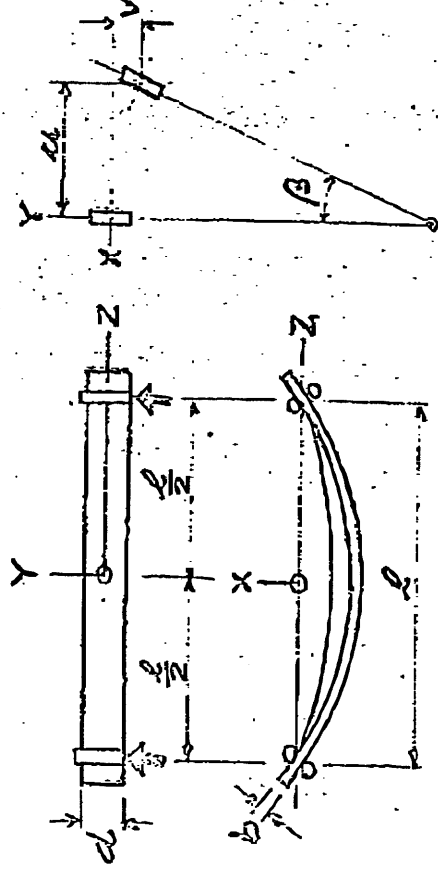


## LATERAL BUCKLING OF BEAMS.

### I. Theory of Buckling. - Simple Beams.



Assumptions:-

Ends of beam can rotate freely about x and y axes, rotation with respect to z axis is prevented. A small lateral buckling has occurred, the critical load is smallest load to keep beam in slightly buckled form.

Fig. 1

1. Critical buckling values obtained by integration of differential equations by infinite series.

a. Narrow rectangular bar subjected to pure bending in its plane.

$$M_{cr} = \frac{\pi}{l} \sqrt{BC} \quad (1)$$

where

$B = EI_y$  Flexural Rigidity

$C = GK$  Torsional Rigidity

$$M_{cr} = \frac{\pi}{l} \sqrt{EI_y GK}$$

$$K = \frac{2}{3} \left( \frac{I_y}{I_x} \right)^{1/3} \quad (2)$$

b. Narrow rectangular bar with load  $P$  applied in the middle, at the centroid of bar.

$$P_{cr} = \frac{16.93}{l^2} \sqrt{BC} \quad (3)$$

with  $M_{cr} = P_{cr} \cdot \frac{l}{4}$

$$M_{cr} = \frac{4.23}{l} \sqrt{BC} \quad (4)$$

2. Critical buckling values obtained by using strain energy method.

a. Narrow rectangular bar with load  $P$  applied in the middle, at the centroid of bar.

### External Work      Internal Work

$$\frac{e}{2} P \int_0^{\frac{L}{2}} \frac{d^2 u}{dz^2} \left( \frac{L}{2} - z \right) dz = \frac{e}{2} \int_0^{\frac{L}{2}} B \left( \frac{d^2 u}{dz^2} \right)^2 dz + \frac{e}{2} \int_0^{\frac{L}{2}} C \left( \frac{d\theta}{dz} \right)^2 dz =$$

Buckling      Bending      Torsion

(5)

where  $\beta = a_1 \cos \frac{\pi z}{L} + a_3 \cos \frac{3\pi z}{L} + a_5 \frac{5\pi z}{L} + \dots$

(6)

Using first term of Eq. 6,  $P_{cr} \sim 1.015 \times P_{cr}$  of Eq. 3  
 using first two terms,  $P_{cr} \sim 1.001 \times P_{cr}$  of Eq. 3

b. I-beam with load  $P$  applied in the middle of the middle of centroid of section.

$$\frac{e}{2} P \int_0^{\frac{L}{2}} \frac{d^2 u}{dz^2} \left( \frac{L}{2} - z \right) dz = \frac{e}{2} \int_0^{\frac{L}{2}} B \left( \frac{d^2 u}{dz^2} \right)^2 dz + \frac{e}{2} \int_0^{\frac{L}{2}} C \left( \frac{d\theta}{dz} \right)^2 dz + \frac{e}{2} \int_0^{\frac{L}{2}} D \left( \frac{d^2 \beta}{dz^2} \right)^2 dz$$

Flange Twist

(7)

where  $D \sim \frac{B}{2} = \frac{EI_y}{2}$  Flexural Rigidity of one Flange

c. I-beam with load  $P$  applied in the middle of the top or the bottom flange.

$$\frac{e}{2} P \int_0^{\frac{L}{2}} \frac{d^2 u}{dz^2} \left( \frac{L}{2} - z \right) dz \pm \frac{Pe}{4} \beta_0^2 = \frac{e}{2} \int_0^{\frac{L}{2}} B \left( \frac{d^2 u}{dz^2} \right)^2 dz + \frac{e}{2} \int_0^{\frac{L}{2}} C \left( \frac{d\theta}{dz} \right)^2 dz + \frac{e}{2} \int_0^{\frac{L}{2}} D \left( \frac{d^2 \beta}{dz^2} \right)^2 dz$$

Rotation

(8)

where  $\beta_0$  = angle of twist at middle of beam

With  $B = EI_y$      $C = \frac{EI_y d^2}{4a^3}$      $D = \frac{EI_y}{2}$

where  $a$  = torsional bending constant

and  $B \frac{d^2 u}{dz^2} = \frac{P}{2} \left( \frac{L}{2} - z \right) \beta$

$$\frac{P^2}{4EI_y} \int_0^{\frac{L}{2}} \beta^2 \left( \frac{L}{2} - z \right)^2 dz \pm \frac{Pe}{4} \beta_0^2 = \frac{e}{2} \int_0^{\frac{L}{2}} B \left( \frac{d^2 u}{dz^2} \right)^2 dz - \frac{e}{4} \int_0^{\frac{L}{2}} \frac{d^2 \beta}{dz^2} \left( \frac{d^2 u}{dz^2} \right)^2 dz = 0$$

