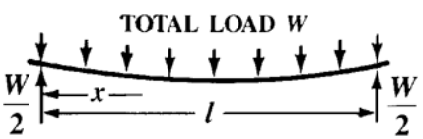
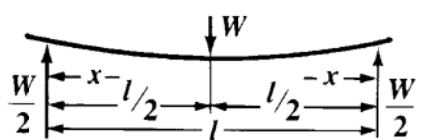
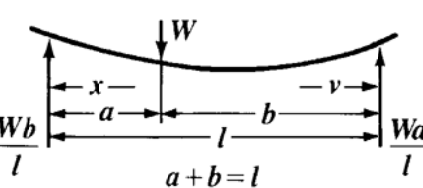
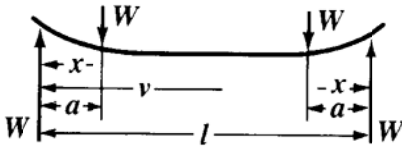
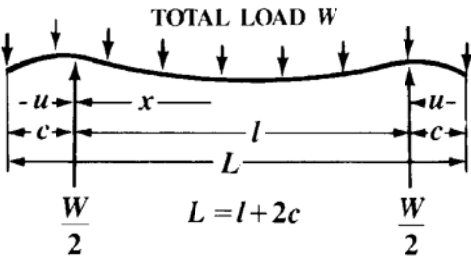


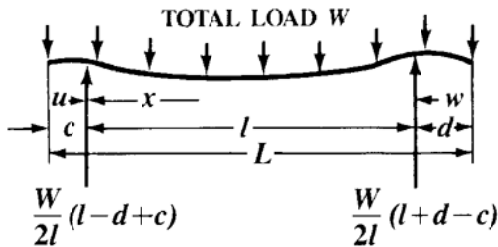
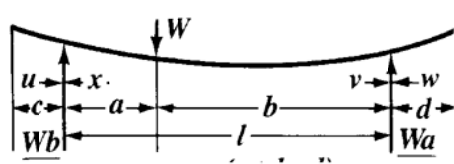
Stresses and Deflections in Beams

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 1. — Supported at Both Ends, Uniform Load				
 <p>TOTAL LOAD W</p>	$s = -\frac{W}{2Zl}x(l-x)$	Stress at center, $-\frac{Wl}{8Z}$ If cross-section is constant, this is the maximum stress.	$y = \frac{Wx(l-x)}{24EI} [l^2 + x(l-x)]$	Maximum deflection, at center, $\frac{5}{384} \frac{Wl^3}{EI}$
Case 2. — Supported at Both Ends, Load at Center				
	Between each support and load, $s = -\frac{Wx}{2Z}$	Stress at center, $-\frac{Wl}{4Z}$ If cross-section is constant, this is the maximum stress.	Between each support and load, $y = \frac{Wx}{48EI} (3l^2 - 4x^2)$	Maximum deflection, at load, $\frac{Wl^3}{48EI}$
Case 3. — Supported at Both Ends, Load at any Point				
	For segment of length a , $s = -\frac{Wbx}{Zl}$ For segment of length b , $s = -\frac{Wav}{Zl}$	Stress at load, $-\frac{Wab}{Zl}$ If cross-section is constant, this is the maximum stress.	For segment of length a , $y = \frac{Wbx}{6EI} (l^2 - x^2 - b^2)$ For segment of length b , $y = \frac{Wav}{6EI} (l^2 - v^2 - a^2)$	Deflection at load, $\frac{Wa^2b^2}{3EI}$ Let a be the length of the shorter segment and b of the longer one. The maximum deflection $\frac{Wav_1^3}{3EI}$ is in the longer segment, at $v = b\sqrt{\frac{1}{3} + \frac{2a}{3b}} = v_1$

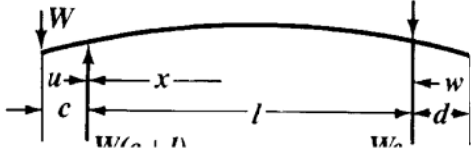
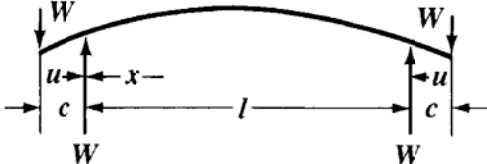
Stresses and Deflections in Beams (Continued)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 4. — Supported at Both Ends, Two Symmetrical Loads				
	<p>Between each support and adjacent load,</p> $s = -\frac{Wx}{Z}$ <p>Between loads,</p> $s = -\frac{Wa}{Z}$	<p>Stress at each load, and at all points between, $-\frac{Wa}{Z}$</p>	<p>Between each support and adjacent load,</p> $y = \frac{Wx}{6EI}[3a(l-a) - x^2]$ <p>Between loads,</p> $y = \frac{Wa}{6EI}[3v(l-v) - a^2]$	<p>Maximum deflection at center,</p> $\frac{Wa}{24EI}(3l^2 - 4a^2)$ <p>Deflection at loads</p> $\frac{Wa^2}{6EI}(3l - 4a)$
