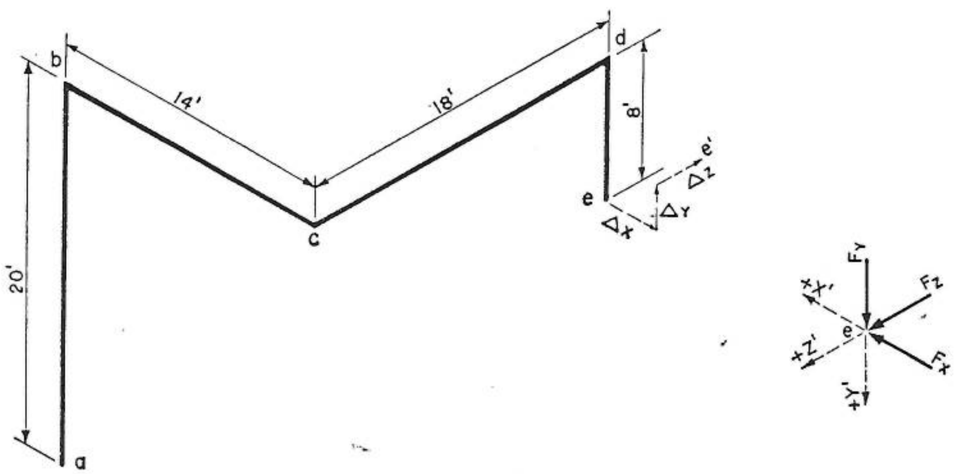
MULTIPLE PLANE SYSTEM



Given: A 10 inch piping system in accordance with the sketch shown above.

Maximum Operating Temperature	750° F
Maximum Operating Pressure, P	350 psi
Piping Specification	A.S.T.M. A-106 Grade A

Data:

t = 0.365 inches, = schedule 40 — from page 2 and 12

d = 10.02 inches, I_P = 160.8 inches^4, S_m = 29.9 inches^3, A_I = 78.9 inches^2, A_M = 11.91 inches^2 — from page 12

$c_{\text{at } 750°}$ = 996 — from page 9

S_A = 17,675 psi — from page 3

Find: Reaction forces F_x, F_y, and F_z, at point e. (At point a reaction forces are equal and opposite.)

Reaction Moments M_{xy}, M_{xz}, and M_{yz} at points a and e.

Amount and location of Maximum Combined Stress, s.

Solution: Assume point a fixed and point e temporarily released. The thermal expansion would then move point e in the direction and by the amounts Δ_x, Δ_y, and Δ_z to a new location e'.

Establish axes $+X'$, $+Y'$, and $+Z'$ at point e opposite to the direction of Δ_x, Δ_y, and Δ_z respectively. Project the piping system into the three planes formed by these axes. The three planes are denoted as the XY, XZ, and YZ planes.

Determine the location of the centroid and calculate the line inertias for each projection.

Calculation of the line inertias results in two moments of inertia for each axis which are added.

$$\text{Total } I_x = 2013 + 3531 = 5544 \text{ feet}^3$$ } See Pg 55 & 56
$$\text{Total } I_y = 2647 + 3077 = 5724 \text{ feet}^3$$
$$\text{Total } I_z = 2889 + 1998 = 4887 \text{ feet}^3$$

Products of inertia from the calculations are:

$$I_{xy} = +1461 \text{ feet}^3$$
$$I_{xz} = +2360 \text{ feet}^3$$
$$I_{yz} = +\ 529 \text{ feet}^3$$

Introduce these values into the following equations and solve for F_x, F_y, and F_z.

$$+F_xI_x - F_yI_{xy} - F_zI_{xz} = L_xcI_P$$
$$-F_xI_{xy} + F_yI_y - F_zI_{yz} = L_ycI_P$$
$$-F_xI_{xz} - F_yI_{yz} + F_zI_z = L_zcI_P$$

