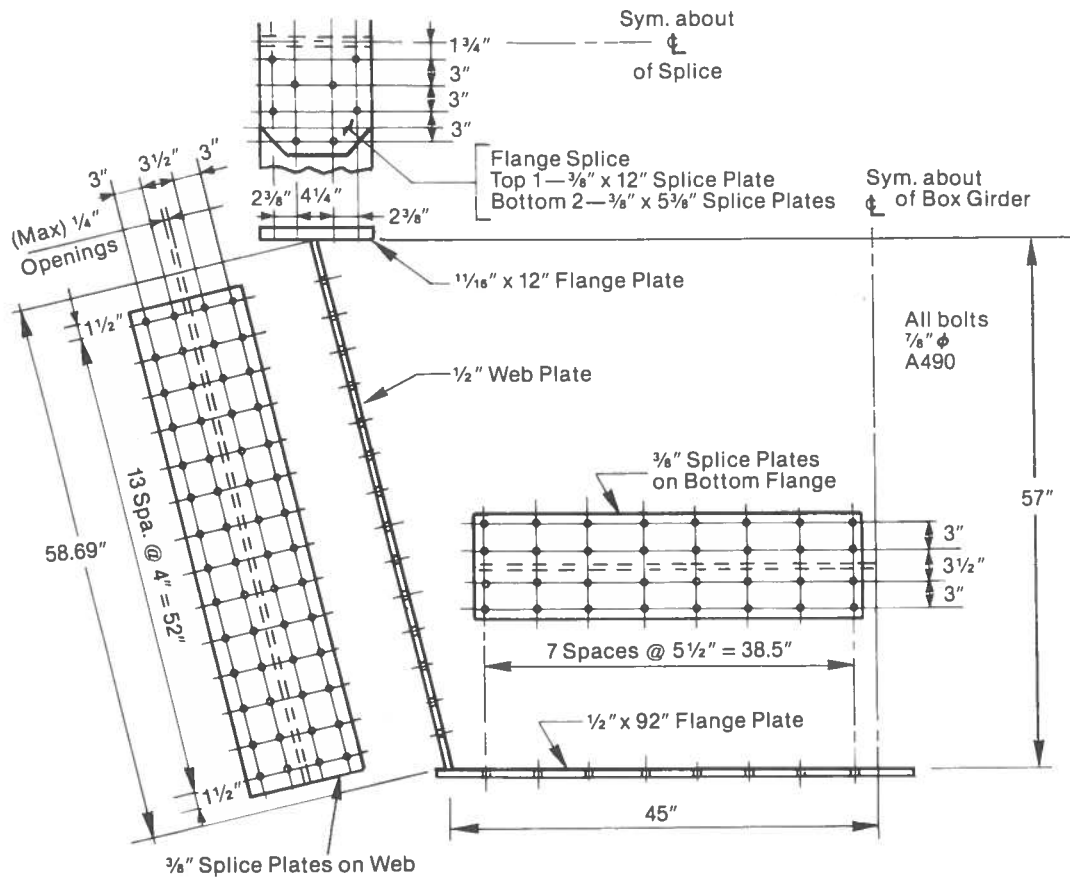
Negative moment that acts only on the steel section.

Positive moment that acts on the composite steel-concrete section.

Negative moment resisted by the steel section and the concrete reinforcement.

Because the effects of positive moment dominate at the splice, splice material is designed for positive moment. Also, to simplify the design procedure, the composite concrete slab is neglected.

Net section properties at the splice are those for the smaller section, on the positive-moment side of the splice.

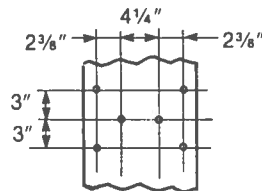


### BOLTED FIELD SPLICE

#### Net Section at Top-Flange Splice

The splice of each top flange is made with  $\frac{7}{8}$ -in.-dia, A490 bolts, arranged staggered in four rows. Pitch of the bolts longitudinally is  $s=3$  in. Gage  $g=2\frac{3}{8}$  in.

$$\frac{s^2}{4g} = \frac{(3)^2}{4 \times 2.375} = 0.947$$



#### BOLT HOLES IN TOP FLANGE

The deduction from the flange width at the section across the flange through two holes equals  $2 \times 1 = 2.00$  in.