Case 5. — Both Ends Overhanging Supports Symmetrically, Uniform Load				
	<p>Between each support and adjacent end,</p> $s = \frac{W}{2Zl}(c - u)^2$ <p>Between supports,</p> $s = \frac{W}{2ZL}(c^2 - x(l - x))$	<p>Stress at each support,</p> $\frac{wc^2}{2ZL}$ <p>Stress at center,</p> $\frac{W}{2ZL}(c^2 - \frac{1}{4}l^2)$ <p>If cross-section is constant, the greater of these is the maximum stress.</p> <p>If l is greater than $2c$, the stress is zero at points $\sqrt{\frac{1}{4}l^2 - c^2}$ on both sides of the center.</p> <p>If cross-section is constant and if $l = 2.828c$, the stresses at supports and center are equal and opposite, and are</p> $\pm \frac{WL}{46.62Z}$	<p>Between each support and adjacent end,</p> $y = \frac{Wu}{24EIL}[6c^2(l + u) - u^2(4c - u) - l^3]$ <p>Between supports,</p> $y = \frac{Wx(l - x)}{24EIL}[x(l - x) + l^2 - 6c^2]$	<p>Deflection at ends,</p> $\frac{Wc}{24EIL}[3c^2(c + 2l) - l^3]$ <p>Deflection at center,</p> $\frac{Wl^2}{384EIL}(5l^2 - 24c^2)$ <p>If l is between $2c$ and $2.449c$, there are maximum upward deflections at points $\sqrt{3(\frac{1}{4}l^2 - c^2)}$ on both sides of the center, which are,</p> $-\frac{W}{96EIL}(6c^2 - l^2)^2$

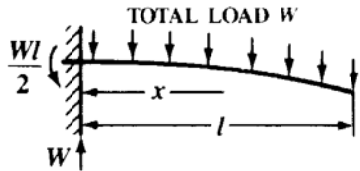
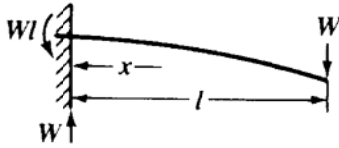
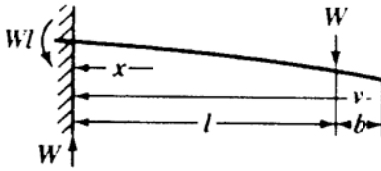
Stresses and Deflections in Beams (Continued)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 6. — Both Ends Overhanging Supports Unsymmetrically, Uniform Load				
	<p>For overhanging end of length c,</p> $s = \frac{W}{2ZL}(c-u)^2$ <p>Between supports,</p> $s = \frac{W}{2ZL} \left\{ c^2 \left(\frac{l-x}{l} \right) + d^2 \frac{x}{l} - x(l-x) \right\}$ <p>For overhanging end of length d,</p> $s = \frac{W}{2ZL}(d-w)^2$	<p>Stress at support next end of length c, $\frac{Wc^2}{2ZL}$</p> <p>Critical stress between supports is at</p> $x = \frac{l^2 + c^2 - d^2}{2l} = x_1$ <p>and is $\frac{W}{2ZL}(c^2 - x_1^2)$</p> <p>Stress at support next end of length d, $\frac{Wd^2}{2ZL}$</p> <p>If cross-section is constant, the greatest of these three is the maximum stress.</p> <p>If $x_1 > c$, the stress is zero at points $\sqrt{x_1^2 - c^2}$ on both sides of $x = x_1$.