(9)

solved for  $P$ , and with solutions for integrals

$P_{cr} = \frac{8EI_y d^2}{L^3} k$  where  $k$  is a coefficient containing the expression  $\frac{L}{a}$

With  $for = P_{cr} \cdot \frac{L}{4} \cdot \frac{d^2 \beta}{dz^2}$

See sh 4 #3 for  $k$  values

$\int_0^{\frac{L}{2}} \frac{I_y}{I_x} \left( \frac{d^2 \beta}{dz^2} \right)^2 dz$

General expression for critical axial stress.

## 3

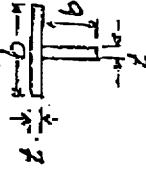
### II. Buckling Formulas. - Simple Beams.

Buckling formulas contain either the torsion constant  $I_T$ , the torsional rigidity  $C$ , or the torsional bending constant  $\alpha$ .  $I_T = M_T / \theta G$ , and may be approximated by assuming the beams to be composed of rectangles, taking the sum of the terms  $b t^{3/3}$  of each rectangle, in which  $b$  equals the width, and  $t$  the thickness of the rectangle, both in inches.

$$C = G K$$

$$\alpha = \frac{E I_T d^2}{4 C} = \frac{E I_T d^2}{4 G K}$$

$I_T$  and  $\alpha$  values for Bethlehem Wide-Flange Beams and American Standard Beams are given in Bethlehem Steel Co Booklet S-57, Torsional Stresses in Structural Beams.



Eq. 10 is analogous to Euler's formula for buckling of compressed struts, correct only so long as  $f < E$  elastic limit.

#### 1. Value of $k$ in Eq. 10 for Pure Bending

$$k = \frac{\pi}{4} \sqrt{\frac{I^2}{\alpha^2} + \pi^2}$$

#### 2. Values of $k$ in Eq. 10 for Uniform and Concentrated Loads obtained by using first two terms of Eq. 6. (Approx.)

Uniformly distributed load

$$\text{on top flange} \quad k = 1.2367 \left[ \sqrt{0.5115 \frac{I^2}{\alpha^2} + 5.990} - 1 \right]$$

$$\text{at centroid} \quad k = 0.8851 \left[ \sqrt{\frac{I^2}{\alpha^2} + 9.796} + 0.0253 \right]$$

$$\text{on bottom flange} \quad k = 1.3061 \left[ \sqrt{0.4573 \frac{I^2}{\alpha^2} + 5.426} + 1 \right]$$

Concentrated load at middle

$$\text{on top flange} \quad k = 1.7954 \left[ \sqrt{0.3444 \frac{I^2}{\alpha^2} + 4.413} - 1 \right]$$

$$\text{at centroid} \quad k = 1.0557 \left[ \sqrt{\frac{I^2}{\alpha^2} + 9.376} + 0.1146 \right]$$

$$\text{on bottom flange} \quad k = 2.0154 \left[ \sqrt{0.2763 \frac{I^2}{\alpha^2} + 3.355} + 1 \right]$$

1592  
08321592  
0832

3. Values of  $k$  in Eq. 10, computed by using first two terms of Eq. 6.

$\frac{L}{a}$	Uniform Load			Concentrated Load			Pure Bending
	Top Flange	Centroid	Bottom Flange	Top Flange	Centroid	Bottom Flange	
2	2.269	3.310	4.824	2.525	3.902	6.272	2.925
3	2.788	3.860	5.341	3.127	4.649	6.885	3.412
4	3.418	4.518	5.969	3.863	5.440	7.632	3.995
5	4.121	5.244	6.669	4.687	6.310	8.469	4.638
6	4.872	6.013	7.417	5.568	7.232	9.365	5.319
7	5.655	6.810	8.197	6.490	8.187	10.30	6.026
8	6.459	7.626	8.999	7.349	9.164	11.26	6.750
9	7.280	8.456	9.817	8.409	10.16	12.24	7.487
10	8.113	9.296	10.65	9.393	11.16	13.24	8.232
11	8.954	10.14	11.49	10.39	12.18	14.24	8.985
12	9.802	11.00	12.33	11.39	13.20	15.26	9.742
13	10.65	11.85	13.18	12.41	14.22	16.28	10.504
14	11.51	12.72	14.04	13.43	15.25	17.30	11.269

Table 1

#### 4. Buckling Formula for Pure Bending

$$for = \frac{I_x}{I_x} \frac{d^2}{L^2} E \frac{\pi}{4} \sqrt{\frac{L^2}{a^2} + \pi^2}$$

$$with \frac{L^2}{a^2} = \frac{4GK L^2}{E I_y d^2}$$

$$for = \frac{I_x}{I_x} \frac{d^2}{L^2} E \frac{\pi}{4} \sqrt{\frac{4GK L^2}{E I_y d^2} + \pi^2}$$

### III. Specification Formulas.

#### 1. Aluminum Alloys. From Eq. 12

$$for = \frac{I_x}{I_x} \frac{d^2}{L^2} E \frac{\pi}{4} \sqrt{\frac{4GK L^2}{E I_y d^2} + \frac{\pi^2 E}{4G}}$$

with  $I_x = S_x \frac{d}{2}$ , and  $\frac{L}{\pi}$  instead of  $L$  for End Fixity

$$for = n \pi E \sqrt{\frac{4GK L^2}{E I_y d^2} + \frac{n^2 \pi^2 E}{4G}}$$

with  $n = 1.33$ ;  $E = 10.6 \times 10^6$  psi;  $G = 4.0 \times 10^6$  psi; Factor of Safety = 2.5

$$for = 10400000 \frac{d^2}{L^2} \sqrt{\frac{4GK L^2}{E I_y d^2} + 11.7}$$

(11)

(12)

(13)

Specifications give charts with values of

$l \div \sqrt{\frac{a I_x}{S_x} \sqrt{\frac{K l^2}{I_y a^2} + 11.7}}$  plotted as abscissa, and  $f$  as ordinates, with reduction for stresses above the elastic limit.

