

PIN-CONNECTED PLATE LINKS

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SYNOPSIS

Results of 106 tests of differently proportioned steel plates are presented in this paper. The specimens were loaded in tension by means of 3-in. steel pins which passed through holes in either end of the plate. The effects of plate thickness, side and bottom edge distances, and pin clearance were studied in relation both to the general yield and the ultimate strength of the links. The "dishing" phenomenon, which greatly reduces the ultimate strength of thin plates unsupported laterally, was given particular attention. Empirical formulas are given for predicting the general yield and ultimate strength of the plates, together with formulas for producing a pin-plate connection of balanced design.

INTRODUCTION

The design of steel eye-bars has developed an accepted practice in regard to the relative proportions and distribution of material in the eye-bar head. The present investigation is concerned with pin-connected plates which do not have reduced width in the body as in the case of the forged eye-bar, but which are made simply by boring pinholes near the ends of structural steel plates. Examples of this type of connection are found in steel sheave blocks, derrick tackle, anchorage bars for cables, and various types of erection equipment. Although the plates tested differ from forged eye-bars, their action around the pinholes during the progress of failure is similar to that in the eye-bar head.

The problem of designing such pin-connected plate links was encountered on a large scale in arranging the hangers for raising the suspended span of the Quebec Bridge.² Tests were made both on model links and on full-sized hangers with 12-in. pins, and the results show a good correlation with those of the present tests. The general relation between plate thickness and "dishing" of thin plates was noted in the Quebec tests.

The bibliography on the subject of stresses in plates around pinholes is extensive, but most of it is concerned primarily with the study of stress dis-

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³"The Quebec Bridge," Vol. 1, pp. 222 to 227, Rept. of Canadian Govt. Board of Engrs., 1919.

tribution in the elastic range. References to various photo-elastic, mathematical, and experimental studies of the stresses in eye-bar heads may be found in the work of Professor S. Timoshenko.³ Mathematical stress analyses require an assumed distribution of bearing pressure between pin and pinhole as a starting point. The actual bearing stress distribution depends on pin clearance and varies somewhat during the application of load. The maximum stresses are localized and, for ductile materials, the initial yielding is restrained from rapid progress by surrounding low-stressed areas and, in the early stages, has no appreciable effect on the elastic behavior of the link as a whole. Furthermore, these localized stresses have practically no effect on the ultimate strength of the link under static load. Hence, the use of a ductile material such as structural steel is very advantageous.

Local yielding in the plate at the contact point, except in the case of tight-fitting pins, will occur even at working loads. The general, or over-all yield at the end of the plate, occurs at a much later stage than initial local yielding. The term, "general yield point," will be used herein to designate the load or average bearing stress at which the slope of the curve of load plotted against deformation between pin and plate (about 3 pin diameters away) is three times the initial slope.

Because of the early local yielding and associated stress redistribution, mathematical formulas for maximum local stresses based on the assumption of elastic behavior have doubtful utility in the actual design of pin-connected steel plates under steady loads. For non-ductile materials, or for ductile materials designed for repeated stress, the elastic stress distribution is of importance. An approximate mathematical solution by J. Beke⁴ has been applied to experimental tests of large pin-connected members in the Hugo-Preuss Bridge, in Berlin, Germany.⁵

Beke's formula is based on the simplest possible assumptions and is applicable in the elastic range. More exact solutions become increasingly complex and, in any case, give little indication as to the actual load carried at the general yield point or ultimate. To have practical value, a mathematical solution should be based on the theory of plasticity, and should include the study of the phenomenon of plastic "dishing" of thin plates. The solution of this problem has not been attempted in the present investigation. Of first importance to the designer are the answers to the questions: "How much load will a structural unit carry without excessive deformation?" and "What is its strength at failure?" In the present instance, the answer is given in the form of empirical equations, validated by a wide range of tests, and concerned with average stresses at general yield and ultimate. The large number of relatively thin plates tested give information on the greatly reduced margin between general yield and ultimate which is caused by the "dishing" type of failure in laterally unrestrained plates.

TEST PROGRAM AND PROCEDURE

The program of 106 tests was designed to cover the critical range of variables. The same size of pin, 3 in. in diameter, was used in all tests. The four

⁴"Theory of Elasticity," p. 120.

⁵"Der Eisenbau," 1921, p. 233.

⁶"Die Bautechnik," February 12, 1932, pp. 79-83.

variables ranged follows: (1) Plate thicknesses varying from $\frac{1}{8}$ to $\frac{1}{2}$ in.; (2) plate widths of 6, 8, and 10 in.; (3) edge distance behind pinhole varying from 1 in. to 3.2 in.; and (4) pin clearance varying from a tight fit to 0.2 in.

The dimensions of the test specimens were measured to the nearest thousandth of an inch by micrometer calipers, and are indicated in Table 1. This table shows in detail the range of variables covered. Fig. 1 shows the notation adopted in regard to pin dimensions and dimension ratios.

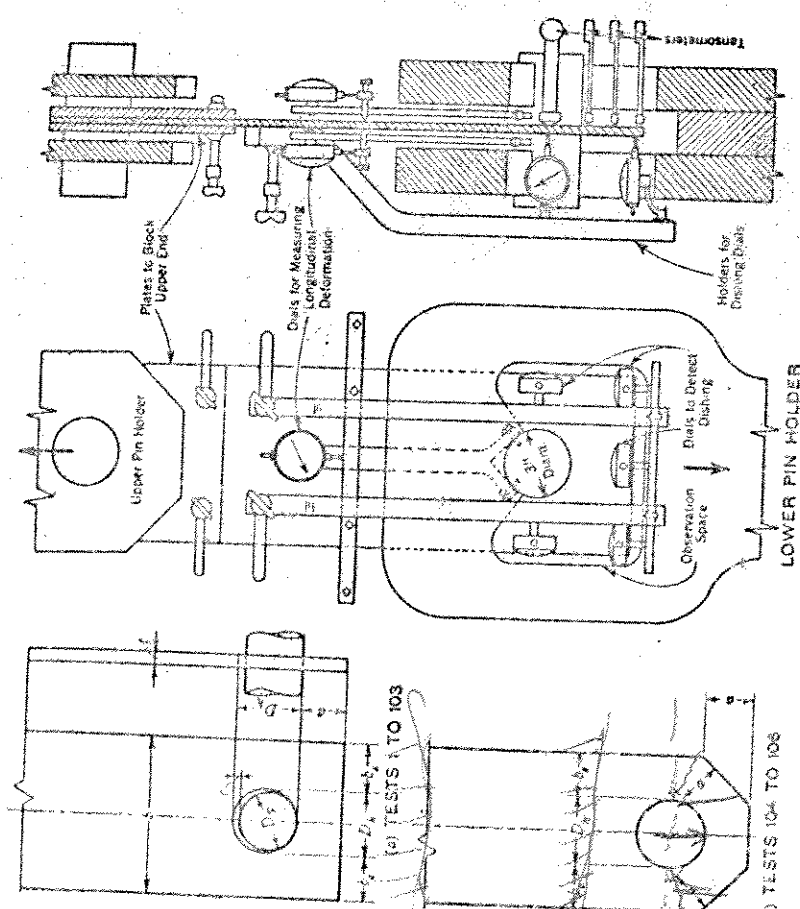


FIG. 1.—APPARATUS FOR TESTING PIN-CONNECTED STEEL PLATE LINKS

The material specified was Universal Mill Plate with rolled edges, a medium grade structural steel free from rust. All the plates of similar width and thickness were cut from the same stock with an additional section cut for tensile test coupons. The ends of the specimens were flame-cut to a length an inch or more longer than required, and were cut at the testing laboratory by hacking to the exact length. Two tensile coupons cut parallel with the axis of rolling and two cut transversely were made from each plate series, the results being given in Table 2.

The arrangement of the apparatus used during a typical test is shown diagrammatically in Fig. 2, and by photograph in Fig. 3. The upper pin-holder

