2005

Strength Analysis of Steel–Concrete Composite Beams in Combined Bending and Shear

Q. Q. Liang
University of New South Wales

B. Uy
University of New South Wales, brianuy@uow.edu.au

M. A. Bradford
University of New South Wales

H. R. Ronagh
University of Queensland

http://ro.uow.edu.au/engpapers/184

Strength Analysis of Steel–Concrete Composite Beams in Combined Bending and Shear

Qing Quan Liang, M.ASCE[1]; Brian Uy, M.ASCE[2]; Mark A. Bradford, M.ASCE[3]; and Hamid R. Ronagh[4]

Abstract: Despite experimental evidences, the contributions of the concrete slab and composite action to the vertical shear strength of simply supported steel–concrete composite beams are not considered in current design codes, which lead to conservative designs. In this paper, the finite element method is used to investigate the flexural and shear strengths of simply supported composite beams under combined bending and shear. A three-dimensional finite element model has been developed to account for geometric and material nonlinear behavior of composite beams, and verified by experimental results. The verified finite element model is than employed to quantify the contributions of the concrete slab and composite action to the moment and shear capacities of composite beams. The effect of the degree of shear connection on the vertical shear strength of deep composite beams loaded in shear is studied. Design models for vertical shear strength including contributions from the concrete slab and composite action and for the ultimate moment–shear interaction are proposed for the design of simply supported composite beams in combined bending and shear. The proposed design models provide a consistent and economical design procedure for simply supported composite beams.

CE Database subject headings: Bending; Composite structures; Finite element method; Shear strength; Beams; Slabs.

Introduction

Steel–concrete composite beams have been extensively used in building and bridge construction. Composite action in a composite beam is achieved by means of mechanical shear connectors. Headed stud shear connectors are usually welded to the top flange of a steel beam to resist longitudinal slip and vertical separation between the concrete slab and the steel beam. Concrete slabs can be either solid slabs or composite slabs incorporating profiled steel sheeting. Composite beams under applied loads are often subjected to combined actions of bending and vertical shear. Despite experimental evidences, the contributions from the concrete slab and composite action to the vertical shear strength of a simply supported composite beam is not considered in current design codes, such as *AS 2327.1* (Standards 1996), *EUROCODE 4* (1994) and LRFD (AISC 1999), which result in conservative designs (Johnson and Anderson 1993). In order to design composite beams consistently and economically, it is necessary to develop new design models for shear strength including contributions from the concrete slab and composite action and for moment-shear interactions.

Experimental studies on the ultimate strength of steel–concrete composite beams in combined bending and shear have been of interest to researchers. Johnson and Willmington (1972) conducted experiments on continuous composite beams in combined negative bending and vertical shear. Their test results indicated that longitudinal steel reinforcement in the concrete slab increases the strength and stiffness in vertical shear of a composite beam. Allison et al. (1982) tested five composite plate girders and one steel plate girder under negative bending and shear to failure. Porter and Cherif (1987) studied experimental behavior of simply supported composite plate girders loaded primarily in shear. They proposed a shear strength model that incorporates contributions from both the concrete slab and the steel plate girder for the design of composite beams.

Research on the behavior of composite beams with web openings indicated that the concrete slab contributes significantly to the vertical shear strength of a composite section at web openings. Tests on short-span composite plate girders with web openings have been carried out by Narayanan et al. (1989) and Roberts and Al-Amery (1991). These tests showed that the shear strength of a composite plate girder is significantly higher than that of a steel plate girder alone if adequate shear connectors are provided in the composite girder. In addition, the composite action under predominantly shear loading depends on the tensile or pullout strength of the shear connectors. Analytical models including a contribution from the concrete slab were proposed for determining the shear strength of composite plate girders. Experiments conducted by Clawson and Darwin (1982) and Donahey and Darwin (1988) indicated that the behavior of composite beams with web openings is largely controlled by the moment–shear ratio at the opening. Darwin and Donahey (1988) proposed an equation to

[1]Australian Postdoctoral Fellow, School of Civil and Environmental Engineering, The Univ. of New South Wales, Sydney, NSW 2052, Australia; Professor, Central South University, Changsha, Hunan, China. E-mail: q.liang@unsw.edu.au

[2]Associate Professor, School of Civil and Environmental Engineering, The Univ. of New South Wales, Sydney, NSW 2052, Australia.

[3]Professor of Civil Engineering, School of Civil and Environmental Engineering, The Univ. of New South Wales, Sydney, NSW 2052, Australia.

[4]Lecturer, Dept. of Civil Engineering, Univ. of Queensland, Brisbane, QLD 4072, Australia.

express the ultimate moment–shear relationship for composite beams with web openings.

Numerical analysis methods have been used to analyze the inelastic behavior of composite beams. Yam and Chapman (1968) presented an iterative numerical method for the inelastic analysis of simply supported composite beams. The inelasticity of steel, concrete and shear connections was taken into account in the analysis. Hirst and Yeo (1980) used a standard finite element program to analyze composite beams with partial and full shear connection. Quadrilateral elements were employed to simulate discrete stud shear connectors. The material properties of stud elements were modified to make them equivalent in strength and stiffness to the actual shear connectors in composite beams. A three-dimensional bar element has been developed by Razaqpur and Nofal (1989) for modeling the nonlinear behavior of shear connectors in composite beams. An empirical shear-slip relationship was used to express the stiffness properties of the bar element.