## 2. Light Gage Steel From Eq. 12

$$fcr = \frac{I_y}{I_x} \frac{d^2}{l^2} E \frac{\pi^2}{4} \sqrt{\frac{4.5 I_x l^2}{\pi^2 E I_y d^2} + 1}$$

discarding first term of square root; with  $\frac{I_y}{I_x} = \frac{r_y^2}{r_x^2}$

$$fcr = \frac{r_y^2}{r_x^2} \cdot \frac{d^2}{l^2} E \frac{\pi^2}{4} \quad \text{or}$$

$$fcr = \frac{\pi^2}{4} E \left( \frac{d}{r_x} \right)^2 \left( \frac{r_y}{l} \right)^2$$

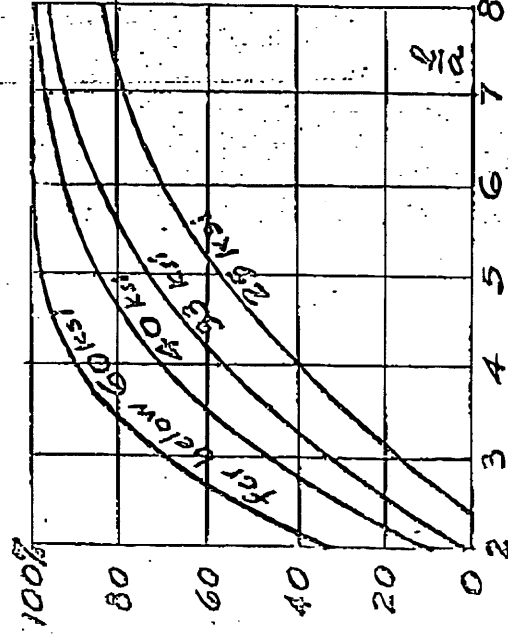
with  $\frac{d}{r_x} = 2.5$ ;  $E = 29.6 \times 10^6$  psi; Factor of Safety = 1.85

$$f = 250000000 \div \left( \frac{l}{r_y} \right)^2$$

(14)

## 3. Structural Steel - Rolled Beams.

To obtain a simplified formula for steel, critical buckling stresses of Eq. 10 were computed, with  $k$ -values of Table 1 for uniform top flange load;  $E = 30000$  ksi, and  $a$ -values from Bethlehem Booklet 5-57; for "wide flange" (WF) sections from 36" to 10" deep, and "light beams" 12" and 10" deep, a total of 132 beams.



Steel	Y.P.	E.L.	Allow.
Nickel	55 ksi	40 ksi	30 ksi
Silicon	45	33	24
Carbon	33	25	18

1)  $E = 30000$  ksi

2) Assumed Elastic Limit

3) Basic in Bending, AREA Spec.

Note: For Aluminum Alloys, with  $E = 10.6$  ksi,  $Y.P. = 53$  ksi, Allow. = 22 ksi; reduce values of  $fcr$  thus  $10.6/30 = 60$  ksi = 21.2 ksi.

Range of  $fcr$  of 132 beams in percentages of total number.

Fig. 2

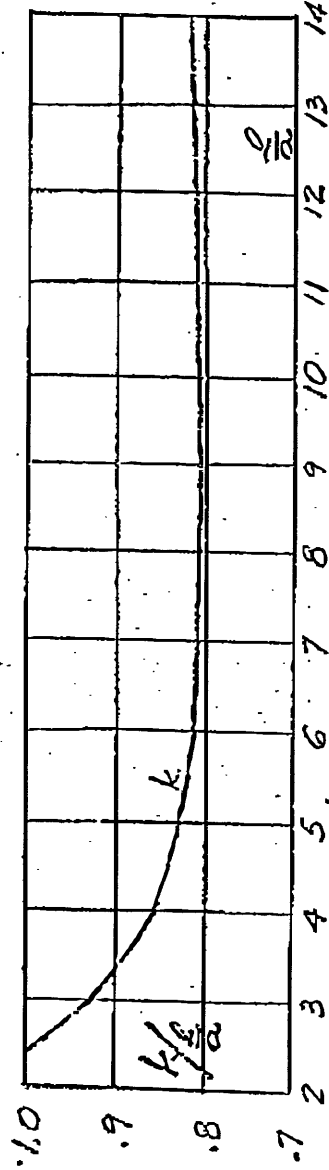
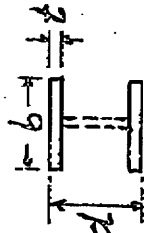


Fig. 3

k-values for Uniform Top Flange Load (Table 1)