TABLE 1.—DIMENSIONS AND TEST DATA; PIN-CONNECTED PLATE LINKS

Test No.	MEASURED DIMENSIONS, IN INCHES				SURFACE SLIP LINES, LOAD, IN KIPE		LOAD, IN KIPE			
	Diameter of hole, D_h	b	t	a	First observed line*	General spread of lines	Local yield*	General yield	Initial dishing*	Ultimate strength*
1	3.015	2.510	0.128	0.637	6.5 BS	8.0	5.6 B	8.5	8.0	14.5 D
2	3.019	2.510	0.128	1.671	7.0 BC	10.0	9.0 B	11.6	12.5 +	18.0 D
3	3.017	2.511	0.128	2.193	10.0 BC	14.5	10.8 B	14.1	14.0	19.3 D
4	3.016	2.510	0.128	2.881	7.0 BC	15.0	11.2 E	15.2	13.0	17.4 D
5	3.015	2.512	0.128	3.204	9.0 BC	11.5 E	15.1	14.0	16.3 D
6	3.015	2.513	0.128	3.204	7.0 BC	12.0 E	14.4	14.0	13.9 D
7	3.069	2.482	0.127	0.970	6.0 BS	5.2 B	6.3	7.5 +	13.2 D
8	3.070	2.482	0.128	1.700	7.5 BC	9.5	7.2 B	9.1	9.8 +	15.2 D
9	3.069	2.482	0.128	2.183	6.0 BC	12.5	6.0 B	12.6	10.0	13.5 D
10	3.060	2.473	0.128	2.352	5.0 BC	12.5	10.5 E	12.4	11.0	13.5 D
11	3.067	2.473	0.128	2.905	5.0 BC	10.3 E	11.1	11.0	12.0 D
12	3.068	2.473	0.127	3.390	4.0 BC	11.5	11.9 B	12.6	11.0	13.2 D
13	3.017	2.511	0.129	0.018	12.0 BS	15.0	13.7 B	16.8	14.5	26.1 D
14	3.014	2.510	0.129	1.760	12.0 BS	15.0	16.4 B	17.7	16.0 +	27.7 D
15	3.019	2.509	0.129	2.999	14.0 BC	22.0	20.3 B	21.3	22.0	26.5 D
16	3.018	2.512	0.130	2.584	14.0 BC	22.0	20.3 B	21.3	22.0	28.8 D
17	3.019	2.511	0.130	2.888	16.0 BC	22.0	21.5	21.5	21.0	26.3 D
18	3.020	2.513	0.130	3.400	12.0 BS	17.0 +	19.0 E	21.5	20.0	26.0 D
19	3.073	2.473	0.130	1.708	4.0 BS	7.5	4.2 B	9.7	7.0	25.3 D
20	3.074	2.478	0.130	1.880	9.0 BC	14.5	12.1 B	14.1	14.5	25.7 D
21	3.074	2.478	0.130	2.306	5.0 BC	16.0	15.8 B	18.5	18.5 +	26.4 D
22	3.074	2.473	0.130	2.906	7.0 BC	18.0	20.0 E	21.5	16.0	22.0 D
23	3.075	2.473	0.130	2.880	7.0 BC	20.0	16.0 E	18.2	16.0	22.4 D
24	3.074	2.480	0.130	3.008	10.0 BC	18.0 +	12.3 B	16.8	20.0	38.0 FB
25	3.017	2.483	0.251	1.697	13.0 BS	20.0	19.1 B	23.2	20.0	42.5 D
26	3.018	2.483	0.251	1.697	13.0 BS	23.0	24.4 B	28.2	27.0	43.4 D
27	3.016	2.480	0.250	2.216	13.0 BC	31.0	26.2 E	29.7	29.0	43.4 D
28	3.019	2.480	0.250	2.216	13.0 BC	31.0	38.3 B	31.0	25.0	43.8 D
29	3.018	2.483	0.250	2.915	13.0 BC	27.0	29.4 E	32.0	34.0	41.0 D
30	3.019	2.481	0.250	3.249	11.0 BC	31.0	18.1 B	19.9	10.9	36.9 FB
31	3.072	2.458	0.256	0.958	7.5 BC	16.0 +	15.2 B	19.5	29.0	49.0 FB
32	3.069	2.460	0.257	1.250	10.0 BS	20.0	21.1 B	23.8	25.0	48.2 D
33	3.069	2.460	0.257	1.251	10.0 BS	26.0 +	24.2 B	27.8	37.0	48.0 D
34	3.072	2.452	0.250	2.590	3.0 BC	24.0 +	28.3 B	28.0	31.0	41.5 D
35	3.073	2.453	0.251	2.908	3.0 BC	24.0	23.8 B	29.0	29.0	39.0 D
36	3.071	2.459	0.250	3.123	3.0 BC	27.0	23.8 B	28.0	29.0	39.0 D
37	3.016	2.493	0.373	0.957	17.0 BS	33.0	15.0 B	21.6	None	53.7 FB
38	3.016	2.493	0.372	1.709	20.0 BC	30.0	24.3 B	29.4	None	53.7 FB
39	3.018	2.480	0.373	2.203	21.0 BC	36.0	30.4 B	36.5	38.0 +	76.5 D
40	3.017	2.491	0.374	2.593	21.0 BC	40.0	34.5 E	40.0	45.0 +	76.5 D
41	3.019	2.489	0.374	2.896	18.0 BC	49.0	37.8 B	42.0	48.0 +	77.4 D
42	3.017	2.492	0.371	3.198	13.0 BC	16.0	39.6 E	43.0	42.0	77.7 D
43	3.065	2.466	0.373	0.959	13.0 BC	25.0	11.6 B	17.0	None	58.9 FB
44	3.065	2.474	0.371	1.681	11.6 BC	25.0	21.7 B	24.5	34.0 +	78.5 D
45	3.066	2.469	0.371	2.193	25.0 BC	32.0	31.2 B	32.5	34.0 +	78.5 D
46	3.066	2.467	0.371	2.594	16.0 BC	38.0	31.2 B	39.5	52.0 +	73.9 D
47	3.067	2.465	0.372	2.910	16.0 BC	48.0	35.5 B	46.5	40.0 +	70.4 D
48	3.066	2.469	0.371	3.200	13.0 BC	30.0	29.1 B	28.5	None	73.4 F
49	3.014	2.500	0.436	1.019	16.0 BC	30.0	36.0 B	47.0	None	101.8 F
50	3.014	2.510	0.493	2.210	33.0 BS	42.0	46.4 B	54.5	Ult.	133.4 D
51	3.016	2.510	0.493	2.210	33.0 BS	42.0	46.4 B	54.5	Ult.	133.4 D
52	3.014	2.511	0.498	2.580	27.0 BC	46.4 B	54.5	Ult.	133.4 D

* B = below pin; C = crushing; D = "dishing"; E = side of pin; F = fracture; and S = shear.
† 1 kip = 1000 lb.

passed through the upper fixed head of the testing machine where it engaged transverse bars by means of lugs which allowed freedom of motion of the holder. The lower holder passed through the moving head of the testing machine and was open at the side to allow observation of the test. The upper end of the test specimen was blocked by plates on either side to ensure failure at the lower pinhole. The holders were built entirely by flame-cutting and welding steel slabs.

TABLE 1.—(Continued)

Test No.	MEASURED DIMENSIONS, IN INCHES				SURFACE SLIP LINES, LOAD, IN KIIPS		LOAD, IN KIIPS†		
	Diameter of hole, D_h	b	t	a	First observed line*	General spread of lines	Local yield*	General yield	Initial "dishing"
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
53	3.014	2.496	0.492	2.000	15.0 BC	46.0	40.0 E	53.5	UL
54	3.014	2.509	0.493	2.213	30.0 BC	52.0	49.8 E	59.0	UL
55	3.007	2.479	0.494	0.933	20.0 BC	22.0	14.5 E	21.0	None
56	3.007	2.471	0.494	1.480	20.0 BC	31.0	28.5 E	33.0	None
57	3.006	2.480	0.491	1.179	20.0 BC	44.0	35.3 E	45.0	UL
58	3.006	2.475	0.493	2.502	20.0 BC	48.0	35.0 E	47.5	UL
59	3.006	2.475	0.493	2.502	20.0 BC	51.0	39.5 E	50.8	UL
60	3.013	2.475	0.493	0.933	30.0 BC	52.0	42.0 E	57.0	UL
61	3.013	2.493	0.495	0.933	30.0 BC	52.0	42.0 E	57.0	UL
62	3.016	2.503	0.497	1.712	30.0 BC	52.0	42.0 E	57.0	UL
63	3.016	2.492	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
64	3.016	2.492	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
65	3.019	2.492	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
66	3.017	2.492	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
67	3.007	2.493	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
68	3.007	2.493	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
69	3.007	2.493	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
70	3.007	2.493	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
71	3.007	2.493	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
72	3.007	2.493	0.497	2.579	30.0 BC	52.0	42.0 E	57.0	UL
73	3.014	2.473	0.497	0.933	15.0 BC	15.0	7.4 E	15.0	UL
74	3.014	2.473	0.497	0.933	15.0 BC	15.0	7.4 E	15.0	UL
75	3.019	2.473	0.497	1.087	10.0 BC	15.0	12.8 E	15.0	UL
76	3.019	2.473	0.497	1.087	10.0 BC	20.0	18.0 E	20.0	UL
77	3.018	2.473	0.497	1.087	10.0 BC	22.0	11.2 E	20.8	UL
78	3.018	2.473	0.497	1.087	10.0 BC	22.0	13.2 E	21.0	UL
79	3.018	2.473	0.497	1.087	10.0 BC	22.0	17.3 E	21.0	UL
80	3.016	2.473	0.497	1.087	10.0 BC	30.0	24.8 E	21.0	UL
81	3.017	2.473	0.497	1.087	10.0 BC	40.0	30.8 E	33.0	UL
82	3.016	2.473	0.497	1.087	10.0 BC	40.0	30.8 E	33.0	UL
83	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
84	3.013	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
85	3.013	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
86	3.013	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
87	3.013	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
88	3.013	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
89	3.023	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
90	3.023	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
91	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
92	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
93	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
94	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
95	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
96	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
97	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
98	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
99	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
100	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
101	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
102	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
103	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
104	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
105	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL
106	3.016	2.473	0.497	1.087	10.0 BC	53.0	38.0 E	53.0	UL

† Corners clipped at 45 degrees.

Four different observations were made at successive loads during each test, as follows:

- (1) Longitudinal deformation between pin and plate was measured by two Ames dials ($\frac{1}{10000}$) attached rigidly to arms which were fixed to the pins by set screws as shown in Fig. 2. The plungers of these dials engaged bars at

TABLE 2.—PHYSICAL PROPERTIES OF MATERIALS BASED ON THE AVERAGE RESULTS OF TWO TESTS

Plate Nos.	LONGITUDINAL					TRANSVERSE				
	Yield point stress $\frac{1}{1000}$	Ultimate strength $\frac{1}{1000}$	Per-centage reduction of area	Per-centage elongation in 2 inches	Modulus of elasticity, E , 10^6	Yield point stress $\frac{1}{1000}$	Ultimate strength $\frac{1}{1000}$	Per-centage reduction of area	Per-centage elongation in 2 inches	Modulus of elasticity, E , 10^6
1-12	41.1	50.6	62.4	40.3	30.8	45.6	62.0	61.1	28.0	31.0
13-24	41.3	57.9	69.0	46.7	30.3	41.0	59.3	61.4	38.2	30.0
25-36	41.3	63.7	62.5	47.5	28.4	40.2	64.4	45.3	34.5	29.5
37-48	38.1	63.4	62.7	50.0	29.1	37.4	63.3	47.5	34.8	30.8
49-60	38.1	63.4	62.7	51.8	29.3	37.4	63.3	47.5	34.8	30.8
61-72	41.3	60.2	60.2	56.8	29.3	31.3	61.5	50.3	31.5	29.1
73-84	42.0	62.6	66.6	42.2	30.6	38.4	63.6	57.4	32.5	30.0
85-96	41.0	59.0	62.6	40.3	29.7	39.3	58.5	57.2	37.5	30.2
97-106	41.0	63.1	60.5	51.3	29.2	39.1	64.3	27.1	38.5	29.8
Average	39.2	61.6	64.1	48.6	29.5	38.7	62.4	47.2	33.8	29.9

tached to the test pieces by pointed screws about 8.5 in. from the center of the lower pin. The graph of the longitudinal deformation, plotted as abscissa with the load as ordinate, was used as a basis for determining the general yield

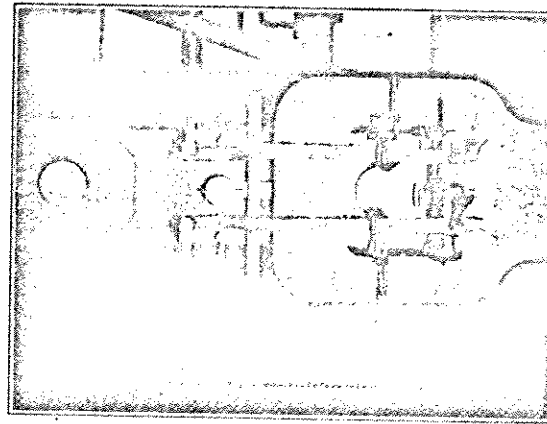


FIG. 3.—GENERAL VIEW OF TEST ARRANGEMENT, WITH DIALS PLACED TO DETECT "DISHING".