</p>	<p>For overhanging end of length c,</p> $y = \frac{Wu}{24EIL} [2l(d^2 + 2c^2) + 6c^2u - u^2(4c - u) - l^3]$ <p>Between supports,</p> $y = \frac{Wx(l-x)}{24EIL} \{ x(l-x) + l^2 - 2(d^2 + c^2) - \frac{2}{l}[d^2x + c^2(l-x)] \}$ <p>For overhanging end of length d,</p> $y = \frac{Ww}{24EIL} [2l(c^2 + 2d^2) + 6d^2w - w^2(4d - w) - l^3]$	<p>Deflection at end c,</p> $\frac{Wc}{24EIL} [2l(d^2 + 2c^2) + 3c^3 - l^3]$ <p>Deflection at end d,</p> $\frac{Wd}{24EIL} [2l(c^2 + 2d^2) + 3d^3 - l^3]$ <p>This case is so complicated that convenient general expressions for the critical deflections between supports cannot be obtained.</p>
Case 7. — Both Ends Overhanging Supports, Load at any Point Between				
	<p>Between supports:</p> <p>For segment of length a,</p> $s = -\frac{Wbx}{Zl}$ <p>For segment of length b,</p> $s = -\frac{Wav}{Zl}$ <p>Beyond supports $s = 0$.</p>	<p>Stress at load,</p> $-\frac{Wab}{Zl}$ <p>If cross-section is constant, this is the maximum stress.</p>	<p>Between supports, same as Case 3.</p> <p>For overhanging end of length c,</p> $y = -\frac{Wabu}{6EI}(l+b)$ <p>For overhanging end of length d,</p> $y = -\frac{Wabw}{6EI}(l+a)$	<p>Between supports, same as Case 3.</p> <p>Deflection at end c,</p> $-\frac{Wabc}{6EI}(l+b)$ <p>Deflection at end d,</p> $-\frac{Wabd}{6EI}(l+a)$

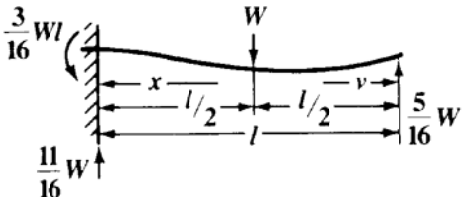
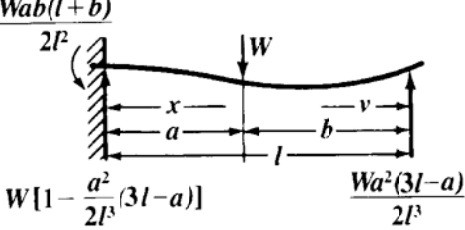
Stresses and Deflections in Beams (Continued)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 8. — Both Ends Overhanging Supports, Single Overhanging Load				
	<p>Between load and adjacent support,</p> $s = \frac{W}{Z}(c - u)$ <p>Between supports,</p> $s = \frac{Wc}{Zl}(l - x)$ <p>Between unloaded end and adjacent supports, $s = 0$.</p>	<p>Stress at support adjacent to load,</p> $\frac{Wc}{Z}$ <p>If cross-section is constant, this is the maximum stress. Stress is zero at other support.</p>	<p>Between load and adjacent support,</p> $y = \frac{Wu}{6EI}(3cu - u^2 + 2cl)$ <p>Between supports,</p> $y = -\frac{Wcx}{6EI}(l - x)(2l - x)$ <p>Between unloaded end and adjacent support, $y = \frac{Wclw}{6EI}$</p>	<p>Deflection at load,</p> $\frac{Wc^2}{3EI}(c + l)$ <p>Maximum upward deflection is at $x = .42265l$, and is $-\frac{Wcl^2}{15.55EI}$</p> <p>Deflection at unloaded end,</p> $\frac{Wcl d}{6EI}$