Al-Amery and Roberts (1990) presented a nonlinear analysis of composite beams with partial shear connection by using a finite difference method. Salari et al. (1998) formulated a composite beam element based on the force analysis method for the nonlinear analysis of composite beams with deformable shear connectors. A distributed spring model was used to simulate shear connectors. Thevendran et al. (1999) utilized the finite element software *ABAQUS* to study the ultimate load behavior of composite beams curved in plan. Shell elements were used to model the concrete slab and the steel beam whilst a rigid beam element was employed to simulate stud shear connectors. Sebastian and Mc-Connel (2000) described a nonlinear finite element program for modeling composite beams. Axial springs with empirical shear-slip relations were used to model discrete shear connectors. A kinematic model was proposed by Fabbrocino et al. (2000) for analyzing continuous composite beams with partial interaction and bond. Baskar et al. (2002) investigated the ultimate strength of composite plate girders under negative bending by using the finite element software *ABAQUS*. Further, Liang et al. (2004a) has undertaken nonlinear finite element analyses on continuous composite beams in combined bending and shear. In their study, design formulas incorporating contributions from the concrete slab and composite action were proposed for the vertical shear strength and the ultimate strength interaction of continuous composite beams.

In this paper, the ultimate flexural and shear strengths of simply supported composite beams in combined bending and shear are investigated by using the finite element analysis method. A three-dimensional finite element model, which accounts for geometric and material nonlinear behavior of composite beams, is described in detail. The finite element model is verified by corresponding experimental results. The verified finite element model
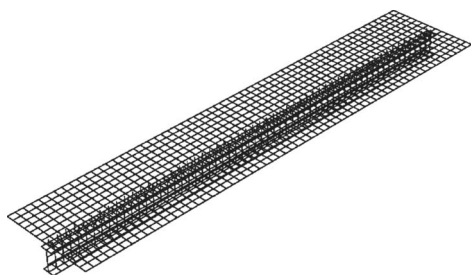
is then used to study the interaction behavior of composite beams subjected to combined actions of bending and shear. The effects of shear connection on the vertical shear strength of composite beams are investigated. Based on the numerical results, design models for vertical shear strength and for moment-shear interactions are developed for the design of simply supported composite beams.

Finite Element Analysis

General

The general-purpose finite element program *ABAQUS* version 6.3 (2002) was used in the present study to investigate the ultimate flexural and shear strengths of composite beams subjected to combined bending and shear. A three-dimensional (3D) finite element model has been developed to account for geometric and material nonlinear behavior of composite beams. The concrete slab, steel flanges, and web were modeled by four-node doubly curved thick/thin shell elements with reduced integration. A 3D beam element was employed to simulate discrete stud shear connectors. The von Mises yield criterion was used in the nonlinear analysis to treat the plasticity of steel material with five integration points through the thickness. A typical finite element discretization of a composite beam used in the present study is shown in Fig. 1.

Steel Modeling

Steel Section
Tests indicate that structural steels in uniaxial tension exhibit strain hardening behavior that is different from the elastic-perfectly plastic assumption (Kemp et al. 2002). The stress–strain curve with strain hardening used in the nonlinear analysis has shown to predict well the behavior of structural steel (Liang and Uy 2000; Liang et al. 2004b). In the present study, structural steel sections were modeled as an elastic–plastic material with strain hardening. A bilinear stress–strain relationship shown in Fig. 2 was used for steel sections in both compression and tension. Material properties, such as the Young's modulus, Poisson's ratio, the yield stress, the ultimate strength, and the ultimate strain, need to be input to define the stress–strain curve. Experimental values of
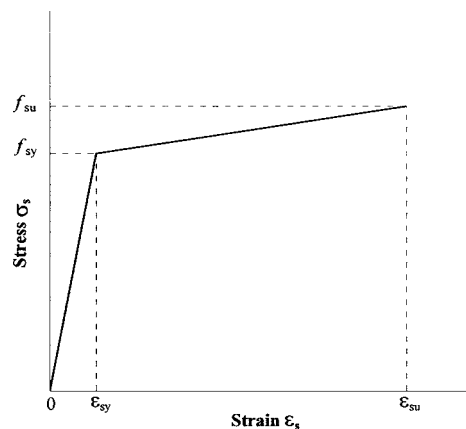

Fig. 1. Typical finite element mesh for the composite beam

Fig. 2. Stress–strain curve for steel with strain hardening

the yield and ultimate strengths were used in the analysis for steel sections. An ultimate strain of 0.25 was assumed for mild structural steel.

Steel Reinforcement

Steel reinforcing bars in concrete slabs were modeled in the present study as smeared layers with a constant thickness in shell elements. The thickness of a steel layer was calculated as the area of a reinforcing bar divided by the spacing of reinforcing bars. In the input data file, reinforcement in a concrete slab was defined by the Rebar Layer option within the shell section that defined the concrete slab. Four layers were used to represent the top and bottom longitudinal and transverse reinforcing bars in the concrete slab in a composite beam. The cross-sectional area of the reinforcing bar, spacing, distance from the midsurface of the concrete slab, material property name, angle to the reference axis and the reference axis were input to define each rebar layer. The material property of reinforcing bars was defined in the material section. The bilinear stress–strain relationship shown in Fig. 2 was also used in the present study for reinforcing bars.