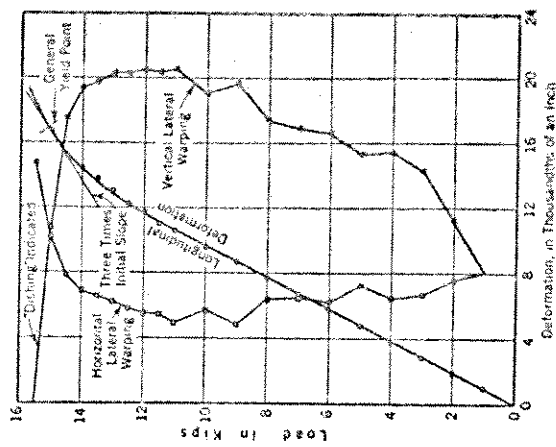


FIG. 4.—LOAD-DEFORMATION GRAPH FOR TEST NO. 5

point of the links. The point of general yield was taken as the load at which the slope of the load-deformation curve was three times that in the elastic range.

(2) Five Ames dials fixed to two main arms and one cross-piece engaged the lower surface of the test piece in order to measure any lateral movement

which would indicate the beginning of "dishing." The arms for holding these dials were fixed rigidly to the center of the test specimen, the arrangement being shown in Fig. 2. There were three points along two vertical lines, therefore, and three across the lower face of the plate. A study of these dial readings during successive load increments indicated any lateral movement along these three lines.

(3) Strains in the pin-plate material were measured along critical sections by means of Huguenberger tensometers, which were calibrated by an interferometer device designed and built by Raymond D. Mindlin, Jun. Am. Soc. C. E. The tensometers served two functions: (1) To obtain the stress distribution across the net section within an elastic range; and (2) to detect stress redistribution as indicated by variations in the slope of the strain curves. This, of course, does not necessarily indicate that the limit of proportionality had been exceeded at the exact point where the strain is measured. The tensometers were also used in determining the modulus of elasticity of the steel in the tensile coupon tests.

(4) The surface slip lines which are visible in the mill scale of structural steel at points of over-strain were accentuated by coating the surfaces of the plate at the lower pinhole with whitewash. The progress of the slip lines was recorded during successive load increments, and photographs were taken of all typical patterns. The whitewash consisted of 35 grams of hydrated lime mixed with 50 cu cm of water.

A similar routine was followed in all the tests, and mimeographed forms were used for recording dimensions, dial and tensometer readings, and the successive development of surface slip-lines.

TEST RESULTS

Types of Failure.—The general yield point, arbitrarily determined in every test from the load-deformation curve, was preceded by surface evidence of local failure below the pin in every instance except Tests Nos. 75 and 77 (see Table 1), these being two of the twelve tests of narrow plates 6 in. wide. Considerable surface slip-line development, extending over a relatively large area of the plate, was usually in evidence before the general yield point.

The type of ultimate failure in the pin-connected plate depends on the relative proportions of the plate at the pinhole. Three general and overlapping classifications may be made: (1) Tension failure in the net section at one side of the pin; (2) crushing and shearing failure below the pin, in some cases followed by a tearing fracture in "hoop" tension after considerable deformation; and (3) "dishing" failure of thin plates which are laterally unrestrained.

Typical Tests.—Failure by "dishing" at very low load is typified by Test No. 5, Table 1. The load-deformation curve, to determine the general yield point, and the curves of lateral and vertical warping indicative of "dishing" are shown in Fig. 4. A photograph of the surface slip-line pattern after "dishing" of Test No. 5 is shown in Fig. 5. This plate "dished" at a load of only 1 200 lb above the general yield point, and, in such cases, the slip lines appeared only on the convex surface. The "dishing" itself is not evident in this side view,

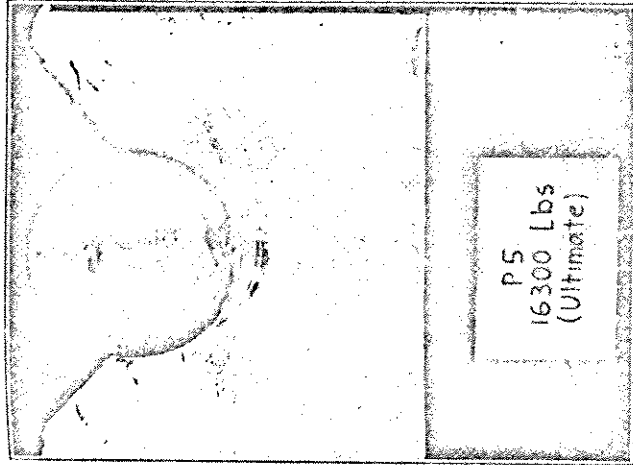


FIG. 5.—TEST NO. 5 AFTER FAILURE BY "DISHING"; SHOWING SLIP-LINES ON CONVEX SURFACE

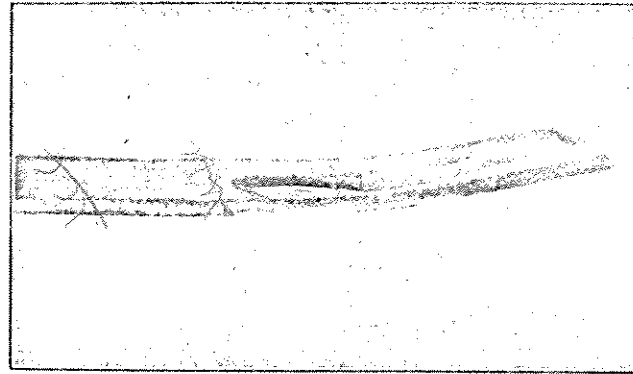


FIG. 6.—SHAPE OF TEST PLATE AFTER FAILURE BY "DISHING"

but is clearly indicated in Fig. 6, showing the shape of the plate after complete failure by "dishing."

The test of Plate No. 32 is typical of failure behind the pinhole, due to insufficient edge distance, *a*. Fig. 7 gives the load-deformation curve for this test and shows sketches similar to those taken in every test to record the progressive growth of surface slip-lines. Fig. 8 illustrates the further spread of the surface slip-lines at a load of 26 kips, and Fig. 9 shows the fracture after failure. The condition of the whitewash in Fig. 9 shows the completely plastic region below the pin prior to failure and the relative absence of surface slip-lines along the sides and above the pinhole. Fig. 10 shows an end fracture in another test of more balanced design and greater plastic elongation. Fig. 11 shows a surface slip-line pattern near the ultimate strength in