L_x = distance in X direction from a to e
= 14 feet

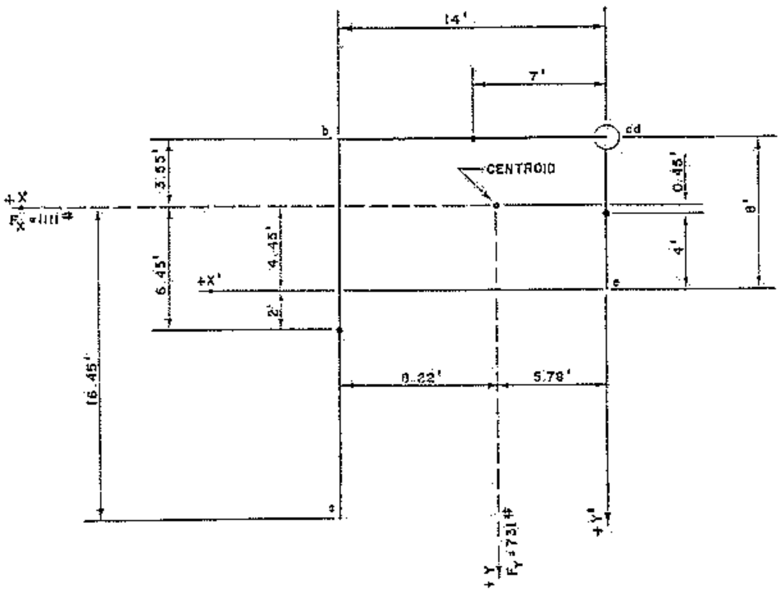
L_y = distance in Y direction from a to e
= 20 − 8 = 12 feet

L_z = distance in Z direction from a to e
= 18 feet

$$L_xcI_P = 14 \times 996 \times 160.8 = 2{,}242{,}195 \text{ lb ft}^3$$
$$L_ycI_P = 12 \times 996 \times 160.8 = 1{,}921{,}882 \text{ lb ft}^3$$
$$L_zcI_P = 18 \times 996 \times 160.8 = 2{,}882{,}822 \text{ lb ft}^3$$

(1) $+F_x\,5544 - F_y\,1461 - F_z\,2360 = 2{,}242{,}195$
(2) $-F_x\,1461 + F_y\,5724 - F_z\ \ 529 = 1{,}921{,}882$
(3) $-F_x\,2360 - F_y\ \ 529 + F_z\,4887 = 2{,}882{,}822$

PROJECTION IN *XY* PLANE



To find c.g. of each segment see page 54.

$$ab = 20'$$
$$bc = 14'$$
$$cd = 18'$$
$$de = 8'$$

Centroid (calculated with origin at e)

	Eq. No.	Length L, Ft	x'	Lx'	y'	Ly'
ab	I	20	+14	+280	+2	+ 40
bc	I	14	+ 7	+ 98	−8	−112
cd	II	$1.3 \times 18 = 23.4$	0	0	−8	−187.2
de	II	8	0	0	−4	− 32
		$\Sigma L = 65.4$		$\Sigma Lx' = +378$		$\Sigma Ly' = -291.2$

$$\bar{x} = \frac{378}{65.4} = +5.78' \qquad \bar{y} = -\frac{291.2}{65.4} = -4.45'$$

I_{xy}

	Eq. No.		
ab	VI	$20 \times 8.22 \times 6.45$	= +1062
bc	VI	$14 \times 1.22 \times (-3.55)$	= − 61
cd	VIII	$1.3 \times 18 \times (-5.78) \times (-3.55)$	= + 481
de	VI	$8 \times (-5.78) \times 0.45$	= − 21
			$I_{xy} = +1461$

I_x

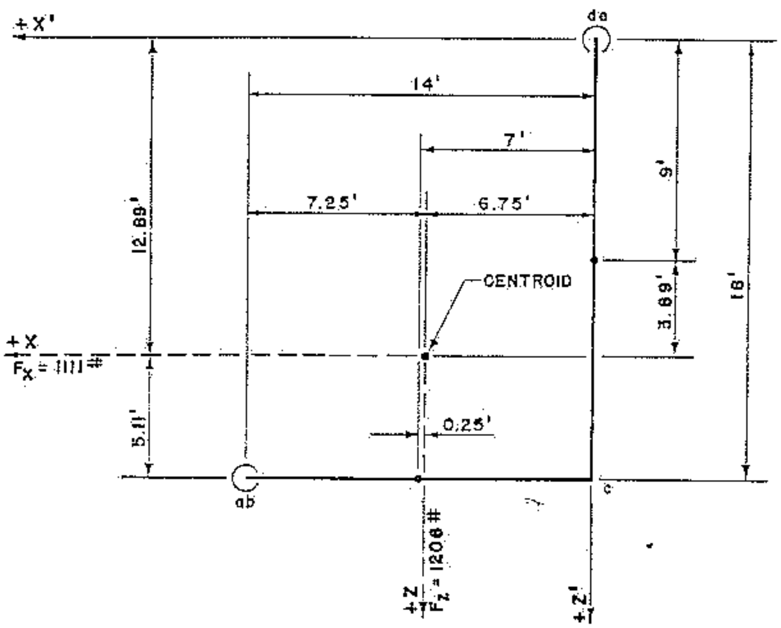
	Eq. No.		
ab	XIV *B*	$\frac{20^3}{12} + 20 \times 6.45^2$	= 1498
bc	XIV *A*	14×3.55^2	= 176
cd	XVI	$1.3 \times 18 \times 3.55^2$	= 295
de	XIV *B*	$\frac{8^3}{12} + 8 \times (0.45)^2$	= 44
			$I_x = 2013$