With an approximate value of  $k = 0.8 \frac{l}{a}$  (from Fig. 3 for the effective range of the beam, see Fig. 2) in Eq. 10, and the following additional approximations:

$$\left. \begin{aligned} I_y &= \frac{tb^3}{6} \\ I_x &= \frac{bt^3}{12} \end{aligned} \right\} \frac{I_y}{I_x} = \frac{1}{3} \cdot \frac{b^2}{t^2}$$



$$\left. \begin{aligned} \frac{E}{G} &= \frac{30.0}{11.5} = 2.6 \\ I_y &= \frac{tb^3}{6} \\ R &= \frac{2}{3} bt^3 \end{aligned} \right\} \begin{aligned} a^2 &= \frac{EI_y d^2}{4GR} \\ a &= \frac{4}{3} \cdot \frac{bd}{t} \\ \frac{l}{a} &= \frac{3}{4} \frac{t}{bd} \end{aligned}$$

$$\left. \begin{aligned} f_{cr} &= \frac{I_y}{I_x} \cdot \frac{d^2}{l^2} E k \\ f_{cr} &= \frac{1}{3} \frac{b^2}{d^2} \cdot \frac{d^2}{l^2} E \cdot 2 \frac{lt}{bd} \\ f_{cr} &= \frac{2}{3} E \frac{bt}{ld} \end{aligned} \right\}$$

$$E = 30000000 \text{ psi}$$

$$f_{cr} = 20000000 \div \frac{ld}{bt}$$

(15)

Eq. 15 then may be considered to be the failure formula of uniform top flange load on a simple beam, applicable for stresses in the elastic range. Fig. 4 shows a comparison of this formula with the critical stresses of the 132 beams.

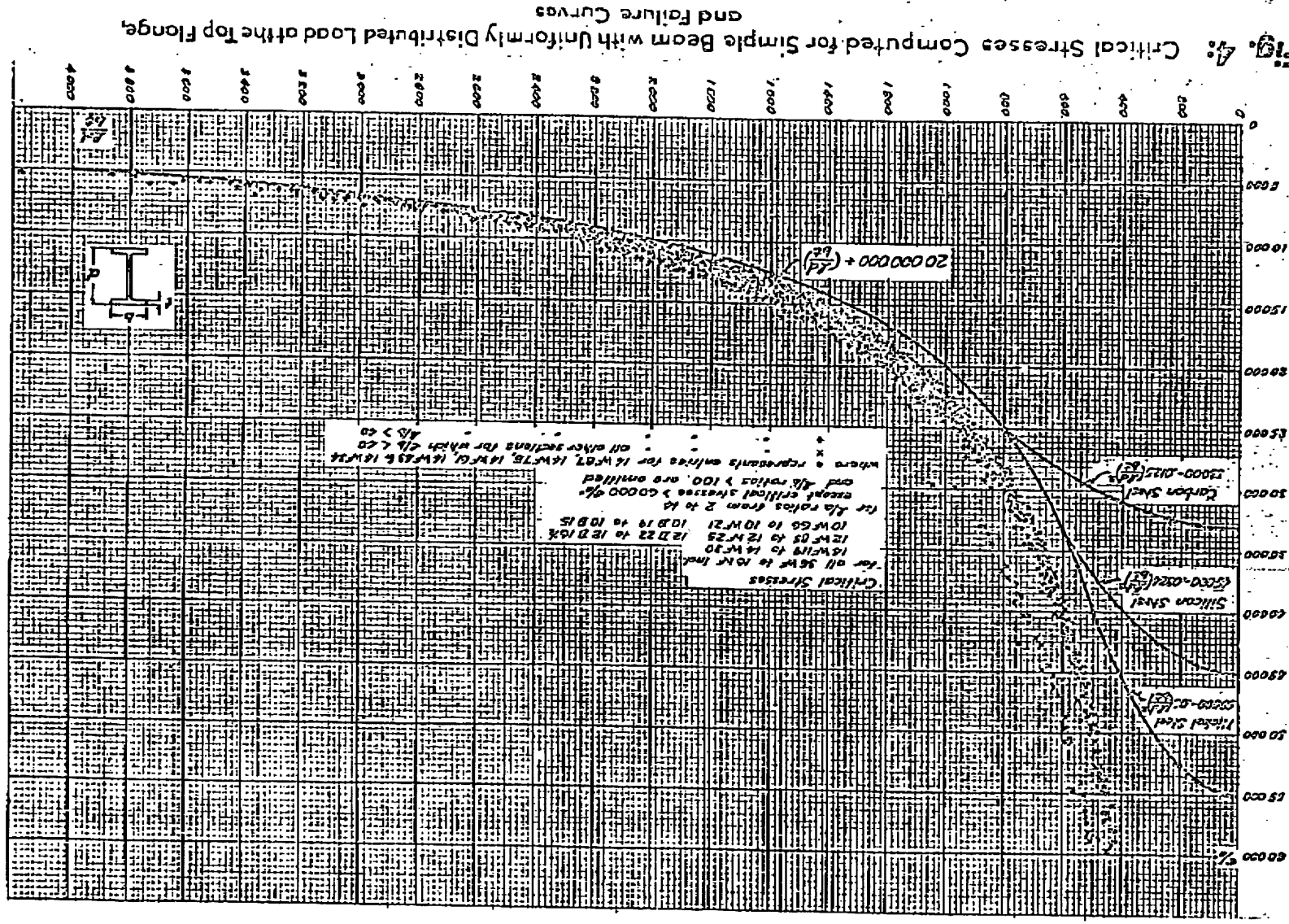
Transition curves between elastic limit and yield point stresses are shown in Fig. 4, in preference to the use of the tangent modulus, for stresses above the elastic limit. Eq. 15 with a factor of safety of 1.67 becomes