The deduction from the flange width at a section through a chain of four holes equals  $4 \times 1 - 2 \times 0.947 = 2.106 > 2.00$  in. Use 2.106 in. for the deduction in computing the net flange area.

### Flange Area and Deductions

$$\text{Gross Area} = 1\frac{1}{16} \times 12 = 8.25 \text{ in.}^2$$

$$\text{Area deducted for bolt holes} = 1\frac{1}{16} \times 2.106 = 1.45$$

$$-15\% \text{ of gross area} = -0.15 \times 8.25 = -1.24$$

$$\text{Net deduction for two flanges} = 0.21 \times 2 = 0.42 \text{ in.}^2$$

### Net Section at Bottom Flange and Stiffener Splices

Assume that the center of gravity of the stiffener coincides with the center of gravity of the bolt holes. Deduct the following areas: 16 holes in the bottom-flange plate, two holes from the flange of the stiffener and two holes from the stiffener stem.

### Flange Area and Deductions

$$\text{Gross area of bottom flange and stiffener} = \frac{1}{2} \times 92 + 7.35 = 53.35 \text{ in.}^2$$

$$\text{Area deducted for bolt holes} = \frac{1}{2} \times 16 + 2 \times 0.622 + 2 \times 0.55 = 10.34$$

$$-15\% \text{ of gross area} = -0.15 \times 53.35 = -8.00 \text{ in.}^2$$

$$\text{Net deduction for bottom flange and stiffener} = 2.34 \text{ in.}^2$$

Properties of the gross cross section of the box girder are obtained from previous calculations for the maximum-positive-moment section. The bolt holes in the flanges are deducted in the computation of properties of the net section, and the ST 7.5×25 properties are added.

### Net Section at the Splice—Steel Section Only

Material	A	d	Ad	AD <sup>2</sup>	I <sub>o</sub>	I
Pos. Mom. Gross Section	121.19		-938			67,641
Top Flg. Bolt Holes	-0.42	28.84	-12	-349		-349
Bot. Flg. Bolt Holes	-2.34	28.75	67	-1,934		-1,934
ST 7.5×25	7.35	-23.25	-171	3,973	41	4,014

$$125.78 \text{ in.}^2$$

$$-963 \text{ in.}^3$$

$$69,372$$

$$d_s = \frac{-963}{125.78} = -7.66 \text{ in.}$$

$$-7.66 \times 963 = -7,377$$

$$I_{NA} = 61,995 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 29.19 + 7.66 = 36.85 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 29.00 - 7.66 = 21.34 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{61,995}{36.85} = 1,682 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{61,995}{21.34} = 2,905 \text{ in.}^3$$

### Design Moments and Shears at the Field Splice

The capacity of the net section is based on the minimum section modulus of the steel section and the allowable stress for the corresponding flange. From above, the lower section modulus is 1,682 in.<sup>3</sup>—for the top flange. Because the effects of positive bending dominate, the allowable stress will be the allowable compressive stress for the top flange. This has previously been calculated for the maximum positive moment section as 36.0 ksi on page 37 in this text.

For  $F_b = 36.0$  ksi, the net section capacity is

$$M_{net} = \frac{36.0 \times 1,682}{12} = 5,046 \text{ kip-ft}$$

$$75\% M_{net} = 0.75 \times 5,046 = 3,785 \text{ kip-ft}$$

With an impact factor of 0.35, the calculated moment is

$$M_{calc} = 13 + 130 + 1.35 \times 3,455 = 4,807 \text{ kip-ft}$$

The average of the calculated moment and the net capacity of the section is

$$M_{av} = \frac{4,807 + 5,046}{2} = 4,920 > 3,785 \text{ kip-ft}$$

The design moment, therefore, is 4,920 kip-ft.