I_y

	Eq. No.		
ab	XIV *A*	20×8.22^2	= 1350
bc	XIV *B*	$\frac{14^3}{12} + 14 \times 1.22^2$	= 250
cd	XVI	$1.3 \times 18 \times 5.78^2$	= 780
de	XIV *A*	8×5.78^2	= 267
			$I_y = 2647$

For equation reference numbers see pages 44 to 48.

PROJECTION IN XZ PLANE



To find c.g. of each segment see page 54.

$ab = 20'$
$bc = 14'$
$cd = 18'$
$de = 8'$

Centroid (calculated with origin at e)

	Eq. No.	Length L, Ft	x'	Lx'	z'	Lz'
ab	II	$1.3 \times 20 = 26$	$+14$	$+364$	$+18$	$+468$
bc	I	14	$+7$	$+98$	$+18$	$+252$
cd	I	18	0	0	$+9$	$+162$
de	II	$1.3 \times 8 = 10.4$	0	0	0	0
		$\Sigma L = 68.4$		$\Sigma Lx' = +462$		$\Sigma Lz' = +882$

$$\bar{x} = \frac{462}{68.4} = +6.75' \qquad \bar{z} = \frac{882}{68.4} = +12.89'$$

I_{xz}

ab	VIII	$1.3 \times 20 \times 7.25 \times 5.11$	$= +964$
bc	VI	$14 \times 0.25 \times 5.11$	$= +18$
cd	VI	$18 \times (-6.75) \times (-3.89)$	$= +473$
de	VIII	$1.3 \times 8 \times (-6.75) \times (-12.89)$	$= +905$
			$I_{xz} = +2360$

I_x

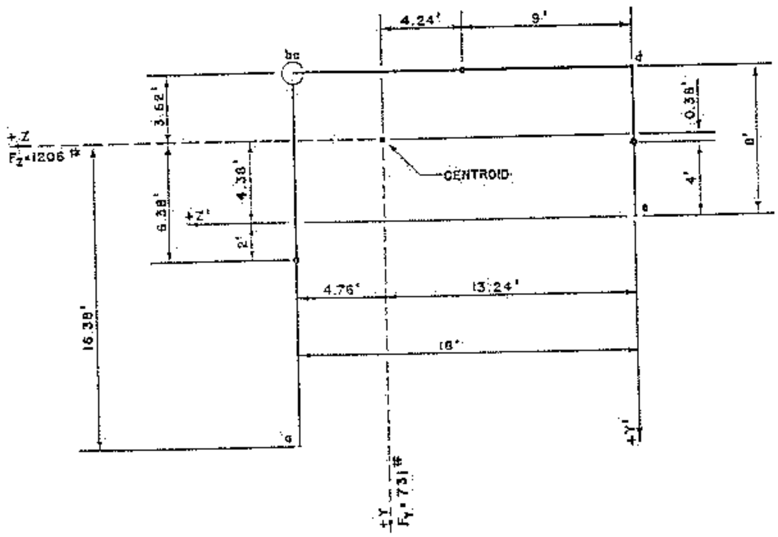
ab	XVI	$1.3 \times 20 \times 5.11^2$	$= 678$
bc	XIV A	14×5.11^2	$= 365$
cd	XIV B	$\frac{18^3}{12} + 18 \times 3.89^2$	$= 758$
de	XVI	$1.3 \times 8 \times 12.89^2$	$= 1730$
			$I_x = 3531$

I_z

ab	XVI	$1.3 \times 20 \times 7.25^2$	$= 1365$
bc	XIV B	$\frac{14^3}{12} + 14 \times (0.25)^2$	$= 230$
cd	XIV A	18×6.75^2	$= 820$
de	XVI	$1.3 \times 8 \times 6.75^2$	$= 474$
			$I_z = 2889$

For equation reference numbers see pages 44 to 48.