$$f = 12000000 \div \frac{ld}{bt}$$

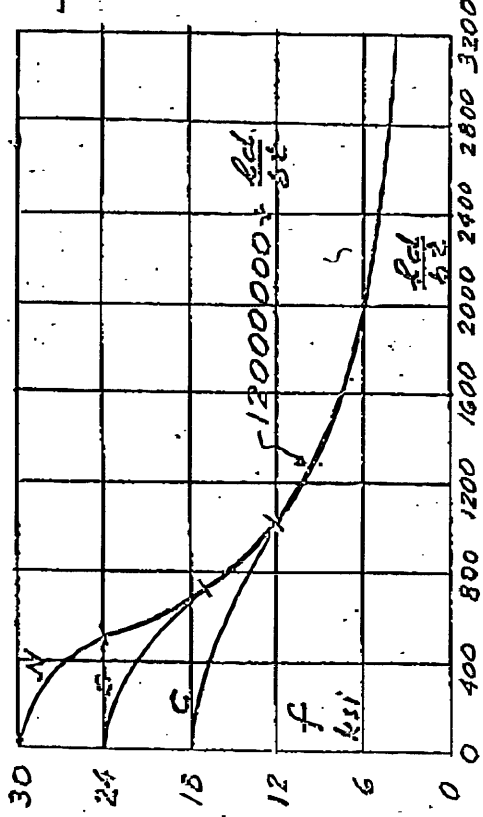
(16)

which is the specification "long beam" formula for structural steels. See Fig. 5.

Uniform top flange load is the most severe type of the six practical load assumptions on p. 3, and Table 1 of p. 4. Failure formulas similar to Eq. 15 with the parameter  $ld/bt$  may be written for the other five types of loading, based on analysis of their critical stresses.



The significance of these formulas is their simple parameter  $\frac{Ld}{bt}$ , their actual values may be changed or determined by tests or other considerations.



Short beam formulas:

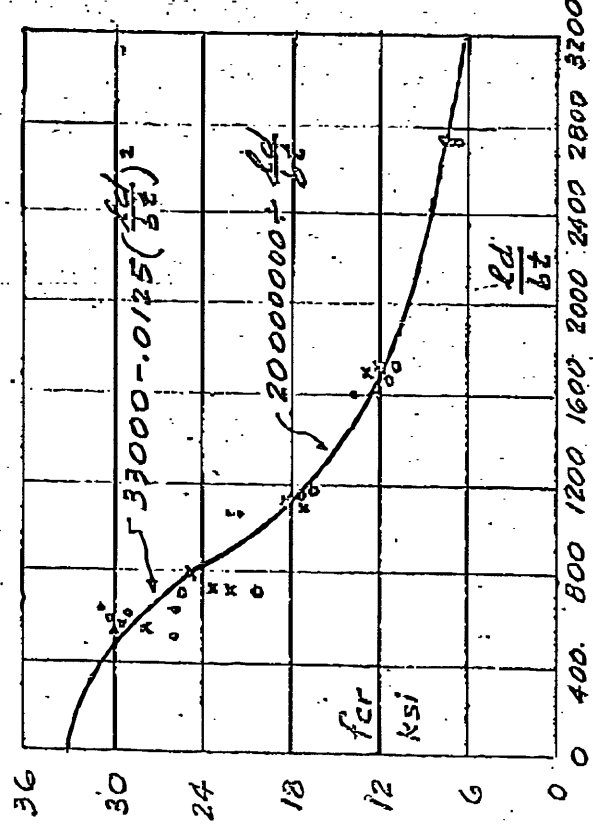
$$N = \text{Nickel Steel} \quad 30000 - .024 \left( \frac{Ld}{bt} \right)^2$$

$$S = \text{Silicon Steel} \quad 24000 - .014 \left( \frac{Ld}{bt} \right)^2$$

$$C = \text{Carbon Steel} \quad 18000 - .006 \left( \frac{Ld}{bt} \right)^2$$

Working Formulas

Buckling tests of I-beams, under the supervision of the Column Research Council, are being conducted at the University of Washington. Fig. 6 shows reported results. The scatter of the shorter beams apparently is due to the test set-up.



Plots in Fig. 6 are for: - 18WF50  $\times$  12WF27  $\times$  10I 25.4

The use of the  $Ld/bt$  parameter in the beam formula implies that beams of the same depths have equal critical buckling stresses for equal flange areas,  $bt$ . Computed critical buckling stresses indicate this to be correct in the long beam range, and that there are deviations in the short beam range.

Results of Lateral Buckling Tests of simply supported beams with two equal loads at quarter points, using their actual  $Ld/bt$  ratios.

C.R.C. - Project No. 14.E

Progress Report of January 1952

Fig. 6



#### 4. Plate Girders and other Built-up Beams

The torsional bending constant  $a$  is computed from the torsion constant  $I_T$ , which for rolled beams with parallel-sided flanges, see Fig. 7, is

$$I_T = \frac{2}{3} b t^3 + \frac{1}{3} (d - 2t) w^3 + 2 a D^4 - 0.42016 t^4 \quad (17)$$

Flanges      Web      Circle      End Lorr

The first term in Eq. 17 is the largest, the last two terms are small.

Eq. 17 used in the critical buckling formulas has led to the use of the flange area as the denominator in the parameter  $I_T / b t$ .