Concrete Modeling

Concrete in Compression

Concrete in compression was modeled as an elastic–plastic material with strain softening. The stress–strain relationship for concrete in uniaxial compression proposed by Carreira and Chu (1985) was adopted in the present study as

$$\sigma_c = \frac{f'_c \gamma (\varepsilon_c / \varepsilon'_c)}{\gamma - 1 + (\varepsilon_c / \varepsilon'_c)^\gamma} \tag{1}$$

where σ_c=compressive stress in concrete; ε_c=strain in concrete; f'_c=cylinder compressive strength of concrete; ε'_c=strain corresponding to f'_c (MPa); and γ is defined by

$$\gamma = \left| \frac{f'_c}{32.4} \right|^3 + 1.55 \tag{2}$$

The strain ε'_c is usually taken as 0.002. A stress–strain curve for concrete with a compressive strength of 42.5 MPa is shown in Fig. 3. In the present study, the stress–strain behavior of concrete in compression was assumed to be linear elastic up to $0.4f'_c$. Beyond this point, it was in the plastic regions in which plastic strain was input to define the stress–strain relationship in the finite element model. The failure ratio option was used to define the failure
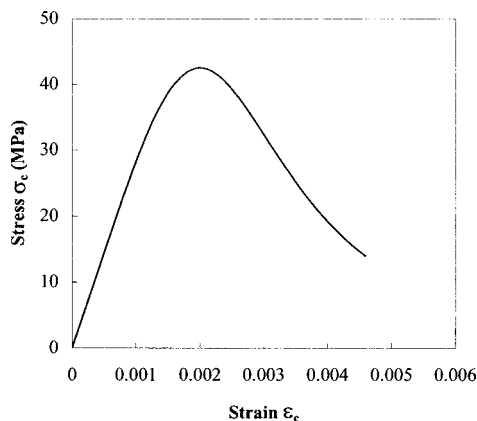
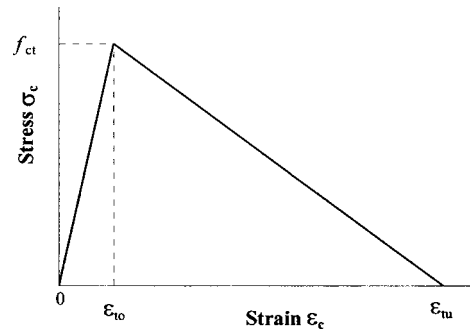**Fig. 3.** Stress–strain curve for concrete in compression

Fig. 4. Stress–strain curve for concrete in tension

surface of concrete. The ratio of the ultimate biaxial compressive stress to the ultimate uniaxial compressive stress was taken as 1.16. The ratio of the uniaxial tensile stress to the uniaxial compressive stress at failure was taken as 0.0836.

Concrete in Tension

The behavior of concrete and reinforcement in a concrete slab was modeled independently. The interaction between the concrete and reinforcing bars was simulated approximately by the tension stiffening model. The model assumes that the direct stress across a crack gradually reduces to zero as the crack opens. Tension stiffening was defined in the present study using stress–strain data. The stress–strain relationship as shown in Fig. 4 assumes that the tensile stress increases linearly with an increase in tensile strain up to concrete cracking. After concrete cracking, the tensile stress decreases linearly to zero as the concrete softens. The value of tension stiffening is an important parameter that affects the solution of a nonlinear analysis of reinforced concrete. Tension stiffening is influenced by the density of reinforcing bars, the bond, the relative size of the aggregate compared to the rebar diameter and the finite element mesh. For heavily reinforced concrete slabs, the total strain at which the tensile stress is zero is usually taken as 10 times the strain at failure in the tension stiffening model. However, it has been found that this value was not adequate for concrete slabs in composite beams (Basker et al. 2002; Liang et al. 2004a). In the present study, a total strain of 0.1 was used for reinforced concrete slabs in composite beams.

Shear Retention

The reduction in shear modulus due to concrete cracking was defined as a function of direct strain across the crack in the shear retention model. The shear modulus of cracked concrete is defined as $G = \varphi G_c$, where G_c=elastic shear modulus of uncracked concrete and φ=reduction factor, which is given by

$$\varphi = \begin{cases} (1 - \varepsilon_c / \varepsilon_{max}) & \text{for } \varepsilon_c < \varepsilon_{max} \quad (3) \\ 0 & \text{for } \varepsilon_c \geq \varepsilon_{max} \quad (4) \end{cases}$$

in which ε_c=direct strain across the crack. The shear retention model states that the shear stiffness of open cracks reduces linearly to zero as the crack opening increases. Parameters $\varepsilon_{max} = 0.005$ and $\varphi = 0.95$ were used in the present study to define the shear retention of concrete, as suggested by Thevendran et al. (1999) and Liang et al. (2004a).

Shear Connector Modeling

Wright (1990) suggested that the stud shear connection should be modeled as a discrete connection to accurately predict the nonlin-

ear behavior of composite beams with partial interaction. A three-dimensional beam element was employed in the present study to model discrete stud shear connectors. Shear connectors were assumed to connect the middle plane of the concrete slab and the top flange of the steel beam. The cross-sectional area of the beam element was modified to make it equivalent in both strength and stiffness to the actual stud shear connector in a composite beam. The bilinear stress–strain relationship illustrated in Fig. 2 was used for the shear connector material. Pin jointed truss elements with an effective stiffness were used in place of shear connectors to transfer direct stress from the concrete slab to the top flange of the steel beam. This model can be used to simulate any degree of shear connection in composite beams.