PROJECTION IN *YZ* PLANE



To find c.g. of each segment see page 54.

$ab = 20'$
$bc = 14'$
$cd = 18'$
$de = 8'$

Centroid (calculated with origin at *e*)

	Eq. No.	Length L, Ft	y'	Ly'	z'	Lz'
ab	I	20	+2	+ 40	+18	+360
bc	II	$1.3 \times 14 = 18.2$	−8	−145.6	+18	+327.6
cd	I	18	−8	−144	+ 9	+162
de	I	8	−4	− 32	0	0
		$\Sigma L = 64.2$		$\Sigma Ly' = -281.6$		$\Sigma Lz' = +849.6$

$$\bar{y} = -\frac{281.6}{64.2} = -4.38 \qquad \bar{z} = \frac{849.6}{64.2} = +13.24$$

I_{yz}

ab	VI	$20 \times 6.38 \times 4.76$	$= +607$
bc	VIII	$1.3 \times 14 \times (-3.62) \times 4.76$	$= -314$
cd	VI	$18 \times (-3.62) \times (-4.24)$	$= +276$
de	VI	$8 \times 0.38 \times (-13.24)$	$= -40$
			$I_{yz} = +529$

I_y

ab	XIV *A*	20×4.76^2	$= 453$
bc	XVI	$1.3 \times 14 \times 4.76^2$	$= 412$
cd	XIV *B*	$\frac{18^3}{12} + 18 \times 4.24^2$	$= 810$
de	XIV *A*	8×13.24^2	$= 1402$
			$I_y = 3077$

I_z

ab	XIV *B*	$\frac{20^3}{12} + 20 \times 6.38^2$	$= 1480$
bc	XVI	$1.3 \times 14 \times 3.62^2$	$= 238$
cd	XIV *A*	18×3.62^2	$= 236$
de	XIV *B*	$\frac{8^3}{12} + 8 \times (0.38)^2$	$= 44$
			$I_z = 1998$

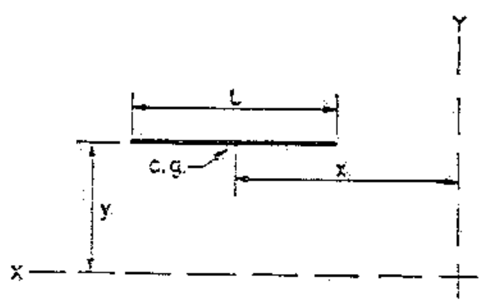
For equation reference numbers see pages 44 to 48.

MOMENT OF INERTIA

The moment of inertia of an element is its length multiplied by the square of its distance from an axis. The moment of inertia of the entire branch is the sum of all these products. The moment of inertia has a positive sign only.

The following formulas give Moment of Inertia for various line segments:

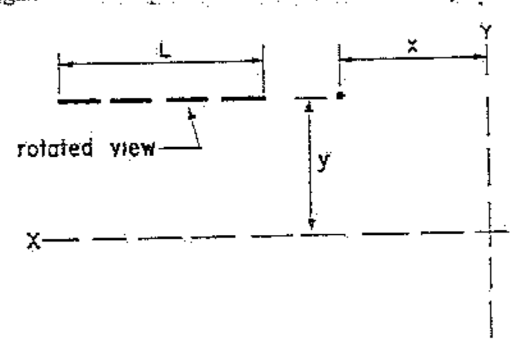
Straight Line in Plane of Projection



$I_x = Ly^2$ (parallel to axis) (Eq. XIVA)

$I_y = \frac{L^3}{12} + Lx^2$ (perpendicular to axis) (Eq. XIVB)

Straight Line Perpendicular to Plane of Projection



$$\left.\begin{aligned} I_x &= 1.3Ly^2 \\ I_y &= 1.3Lx^2 \end{aligned}\right\} \quad \text{(Eq. XVI)}$$