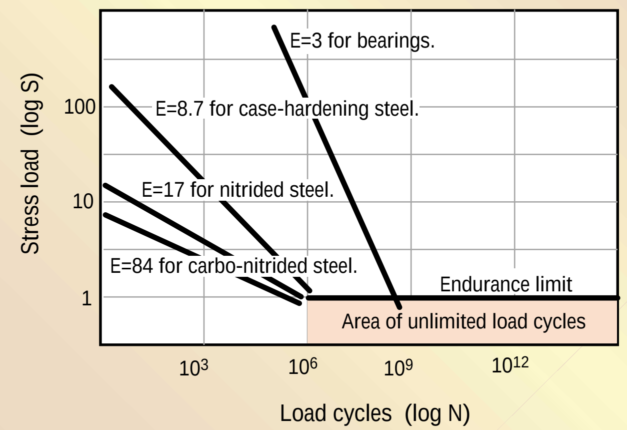
Determining endurance limit slope



tionship is usually manipulated and plotted in its logarithmic form: *log S = -1/E x log N* or *log N = -E x log S*.

• The *area of unlimited load cycles* is the horizontal portion of the S-N curve.

The exponent *E* determines steepness of the S-N relationship slope. It depends on the alloy, heat treatment conditions, and loading type. *E* covers a wide range — about six to 80. A comparison in the logarithmic scale allows the slope to appear as a constant linear slope. For a range or set of different stress levels and its frequency of stress level occurrence, damage accumulation calculation methods are used, including the well-known Palmgren-Miner rule.

However, keep in mind if the number of the load cycles is above the 2 x 10^6 the stress load has to be at or below the endurance limit; otherwise the component will fail. As shown in the example, for the number of load cycles a sun-gear endures over eight hours (2.88 x 10^6) the components must be sized and designed to endure unlimited load cycles. Gearbox rated torque must equal the torque level that loads all components at or below their endurance limits.

Unfortunately there is another phenomenon that makes otherwise straightforward sizing less transparent — the bearing. Rolling element bearings behave differently than other components; specifically, they do not have a clearly definable endurance limit. The high-pressure loading between rolling elements and races do not follow the above described endurance behavior. For this reason bearings are typically sized, designed, and rated for a lifetime (say 10,000 hours) with assigned a certain statistical probability of failure (say L10 life, or a 10% chance of failure.)

As noted, the relationship between the load and number of load cycles is exponential, with an exponent of three to 10/3, but with no defined endurance limit. The familiar exponent three is applied to bearing calculations and also to Root Mean Cube or *RMC* calculations used to analyze complex load cycles consisting of different loads of varying duration.

$$T_{RMC} = \sqrt[3]{\frac{N_1t_1T_1^3 + N_2t_2T_2^3 + ... + N_it_iT_i^3}{N_1t_1 + N_2t_2 + ... + N_it_i}}$$

The exponent of three in this formula is valid for bearings, but isn't really applicable for the other gearbox components. Specially tailored calculations help identify the exponents valid for teeth, shafts, and other parts.

Gearbox torque rating

The majority of real-world gear applications far exceed 2 x 10^6 load cycles. For this reason, the recommendation of nearly all gear rating standards — AGMA, ISO, DIN — is to base gearbox torque ratings on endurance limits and minimum bearing life. For industrial gearboxes, AGMA recommends 5,000 to 10,000 hours.

_E_ mainly depends upon material, heat treatments, and loading type. With it, life expectancies for different stress levels can be found.

There is only one true rated torque for a gearbox — for continuous duty. Unfortunately for marketing reasons sometimes ill-defined acceleration torques, peak torques, or emergency-stop torques are used as references. Still, only in applications where the number of load cycles is below 2x10^6 are loads exceeding rated load permissible. The majority of real-world automation applications reach this number of load cycles in a few days, weeks, or in a best-case scenario, after some months of operation. As a rule of thumb:

If the peak load cycle is part of the designed standard working duty cycle of the machinery, the peak load should not be higher than the rated torque unless the machinery is only working a very limited time. An hour of operation a day qualifies.

Another situation where peak loads can exceed rated torque is if the user and OEM do not expect extended, maintenance-free life. Since simple RMC calculations are not applicable to other internal gearbox components in such cases, data should be submitted to the manufacturer.