The design vertical shear is determined by multiplying the calculated vertical shear for the design loads by the ratio of the design moment to the calculated moment on the section. With an impact factor of 0.50,

$$V_{calc} = 71.4 + 182 + 1.50 \times 83.6 = 215.0 \text{ kips}$$

Hence, the design vertical shear is

$$V_v = 215.0 \times \frac{4,920}{4,807} = 220 \text{ kips}$$

In the plane of each web,

$$V_v = \frac{220}{2} \times \frac{58.69}{57} = 113 \text{ kips}$$

The design torque similarly is obtained by multiplying the calculated torque by the ratio of the design moment to the calculated moment. The design torque is resolved into a torque acting on the noncomposite section and a torque acting on the composite section.

#### Design Torque, Kip-Ft

$$\text{For } DL_1: T = 175.4 \times \frac{4,920}{4,807} = 179.5$$

$$\text{For } DL_2: T = 48.1 \times \frac{4,920}{4,807} = 49.2$$

$$\text{For } L+I: T = 245.7 \times 1.50 \times \frac{4,920}{4,807} = 377.2$$

605.9 ft-kips

The design torsional shear on each web then is

$$V_T = \frac{12 \times 57}{2} \left( \frac{179.5}{5,230} \times \frac{49.2}{6,735} \times \frac{377.2}{6,735} \right) = 33 \text{ kips}$$

and in the plane of each web,

$$V_T = 33 \times \frac{58.69}{57} = 34 \text{ kips}$$

The Maximum Design Vertical and Torsional Shear then is

$$V = V_v + V_T = 113 + 34 = 147 \text{ kips}$$

### Web Splice

The web splice plates must carry the design vertical shear, design torsional shear, moments due to the eccentricities of the shears and a portion  $M_w$  of the design moment on the section. The portion of the design moment to be resisted by the web is obtained by multiplying the design moment by the ratio of the moment of inertia of the web to the net moment of inertia of the entire section. The gross moment of inertia is obtained from the earlier calculation of section properties and adjusted for the change in position of the centroidal axis because of deductions for bolt holes in the flanges.

$$I_w = 15,891 \geq 58.69(7.66)^2 = 19,335 \text{ in.}^4$$

### Web Moments for Design Loads

The bending moment due to eccentricity of the vertical shear is

$$M_{vv} = \frac{220 \times 3.25}{12} = 60 \text{ kip-ft.}$$

The moment due to eccentricity of the torsional shear is

$$M_{vT} = \frac{33 \times 2 \times 3.25}{12} = 18 \text{ kip-ft}$$

The portion of the design moment resisted by the webs is

$$M_{ww} = 4,920 \times \frac{19,335}{61,995} = 1,534 \text{ kip-ft}$$

The total web moment then is

$$M_w = 60 + 18 + 1,534 = 1,612 \text{ kip-ft, or } 806 \text{ kip-ft per web}$$

Try two  $\frac{3}{8} \times 55$ -in. web splice plates. Assume two columns of  $\frac{7}{8}$ -in.-dia, A490 bolts, with 14 bolts per column, on each side of the joint. The area of one hole is 0.375 in.<sup>2</sup> The holes remove from each splice plate the following percentage of its cross-sectional area:

$$\% \text{ of plate} = \frac{14 \times 0.375}{0.375 \times 55} \times 100 = 25.5\%$$

Consequently, the fraction of the hole area that must be deducted in determination of the net section is

$$\frac{25.5 - 15.0}{25.5} = 0.41$$

With 4-in. spacing of bolts along the slope of the web,

$$d^2 \text{ for holes} = 2^2 + 6^2 + 10^2 + 14^2 + 18^2 + 22^2 + 26^2 = 1,820$$

$$\Sigma Ad^2 = 4 \times 0.41 \times \frac{3}{8} \times 1,820 = 1,119 \text{ in.}^4$$

or, with respect to a horizontal axis

$$\Sigma Ad^2 = 1,119 \left( \frac{57}{58.69} \right)^2 = 1,055 \text{ in.}^4$$

Assume that the neutral axis of the splice coincides with the neutral axis of the net section of the box girders. The bending properties of the web splice plates with respect to a horizontal axis are then computed as follows:

$$\text{The area of two bolts holes to be deducted equals } 2 \times 4 \times 0.375 \times 0.41 = 4.31 \text{ in.}^2$$