Solution Method

The nonlinear response of structural concrete is highly discontinuous due to cracking. To prevent the discontinuity, the controls, analysis=discontinuous option was specified in the nonlinear analysis of composite beams. Local instabilities often occur in the nonlinear analysis of reinforced concrete elements because of the large amounts of cracking. The modified Riks method was therefore used in the present study to prevent the local instabilities. The automatic load control scheme was employed. The deflection at midspan of the composite beam was monitored in the analysis.

Validation of Finite Element Models

The finite element model developed herein has been used to analyze a simply supported composite beam (E1) tested by Chapman and Balakrishnan (1964) and the results are compared with corresponding experimental data in this section. The span of the composite beam under a point load was 5.5 m. The cross section of the composite beam is shown in Fig. 5. Material properties of the composite beam are given in Table 1. The finite element discretization of the composite beam is shown in Fig. 1. The concrete slab was modeled with 13×61 elements. The flange of the steel beam was modeled using 2×60 elements while the steel web was modeled with 3×60 elements. The load–deflection curve of the composite beam obtained by the present study is compared with that obtained by experiments in Fig. 6. It can be observed from Fig. 6 that the initial stiffness of the composite beam predicted by the finite element model is the same as that of the experimental one. The ultimate load obtained by the present study was 494 kN, which is 95.3% of the experimental value. The nonlinear finite element analysis conformed the experimental observation that the composite beam failed by crushing of the top concrete slab at midspan. It can be concluded that the finite ele-
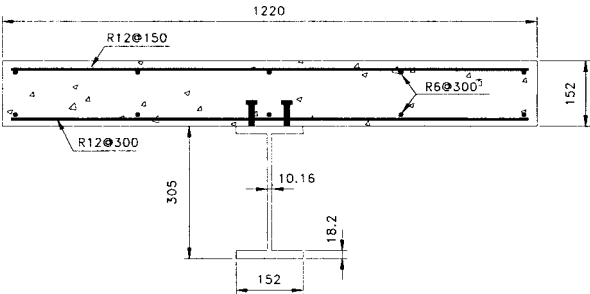
Table 1. Material Properties Used in the Analysis of Composite Beams

Material	Property	Value
Structural steel	Yield stress, f_{sy} (MPa)	265
	Ultimate strength, f_{su} (MPa)	410
	Young's modulus, E_s (MPa)	205×10^3
	Poisson's ratio, ν	0.3
	Ultimate strain, ε_{su}	0.25
Reinforcing bar	Yield stress, f_{sy} (MPa)	250
	Ultimate strength, f_{su} (MPa)	350
	Young's modulus, E_s (MPa)	200×10^3
	Poisson's ratio, ν	0.3
	Ultimate strain, ε_{su}	0.25
Concrete	Comparessive strength, f'_c (MPa)	42.5
	Tensile strength, f_{ct} (MPa)	3.553
	Young's modulus, E_c (MPa)	32,920
	Poisson's ratio, ν	0.15
	Ultimate compressive strain, ε_{cu}	0.0045
Stud shear connector	Spacing (mm)	110
	Number of rows	2
	Yield stress, f_{sy} (MPa)	435
	Ultimate strength, f_{su} (MPa)	565
	Young's modulus, E_s (MPa)	200×10^3
	Poisson's ratio, ν	0.3
	Ultimate strain, ε_{su}	0.25

ment model developed herein is reliable and conservative in predicting the ultimate strength of composite beams.

Load–Deflection Behavior

The finite element model developed has been used to investigate the ultimate load behavior of simply supported composite beams with various moment/shear ratios (α) under combined actions of bending and shear. A point load was applied to the midspan of all composite beams on the analysis. The span of the composite beam (E1) tested by Chapman and Balakrishnan (1964) was varied to give different combinations of moment and shear whereas other conditions of the composite beam were unchanged. The moment/shear ratios used in the analysis were 0.4, 0.5, 0.75, 1.0,
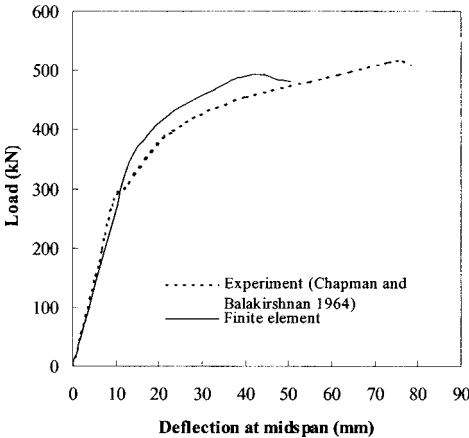
Fig. 5. Cross section of the composite beam

Fig. 6. Comparison of results by finite element modeling with experimental data
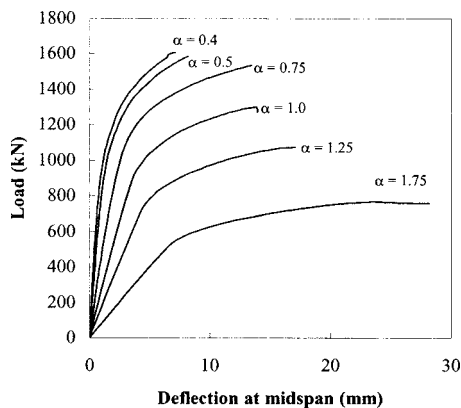
Fig. 7. Load–deflection curves of composite beams with various moment/shear ratios

Fig. 8. Moment–shear interaction of composite beams

1.25, 1.75, and 2.75 m, which correspond to the spans of 0.8, 1.0, 1.5, 2.0, 2.25, 3.5, and 5.5 m, respectively. Since these composite beams with various spans were actually cut short from the one tested by Chapman and Balakrishnan (1964), the degree of shear connection was approximately same for all cases. Material properties given in Table 1 were used for all cases.