It has been proved by tests that the  $I_T$ -values of welded beams consisting of three plates, and of those consisting of more than three plates, such as shown in Fig. 8a, if

they have adequate continuous welds, may considered to be equal to the  $I_T$ -values of identical solid sections. Since the critical buckling formula contains the value

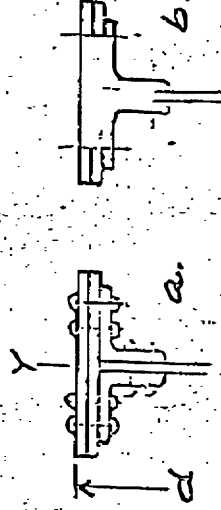
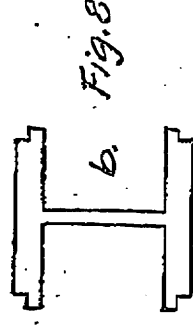
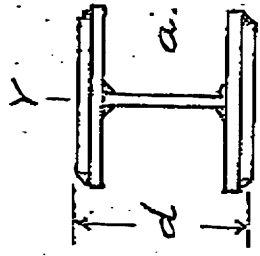
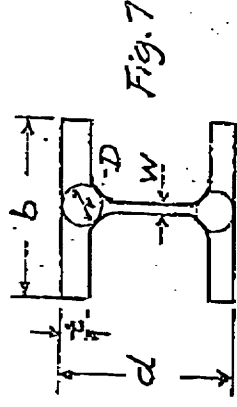
$I_y$ , which in the derivation of Eq. 15 for rolled beams is assumed as  $I_y = \frac{1}{6} b^3 t$ , and which does not hold for beams other than the rolled beams and its welded equal, it will be necessary to use for the welded beam with irregular flanges, such as shown in Fig. 8, in the  $I_T / b t$  parameter the equivalent flange area

$$b t = \frac{6 I_y}{b^2} \quad (18)$$

If welded beams do not have adequate welds, then the equivalent flange areas will have to be reduced.

b. In the computation of  $I_T$  of

Eq. 17 for the riveted plate girder flange shown in Fig. 9a, the flange may be assumed solid between the



b. Fig. 9

laminated beyond these lines, see Fig. 9b, a condition which has been proved by tests to exist, and which will reduce the  $K$ -value of its identical solid section in such manner that for riveted plate girders and other similarly riveted built-up beams the equivalent flange area may be assumed as

$$b t = \frac{5 I_y}{b^2}$$

(19)

Simple basic expressions for determining lateral buckling then are established by theory and test, but further assumptions and approximations have to be made to allow the solution of some frequently occurring practical problems, a few of which will be discussed in the following.

c. Parts of the flanges which do not extend

along the full length of the beam are accordingly evaluated. For the partial-length covers of the plate girder of Fig. 10, an equivalent average thickness of covers may be assumed, then

$$\begin{aligned} \frac{7}{8} \times 80/100 &= .700 & 4-L 8 \times 8 \times \frac{7}{8} &= 651.1 & b t &= \frac{5 \times 2084.5}{20^2} \\ \frac{3}{4} \times 50/100 &= .375 & 2-Cov. 20 \times \frac{7}{8} &= 14.33.4 & \Delta t &= 26.06 \text{ in.}^2 \\ \text{effect. } t &= 1.075" & I_y &= 2084.5 \text{ in.}^4 \end{aligned}$$

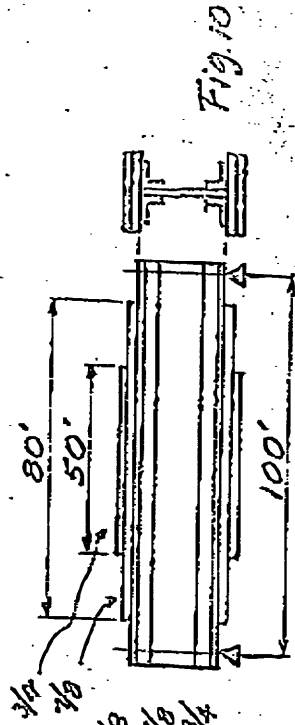


Fig. 10

Eqs. 18 and 19 apply only to beams which are symmetrical about both axes. The flanges of such beams being equal, the equivalent flange area may be expressed in terms of  $I_{yc}$ , the moment of inertia of the compression flange about the vertical axis of the beam, and Eq. 19 then becomes

$$b t = \frac{10 I_{yc}}{b^2}$$

(20)

d. Investigations of beams which are unsymmetrical about their horizontal axis have resulted in complicated formulas which lead to questionable results in their application. A simple and apparently satisfactory

formula for beams of different compression and tension flanges may be written thus

$$b_t = \frac{2}{3} (b_t)_{\text{Compr}} + \frac{1}{3} (b_t)_{\text{Tens.}} \quad (21)$$

$$\text{or} \quad b_t = \frac{2}{3} \left[ \frac{10 I_{yc}}{b_c^2} \right] + \frac{1}{3} \left[ \frac{10 I_{yt}}{b_t^2} \right] \quad (22)$$

where  $I_{yc}$  and  $I_{yt}$  are the moment of inertia about the vertical axis of the beam, and  $b_c$  and  $b_t$  the widths of the compression and the tension flange, respectively.