### Web-Splice Section

Material	$A$	$d$	$Ad^2$	$I_o$	$I$
2 Splice Pl. $\frac{3}{8} \times 55$	41.25	7.66	2,420	9,807	12,227
Area of Holes	-4.31	7.66	-253	-1,055	-1,308

$$10,919 \text{ in.}^4$$

$$d_{\text{Top of splice}} = 27.50 + 7.66 = 35.16 \text{ in.}$$

$$d_{\text{Bot. of splice}} = 27.50 - 7.66 = 19.84 \text{ in.}$$

$$S_{\text{Top of splice}} = \frac{10,919}{3,516} = 311 \text{ in.}^3$$

$$S_{\text{Bot. of splice}} = \frac{10,919}{19.84} = 550 \text{ in.}^3$$

The maximum bending stress in the plates for the Maximum Design Load therefore is

$$f_b = \frac{806 \times 12}{311} = 31.1 < 36 \text{ ksi}$$

The plates are satisfactory for bending. The allowable shear stress is

$$F_v = 0.58F_y = 0.58 \times 36 = 20.9 \text{ ksi}$$

The shear stress for the Maximum Design Vertical and Torsional Shear is

$$f_v = \frac{147}{41.25} = 3.56 < 20.9 \text{ ksi}$$

The  $\frac{3}{8} \times 55$ -in. web splice plates are satisfactory for Maximum Design Load requirement. The plates are next checked for fatigue under service loads.

The range of moment on the section is

$$M_r = 1.35 \times 1,595 + 135 \times 1,000 = 2,153 + 1,350$$

The range of moment carried by the web equals

$$M_w = (2,153 + 1,350) \frac{19,335}{61,995} = 1,093, \text{ or } \frac{1,093}{2} = 547 \text{ kip-ft per web}$$

The maximum bending-stress range in the gross section of the web splice plate then is

$$f_{br} = \frac{547 \times 12 \times 35.16}{12,227} = 18.9 \text{ ksi}$$

### Check for Fatigue

Fatigue in base metal adjacent to friction-type fasteners is classified by AASHTO as Category B. For 500,000 cycles of truck loading, the associated allowable stress range is 27.5 ksi. The splice plates therefore are satisfactory for fatigue.

Use two  $\frac{3}{8} \times 55$ -in. web splice plates.

### Web Bolts

The 28 bolts in the web splice must carry the vertical and torsional shears, the moment due to the eccentricities of these shears about the centroid of the bolt group, and the portion of the beam moment taken by the web. These forces are induced by the Overload  $D + 5/3(L + I)$ . The allowable load in double shear was previously computed to be  $P = 32.0$  kips per bolt.

The polar moment of inertia of the bolt group about the assumed location of the neutral axis is

$$I = 2 \times 2 \times 1,820 + 28 \left( 7.66 \times \frac{58.69}{57} \right)^2 + 28(1.5)^2 = 9.085 \text{ in.}^4$$

### Web Shears for Overload

From the previous tabulation of shears for the section 38 ft from the interior support, the vertical shear per web for Overload is, with an impact factor of 0.50,

$$V_v = \frac{1}{2}(54.9 + 14.0 + 1.5 \times 64.3) = 82.7 \text{ kips}$$

Also, at the section 38 ft from the interior support, the torsional shear for Overload is

$$V_T = \frac{12 \times 57}{2} \left( \frac{134.9}{5,230} + \frac{37.0}{6,735} + 1.50 \times \frac{189.0}{6,735} \right) = 25.1 \text{ kips}$$