The load–deflection curves obtained from the results of the nonlinear finite element analysis on composite beams with various moment/shear ratios are shown in Fig. 7. It can be seen from Fig. 7 that the response of composite beams to applied loads is initially linear. After concrete cracking, steel yielding and large deformations, the nonlinear load–deflection behavior is observed. It is seen that the strength and stiffness of composite beams decrease with an increase in the moment/shear ratio. The ultimate load of composite beams decreases with an increase in the moment/shear ratio. This is justified by the fact that for the same composite section, increasing the span of the composite beam will reduce the load carrying capacity of the composite beam. When the moment/shear ratio was high, the composite beam failed by flexure. In contrast, the composite beam failed by shear when the moment/shear ratio was low such as the beam with a $\alpha = 0.4$ or 0.5, as indicated in Fig. 7. The ultimate loads of these two beams are almost the same as they reach the same ultimate shear strength of the same composite section.

Moment–Shear Interaction

The behavior of composite beams depends on the moment/shear ratios. The moment–shear interaction strength of composite beams has been investigated by undertaking nonlinear analyses on composite beams with various spans and with the same cross section and material properties, as discussed in the previous section. The ultimate moment and shear capacities of these composite beams can be calculated from the ultimate loads predicted by the nonlinear finite element analysis, and are shown in Fig. 8. The ultimate moment capacity is almost not affected by the vertical shear when the moment/shear ratio is high such as the point with the maximum moment capacity shown in Fig. 8. When the vertical shear exceeds half of the ultimate shear strength of the composite section, the ultimate flexural strength of the composite beam is reduced with an increase in the vertical shear. A weak interaction between moment and shear strength is observed when the moment/shear ratio is low. This means that the composite beam can withstand similar maximum shear forces, whereas the
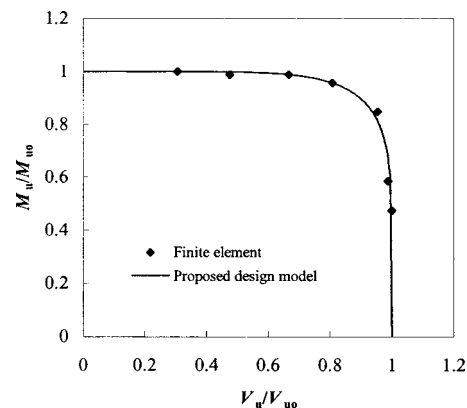
applied moments are quite different. The maximum ultimate shear strength of the composite beam obtained from the results of the finite element analysis is 804 kN, whereas it is only 439 kN according to *AS 2327.1* (Standards 1996). Through composite action, the concrete slab increases the maximum shear strength of the composite beam by 85%.

Effect of Shear Connection on Vertical Shear Strength

The effect of the degree of shear connection on the ultimate moment capacities of simply supported composite beams is reflected in design codes, such as *AS 2327.1* (Standards 1996), *EUROCODE 4* (1994) and LRFD (AISC 1999). The codes assume that the web of the steel beam resists the entire vertical shear, and do not consider the effect of shear connection on the vertical shear strength of composite beams. This assumption allows for a simple model to be given but results in conservative designs. In real composite construction, the vertical shear strength of a composite beam is in fact a function of the degree of shear connection (Donahey and Darwin 1988). To quantify this effect, a simply supported composite beam with a span of 0.8 m and with various degrees of shear connection has been analyzed. This deep composite beam is a nonflexural member where the shear load is transferred to the supports by a strut-and-tie model, as reported by Liang et al. (2000, 2002). The composite beam was a shortened version of the one tested by Chapman and Balakrishnan (1964). The cross section of the composite beam is shown in Fig. 5. Only the cross-sectional area of stud shear connectors was modified to give different degrees of shear connection while other conditions of the composite beam were unchanged. Material properties given in Table 1 were used in the analysis.

Fig. 9 shows the ultimate shear strength of the composite beam with various degrees of shear connection obtained from the finite element analysis. It can be observed from Fig. 9 that the vertical shear strength of the composite beam increases with an increase in the degree of shear connections (β). This confirms experimental findings presented by Donahey and Darwin (1988). When $\beta > 1$, the vertical shear strength is not affected by the degree of shear connection. This indicates that the composite beam exhibits full shear connection. It is also observed from Fig. 9 that the vertical shear strength of a composite beam with full shear connection is 29.5% higher than that of the one without composite action.
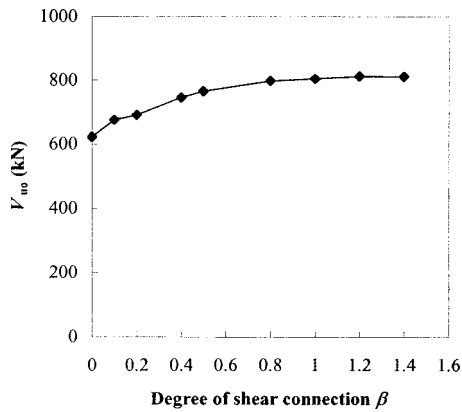
Fig. 9. Effects of shear connection on vertical shear strength of composite beams