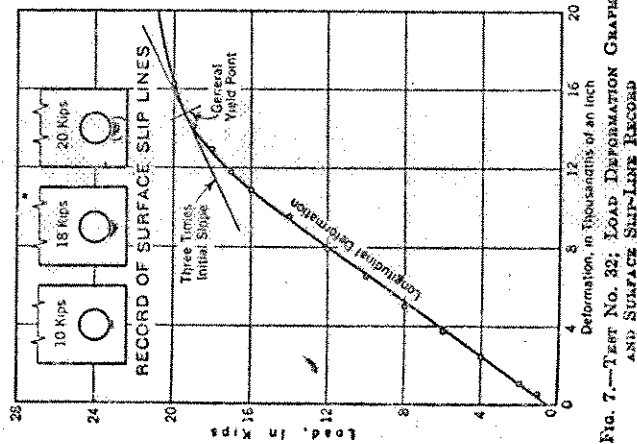


FIG. 7.—TEST NO. 32; LOAD-DEFORMATION GRAPH AND SURFACE SLIP-LINE RECORD

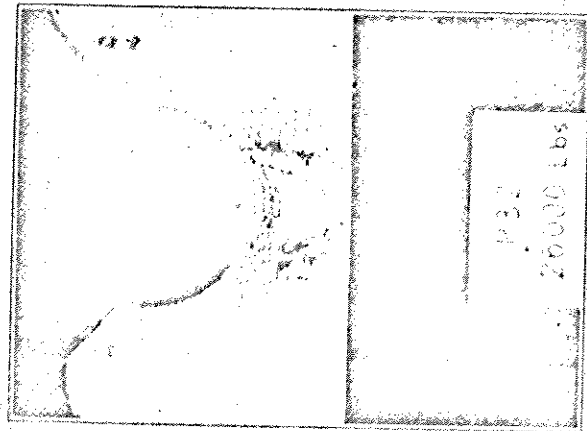


FIG. 8.—TEST NO. 32 AT 20 KIPS

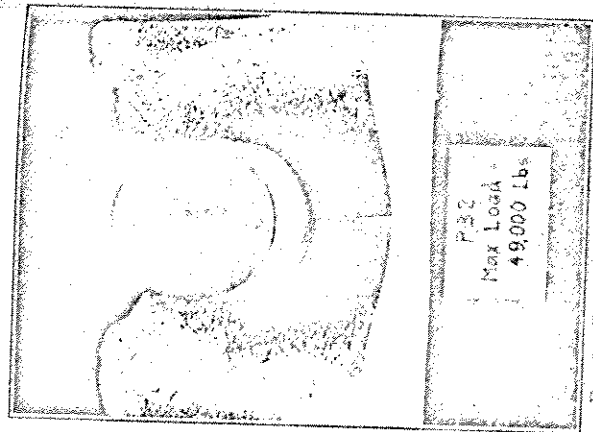


FIG. 9.—TEST NO. 32 AFTER FRACTURE BEHIND THE PIN

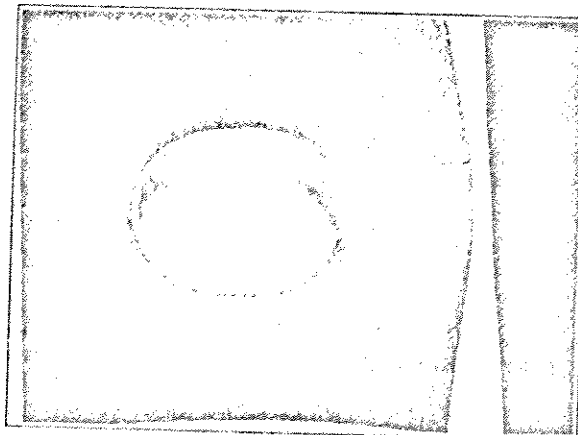


FIG. 10.—TEST NO. 64 AFTER PARTIAL FAILURE BEHIND THE PIN

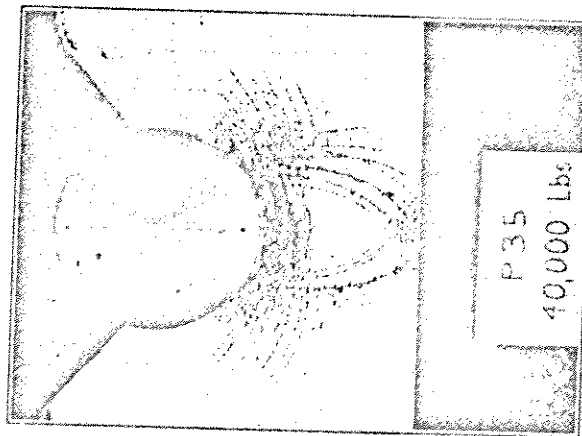


FIG. 11.—TEST NO. 35 AT 40 KIPS AND JUST PRIOR TO "DISHING"

Plate No. 35, similar to Plate No. 32, but of more balanced design. This plate was the same in every respect as Plate No. 32 with the exception of greater edge distance, *a*, with the result that considerable local crushing below the pin developed before the final spread of the slip-lines. The slip-lines indicated in Fig. 11 were duplicated on both sides of the plate. At a load of 41.5 kips, the plate "dished" causing a slip-line pattern similar to that in Fig. 5 to be superposed on the convex side. Unlike Test No. 5, however, Plate No. 35 was thick enough to develop considerable reserve strength between the general yield stress and ultimate failure by "dishing."

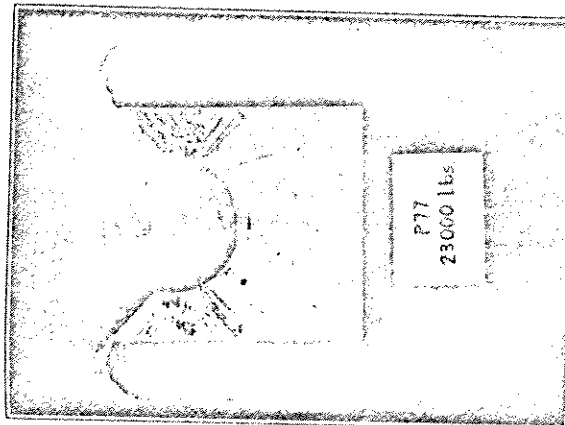


FIG. 12.—TEST NO. 77 AT 23 KIPS, SHOWING YIELDING AT THE SIDE

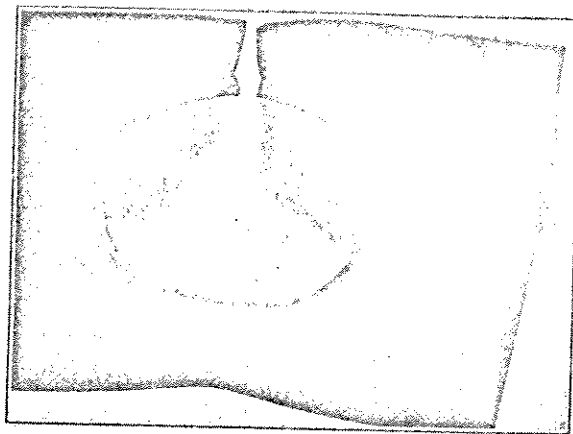


FIG. 13.—TEST NO. 82 AFTER FRACTURE AT SIDE OF PINHOLE

With one exception the series of plates which were 6 in. wide failed either in the net section in tension, or by "dishing." Fig. 12 shows the slip-line pattern soon after the general yield point of a 6-in. by $\frac{1}{2}$ -in. plate prior to ultimate failure by "dishing," and Fig. 13 shows the side fracture, at ultimate strength of a 6-in. by $\frac{1}{2}$ -in. plate.

Stress Distribution.—The stress distribution in the plate around the pinhole is complex, and may best be studied within the elastic range by means of photo-elastic tests. Complete studies of the stress pattern and distribution in a plate around a pinhole have been made by previous investigators.*

The maximum stresses are the localized contact compressive stresses at the point of bearing of pin on plate, and the concentration of tensile stress at each side of the pinhole. Except in the case of tight-fitting pins, the localized contact stresses produce the first surface evidence of local yielding. For a discussion of the subject of contact stresses reference may be made to an in-

*Journal, Franklin Inst., Vol. 199, 1925, p. 289; and Bulletin, Aero. Research Inst., Tokyo Univ., September 12, 1926.

vestigation of stresses under bridge rollers reported¹ in 1936, by V. P. Jensen, Assoc. M. Am. Soc. C. E.

Insufficient clearance around the pin made it impossible to place tensometers for measuring strains at the exact locations of maximum stress. The tensometers were useful in all tests, however, in detecting initial stress redistribution which indirectly determined loads at which local yielding had occurred at points of maximum stress. Additional tensometers were used in some of the tests to determine the strain distribution along vertical and horizontal lines (see Fig. 2). The results are indicated in Fig. 14 for six tests on 8-in. by 0.5-in. plates with a varying distance, a , and two different pin clearances. The values indicated in Fig. 14 are the products of the strain in one direction multiplied by the modulus of elasticity, and are not true stresses, although very nearly so along Line 1-1, where the normal horizontal stresses are negligible. The maximum value of the vertical strain times the modulus of elasticity, E , normal to Section 1-1 adjacent to the pin was determined indirectly by extending from the measured values a stress area that would be in equilibrium with the known total load on the section. The contact stresses below the pin vary rapidly and could not have been measured satisfactorily over the half-inch gage length available in a Hugenberger tensometer, even if clearance had been available. The strains normal to the vertical section, 2-2, were measured at as many points as possible and the products of these strains and the modulus of elasticity, E , are also indicated in Fig. 14. The true stresses normal to Section 2-2 in the vicinity of the pin are considerable larger than the values of strain times the modulus of elasticity, E , because of the unmeasured contact stresses.

It is important to note that considerable local yielding, either below or to the side of the pinhole, was always evidenced before the general yield point of the plate was noted, and that stress concentrations had little effect upon the ultimate strength.

Pin Clearance.—A special series of tests was made in which clearance between pin and pinhole was the only variable. The load-deformation curves were extended far beyond the general yield point in these tests and three typical curves are shown in Fig. 15. The results of this series of tests are discussed in the general "Summary of Test Results" which follows.

Summary of Test Results.—Table 1 presents an abbreviated summary of the results of all the tests. A detailed explanation of the various columns in this table follows: Columns (1), (2), (3), and (4) give measured dimensions of the specimens before test, at the end near the observed pinhole, based on the average value of micrometer readings made to the nearest 0.001 in.

Column (5) indicates the load at which the first surface slip-line was noted, together with its location and type. The first letter, B or E , indicates whether the line was below or to one side of the pin. The second letter (S or C) indicates whether the flaking was a definite line, indicating slip or shear along a certain path, or general flaking below the pin, indicative of compression yield by crushing (see Fig. 11 for illustrations of both types). In this column, of which only a qualitative interpretation may be made, the trend of the lowest

¹ Bulletin, Iowa Eng. Soc., Vol. XI, No. 4, October, 1936, p. 9.

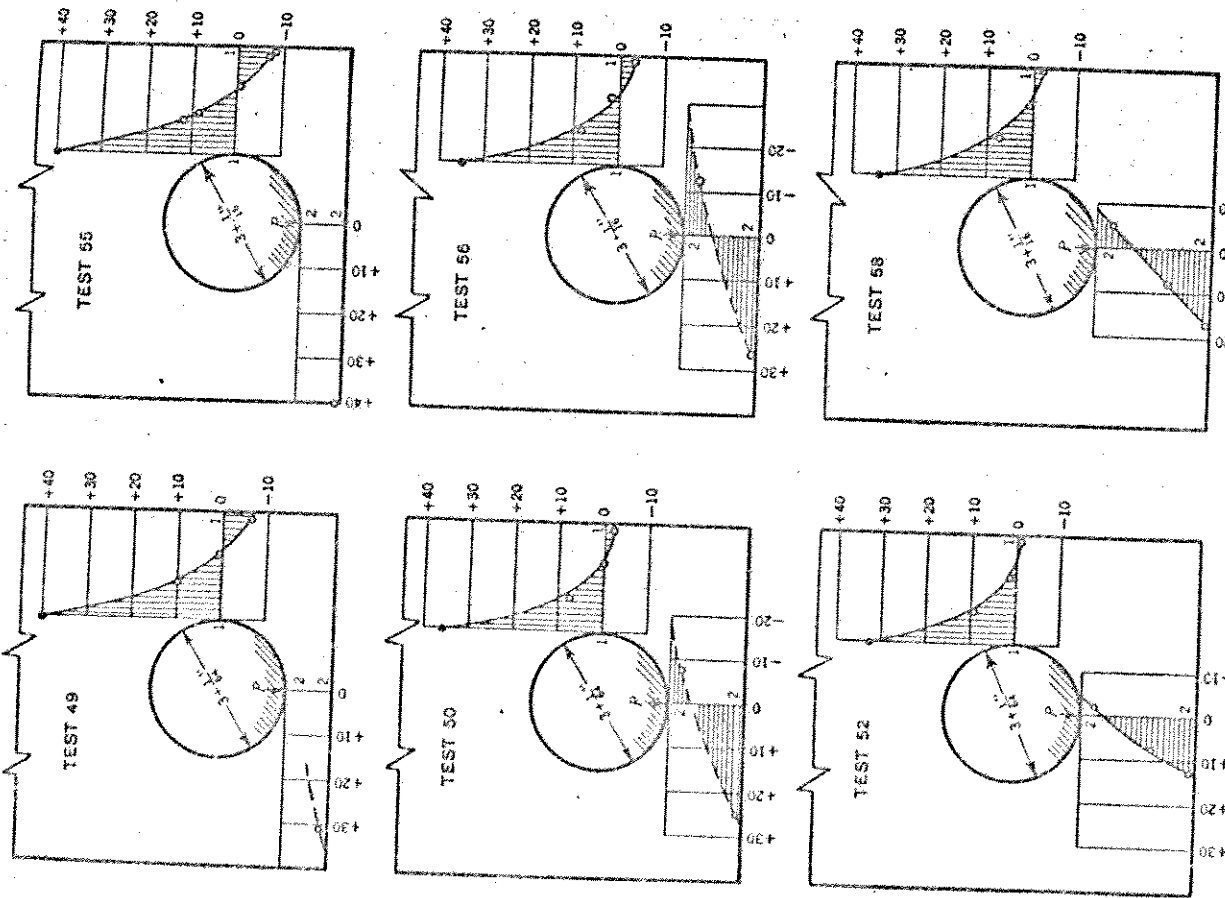


FIG. 14.—STRAIN TIME MODULUS OF ELASTICITY NORMAL TO THE HORIZONTAL AND VERTICAL LINES BY TENSOMETERS ($P = 20$ KIIPS)

noted loads is nearly correct, since the whitewash at times had enough cohesion to remain in place after yielding had commenced.