The Overload Vertical and Torsional Shear then is

$$V_u = V_v + V_T = 82.7 + 25.1 = 107.8 \text{ kips}$$

### Web Moments for Overload

The bending moment due to eccentricity of the vertical shear is

$$M_{vv} = \frac{82.7 \times 3.25}{12} = 22 \text{ kip-ft}$$

The moment due to eccentricity of the torsional shear is

$$M_{vT} = \frac{25.1 \times 3.25}{12} = 7 \text{ kip-ft}$$

The direct bending moment at the section 38 ft from the interior support is, with an impact factor of 0.35,

$$M = 10 + 100 + 1.35 \times 2,658 = 3,698 \text{ kip-ft}$$

The portion of this moment to be resisted by each web is

$$M_w = \frac{1}{2} \times 3,698 \times \frac{19,335}{61,995} = 577 \text{ kip-ft}$$

The total moment due to Overload then is

$$M_w = 577 + 22 + 7 = 606 \text{ kip-ft per web}$$

Load per bolt due to shear is

$$P_s = \frac{107.8}{28} \times \frac{58.69}{57} = 3.96 \text{ kips}$$

Load on the outermost bolt due to moment is

$$\text{Vertical in-plane component} = \frac{606 \times 12 \times 1.5}{9,085} \times \frac{58.69}{57} = 1.24 \text{ kips}$$

$$\text{Horizontal in-plane component} = \frac{606(58.69/57)12(26 + 7.66 \times 58.69/57)}{9,085} = 30.75 \text{ kips}$$

Therefore, the total load on the outermost bolt is the resultant

$$P = \sqrt{(3.96 + 1.24)^2 + (30.75)^2} = 31.2 < 32.0 \text{ kips}$$

Use fourteen  $\frac{7}{8}$ -in.-dia, A490 bolts in two rows.

### Flange-Splice Design

The flange splice plates are proportioned for the Maximum Design Load and checked for fatigue.

The average stress in the top flange under the Maximum Design Load is

$$f_{b \text{ Top}} = \frac{4,920 \times 12(28.84 + 7.66)}{61,995} = 34.8 \text{ ksi}$$

The total flange force is determined by multiplying the average stress by the net flange area.

$$P_{\text{Top}} = 34.8 \left( \frac{16.50 - 0.42}{2} \right) = 280 \text{ kips}$$

The required net area of the top-flange splice plates then becomes

$$A_{\text{Top}} = \frac{280}{36} = 7.78 \text{ in.}^2$$

This value exceeds 75% of the net area of the top flange:

$$0.75 \left( \frac{16.50 - 0.42}{2} \right) = 6.03 < 7.78 \text{ in.}^2$$

Try a  $\frac{3}{8}$ -in. outer splice plate and two  $\frac{3}{8} \times 5\frac{3}{8}$ -in. inner splice plates. The net area of these plates after deduction of bolt holes in excess of 15% of the plate area is

$$\begin{aligned} \text{Top plate} &= (\frac{3}{8} \times 12) - 2.106(1 \times \frac{3}{8}) + 0.15(\frac{3}{8} \times 12) = 4.38 \\ \text{Bot. plate} &= 2[(\frac{3}{8} \times 5\frac{3}{8}) - 1.0531(1 \times \frac{3}{8}) + 0.15(\frac{3}{8} \times 5\frac{3}{8})] = \underline{3.85} \\ &8.23 > 7.78 \text{ in.}^2 \end{aligned}$$

The average stress in the bottom flange under the Maximum Design Load is

$$f_b \text{ Bot.} = \frac{4,920 \times 12(28.75 - 7.66)}{61,995} = 20.1 \text{ ksi}$$

The total flange force is

$$P_{\text{Bot.}} = 20.1[46.00 - 16 \times \frac{1}{2} + 0.15 \times 46.00] = 20.1 \times 44.90 = 902 \text{ kips}$$

The design torsional shear across the bottom flange is

$$V_T = \frac{12 \times 90}{2} \left( \frac{179.5}{5,230} + \frac{49.2}{6,735} + \frac{373.2}{6,735} \right) = 53 \text{ kips}$$

For the bottom-flange splice, a trial is made with two  $\frac{3}{8} \times 41\frac{1}{2}$ -in. outer plates and two  $\frac{3}{8} \times 41\frac{1}{2}$ -in. inner plates. For one pair of plates (half box), assume two rows of bolts with 8 bolts per row, on each side of the joint. The area of one bolt hole is 0.375 in.<sup>2</sup> The holes remove from each splice plate the following percentage of its cross-sectional area:

$$\% \text{ of plate} = \frac{8 \times 0.375}{0.375 \times 41.5} \times 100 = 19.3\%$$