Proposed Design Models

Design Models for Vertical Shear Strength

Experiments and nonlinear finite element analyses indicated that the concrete slab and composite action make significant contributions to the vertical shear strength of a composite beam. To take advantage of composite actions, a design model for the vertical shear strength of simply supported composite beams with any degree of shear connection is proposed as

$$V_{uo} = V_o(1 + 0.295\sqrt{\beta}) \quad (0 \leq \beta \leq 1) \tag{5}$$

where V_{uo}=ultimate shear strength of the composite beam in pure shear; V_o=ultimate shear strength of the noncomposite beam in pure shear (with zero degree of shear connection); and β=degree of shear connection. It should be noted that the pullout failure of stud shear connectors results in the damage of composite action. If this occurs, the ultimate shear strength of the damaged composite beam (V_{uo}) should be taken as V_o for safety. The proposed design model for vertical shear strength is compared with the results obtained from the nonlinear finite element analysis in Fig. 10. It is shown that the design model agrees very well with numerical predictions.

If no shear connection is provided between the concrete slab and the steel beam, the two components will work independently to resist vertical shear. The superposition rule can be applied to
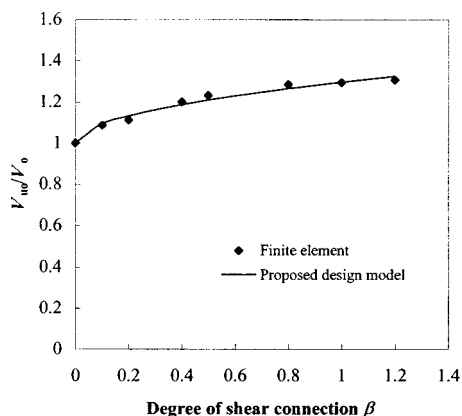

Fig. 10. Proposed design model for vertical shear strength of composite beams

the vertical shear strength of the noncomposite section. The vertical shear strength of a noncomposite beam can be expressed by

$$V_o = V_c + V_s \tag{6}$$

where V_c=contribution of the concrete slab and V_s=shear capacity of the web of the steel beam. Tests indicated that the pullout failure of stud shear connectors in composite beams might occur (Narayanan et al. 1989). This failure mode may reduce the shear resistance of the concrete slab. Therefore, the contribution of the concrete slab (V_c) should be taken as the lesser of the shear strength of the concrete slab V_{slab} and the pullout capacity of stud shear connectors T_p. The shear strength of the concrete slab is proposed as

$$V_{slab} = 1.16(f_c')^{1/3}A_{ec} \tag{7}$$

where f_c'=compressive strength of the concrete (MPa) and A_{ec}=effective shear area of concrete. The effective shear area of concrete in a solid slab can be evaluated as $A_{ec}=(b_f+D_c)D_c$, in which b_f=width of the top flange of the steel beam and D_c=total depth of the concrete slab. For a composite slab with profiled steel sheeting orientated perpendicular to the steel beam, A_{ec} can be taken as $(b_f+h_r+D_c)(D_c-h_r)$, in which h_r=rib height of the profiled steel sheeting. The effect of longitudinal steel reinforcement in the concrete slab is not considered in Eq. (7). The model gives a good estimate to the shear strength of the concrete slab in which there is little longitudinal steel reinforcement passing through the effective shear area in a composite section in the positive moment region.

The pullout capacity of stud shear connectors in composite beams with solid slabs can be expressed as

$$T_p = [\pi(d_s + h_c) + 2s]h_c f_{ct} \quad \text{(pair studs)} \tag{8}$$

$$T_p = \pi(d_s + h_c)h_c f_{ct} \quad \text{(single stud)} \tag{9}$$

where d_s=head diameter of the stud; h_c=total height of the stud; s=transverse spacing of studs; and f_{ct}=tensile strength of concrete (MPa). The pullout capacity of stud shear connectors in composite slabs incorporating profiled steel sheeting should be calculated using the effective pullout failure surface in Eqs. (8) and (9).

The shear capacity of the web of the steel beam can be determined by (Trahair and Bradford 1991)

$$V_s = 0.6\alpha_w f_{yw}d_w t_w \tag{10}$$

where f_{yw}=yield strength of the steel web (MPa); d_w=depth of the steel web; t_w=thickness of the steel web; and α_w=reduction factor for slender webs in shear buckling. The reduction factor α_w is equal to 1.0 for stocky steel webs without shear buckling.

Design Model for Strength Interaction

Both the ultimate moment and shear capacities of a composite beam under combined actions of bending and shear are a function of the degree of shear connection. The effect of the vertical shear on the ultimate moment capacity of composite beams is considered in *AS 2327.1* (Standards 1996) and *EUROCODE 4* (1994) by using interaction equations. However, design codes allow only the shear strength of the steel web to be considered in the interaction equations. To determine the flexural and shear strengths of simply supported composite beams, design model for strength interactions is proposed as

$$\left(\frac{M_u}{M_{uo}}\right)^6 + \left(\frac{V_u}{V_{uo}}\right)^6 = 1 \qquad (11)$$

where M_u=ultimate moment capacity of the composite beam in combined bending and shear; M_{uo}=ultimate moment capacity of the composite beam in pure bending; V_u=ultimate shear strength of the composite beam in combined bending and shear; and V_{uo}=ultimate shear strength of the composite beam in pure shear. The proposed design model for strength interaction is compared with the results from the finite element analysis in Fig. 8. It is seen from Fig. 8 that the proposed design model agrees very well with the numerical results.