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Course audit

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Sizing gearboxes under dynamic loading

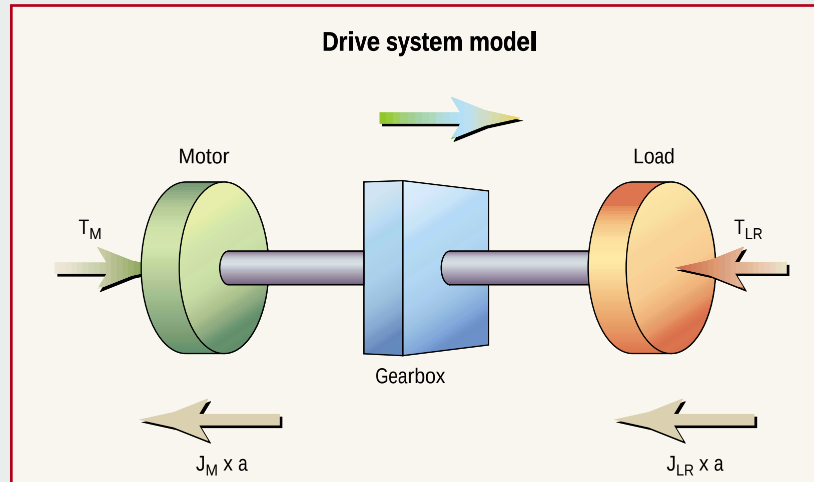
Gerhard G. Antony, PhD
Neugart USA LP

Day 1
True gearbox rating

Topics of discussion:

- ✓ Servomotor automation
- ✓ Repeated dynamic load cycles
- ✓ Selection of gears

Blindly assuming the dynamic peak torques listed on servomotor data sheets for a design almost always results in over-sized, inappropriately expensive gearboxes. A motor's acceleration peak torque is never transmitted to its gearbox. In most real applications, a substantial portion of the motor peak acceleration torque is consumed while accelerating the motor rotor. Luckily, for most situations the proper selection of a gearbox or gearmotor is simple. Here is one proven method; its simple formulas and minimal required input data facilitate programming and online sizing routines.



A model *in which all parameters are reflected to the motor axis* helps find the torque transmitted through the gearbox.

Before using this method for sizing, two conditions should be met: First, the true torque rating must be known. Here, *true rating* refers to a gearbox's rating based on endurance limits of its main components — not some ill-defined peak or acceleration torque. Second, the gearbox application shouldn't involve frequent jams, emergency stops, or other sudden, heavy load-side shocks; these situations require more involved analysis.

Trial run

Consider a typical servo system with an inertia ratio (reflected load Inertia / motor inertia) of 1:1. Because this inertia ratio has the best balance of control characteristics and economics, most system designers aim for this optimal value.

Assuming there are only dynamic inertial loads (T_{LR}=0), with the inertia ratio J_{LR} / J_M = 1 the inertia parameter k becomes 1/(1+1) = 0.5, which means T_{GR} = 0.5 x T_M.

This implies that in a system with a 1:1 inertia ratio, 50% of the motor peak torque will be consumed while accelerating the motor rotor, leaving only the remaining 50% to travel through the gearbox to accelerate the load. Using the above equation, an appropriate gearbox can be selected based on the calculated maximum output torque requirement of *TGR x i* instead of selecting one based on the full peak motor torque T_M *x i*.

Load cycles

Automation, motion control, and positioning applications are characterized by the repeated acceleration and deceleration cycles of starts, stops, and reversals. The resulting characteristic load cycle usually consists of several load peaks of varied intensity and duration, which makes the exact cycle difficult to predict. However, it can be measured and a statistically representative load characteristic cycle compiled.

Typically, when a gearbox must be selected for a servomotor, the exact load cycle is unknown. Having basic load-related data — **namely the load inertia and the non-dynamic load torque *(friction and gravity loads)*** — makes reliable and realistic estimations of the required gearbox torque rating possible.

Dynamic torque load equilibrium in a drive train: *motor—gear—load*

Knowing the peak torque a gearbox is subjected to eases sizing. Consider a system consisting of a motor, gearbox, and load. All system parameters are reflected to the motor axis so that

$J_{LR} = J_L/i^2$ and

$T_{LR} = T_L/i$

where i is the reduction ratio, J_L the load inertia at the gearhead output, J_{LR} is the reflected inertia at the input (motor axis). T_L is the load torque at the output and T_{LR} is the corresponding reflected torque at the motor axis. This *non-inertial* friction and/or gravity-born torque, is present in the system under steady state as well as under the acceleration phases.