Consequently, the fraction of the hole area that must be deducted in determination of the net section is

$$\frac{19.3 - 15.0}{19.3} = 0.22$$

With  $5\frac{1}{2}$ -in. spacing of bolts,

$$d^2 \text{ for holes} = 2.75^2 + 8.25^2 + 13.75^2 + 19.25^2 = 635$$

$$\Sigma A d^2 = 4 \times 0.22 \times \frac{3}{8} \times 635 = 210 \text{ in.}^4$$

The net section modulus of the pair of splice plates is

$$S_{\text{net}} = \frac{41.5}{2} \left( 2 \times \frac{1}{12} \times \frac{3}{8} \times 41.5^3 - 210 \right) = 205 \text{ in.}^3$$

The net area of each pair of splice plates is

$$A_{\text{net}} = 2 \times \frac{3}{8} \times 41.5 - 8 \times 2 \times \frac{3}{8} \times 0.22 = 31.13 - 1.32 = 29.81 \text{ in.}^2$$

The splice plates must resist the direct flange load  $P$ , the torsional shear  $V$ , and the moment due to eccentricity of the torsional shear  $M_v$ . These are computed as follows:

$$P = \frac{902}{2} = 451 \text{ kips}$$

$$V = \frac{53}{2} = 27 \text{ kips}$$

$$M_v = \frac{27 \times 3.25}{12} = 7 \text{ kip-ft}$$

The maximum direct stress in the splice plates then is

$$f_t = \frac{451}{29.81} + \frac{7 \times 12}{205} = 15.1 + 0.4 = 15.5 < 36 \text{ ksi}$$

and the maximum shear stress on the gross area of a pair of plates is

$$f_v = \frac{27}{31.13} = 0.87 < 19.8 \text{ ksi}$$

#### Check of Flange Splices for Fatigue

The flange splice plates are then checked for fatigue under Service Loads. The range of live-load moment at the splice equals

$$M_{Lr} = 1.35[1,595 - (-1,000)] = 3,503 \text{ kip-ft}$$

And the range of average stress in the flanges, disregarding the relatively small effect of torsion, is

$$\text{Top Flange: } f_{sr} = \frac{3,503 \times 12(28.84 + 7.66)}{61,995} = 24.7 \text{ ksi}$$

$$\text{Bot. Flange: } f_{sr} = \frac{3,503 \times 12(28.75 - 7.66)}{61,995} = 14.3 \text{ ksi}$$

The corresponding range of stress in the gross section of the flange splice plates is

$$\text{Top Flange: } f_{sr} = \frac{24.7(\frac{1}{2})(16.50 - 0.42)}{12 \times \frac{5}{16} + 2 \times 5.375 \times \frac{3}{8}} = 18.9 < 27.5 \text{ ksi}$$

$$\text{Bot. Flange: } f_{sr} = \frac{14.3 \times 44.90}{4 \times 41.5 \times \frac{3}{8}} = 7.65 < 27.5 \text{ ksi}$$

The flange splice plates, therefore, are satisfactory.

#### Flange Bolts

The number of bolts required in the flange splice is determined by the capacity needed for transmitting the flange force under the Overload  $D + \frac{5}{3}(L + I)$ . The total moment on the section is 3,698 kip-ft (see Web Moments for Overload).

The average stress in the top flange is

$$f_b = \frac{3,698 \times 12(28.84 + 7.66)}{61,995} = 26.2 \text{ ksi}$$

And the flange force becomes

$$P_{\text{Top}} = 26.2 \left( \frac{16.50 - 0.42}{2} \right) = 211 \text{ kips}$$

For this flange force, the number of bolts required is

$$\frac{211}{32.0} = 6.6 \text{ bolts}$$

Use 8 bolts.