The ultimate moment capacity (M_{uo}) of a composite section with any degree of shear connections in pure bending can be determined by the rigid plastic analysis method (Oehlers and Bradford 1999; Standards 1996; *EUROCODE 4*, 1994; AISC 1999). The ultimate shear strength (V_{uo}) of a composite beam under pure shear loading can be evaluated by Eq. (5). Any point (M_u, V_u) on the moment–shear interaction curve shown in Fig. 8 corresponds to the applied moment/shear ratio that defines the load path. This means that the ultimate moment/shear ratio (M_u/V_u) is also equal to α. If the applied moment/shear ratio is known, the ultimate moment and shear capacities of a composite beam in combined bending and shear can be determined by solving the only unknown in Eq. (11).

Conclusions

The ultimate flexural and shear strengths of simply supported composite beams under combined bending and shear have been investigated by using the finite element method in this paper. A three-dimensional finite element model, which incorporates geometric and material nonlinear behavior of the reinforced concrete slab, stud shear connectors and the steel beam in a composite beam, has been presented for the nonlinear analysis of composite beams with any degrees of shear connection. The effects of the concrete slab on the flexural and shear strengths were taken into account in the analysis. The load–deflection behavior of composite beams with various moment/shear ratios has been demonstrated. The effects of the degree of shear connection on vertical shear strength of composite beams have also been studied. Design models for the vertical shear strength and for ultimate moment–shear interactions have been developed for the design of simply supported composite beams under combined actions.

The finite element models developed in this study predict well the ultimate strength of composite beams in combined bending and shear. Numerical results indicate that the vertical shear strength of composite beams increases with an increase in the degree of shear connection. The design model for vertical shear strength is proposed as a function of the shear capacity of the noncomposite section and the degree of shear connection. The proposed shear strength equation for the noncomposite section comprises contributions from the concrete slab and the steel beam. The behavior of composite beams depends on the moment/shear ratio. If the applied moment and shear force at the cross section of a composite beam is known, the moment–shear interaction equation developed can be used to determine the ultimate moment and shear capacities of the composite beam. Although the proposed design models have been based on the nonlinear analysis of the tested composite beam, similar design models for continuous composite beams have been verified by experimental results (Liang et al. 2004a). The design models presented in this paper are applicable to simply supported composite beams with any section. The proposed design models take account of the effects of the concrete slab and composite action on both the ultimate moment and shear capacities of composite beams, and thus provide a consistent and economical design procedure for simply supported composite beams.

Notation

The following symbols are used in this paper:

A_{ec} = effective shear area of concrete;
b_f = width of the top flange of steel beam;
D_c = total depth of the concrete slab;
d_s = diameter of the head of headed stud shear connector;
d_w = depth of the web of steel beam;
E_c = Young's modulus of concrete;
E_s = Young's modulus of steel;
f_c' = cylinder compressive strength of concrete;
f_{ct} = concrete tensile strength;
f_{su} = ultimate strength of steel;
f_{sy} = yield strength of steel;
f_{yw} = yield stress of the web of steel beam;
G = shear modulus of cracked concrete;
G_c = elastic shear modulus of uncracked concrete;
h_c = height of shear connector;
h_r = rib height of profiled steel sheeting;
M_u = ultimate moment capacity of composite beam;
M_{uo} = ultimate moment capacity of composite beam in pure bending;
T_p = pullout capacity of stud shear connectors;
t_w = thickness of steel web;
V_c = shear contribution of the concrete slab;
V_o = ultimate shear strength of noncomposite beam;
V_s = ultimate shear strength of the steel web;
V_{slab} = shear strength of the concrete slab;
V_u = ultimate shear strength of composite beam in combined bending and shear;
V_{uo} = ultimate shear strength of noncomposite beam in pure shear;
α = moment/shear ratio, $\alpha = M/V$;
α_w = reduction factor for slender web;
β = degree of shear connection;
γ = parameter used to define stress-strain curve for concrete;
ε_c = strain in concrete;
ε_c' = strain in concrete corresponding to f_c';
ε_{max} = maximum direct strain;
ε_s = strain in steel;
ε_{su} = ultimate strain in steel;
ε_{sy} = yield strain in steel;
ν = Poisson's ratio;
σ_c = compressive stress in concrete;
σ_s = stress in steel; and
φ = reduction factor.

References

ABAQUS User's Manual, Version 6.3. (2002). Hibbitt, Karlsson and Sorensen, Inc., Pawtucket, R.I.

Al-Amery, R. I. M., and Roberts, T. M. (1990). "Nonlinear finite difference analysis of composite beams with partial interaction." *Comput. Struct.*, 35(1), 81–87.

Allison, R. W., Johnson, R. P., and May, I. M. (1982). "Tension field action in composite plate girders." *Proc.-Inst. Civ. Eng.*, *Part 2*, 73(2), 255–276.

American Institution of Steel Construction (AISC). (1999). *Load and resistance factor design specification for steel buildings*, Chicago.

