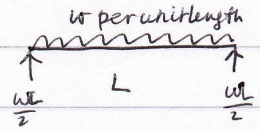
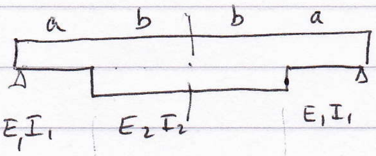
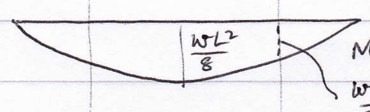
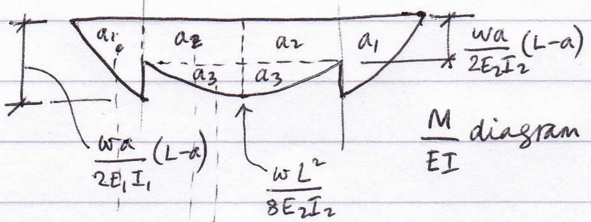




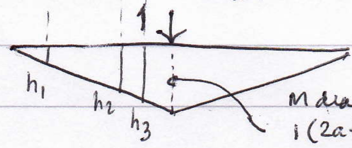


M diagram

$\frac{waL}{2} - \frac{wa^2}{2} = \frac{wa}{2}(L-a)$



$\frac{M}{EI}$ diagram



M diagram for unit load

$1(2a+2b)/4 = \frac{1}{2}(a+b)$

h_i below centroids of a_i

Central deflection $= 2\times(a_1h_1 + a_2h_2 + a_3h_3)$

where $h_1 = \frac{a}{2}\frac{(4L-3a)}{(3L-2a)} \cdot \frac{(a+b)}{(a+b).2} = \frac{a}{4}\frac{(4L-3a)}{(3L-2a)}$

$h_2 = \frac{a+\frac{b}{2}}{a+b} \cdot \frac{1}{2}(a+b) = \frac{a}{2} + \frac{b}{4} = \frac{1}{4}(2a+b)$

$h_3 = \frac{a+\frac{5}{8}b}{a+b} \cdot \frac{a+b}{2} = \frac{a}{2} + \frac{5}{16}b = \frac{1}{16}(8a+5b)$

$\Delta = 2\times\left[a_1h_1 + a_2h_2 + a_3h_3\right]$

$= 2\times\left[\frac{wa^2}{12E_1I_1}(3L-2a) \cdot \frac{a}{4}\frac{(4L-3a)}{(3L-2a)}\right] + \cancel{2}\frac{wab}{\cancel{2}E_2I_2}(L-a) \cdot \frac{1}{4}(2a+b)$

$+ 2 \cdot \frac{2}{3}\left[\frac{wb}{8E_2I_2}(L-a)^2\right]\frac{1}{16}(8a+5b)$

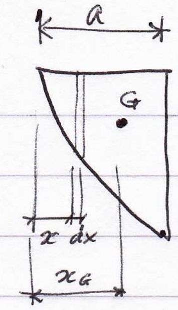
$= \ldots$

$= \frac{wa^3}{24E_1I_1}(4L-3a) + \frac{wab}{4E_2I_2}(L-a)(2a+b)$

$+ \frac{wb}{96E_2I_2}(L-a)^2(8a+5b)$

Check $a=0$ $b=\frac{L}{2} \Rightarrow \Delta = \frac{5}{384}wL^4$ ✓

Check $b=0$ $a=\frac{L}{2} \Rightarrow \Delta = \frac{5}{384}wL^4$ ✓



$\frac{wa}{2}\left[L-a\right]\frac{1}{E_1I_1}$ Zone 1 of $\frac{M}{EI}$ diagram

Eqn of curve is $\frac{1}{E_1I_1}\left(\frac{wLx}{2} - \frac{wx^2}{2}\right)$

area $a_1 = \frac{1}{E_1I_1}\int_{x=0}^{x=a}\left(\frac{wLx}{2} - \frac{wx^2}{2}\right)dx$

$= \frac{1}{E_1I_1}\left[\frac{wLx^2}{4} - \frac{wx^3}{6}\right]_0^a$

$= \frac{1}{E_1I_1}\left[\frac{wLa^2}{4} - \frac{wa^3}{6}\right] - 0$

$= \frac{wa^2}{12E_1I_1}(3L-2a)$

1st moment of area about LH end

$= \frac{1}{E_1I_1}\int_{x=0}^{x=0}\left(\frac{wLx^2}{2} - \frac{wx^3}{2}\right)dx$

$= \frac{1}{E_1I_1}\left[\frac{wLx^3}{6} - \frac{wx^4}{8}\right]_0^a$

$= \frac{1}{E_1I_1}\left[\frac{wLa^3}{6} - \frac{wa^4}{8}\right] - 0$

$= \frac{wa^3}{24E_1I_1}(4L-3a)$

$\therefore x_{G1} = \frac{\cancel{w}a^3(4L-3a)}{24\cancel{E_1I_1}} \times \frac{12E_1I_1}{\cancel{w}a^2(3L-2a)}$

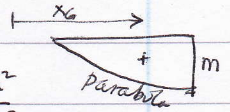
$= \frac{a}{2}\left(\frac{4L-3a}{3L-2a}\right)$

$a_2 = \frac{wab}{2E_2I_2}(L-a)$

$x_{G2} = a + \frac{b}{2}$ from LH end

---

a_3 is a standard case



$m = \frac{wL^2}{8E_2I_2} - \frac{waL}{2E_2I_2} + \frac{wa^2}{2E_2I_2}$

$= \frac{w}{8E_2I_2}\left(L^2 - 4aL + 4a^2\right) = \frac{w}{8E_2I_2}(L-2a)^2$

$x_G = a + \frac{5}{8}b$

$a_3 = \frac{2}{3}\left(\frac{wb}{8E_2I_2}(L-2a)^2\right)$