The average stress in the bottom flange is

$$P_{\text{Bot.}} = 15.1 \times 44.90 = 677 \text{ kips}$$

For this flange force, the number of bolts required is

$$\frac{677}{32.0} = 21.2 \text{ bolts}$$

For detail purposes, 32 bolts are used. With this substantial margin of additional bolts, the torsional effects on the bolt pattern may be neglected.

### Stiffener Splice

Next, the splice is designed for the ST7.5×25, longitudinal, bottom-flange stiffener. A splice of the stiffener is desirable to assure that the interruption of the stiffener at the field splices does not become a node for buckling. The splice is designed for the axial-load capacity of the ST7×25. This capacity equals the product of the allowable compression stress for the bottom flange and the area of the stiffener.

The allowable compression is a function of the torsional shear stress  $f_v$ , the coefficient  $K_s$ , and the buckling coefficient  $K$  furnished by the combination of the longitudinal stiffener and the bottom flange. As previously calculated for the field splice location, the torsional shear stress  $f_v = 1.1$  ksi. The other parameters required for calculation of the allowable compression stress in the bottom flange are determined as follows:

$$K = \sqrt[3]{\frac{I_s}{0.125t^3b}} = \sqrt[3]{\frac{243.2}{0.125(0.5)^3 45}} = 7.0 > 4 \text{ Use 4.}$$

$$K_s = \frac{5.34 + 2.84 \left( \frac{243.2}{45(0.5)^3} \right)^{1/3}}{(1+1)^2} = 3.83$$

$$\Delta = \sqrt{1 - 3 \left( \frac{1.1}{36} \right)^2} = 0.9986$$

With the use of the preceding results,

$$R_1 = \frac{97.08\sqrt{4}}{\sqrt{\frac{1}{2} \left[ 0.9986 + \sqrt{(0.9986)^2 + 4 \left( \frac{1.1}{36} \right)^2 \left( \frac{4}{3.83} \right)^2} \right]}} = 194.2$$

$$R_2 = \frac{210.3\sqrt{4}}{\sqrt{\frac{1}{1.2} \left[ 0.9986 - 0.4 + \sqrt{(0.9986 - 0.4)^2 + 4 \left( \frac{1.1}{36} \right)^2 \left( \frac{4}{3.83} \right)^2} \right]}} = 420.4$$

Because  $w\sqrt{F_y}/t = 45\sqrt{36}/0.5 = 540 > (R_2 = 420.4)$ , the allowable compression stress in the bottom flange is

$$F_b = 26,210 \times 4 \left( \frac{0.5}{45} \right)^2 - \frac{(1,100)^2 4}{26,210(3.83)^2 \left( \frac{0.5}{45} \right)^2} = 12.9 \text{ ksi}$$

With an allowable compression stress of 12.9 ksi for the bottom flange, the force on the stiffener is

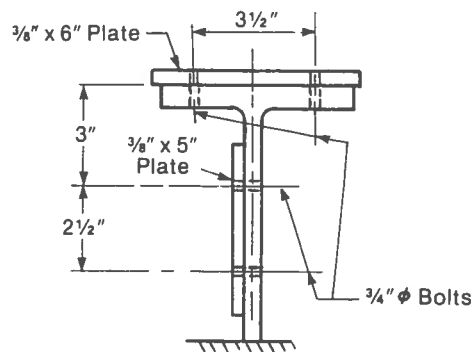
$$P_{st} = 12.9 \times 7.35 = 94.8 \text{ kips}$$

The stiffener splice also must be designed to resist lateral bending. The lateral-bending moment is taken as that associated with the bottom-flange stress of 12.9 ksi and is computed with the theory presented in General Design Considerations under Lateral Bending Stresses under  $DL_1$ . The stress at the top of the ST7.5 stiffener, with  $y_b = d_{\text{Bot. of steel}}$  for the net steel section at the splice, is

$$f_s = \frac{y_b - y_s}{y_b} f_b = \frac{21.34 - 7.50}{21.34} \times 12.9 = 8.4 \text{ ksi}$$