Column (6), Table 1, gives the load at which surface indication of failure had become general. By this is meant either the presence of several lines extending from the pinhole through to the end or side of the plate, or a general network of lines developed in the vicinity of the pinhole. The same qualifica-

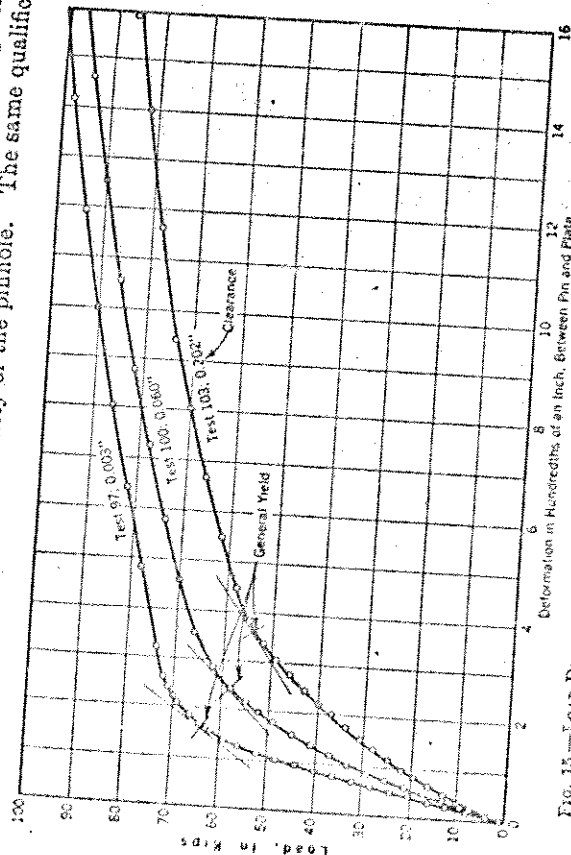


FIG. 15.—LOAD-DEFORMATION CURVES WITH PIN CLEARANCE AS THE ONLY VARIABLE

ns must be made regarding interpretation as in the case of Column (5). Column (7) gives the first indication of general stress redistribution due to axial yielding, either to one side (*E*) or below (*B*) the pinhole. The noted in each case is the limit of proportionality of the tensometer readings plotted against load.

Column (8), Table 1, gives the load at the arbitrarily determined "general strength" at which point the longitudinal deformation between pin and plate was progressing at three times the initial rate; Column (9) indicates load at the start of "dishing," as detected by the limit of proportionality of the readings of the lateral dials, plotted against load; Column (10) gives the maximum load either at fracture or "dishing," in the case, just prior to large falling off of load during the progress of "dishing." In Tests Nos. 49 to 106, the set readings were taken at five or more different intervals to give a complete curve of set plotted against load. These are omitted to conserve space but the permanent set in all tests followed by the curve indicated in Fig. 19(b).

Graphical Summary of Test Results.—Average bearing stress at the general point, and the ultimate strength for the plates 8 in. wide (Tests Nos. 1 to 16) are plotted in Figs. 15(c) to 16(d), with the ratio, $\frac{c}{D_p}$, as abscissa. Figs.

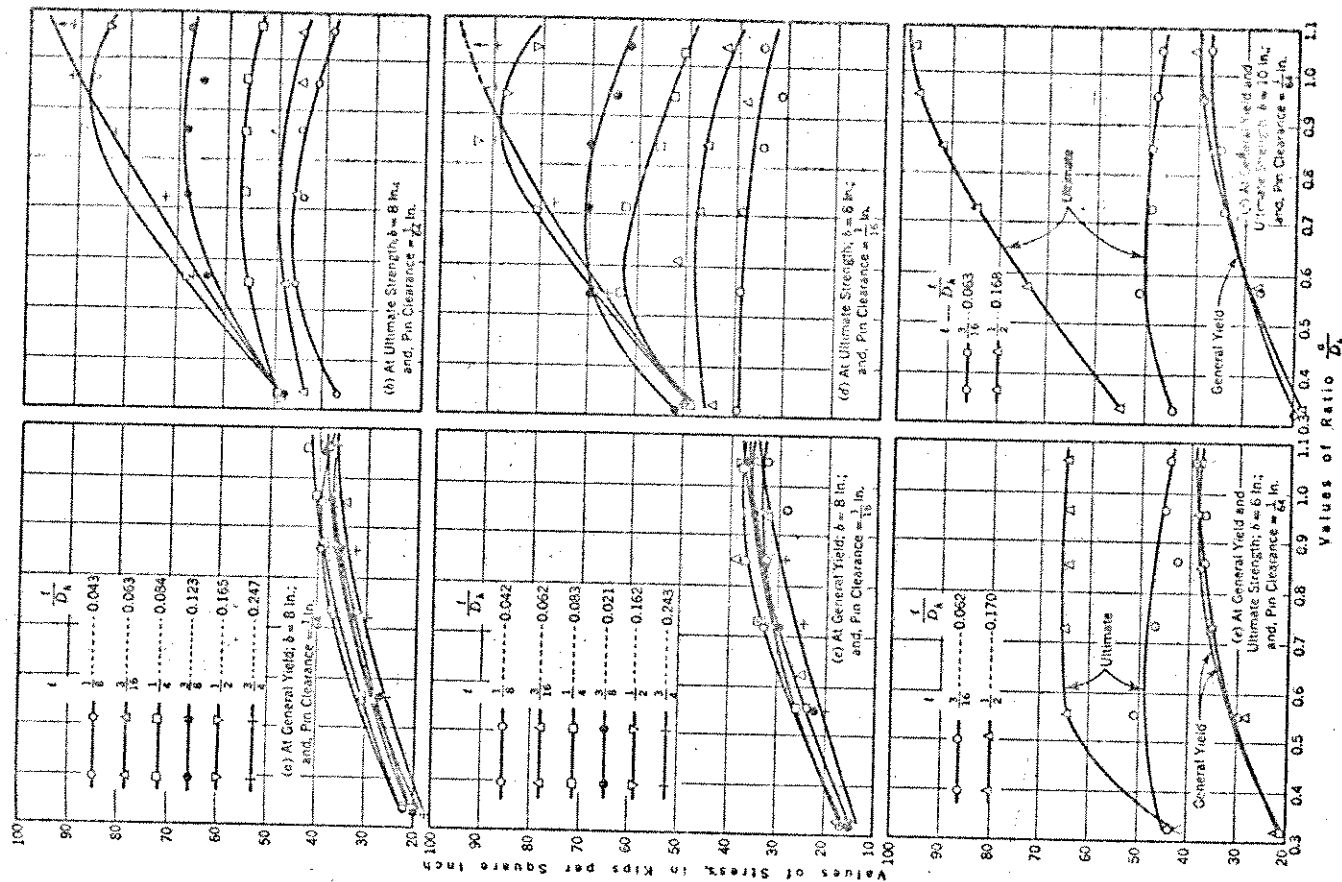


FIG. 16.—AVERAGE BEARING STRESSES CORRESPONDING TO VALUES OF THE RATIO, $\frac{c}{D_p}$.

Fig. 15(e) and 16(f) give the same results for the 6-in. and 10-in. plates, Tests Nos. 73 to 96, inclusive.

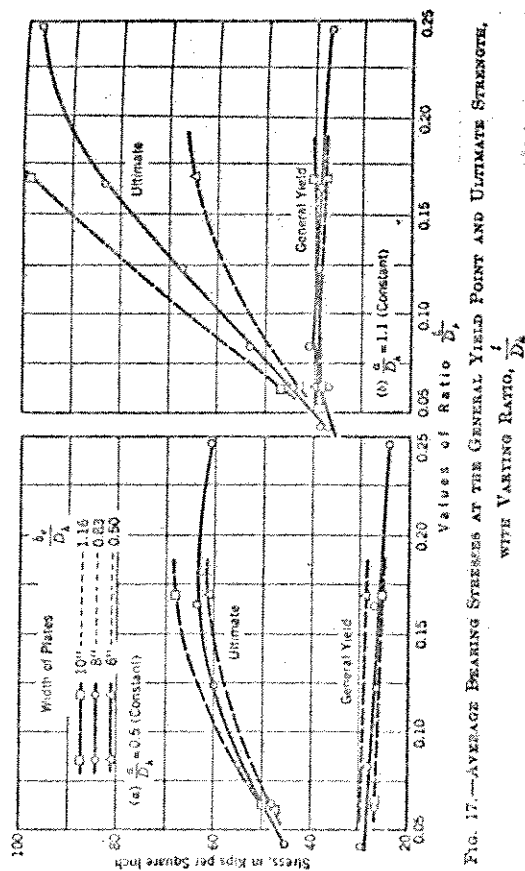


FIG. 17.—AVERAGE BEARING STRESSES AT THE GENERAL YIELD POINT AND ULTIMATE STRENGTH, WITH VARYING RATIO, $\frac{D_A}{t}$.