The torque equilibrium equation during the acceleration can be written as:

$$T_M - T_{LR} - J_M \times a - J_{LR} \times a = 0$$

$$a = \frac{T_M - T_{LR}}{J_M + J_{LR}}$$

where

T_M = maximum available motor peak torque

J_M = motor rotor inertia

a = angular acceleration

The torque traveling through the gearbox is:

$$T_{GR} = T_M - J_M \times a$$
$$T_{GR} = T_{LR} + J_{LR} \times a$$

Substituting (1) in (2) gives

$$T_{GR} = T_M \times \frac{1 - J_M}{J_{LR} + J_M} + T_{LR} \times \frac{J_M}{J_{LR} + J_M}$$

The *inertia parameter*, which is a function of the system inertia, is defined as $k = J_M/(J_{LR} + J_M)$.

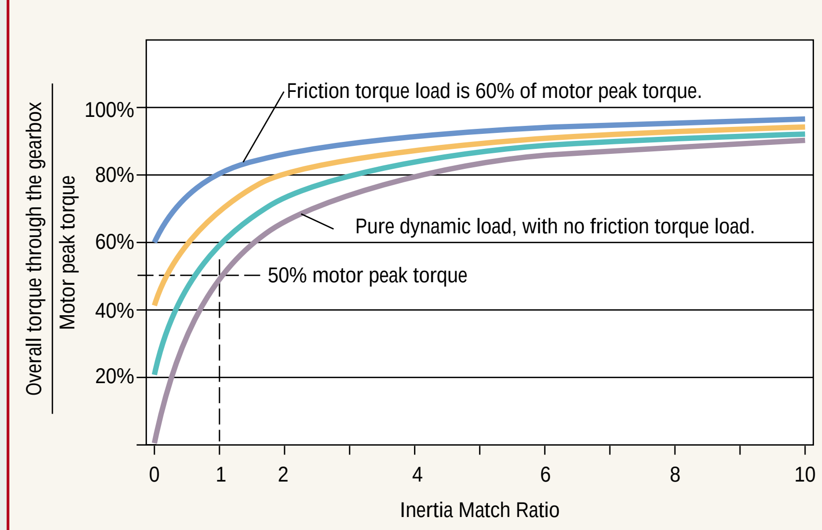
By inserting k into (3), torque through the gearbox can be written:

$$T_{GR} = (T_M - T_{LR}) \times (1 - k) + T_{LR}$$

This easy-to-handle formula is valid for all motor torque inertia and frictional torque combinations.

For long, maintenance-free gearbox life, the calculated gearbox peak load T_{GR} x i should be equal to or below the gearbox torque rating. ●

The k—IMR relationship



Here, each curve represents the overall (dynamic *plus* non-dynamic steady-state) torque load through the gearbox as a percentage of the motor peak torque. Each curve is for a different static load of 0, 20, 40, or 60%.

The inertia parameter k is very closely related to the well-known and (in servo drive system sizing) frequently used **Inertia Match Ratio.** The IMR is defined as ratio of the reflected load inertia to the motor rotor inertia:

$$IMR = \frac{J_{LR}}{J_M}$$

The inertia parameter k can be easily expressed in terms of the IMR:

$$k = \frac{1}{IMR + 1}$$

With the direct relationship between k and the Inertia Match Ratio, it is easy to find the motor torque transmitted through the gearbox.

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Precision gearhead torque rating for automation and robots

Gerhard Antony, PhD
Neugart USA, Bethel Park, Pa.
www.neugartusa.com

Day 2

Topics of discussion:

- Ferrous material fatigue
- Gearbox torque rating

The basic limiting factor for electrical devices is prolonged exposure to high temperatures which deteriorate insulation layers, boundary layers in transistors, and other parts. Since generated heat equals $R \times I^2 \times t$ the rating limitation of a motor, relay, or inverter is the RMS value of current. Similarly, the limiting factors of mechanical devices are mechanical stresses — tension, compression, bending shear, and Herzian pressure. The exponent here is not two, but ranges from 3 to 50 and above.