Baskar, K., Shanmugam, N. E., and Thevendran, V. (2002). "Finite-element analysis of steel-concrete composite plate girder." *J. Struct. Eng.*, 128(9), 1158–1168.

Carreira, D. J., and Chu, K. H. (1985). "Stress-strain relationship for plain concrete in compression." *ACI Struct. J.*, 82(11), 797–804.

Chapman, J. C., and Balakrishnan, S. (1964). "Experiments on composite beams." *Struct. Eng.*, 42(11), 369–383.

Clawson, W. C., and Darwin, D. (1982). "Tests of composite beams with web openings." *J. Struct. Div. ASCE*, 108(1), 145–162.

Darwin, D., and Donahey, R. C. (1988). "LRFD for composite beams with unreinforced web openings." *J. Struct. Eng.*, 114(3), 535–552.

Donahey, R. C., and Darwin, D. (1988). "Web openings in composite beams with ribbed slabs." *J. Struct. Eng.*, 114(3), 518–534.

EUROCODE 4. (1994). "Design of composite steel and concrete structures, part 1.1, general rules and rules for buildings." British Standards Institution, London.

Fabbrocino, G., Manfredi, G., and Cosenza, E. (2000). "Analysis of continuous composite beams including partial interaction and bond." *J. Struct. Eng.*, 126(11), 1288–1294.

Hirst, M. J. S., and Yeo, M. F. (1980). "The analysis of composite beams using standard finite element programs." *Comput. Struct.*, 11(3), 233–237.

Johnson, R. P., and Anderson, D. (1993). *Designer's Handbook to Eurocode 4*, Thomas Telford, London.

Johnson, R. P., and Willmington, R. T. (1972). "Vertical shear in continuous composite beams." *Proc.-Inst. Civ. Eng.*, Part 2, 53, 189–205, September.

Kemp, A. R., Byfield, M. P., and Nethercot, D. A. (2002). "Effect of strain hardening on flexural properties of steel beams." *Struct. Eng.* 80(8), 29–35.

Liang, Q. Q., and Uy, B. (2000). "Theoretical study on the post-local buckling of steel plates in concrete-filled box columns." *Comput. Struct.*, 75(5), 479–490.

Liang, Q. Q., Uy, B., Bradford, M. A., and Ronagh, H. R. (2004a). "Ultimate strength of continuous composite beams in combined bending and shear." *J. Constr. Steel Res.*, 60(8), 1109–1128.

Liang, Q. Q., Uy, B., and Steven, G. P. (2002). "Performance-based optimization for strut-tie modeling of structural concrete." *J. Struct. Eng.*, 128(6), 815–823.

Liang, Q. Q., Uy, B., Wright, H. D., and Bradford, M. A. (2004b). "Local buckling of steel plates in double skin composite panels under biaxial compression and shear." *J. Struct. Eng.*, 130(3), 443–451.

Liang, Q. Q., Xie, Y. M., and Steven, G. P. (2000). "Topology optimization of strut-and-tie models in reinforced concrete structures using an evolutionary procedure." *ACI Struct. J.*, 97(2), 322–330.

Narayanan, R., Al-Amery, R. I. M., and Roberts, T. M. (1989). "Shear strength of composite plate girders with rectangular web cut-outs." *J. Constr. Steel Res.*, 12(2), 151–166.

Oehlers, D. J., and Bradford, M. A. (1999). *Elementary behaviour of composite steel and concrete structural members*, Butterworth-Heinemann, Oxford, U.K.

Porter, D. M., and Cherif, Z. A. E. (1987). "Ultimate shear strength of thin webbed steel and concrete composite girders." *Proc., Int. Conf. on Steel and Aluminium Structures*, Cardiff, U.K., R. Narayanan et al., eds., Elsevier, London, 55–64.

Razaqpur, A. G., and Nofal, M. (1989). "A finite element for modeling the nonlinear behaviour of shear connectors in composite structures." *Comput. Struct.*, 32(1), 169–174.

Roberts, T. M., and Al-Amery, R. I. M. (1991). "Shear strength of composite plate girders with web cutouts." *J. Struct. Eng.*, 117(7), 1897–1910.

Salari, M. R., Spacone, E., Shing, B., and Frangopol, D. M. (1998). "Nonlinear analysis of composite beams with deformable shear connectors." *J. Struct. Eng.*, 124(10), 1148–1158.

Sebastian, W., and McConnel, R. E. (2000). "Nonlinear FE analysis of steel-concrete composite structures." *J. Struct. Eng.*, 126(6), 662–674.

Standard Australia (1996). "Composite structures, part I: Simply supported beams." *AS 2327.1*, Sydney.

Thevendran, V., Chen, S., Shanmugam, N. E., and Liew, J. Y. R. (1999). "Nonlinear analysis of steel-concrete composite beams curved in plan." *Finite Elem. Anal. Design*, 32(3), 125–139.

Trahair, N. S., and Bradford, M. A. (1991). *The behaviour and design of steel structures*, Chapman and Hall, London.

Wright, H. D. (1990). "The deformation of composite beams with discrete flexible connection." *J. Constr. Steel Res.*, 15(1–2), 49–64.

Yam, L. C. P., and Chapman, J. C. (1968). "The inelastic behaviour of simply supported composite beams of steel and concrete." *Proc.-Inst. Civ. Eng.*, 41(1), 651–683.