Fig. 17 gives average bearing stresses plotted against the variable thickness ratio, $\frac{D_A}{t}$, for Tests Nos. 1 to 96, Table 1, and for two different ratios, $\frac{c}{D_A}$, of 0.5 and 1.1. The values plotted were taken from the curves in Fig. 16.

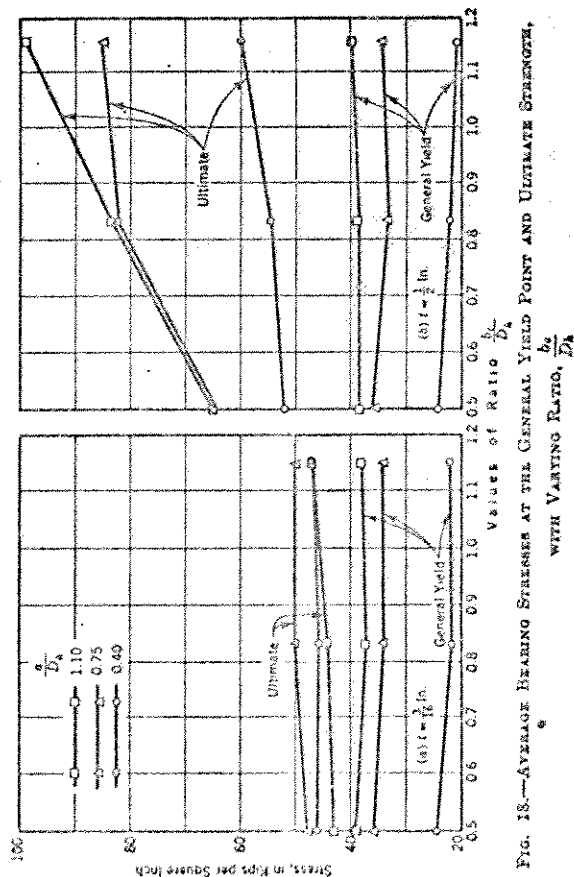


FIG. 18.—AVERAGE BEARING STRESSES AT THE GENERAL YIELD POINT AND ULTIMATE STRENGTH, WITH VARYING RATIO, $\frac{b_A}{D_A}$.

In Fig. 18 the average bearing stress is plotted against the ratio, $\frac{b_A}{D_A}$, as abscissa for all the tests of $\frac{1}{16}$ -in. and $\frac{1}{8}$ -in. plates.

Fig. 19(a) gives the average bearing stress at the general yield point and ultimate strengths for Tests Nos. 97 to 106 (Table 1), in which the pin clear-

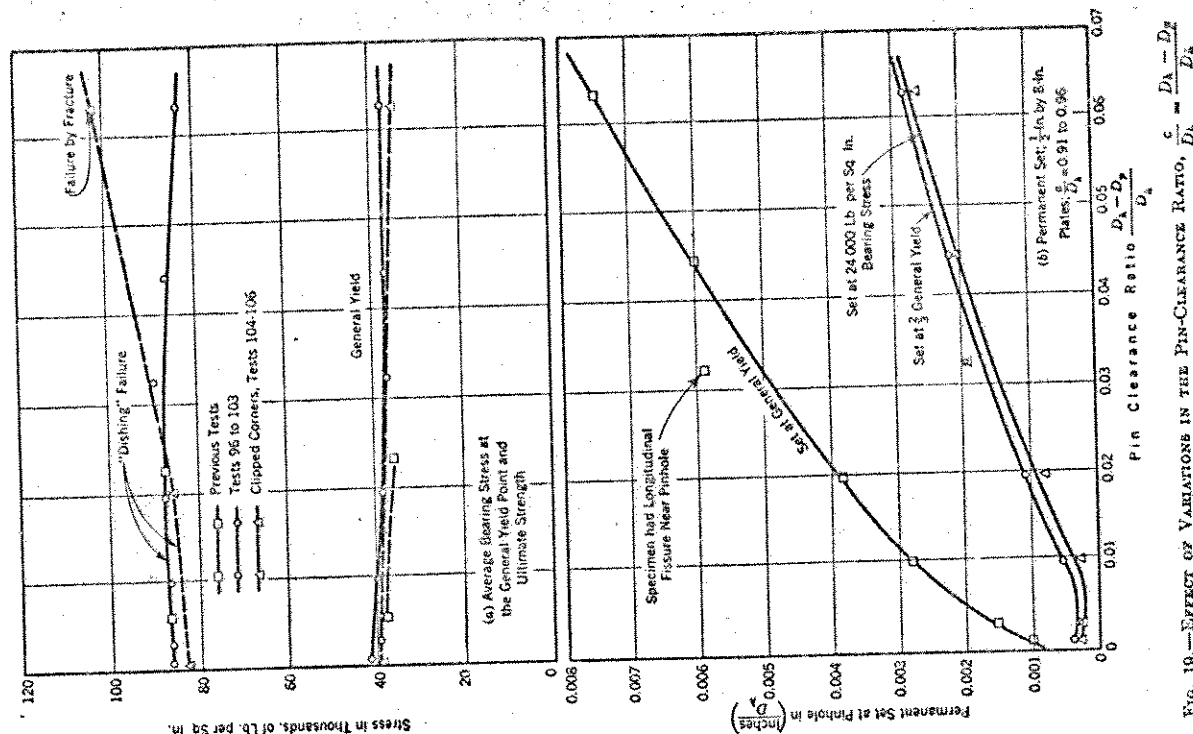


FIG. 19.—EFFECT OF VARIATIONS IN THE PIN-CLEARANCE RATIO, $\frac{c}{D_A} = \frac{D_A - D_g}{D_A}$.

ances are the variable, and are plotted as abscissa. Fig. 19(b) shows the relation between permanent set and variable pin clearance for the same series at different bearing stresses and at the general yield.

RELATION OF TEST RESULTS TO DESIGN

This paper includes no recommendations as to proper working stresses, factors of safety, or allowable permanent sets, which are properly the subject of specifications set up by engineering groups. In the following text, empirical formulas are presented for predicting the average bearing stress at the general yield and ultimate strength of pin-connected plate links similar to those tested. In order to study the four variable dimension ratios which affect the strength of the plates in the regions of greatest variation, many of the test pieces have dimensions outside the usual range of design. As possible design proportions are approached, however, the test results show a uniformity of trend which warrants the application of the formulas to all usual design proportions. Definite limits within which the formulas are applicable are noted in connection with each. The formulas agree closely with the average test results; and, in case of variation of more than a small percentage, they are adjusted to agree with the lowest trend of test results. In applying these formulas to design, these further restrictions are emphasized:

- (1) Material should be structural grade steel for bridge work as covered by standard specifications.*
- (2) Loads are static and pins do not rotate. For repeated stress conditions or for rotating pins appropriate allowances must be made.
- (3) No rivet or bolt-holes may be drilled or punched in the vicinity of the pinhole. Previous tests² have noted the weakening effect of such small holes, either below or above the pinhole.
- (4) The body of the plate shall have the same dimensions as the gross section at the pinhole.

Notation.—Reference should be made to Fig. 1 for the notation in regard to dimensions of plate. Ratios to which reference has already been made are:

$$\frac{t}{D_A} = \frac{\text{Plate thickness}}{\text{Pinhole diameter}}; \frac{b_c}{D_A} = \frac{\text{Edge distance to side of pinhole}}{\text{Pinhole diameter}}; \frac{a}{D_A} = \frac{\text{Edge distance below pinhole}}{\text{Pinhole diameter}}; \frac{c}{D_A} = \frac{D_A - D_P}{D_A} = \frac{\text{Pin clearance}}{\text{Pinhole diameter}}$$

The unit stresses are: s_{gy} = average bearing stress between pin and plate at general yield of plate; s_u = average bearing stress between pin and plate at ultimate strength of plate; s_y = tensile yield point of material as determined by standard tension test; and s_t = ultimate tensile strength of material as determined by standard tension test.

General Yield Point.—The average bearing stress at the general yield point is not appreciably influenced by plate thickness ratios, $\frac{t}{D_A}$ greater than $\frac{1}{20}$, nor by the edge distance ratio, $\frac{b_c}{D_A}$, for values greater than $\frac{1}{2}$. The test data do not

give results for $\frac{b_c}{D_A}$ greater than 1.2, but the bearing stress in such cases at the

* Specification A7-34, Am. Soc. for Testing Materials.

general yield point is probably not influenced greatly for values of $\frac{b_c}{D_A}$ within the usual range of design. The clearance between pin and pinhole, as defined by the ratio, $\frac{c}{D_A}$, has a minor effect on the general yield point, as indicated in Fig. 10(a). The general yield point of the plate is induced principally by a yielding of the material behind the pin. This yielding is preceded by a crushing failure below the pin which progresses until all the metal behind it starts to give way in a plastic state. In very thin plates the general yield is followed quickly by "dishing" and ultimate failure, whereas in thicker plates there is a large plastic reserve before ultimate failure occurs.

The following formula gives the average bearing stress at the general yield point:

$$s_{gy} = \frac{s_y}{2} \left[3 \left(\frac{a}{D_A} \right) - \left(\frac{a}{D_A} \right)^2 - 2 \left(\frac{c}{D_A} \right) \right] \dots \dots \dots (1)$$

Equation (1) applies within the following limits:

$\frac{a}{D_A}$ between 0.3 and 1.2

$\frac{c}{D_A}$ between 0 and 0.07

$\frac{b_c}{D_A}$ greater than 0.5 (otherwise general yield will be entirely in net section)

$\frac{t}{D_A}$ greater than 0.05 (otherwise ultimate failure by "dishing" may precede any yielding)

Equation (1) is based on the test average of s_y equal to 39 kips per sq in. Based on the minimum specification requirement of 33 kips per sq in. for structural steel for plates, shapes, and bars in bridges (A. S. T. M. A7-34) and using ratios of $\frac{a}{D_A} = 1.2$ and $\frac{c}{D_A} = \frac{1}{250} = \frac{1}{50}$ (in. clearance for a pin 5 in. in diameter), by Equation (1), s_y is 35.4 kips per sq in. If a factor of safety of 1.5 were suitable with respect to the general yield point, the allowable working stress would be 23.6 kips per sq in., which corresponds closely with the frequently specified allowable stress of 24 kips per sq in. If such a working stress is suitable, it should only be used when the plate is thick enough to prevent ultimate failure at a low load by "dishing." The tests will be examined later to develop criteria of proportions which will insure against "dishing" failure and at the same time give a balanced design.