Case 9. — Both Ends Overhanging Supports, Symmetrical Overhanging Loads				
	<p>Between each load and adjacent support,</p> $s = \frac{W}{Z}(c - u)$ <p>Between supports,</p> $s = \frac{Wc}{Z}$	<p>Stress at supports and at all points between,</p> $\frac{Wc}{Z}$ <p>If cross-section is constant, this is the maximum stress.</p>	<p>Between each load and adjacent support,</p> $y = \frac{Wu}{6EI}[3c(l + u) - u^2]$ <p>Between supports,</p> $y = -\frac{Wcx}{2EI}(l - x)$	<p>Deflections at loads,</p> $\frac{Wc^2}{6EI}(2c + 3l)$ <p>Deflection at center,</p> $-\frac{Wcl^2}{8EI}$
<p>The above expressions involve the usual approximations of the theory of flexure, and hold only for small deflections. Exact expressions for deflections of any magnitude are as follows:</p> <p>Between supports the curve is a circle of radius</p> $r = \frac{EI}{Wc}; y = \sqrt{r^2 - \frac{1}{4}l^2} - \sqrt{r^2 - (\frac{1}{2}l - x)^2}$ <p>Deflection at center, $\sqrt{r^2 - \frac{1}{4}l^2} - r$</p>				

Stresses and Deflections in Beams (*Continued*)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 10. — Fixed at One End, Uniform Load				
 <p>TOTAL LOAD W</p> <p>Reaction at fixed end: $\frac{Wl}{2}$</p> <p>Length: l</p> <p>Point x from fixed end</p>	$s = \frac{W}{2Zl}(l-x)^2$	Stress at support, $\frac{Wl}{2Z}$ If cross-section is constant, this is the maximum stress.	$y = \frac{Wx^2}{24EI} [2l^2 + (2l-x)^2]$	Maximum deflection, at end, $\frac{Wl^3}{8EI}$
Case 11. — Fixed at One End, Load at Other				
 <p>Reaction at fixed end: Wl</p> <p>Length: l</p> <p>Point x from fixed end</p> <p>Point load W at free end</p>	$s = \frac{W}{Z}(l-x)$	Stress at support, $\frac{Wl}{Z}$ If cross-section is constant, this is the maximum stress.	$y = \frac{Wx^2}{6EI}(3l-x)$	Maximum deflection, at end, $\frac{Wl^3}{3EI}$
Case 12. — Fixed at One End, Intermediate Load				
 <p>Reaction at fixed end: Wl</p> <p>Length: l</p> <p>Point x from fixed end</p> <p>Point load W at distance v from free end</p> <p>Distance from fixed end to load: $l-v$</p>	Between support and load, $s = \frac{W}{Z}(l-x)$ Beyond load, $s = 0$.	Stress at support, $\frac{Wl}{Z}$ If cross-section is constant, this is the maximum stress.	Between support and load, $y = \frac{Wx^2}{6EI}(3l-x)$ Beyond load, $y = \frac{Wl^2}{6EI}(3v-l)$	Deflection at load, $\frac{Wl^3}{3EI}$ Maximum deflection, at end, $\frac{Wl^2}{6EI}(2l+3b)$