Ferrous material fatigue

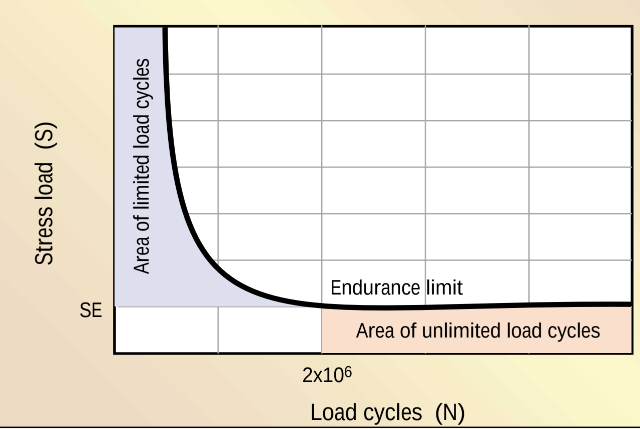
Fatiguing of metallic components is well documented. Cyclically loaded parts fail even when the stress load magnitude never exceeds static strength (which parts can easily endure without damage.) Another well-known phenomenon: If the magnitude of the cyclic stress load is decreased enough, parts can endure unlimited load cycles. This stress level is called the *endurance limit.* Described by *S-N Curve*, this behavior is defined in terms of stress and number of cycles.

All major gearbox components are subjected to cyclic stresses, even if external loads are constant. For example, consider gear teeth. With every wheel rotation each tooth becomes fully loaded on engagement; then it passes through the action area, becomes unloaded, and completes a dynamic load cycle. In the case of a sun gear in a three-planet gearbox, this actually occurs three times per revolution due to multiple contacts. Assuming the gearbox is driven by a servomotor at a moderate 2,000 rpm input speed, in an eight-hour period each sun gear tooth endures 2,000 x 3 x 8 x 60 = 2,880,000 load cycles.

Countless tests confirm that for ferrous materials, the exponential relationship between stress S and the number of endured load cycles N **that result in no damage** actually levels out at the endurance limit — at about 2 x 10^6 to 10^7 cycles. This is valid if the part is subjected to bending, shear, tension, or compression. Based on this we can distinguish two distinct areas of the S-N relationship:

• The *area of limited load cycles* — plots situations resulting in limited life. This area shows and exponential relationship between S and N, mathematically described as $S = 1 / N^{1/E} = N^{1/E}$ or $N = 1/S^E$. Because of the large range involved, the rela-

Characteristic SN curve for ferrous metals



The constant, horizontal part of the S-N curve is the endurance limit value, at which a component can withstand unlimited cycles

FORMULAS

TO DETERMINE SELECTION

1) How to calculate torque when horsepower and speed are known

$$\text{torque ft lb} = \frac{5252 \text{ x horsepower x service factor}}{\text{speed}} \qquad T = \frac{5252 \text{ x hp x k}}{n}$$

2) Inertia - How to determine inertia when material and shape are known.

(Total system inertia is total inertia of all the components. If the components are not simple shafts or flanges, break down each of the components into its basic shape and calculate inertia of that individual component. When inertia is being calculated in relation to the clutch or brake, remember to adjust for reflected inertia amounts which may have a significant increase or decrease on the inertia that the clutch has to handle based upon a speed differential.)

(Inertia constants lb. in.^{3})

ρ (aluminum) = 0.0924

ρ (bronze) = 0.321

ρ (cast iron) = 0.26

ρ (steel) = 0.282

Values

wk^2 = lb. ft.^{2}

D, D_0, D_1, L = in.

Formula to determine inertia of a solid shaft

wk^2 = .000681 x p x Length x Diameter^4

wk^2 = .000681 x p x L x D^4

Formula to determine inertia of a hollow shaft

wk^2 = .000681 x p x length x (outer diameter^4 - inner diameter^4)

wk^2 = .000681 x P x L x ($D_O{}^4$ -$D_I{}^4$)

Reflected inertia via gears, chain or belt

reflected inertia = load inertia divided by the square of the speed ratio

$$wk^2_R = \frac{wk^2_L}{r^2}$$

3) How to calculate the amount of torque required to accelerate or decelerate a load when inertia value is known (t = time to speed or time to stop depending if you are using a clutch or a brake.)

$$\text{torque ft lb} = \frac{\text{(inertia x the change in rpm)}}{308 \text{ x the time required}} \qquad T = \frac{wk^2 \text{ x } \Delta \text{ rpm}}{308t}$$

4) Heat Dissipation

Quick reference for determining slip watts for magnetic particle applications. (Magnetic particle clutches are normally limited to heat dissipation rather than torque when they are involved in a constant slip application.)

watts = .0118 x torque in inch lbs. x the change in rpm $\qquad$ W = .0118 x t x Δ rpm

5) Linear Speed to Rotational Speed

$$\text{RPM} = \frac{\text{speed in feet per minute}}{3.14 \text{ x diam. in feet}} \qquad \text{RPM} = \frac{\text{FPM}}{3.14 \text{ x D}}$$