The computations of  $b_t$ , then, for the plate girder of Fig. 11, which is similar to that of Fig. 10, except that the top covers are extended over the full length of the beam, are thus

$$2 - 2 \times 8 \times 8 \times \frac{7}{8} = 325.6$$

$$1 - \text{Cov. } 20 \times 15 \times \frac{1}{8} = \frac{1083.4}{I_{yc} = 1409.0 \text{ in}^4}$$

$$2 - 2 \times 8 \times 8 \times \frac{7}{8} = 325.6$$

$$1 - \text{Cov. } 20 \times 1.075 = \frac{716.7}{I_{yt} = 1042.3 \text{ in}^4}$$

$$(b_t)_c = \frac{10 \times 1409.0}{20^2} = 35.23 \text{ in}$$

$$(b_t)_t = \frac{10 \times 1042.3}{20^2} = 26.06 \text{ in}$$

$$b_t = \frac{2}{3} (35.23) + \frac{1}{3} (26.06)$$

$$b_t = 32.17 \text{ in}$$

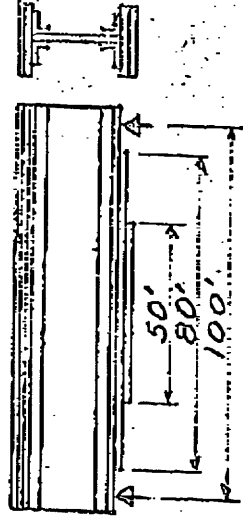


Fig. 11

The length of the beam,  $L$ , in the parameter  $Ld/bt$ , is the laterally unsupported length of the compression flange;  $d$  is the depth of the beam between its extreme fibers. In most structures, compression flanges are continuously supported; are intermittently supported by bracing systems; or are stayed against rotation at several intermediate points, or at one point near their centers; such beams will require no, or a very small reduction from the basic allowable bending stress, since the design stress of these beams generally falls within the range of the short beam formulas, see Fig. 5.

e. The long beam failure formula of Eq. 15, based on the  $k$ -factors for uniform top flange loading of Table 1 in Eq. 10, and as shown to be applicable for that loading in Fig. 4, has been used as the basis for the working

SIX

SIX

formulas of Fig. 5. Uniform top flange load application is a rare case; nevertheless, it is proposed for general use to cover incidental eccentricities of loading. To permit for special cases, in which the loading is known and where incidental eccentricities of loading are known not to exist, the use of simplified formulas,  $Ld/bt$  formulas have been developed to cover the load applications indicated in Table I, except that of pure bending which is considered not to be a practical one. Long beam failure formulas for the six cases (including that for uniform top flange load) are:

Uniformly distributed load	Top flange	$20000000 \div Ld/bt$	(23)
	Centroid	$24000000 \div Ld/bt$	(24)
Concentrated load at the center	Bottom Flange	$27000000 \div Ld/bt$	(25)
	Top Flange	$23000000 \div Ld/bt$	(26)
	Centroid	$28000000 \div Ld/bt$	(27)
	Bottom Flange	$34000000 \div Ld/bt$	(28)

8. A safety factor of 1.6 applied to the failure formulas or to the critical stresses of Eq. 10, generally will keep the extreme fiber stress within the usual stress limitations for structural steels; a lower safety factor may only be used if the actual stresses are computed with great accuracy, and if the stresses from impact and wind can be kept at their lowest levels.

Long beams, which in the finished structure have filler beams, struts or bracing, always have presented a problem to the engineer who wants to evaluate their expected behavior during erection; it was found that the  $L/b$  beam formula when plotted gave "zero" values for the beam lengths in question, and as the buckling theory became known, that its application was too involved to be of practical use. The erection problem has become acute ever since the beams to be handled are being made longer and heavier. The  $Ld/bt$  formula has here its most important field of application and apparently it gives very satisfactory results.

9. In erecting the beam is loaded with its own weight,

that means it is subjected to uniform centroid load and its failure formula is expressed in Eq. 23,  $24000000 \div 240/32$ . Picking of the beam from the ground mostly can be done without causing too much impact or vibration. After being placed, if not sufficiently strong to resist unexpected wind loads, the beam will have to be held laterally at intermediate points by bracing to adjacent beams or in some other way, before such winds occur and always before nightfall. All long beams have to be held against tipping at their points of support when placed.

The best practice seems to be to compute the actual stresses in the beam and to determine its factor of safety as compared with the failure formula, Eq. 23.

h. Common erection conditions of long beams are indicated in Fig. 12. Fig. 12a shows a simple span placed, in Fig. 12b the placed beam is a part of a continuous or cantilever structure. Fig. 12c shows a beam being picked at its ends, Fig. 12d a beam picked away from its ends. In Fig. 12e, f & g the picking is done at one point, above the center of gravity of the beam; a spreader is used in Fig. 12e to reduce the length of the cantilever.

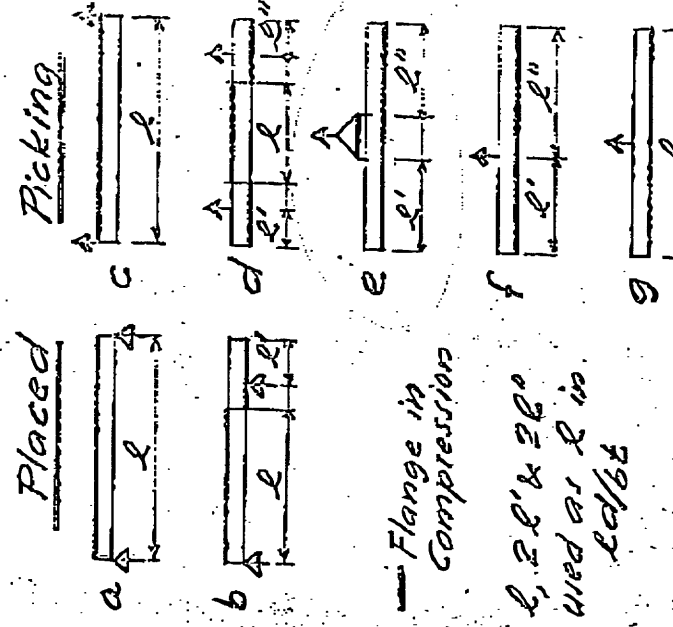


Fig. 12

Beams are always picked at their top flanges.  $l$  in  $240/32$  is the length between supports or to the points of counterflexure, as indicated, and twice the length of cantilevers.

i. If it is found that a beam when placed is unsafe, then the critical stresses may be increased, the actual stresses decreased, and a larger safety factor may be obtained by one or a combination of the following schemes: Non intermediate support, or falsework beam, may be introduced. A horizontal spreader may be supplied when the

beam is being placed, before it is fully released from the falls; 3) cover plates on the compression flange may be extended; 4) cover plates on the tension flange may be extended; 5) cover plates or other flange material may be added; and 6) beams or trusses may be used to give the beam temporarily the necessary lateral stiffness.