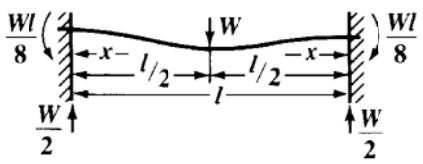
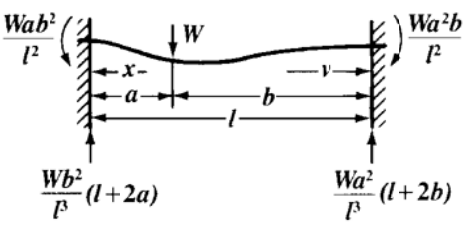
Stresses and Deflections in Beams (*Continued*)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 13. — Fixed at One End, Supported at the Other, Load at Center				
	<p>Between point of fixture and load,</p> $s = \frac{W}{16Z}(3l - 11x)$ <p>Between support and load,</p> $s = -\frac{5}{16}\frac{Wv}{Z}$	<p>Maximum stress at point of fixture, $\frac{3}{16}\frac{Wl}{Z}$</p> <p>Stress is zero at $x = \frac{3}{11}l$</p> <p>Greatest negative stress at center, $-\frac{5}{32}\frac{Wl}{Z}$</p>	<p>Between point of fixture and load,</p> $y = \frac{Wx^2}{96EI}(9l - 11x)$ <p>Between support and load,</p> $y = \frac{Wv}{96EI}(3l^2 - 5v^2)$	<p>Maximum deflection is at $v = 0.4472l$, and is $\frac{Wl^3}{107.33EI}$</p> <p>Deflection at load,</p> $\frac{7}{768}\frac{Wl^3}{EI}$
Case 14. — Fixed at One End, Supported at the Other, Load at any Point				
<p>$m = (l + a)(l + b) + al$ $n = al(l + b)$</p> 	<p>Between point of fixture and load,</p> $s = \frac{Wb}{2Zl^3}(n - mx)$ <p>Between support and load,</p> $s = -\frac{Wa^2v}{2Zl^3}(3l - a)$	<p>Greatest positive stress, at point of fixture,</p> $\frac{Wab}{2Zl^2}(l + b)$ <p>Greatest negative stress, at load,</p> $-\frac{Wa^2b}{2Zl^3}(3l - a)$ <p>If $a < 0.5858l$, the first is the maximum stress. If $a = 0.5858l$, the two are equal and are $\pm \frac{Wl}{5.83Z}$ If $a > 0.5858l$, the second is the maximum stress.</p> <p>Stress is zero at $x = \frac{n}{m}$</p>	<p>Between point of fixture and load,</p> $y = \frac{Wx^2b}{12EI l^3}(3n - mx)$ <p>Between support and load,</p> $y = \frac{Wa^2v}{12EI l^3}[3l^2b - v^2(3l - a)]$	<p>Deflection at load,</p> $\frac{Wa^3b^2}{12EI l^3}(3l + b)$ <p>If $a < 0.5858l$, maximum deflection is $\frac{Wa^2b}{6EI}\sqrt{\frac{b}{2l+b}}$ and located between load and support, at $v = l\sqrt{\frac{b}{2l+b}}$</p> <p>If $a = 0.5858l$, maximum deflection is at load and is $\frac{Wl^3}{101.9EI}$</p> <p>If $a > 0.5858l$, maximum deflection is $\frac{Wbn^3}{3EI m^2 l^3}$ and located between load and point of fixture, at</p> $x = \frac{2n}{m}$

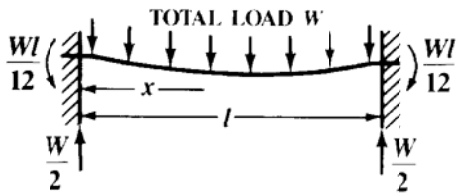