Ultimate Failure.—The three general types of ultimate failure have been described previously (see heading, "Test Results: Types of Failure"). Three empirical formulas (in close agreement with test results) will be presented to serve as indices of the average bearing stress at the ultimate strength in the case of failure by any of the three types. Applied to a plate of any proportions,

the lowest value obtained from the three equations indicates the expected ultimate strength and the type of failure. All the equations are written in terms of average bearing stress between pin and plate, as follows:

Failure by fracture at side of pinhole,

$$s_{st} = 2s_t \left(\frac{b_e}{D_h} \right) \dots \dots \dots (2)$$

Failure by fracture below pinhole,

$$s_{st} = s_t \left[1.13 \left(\frac{a}{D_h} \right) + \frac{0.92 \left(\frac{b_e}{D_h} \right)}{1 + \left(\frac{b_e}{D_h} \right)} \right] \dots \dots \dots (3)$$

Failure by "dishing,"

$$s_{st} = 20 + 315 \left(\frac{t}{D_h} \right) + 75 \left(\frac{b_e}{D_h^2} \right) + 20 \left(\frac{a}{D_h} \right) - 20 \left(\frac{a}{D_h} \right)^2 \dots \dots (4)$$

In correlating Equations (2) and (3) with the test results, the test coupon average of $s_t = 62$ kips per sq in. applies. In applications to design the minimum specification requirement for s_t should be used. The following limitations as to dimensions are made: $\frac{a}{D_h}$ between 0.3 and 1.2; and, $\frac{c}{D_h}$ between 0 and 0.07.

The first limitation automatically limits the other dimensions (whenever they are determining factors) to values close to the test range. The second limitation indicates the test range but does not enter the equations because the clearance was not found to influence the ultimate strength appreciably. Although $\frac{c}{D_h}$ varied from 0.325 to 1.05 in the tests, the extension to 1.2 is based on the fact that the test results approached a uniform trend in the upper limit of $\frac{a}{D_h}$. Further remarks on each type of failure follow.

Failure by Fracture at Side of Pinhole.—This type of failure occurred only in the 6-in. plates. The test results showed that the material in the net section did not fail until the average stress in that section reached the ultimate tensile strength of the material as determined by a standard tension test. This indicates that the stress concentrations near a circular hole, which exist in the elastic range, have no effect on the ultimate strength of structural steel under static loads.

Equation (2) simply states that if failure is by tension in the net section, the average stress through this section will be the same at failure as in the case of a standard tension test. The stress actually developed, based on original areas and a test average of $s_t = 62$ kips per sq in., was slightly higher than that determined by Equation (2), due probably to the fact that the lower stressed material above and below the minimum section did not allow so much "necking down" and reduction of area as in the case of the straight tensile coupons.

Ultimate Failure Below Pinhole.—A study of the plates that failed in this manner shows that the ultimate strength of the plate is dependent both on the

material below and to the side of the pinhole, the results agreeing with Equation (3) for $s_t = 62$ kips per sq in.

Ultimate Failure by "Dishing."—Equation (4) is for plates unsupported laterally at the ends. If the plate is blocked securely against "dishing" at the end, either Equation (2) or Equation (3) may determine the ultimate strength. (Although no blocked plates were tested in the present investigation, this statement is verified by tests made on the pin-plates of the Quebec Bridge (see Table 3)). Equation (4) is not written in terms of s_t , as are Equations (2) and (3), because the "dishing" is primarily a function of stability rather than strength.

TABLE 3.—COMPARISON OF TEST DATA, MAUMEE BRIDGE AND QUEBEC BRIDGE

DIMENSIONS, IN INCHES				RATIOS		BEARING STRESSES, IN KIIPS PER SQUARE INCH	
Diam- eter of hole, D_h	Width of Plate b		Thick- ness, t	Edge Clearance		At failure*	Predicted Average Ultimate Strengths
	At pin	Body		At bot- tom, a	At sides, b_s		
(c) MAUMEE BRIDGE							
8.00	22.0	10.0	1.375	7.00	7.00	68.6 D, FB	87.8
8.00	22.0	10.0	1.375	7.00	7.00	61.4 D, FB	87.8
8.00	20.0	10.0	1.375	7.00	6.00	69.2 D, FB	86.1
8.00	20.0	10.0	1.375	7.00	6.00	67.1 D, FB	86.1
8.00	22.0	10.0	1.375	8.00	7.00	75.0 D, FB	86.6
8.00	22.0	10.0	1.375	8.00	7.00	77.3 D, FB	86.6
8.00	22.0	14.0	1.375	8.00	7.00	80.5 D, FB	86.6
8.00	22.0	14.0	1.375	8.00	7.00	79.5 D, FB	86.6
(b) QUEBEC BRIDGE							
5.03	13.0	13.0	0.375	3.75	4.00	57.9 D	51.2
5.03	13.0	13.0	0.375	3.75	4.00	77.4 FB	51.2
5.03	11.0	11.0	0.375	2.75	3.00	52.8 D	40.8
5.03	9.0	9.0	0.375	2.75	2.00	52.8 D	40.7
5.03	9.0	9.0	0.375	3.75	2.00	52.8 D	40.7
12.14	28.3	28.3	2.000	12.1	8.08	72.2 FB	82.3
12.14	28.3	28.3	2.000	12.1	8.08	78.4 FB	82.3
12.14	26.3	26.3	1.500	10.5	7.10	70.5 D, FE	82.7
12.14	26.3	26.3	1.500	10.5	7.10	72.4 D, FB	82.7

* D = "dishing"; F = fracture; B = below pin; and, E = side of pin.

† End of plate blocked to prevent "dishing."

Balanced Design.—Assuming that the plate is thick enough, or that it is blocked laterally, so as to prevent "dishing," a comparison of Equations (2) and (3) yields the following criteria for equal strength or balanced design with respect to the edge distances to the side and below the pin:

$$a = \frac{1 + 1.75 \left(\frac{b_e}{D_h} \right)}{b_e} \dots \dots \dots (5)$$

Equation (5) gives a slight excess of a distance.

What plate thickness is required to prevent "dishing"? Assuming that the design is balanced in other respects according to Equation (5), the following thickness ratio, obtained from a comparison of Equations (2) and (4), gives the required minimum allowable thickness ratio, $\frac{t}{D_h}$:

$$\frac{t}{D_h} = \frac{-20 + 123\left(\frac{b_c}{D_h}\right) - 20\left(\frac{a}{D_h}\right) + 20\left(\frac{a}{D_h}\right)^2}{315 + 75\left(\frac{b_c}{D_h}\right)} \dots\dots\dots (6)$$

Although Equation (6) gives sufficient thickness to prevent "dishing" in an otherwise balanced design, it also gives more than sufficient thickness for plates which are not balanced according to Equation (5). The "dishing" is more nearly a direct function of the ratio, $\frac{a}{t}$, than $\left(\frac{t}{D_h}\right)$, and Fig. 20 shows the maximum allowable ratio, $\frac{a}{t}$, to prevent "dishing" for plates balanced in design according to Equations (5) and (6).

In general, the tests indicate that the most efficiently designed pin-plate connectors are those narrow in width, thick enough to prevent "dishing," and having material below the pinhole in the proportions specified by Equation (5).

Comparison with Previous Tests.—The results of two previous series of tests on pin-connected plates of larger dimensions than those of the present investigation are presented in Table 3. The second group of ten tests is taken from the report on the Quebec Bridge. Six of these tests were models, using pins 5 in. in diameter, and the remaining four were full-sized hangers with 12-in. pins, and plate widths of 26 in. or 28 in. The results of these tests check well with the strengths predicted by Equations (2) to (4), only two of them falling below, with a maximum difference of about 4%, and the remainder above, by as much as 10% in some instances. The material used in these pin-plates had slightly higher ultimate strength than that used in the present study.

The results of tests of eight pin-connected plates to determine the best design for anchorage bars for the Maumee River Suspension Bridge are also presented in Table 3. These bars had a reduced width in the main section between pinholes. All the test strengths of these bars fall below the values

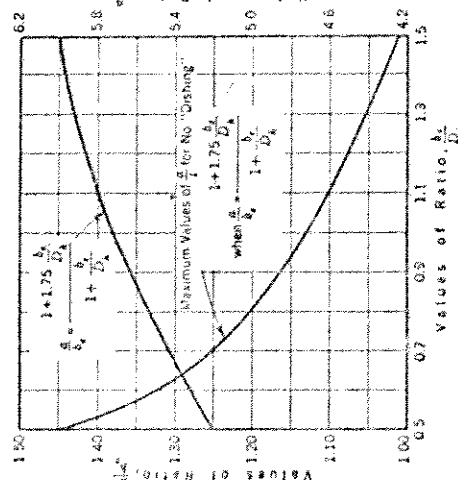


FIG. 20.—BALANCED DESIGN OF ULTIMATE STRENGTH

given by Equation (2), Equation (3), or Equation (4). Apparently, the reduced width in the body of the bar increased the tendency toward "dishing." It and reduced their strength, although this may not necessarily be the cause. It is to be noted that the two tests with largest plate width in the body are only 7% less than the strengths indicated by Equation (5). Fracture failures below the pin were reported in these tests but this was combined at the same time, with "dishing." Since the exact dimensions of the specimens are not known and their number is limited, no final conclusion can be given in connection with these tests.