J. Temporary beams, such as the horizontal beam of Fig. 13 and the horizontal truss of Fig. 14, are used on either or both flanges; they are bolted at intervals of, say, 4 feet to the beam, and will therefore not contribute to its vertical strength. Mostly, they are placed on fills on account of partial length covers, rivet heads or other details; or several inches away from the flange when the beam has details such as shear lugs (composite beams). They are considered as struts to hold the beam flange laterally, and are evaluated by assuming an equivalent flange area

$$(bt)_c \text{ or } (bt)_t = \sqrt{I_y}, \text{ respectively}$$

where  $I_y$  is the moment of inertia of the temporary beam about the y-axis of the beam.

(28)



Fig. 13

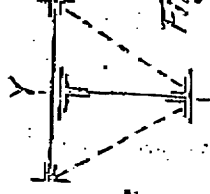


Fig. 14

K. A symmetrical beam which is safe for placing, Fig. 12a, is also safe for picking, Figs. 12c and g. A beam having been made safe for placing may have become safe also for picking. If a beam is to be picked at two points and unsafe for picking at its ends, see Fig. 12c, then picking away from its ends, see Fig. 12d, frequently will permit safe handling. If a beam is to be picked at one point, see Figs. 12f and g, and unsafe, then picking with a spreader see Fig. 12e, frequently will permit safe handling. The maximum length of a spreader probably is 50 feet, but more often is dictated by conditions in the field.

References:

- "Theory of Elastic Stability" by S. Timoshenko, 1936.
- "Structural Beams in Torsion" by Inge Lye and Bruce G. Johnston, Trans. ASCE, Vol. 101, 1936.
- "Strength of Beams as Determined by Lateral Buckling" by Karl de Vries, Trans. ASCE, Vol. 112, 1947.
- "Torsional Stresses in Structural Beams", Booklet 5-57, Bethlehem Steel Company, 1950.
- "Torsion of Plate Girders" by F.K. Chang and Bruce G. Johnston, ASCE Separate No 125, April 1952.
- "Specifications for Heavy Duty Structures of High-Strength Aluminum Alloy" Progress Report, ASCE Vol. 117, 1952.
- "Specifications for the Design of Light Gauge Steel Structural Members", American Iron and Steel Institute, April 1946.

Bethlehem, Pa.  
April 9, 1953

Karl de Vries



INTER-OFFICE CORRESPONDENCE

# BETHLEHEM STEEL

Bethlehem, Pa. January 29, 1954

FILE REF. CHL-233-2

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## INSTRUCTIONS - ENGINEERING

### LATERAL STABILITY OF GIRDERS

For many years we have been checking the lateral stability of girders using the method developed by Mr. Karl de Vries in his article dated April 9, 1953. Experience has shown that changes are necessary when computing stability of welded girders and girders which are unsymmetrical about their horizontal axis (composite design).

Our Engineering Department has been asked to study this problem. Meanwhile, the following changes in method of computing stability are to be followed.

#### Welded Girders--

$$\text{Symmetrical Girders, } bt = \frac{5I_y}{b^2} \quad (A)$$

In which  $I_y$  is total of both flanges.

To be used instead of Equation 13 ( $bt = \frac{6I_y}{b^2}$ ) in

Mr. de Vries' article which was developed for rolled sections and used for welded sections. However, because of the torsional weakness of the flange to web connection, and the high residual stresses from welding, the equation is made the same as for riveted girders, Equation 19.

#### Riveted and Welded Girders Not Symmetrical About Their Horizontal Axis--

$$\text{For one flange, } bt = \frac{10I_y}{b^2} \quad (B)$$

This is Equation 20 in Mr. de Vries' article.

When the stronger flange is the compression flange continue to use:

$$\text{For Girder, } bt = 2/3(bt)_{comp} + 1/3(bt)_{ten}. \quad (C)$$

Which is Equation 21 in Mr. de Vries' article.



-2-

January 29, 1934  
CIL-133-9

However, when the weaker flange is the compression flange the equation is changed to:

For Girder,  $b_t - 3/4(b_t)_{comp.} + 1/4(b_t)_{ten.}$  (D)

The only changes in method of computation are use of Equations A and B for welded girders and Equation D for unsymmetrical riveted and welded girders. Critical stresses are determined as previously and the factor of safety must be at least 1.6.

There has been some misunderstanding as to the proper equation to be used to determine the critical stress when picking girders. See page 11 of Mr. de Vries' article. When picking with dogs, top flange hitches, or choked clips, Equation 23 (centroid loading) is to be used. When picking with a basket type hitch Equation 15 (top flange loading) is to be used.

Also, as shown in Fig. 1 and stated on page 12 of Mr. de Vries' article, all long beams of girders must be held against tipping at their points of support when placed.

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Engineer of Erection

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