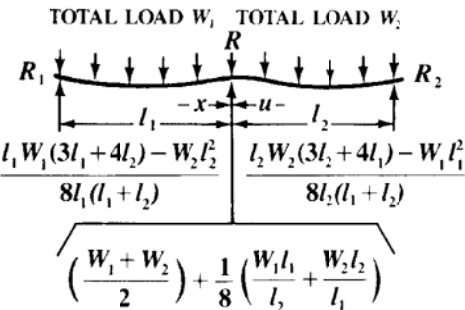
Stresses and Deflections in Beams (*Continued*)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 15. — Fixed at One End, Supported at the Other, Uniform Load				
<p>TOTAL LOAD W</p> <p>At fixed end: $\frac{Wl}{8}$ (moment), $\frac{5W}{8}$ (upward reaction)</p> <p>At support: $\frac{3W}{8}$ (upward reaction)</p> <p>Length: l, coordinate: x</p>	$s = \frac{W(l-x)}{2Zl} (\frac{1}{4}l - x)$	<p>Maximum stress at point of fixture, $\frac{Wl}{8Z}$</p> <p>Stress is zero at $x = \frac{1}{4}l$.</p> <p>Greatest negative stress is at $x = \frac{5}{8}l$ and is $-\frac{9}{128} \frac{Wl}{Z}$</p>	$y = \frac{Wx^2(l-x)}{48EI} (3l - 2x)$	<p>Maximum deflection is at $x = 0.5785l$, and is $\frac{Wl^3}{185EI}$</p> <p>Deflection at center, $\frac{Wl^3}{192EI}$</p> <p>Deflection at point of greatest negative stress, at $x = \frac{5}{8}l$ is $\frac{Wl^3}{187EI}$</p>
Case 16. — Fixed at One End, Free but Guided at the Other, Uniform Load				
<p>TOTAL LOAD W</p> <p>At fixed end: $\frac{Wl}{3}$ (moment), W (upward reaction)</p> <p>At guided end: $\frac{W}{6}$ (horizontal reaction)</p> <p>Length: l, coordinate: x</p>	$s = \frac{Wl}{Z} \left\{ \frac{1}{3} - \frac{x}{l} + \frac{1}{2} \left(\frac{x}{l} \right)^2 \right\}$	<p>Maximum stress, at support, $\frac{Wl}{3Z}$</p> <p>Stress is zero at $x = 0.4227l$</p> <p>Greatest negative stress, at free end, $-\frac{Wl}{6Z}$</p>	$y = \frac{Wx^2}{24EI} (2l - x)^2$	<p>Maximum deflection, at free end, $\frac{Wl^3}{24EI}$</p>
Case 17. — Fixed at One End, Free but Guided at the Other, with Load				
<p>At fixed end: $\frac{Wl}{2}$ (moment), W (upward reaction)</p> <p>At guided end: $\frac{W}{2}$ (horizontal reaction)</p> <p>Length: l, coordinate: x</p>	$s = \frac{W}{Z} (\frac{1}{2}l - x)$	<p>Stress at support, $\frac{Wl}{2Z}$</p> <p>Stress at free end $-\frac{Wl}{2Z}$</p> <p>These are the maximum stresses and are equal and opposite.</p> <p>Stress is zero at center.</p>	$y = \frac{Wx^2}{12EI} (3l - 2x)$	<p>Maximum deflection, at free end, $\frac{Wl^3}{12EI}$</p>

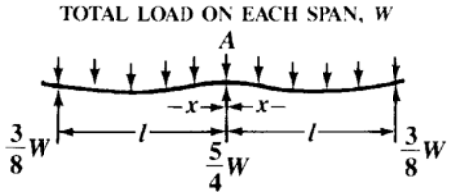
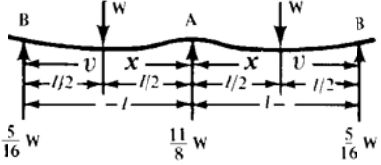
Stresses and Deflections in Beams (Continued)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 18. — Fixed at Both Ends, Load at Center				