Hence, the lateral bending moment is

$$M_{Lat} = \frac{f_s b t d^2}{10R} = \frac{8.4 \times 5.64 \times 0.622 (12.31 \times 12)^2}{10 \times 410.38 \times 12} = 13.1 \text{ kip-in.}$$

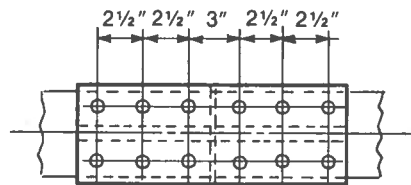


**SPLICE OF ST 7.5 X 25**

With  $3/4$ -in.-dia, A325 bolts and an allowable load in single shear of  $0.442 \times 21 = 9.3$  kips per bolt, the number of bolts required for direct load is

$$\frac{94.8}{1.3 \times 9.3} = 7.8 \text{ bolts}$$

Try 10 bolts, four in the stem splice and six in the flange splice.



**STIFFENER-FLANGE SPLICE**

The polar moment of inertia of the flange bolt group is

$$I = 2 \times 3(1.75)^2 + 2 \times 2(2.5)^2 = 43.38 \text{ in.}^4$$

The force on the outermost bolt due to lateral bending and direct load is computed and found to be within the allowable value:

$$\text{Longitudinal force from direct load} = \frac{94.8}{1.3 \times 10} = 7.29 \text{ kips}$$

$$\text{Longitudinal component from moment} = \frac{12.9 \times 1.75}{1.3 \times 43.38} = 0.40 \text{ kips}$$

$$\text{Lateral component from moment} = \frac{12.9 \times 2.5}{1.3 \times 43.38} = 0.57 \text{ kips}$$

$$\text{Resultant total bolt load} = \sqrt{(7.29 + 0.40)^2 + (0.57)^2} = 7.7 < 9.3 \text{ kips}$$

The area required for the splice plates for direct load is

$$A_{st} = \frac{94.8}{36} = 2.63 \text{ in.}^2$$

Try a  $\frac{3}{8} \times 6$ -in. splice plate on top of the flange and a  $\frac{3}{8} \times 5$ -in. plate on the stem, each with two longitudinal rows of bolts. The net area of the plates is

$$\text{Flange: } 6 \times \frac{3}{8} - 2(\frac{7}{8} \times \frac{3}{8}) + 0.15(6 \times \frac{3}{8}) = 1.93$$

$$\text{Stem: } 5 \times \frac{3}{8} - 2(\frac{7}{8} \times \frac{3}{8}) + 0.15(5 \times \frac{3}{8}) = 1.50$$

$$3.43 > 2.63 \text{ in.}^2$$

In this net section of the flange splice plate, the bolt holes remove the following percentage of the area:

$$\frac{2(\frac{3}{8})(\frac{7}{8})}{6 \times \frac{3}{8}} \times 100 = 29.2\%$$

Hence, the fraction of hole to be deducted is

$$\frac{29.2 - 15.0}{29.2} = 0.49$$

The moment of inertia of the splice plate thus is

$$I = \frac{1}{12} \times \frac{3}{8} (6)^3 - 0.49 \times \frac{3}{8} \times \frac{7}{8} (1.75)^2 = 6.26 \text{ in.}^4$$

The total stress in the flange splice then is

$$F_b = \frac{94.8}{3.43} + \frac{12.9 \times 3}{6.26} = 33.8 < 36 \text{ ksi}$$

The splice design therefore is satisfactory.

## COMPARISON WITH STRAIGHT BRIDGE

The curved box-girder bridge of this example requires about 17% more structural steel than its straight counterpart of Chapter 7. About half of the additional steel is attributable to the top lateral bracing and to the change from a rigid frame support to a continuous support. A quarter of the additional steel is that from intermediate crossframes. The remainder results from use of lower allowable stresses, higher live-load impact factors and other provisions by which the Guide Specifications account for curvature.

Other effects of curvature include:

A490 bolts instead of A325 bolts for field splice of main girder material

15% more shear connectors than those required for the straight bridge

## FINAL DESIGN

Drawings of the curved box-girder bridge of the example are shown on the following sheets.