Illustrative Example.—Assume that clearance requirements make it necessary to use a pin-plate 1 in. thick to carry a load of 200 kips at a working stress in bearing of 24 kips per sq in. (A. S. T. M. Specification A7-34). The required pin diameter at 24 kips per sq in. = $\frac{200}{1 \times 24} = 8.33$ in., which dictates the use of a pin with a diameter of 8.5 in. The required net edge-distance at the pinhole, at 18 kips per sq in., is $b_c = \frac{200}{2 \times 1 \times 18} = 5.55$ in. With $b_c = 6$ in., this makes a total width of 20.5 in. required, and the ratio, $\frac{b_c}{D_h} = \frac{6}{8.5} = 0.706$. From Fig. 20, for balanced design: $\frac{a}{D_h} = 0.75 \times 1.31 = 0.985$, from which $a = 0.985 \times 8.5 = 8.4$ in. (use $a = 9$ in.). By actual measurement, $\frac{a}{D_h} = 1.06$; $\frac{t}{D_h} = \frac{1}{8.5} = 0.118$; and, $\frac{a}{t} = \frac{1.06}{0.118} = 9.0$, which is more than the 5.27 allowable maximum for no "dishing," for the ratio, $\frac{a}{D_h} = 1.06$, from Fig. 20. The plate, therefore, will fail by "dishing." By Equation (4), the average bearing stress at ultimate strength is, $s_u = 20 + 315 \times 0.118 + 75 \times 0.118 \times 0.75 + (1.06 - 1.06^3) 20 = 62.63$ kips per sq in. The ultimate strength is $62.63 \times 8.5 \times 1 = 532$ kips; and, the safety factor with respect to the ultimate = $\frac{532}{200} = 2.66$.

As an alternative, suppose that the plate is securely blocked to prevent "dishing." The ultimate strength will then be given either by Equation (2) for side failure, or by Equation (3) for end failure. For $s_t = 60$, the minimum by A. S. T. M. Specification A7-34, Equation (2) gives $s_u = 120 \times 0.75 = 90.0$ kips per sq in.; and, by Equation (3), $s_u = 60 \left[\frac{1.13 \times 1.06 + \frac{0.92 \times 0.75}{1 + 0.75}}{1} \right] = 96$ kips per sq in. The ultimate strength, then, if the plate is blocked = $90 \times 8.5 \times 1 = 765$ kips; and the factor of safety with respect to the ultimate = $\frac{765}{200} = 3.82$.

General Yield Point of Plate.—If a pin clearance is $\frac{1}{32}$ in., the pin-clearance ratio, $\frac{b_c}{D_h} = \frac{1}{32 \times 8.5} = 0.00367$. By Equation (1), the average bearing stress

at the yield point will be: $s_y = 33 \left[\frac{3 \times 1.06 - 1.06^2 - 2 \times 0.00367}{2} \right] = 32.8$ kips per sq in.; the total strength at the yield point = $32.8 \times 3.5 \times 1 = 283$ kips; the safety factor with respect to the yield point = 1.44 ; and the permanent set for a stress of 24 kips per sq in. (from Fig. 19(b)) = $0.0025 \times 8.5 = 0.002$ in.

It should be noted that the general yield point may be raised by arbitrarily increasing the end ratio, $\frac{b_c}{D_h}$, but this will also increase the tendency to "dish." In cases where "dishing" is eliminated by suitable plate thickness or by side blocking, a low factor of safety with respect to the general yield point may be justified, in the opinion of the engineer, due to the high plastic reserve strength prior to ultimate failure.

Clipped Corners.—As indicated by previous tests and a very limited number of the present tests (three in all), the corners of the plate below the pin may be clipped with only slight loss of strength. It is recommended that 20% more net section be maintained on a 45° line through the corner than the minimum at the side or below.

SUMMARY

Within certain well-defined limits as to material and shape, this paper has presented empirical equations for predicting the general yield point, ultimate strength, type of failure, and balanced proportions of pin-connected steel plates. These equations agree closely with the 106 tests reported, in which the pin diameter was 3 in., and are further validated by tests on similar pin-connected links used in the erection of the Quebec Bridge in which the pin diameter was as much as 12 in. Although the results do not check well with a number of tests of pin-connected anchorage bars for the Maumee River Suspension Bridge, this may possibly be accounted for by reason of the reduced width in the body of these bars. Hence, the results of this paper are not intended to apply directly to pin-connected plates having the shape of standard eye-bars unless tests on such eye-bars show agreement with the present results.

ACKNOWLEDGMENT

The tests were conducted as a research project of the Department of Civil Engineering, of which James K. Finch, M. Am. Soc. C. E., is Executive Officer. The author is indebted for the close co-operation of the late Albin H. Beyer, M. Am. Soc. C. E., who was Director of the Testing Laboratories, and to the present Director, W. J. Krefeld, M. Am. Soc. C. E. Leon Krantz assisted in setting up the apparatus and operating the tests. The test program was originally suggested by Sterling Johnston, M. Am. Soc. C. E., and was conducted in co-operation with the American Institute of Steel Construction. Engineers of the American Bridge Company, the Bethlehem Steel Company, and the Chicago Bridge and Iron Works, contributed valuable criticism both as to the proper test program and the conclusions to be drawn from the results.

DISCUSSION

HAROLD D. HUSSEY,* M. Am. Soc. C. E. (by letter).—This paper is of much value in its field of structural design and especially in giving an empirical solution to a very complicated problem in buckling of plates; that is, the buckling of plates behind the pin in a pin-connected tension member. The tests which are reported were made on plain plates of uniform width and the author does not claim that his conclusions will apply to members of non-uniform width, such as eye-bars. The writer wishes to call attention, however, to some tests on eye-bars that agree very closely with the results obtained by Mr. Johnston.

When the width of the plate is given, the thickness required to prevent "dishing" can be determined from Equations (5) and (6) as follows: Let

$$\frac{a}{b_c} = M; \text{ and, } \frac{a}{t} = N. \text{ Solving for } t, \\ t = \frac{M}{N} b_c \dots \dots \dots (61)$$

Dividing both sides of Equation (61) by D_h and letting $\frac{t}{D_h} = t_D$; and,

$$\frac{b_c}{D_h} = b_D; \\ t_D = \frac{M}{N} b_D \dots \dots \dots (62)$$

Calculations show that the factor, $\frac{M}{N}$, has an approximate value of $0.3 \sqrt{b_D}$; therefore $t_D = 0.3 b_D \sqrt{b_D}$; and,

$$t = 0.3 b_c \sqrt{\frac{b_c}{D_h}} \dots \dots \dots (63a)$$

Solving Equation (63a) for b_c ,

$$b_c = 2.23 t \sqrt{\frac{D_h}{t}} \dots \dots \dots (63b)$$

The value of a is determined from Equation (5).

The use of Equations (63a), (63b), and (5) will result in a design which has equal resistance to failure by fracture at the side of a pin-hole, fracture below a pin-hole, and "dishing." Specimens that failed by fracture at the side of a pin-hole did so at an average stress in that section equal to, or slightly higher than, the ultimate tensile strength of the material as determined by standard tensile tests. A common specification is that the net section through a pin-hole shall be at least 40% greater than the net section of the member.¹⁰ This shows that standard design requirements are on the side of safety.

In the "Summary," the author states, "the results of this paper are not intended to apply directly to pin-connected plates having the shape of

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¹⁰ A. R. E. A. Specification for Steel Railway Bridges, 1935.

Standard eye-bars unless tests on such eye-bars show agreement with the present results." Records of tests of standard eye-bars which broke in the head are extremely rare because of the fact that eye-bars are usually made to certain minimum requirements, one of which is that the diameter of the pin shall be not less than seven-eighths the width of the bar.

Occasionally, however, eye-bars are made, which vary from standard practice, and they sometimes fail in the head. The latest case of this kind, tested to destruction. The two heads were 16.5 in. in diameter and the pin-holes were 7 in. and 5.25 in. in diameter, respectively. The eye-bar failed by "dishing" and fracture in the head having the smaller pin-hole.

An analysis of this eye-bar head according to Equations (2), (3), and (4) indicates that failure should take place by "dishing." Solving Equation (4), $S_A = 93.8$ kips.

This means that a 16.5-in. rectangular plate with the pin-holes in the two ends should have failed by "dishing" back of the 5.25-in. pin when the bearing stress between the pin and plate reached the foregoing value. The total tension on the plate would then be 492 450 lb. The eye-bar in question failed by "dishing" and fracture back of the 5.25-in. pin under a load of 492 000 lb (reading to the nearest kip), indicating agreement between this eye-bar test and the author's work. This is further verified by the fact that another eye-bar, which was a duplicate of the one noted, had been tested a few months earlier, and the failure had occurred in the body of the bar. This appears contradictory, but when it is noted that this eye-bar failed under a tension of 470 kips it is seen that the material failed just before the head of the eye-bar was about to fail by "dishing." These tests indicate that the author's results will apply to standard eye-bars.

If standard practice had been followed when making these eye-bars the minimum size of the pin would have been $6\frac{1}{8}$ in. in diameter (seven-eighths of 7 in.). A re-test of two more 7-in. by 1-in. eye-bars was made, but instead of using a 5.25-in. pin at one end the pin was made 6.25 in. in diameter in order to determine whether this increase would have any influence on the result. Both these eye-bars broke in the body of the bar, as would be expected from the author's results (a solution of Equation (4), using a 6.25-in. pin, shows that it would require a total tension of 520 kips to cause this head to fail by "dishing," whereas the eye-bars failed under tensions of 474 kips and 470 kips, respectively).

The author refers (in the "Introduction") to "forged" eye-bars. The writer wishes to emphasize that, although the two heads are produced by upsetting (or forging), the main body of the eye-bar is a plain rolled plate, or bar. For example, an eye-bar 65 ft from center to center of end pins has nearly 60 ft of rolled material with 4 ft to 5 ft at each end that has been forged. When a "forging" is specified, the material is manufactured to A.S.T.M. Specification A18, whereas A.S.T.M. Specification A7 gives complete requirements for annealed eye-bars. There is a considerable difference between these two materials. The latter specification is generally used for eye-bars, whereas

the former never is; and when the term, "forged," is introduced, it leads to confusion.

BRUCE G. JOHNSTON, "Assoc. M. Am. Soc. C. E. (by letter).—The discussion presented by Mr. Hussey is entirely constructive in nature. The close agreement between the tests of two eye-bars and the strength and type of failure predicted by Equations (2), (3), and (4) is very gratifying. It is unfortunate that these are the only two tests of eye-bars brought to light in discussion. Equations (63a) and (63b), used with Equation (5), will give a well-balanced design entirely safe with respect to failure by dishing. Equations (2), (3), and (4) should be used when close clearances or other special requirements make a balanced design impossible.

In connection with the application of this paper to the design of eye-bars it has been brought to the writer's attention that Section 411 of the A. R. E. A. Specification for Steel Railway Bridges has been changed to recommend a maximum net width-to-thickness ratio at the pin-hole of 8 instead of 12.

The writer wishes to express his appreciation to Mr. Hussey for his courtesy in discussing the paper.

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