	<p>Between each end and load,</p> $s = \frac{W}{2Z}(\frac{1}{4}l - x)$	<p>Stress at ends $\frac{Wl}{8Z}$</p> <p>at load $-\frac{Wl}{8Z}$</p> <p>These are the maximum stresses and are equal and opposite.</p> <p>Stress is zero at $x = \frac{1}{4}l$</p>	$y = \frac{Wx^2}{48EI}(3l - 4x)$	<p>Maximum deflection, at load,</p> $\frac{Wl^3}{192EI}$
Case 19. — Fixed at Both Ends, Load at any Point				
	<p>For segment of length a,</p> $s = \frac{Wb^2}{Zl^3}[al - x(l + 2a)]$ <p>For segment of length b,</p> $\frac{Wl}{8Z}$	<p>Stress at end next segment of length a, $\frac{Wab^2}{Zl^2}$</p> <p>Stress at end next segment of length b, $\frac{Wa^2b}{Zl^2}$</p> <p>Maximum stress is at end next shorter segment.</p> <p>Stress is zero at</p> $x = \frac{al}{l + 2a}$ <p>and</p> $v = \frac{bl}{l + 2b}$ <p>Greatest negative stress, at load, $-\frac{2Wa^2b^2}{Zl^3}$</p>	<p>For segment of length a,</p> $y = \frac{Wx^2b^2}{6EI l^3}[2a(l - x) + l(a - x)]$ <p>For segment of length b,</p> $y = \frac{Wv^2a^2}{6EI l^3}[2b(l - v) + l(b - v)]$	<p>Deflection at load, $\frac{Wa^3b^3}{3EI l^3}$</p> <p>Let b be the length of the longer segment and a of the shorter one. The maximum deflection is in the longer segment, at</p> $v = \frac{2bl}{l + 2b} \text{ and is}$ $x = \frac{l_1}{W_1}(W_1 - R_1)$

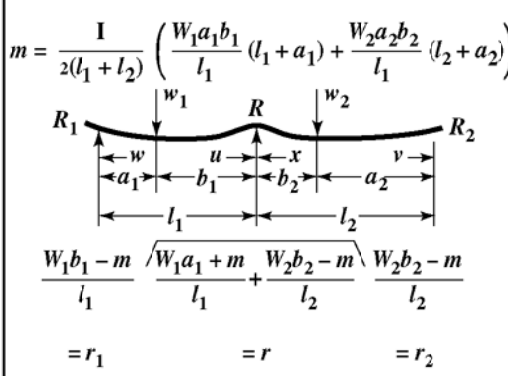
Stresses and Deflections in Beams (Continued)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 20. — Fixed at Both Ends, Uniform Load				
	$s = \frac{Wl}{2Z} \left[\frac{1}{6} - \frac{x}{l} + \left(\frac{x}{l} \right)^2 \right]$	<p>Maximum stress, at ends, $x = \frac{2n}{m}$ and is $\frac{Wbn^3}{3EIm^2l^3}$</p> <p>Stress is zero at $x = 0.7887l$ and at $x = 0.2113l$</p> <p>Greatest negative stress, at center, $-\frac{Wl}{24Z}$</p>	$y = \frac{Wx^2}{24EI}(l-x)^2$	<p>Maximum deflection, at center, $\frac{Wl^3}{384EI}$</p>
Case 21. — Continuous Beam, with Two Unequal Spans, Unequal, Uniform Loads				
	<p>Between R_1 and R,</p> $s = \frac{l_1 - x}{Z} \left\{ \frac{(l_1 - x)W_1}{2l_1} - R_1 \right\}$ <p>Between R_2 and R,</p> $s = \frac{l_2 - u}{Z} \left\{ \frac{(l_2 - u)W_2}{2l_2} - R_2 \right\}$	<p>Stress at support R,</p> $\frac{W_1 l_1^2 + W_2 l_2^2}{8Z(l_1 + l_2)}$ <p>Greatest stress in the first span is at</p> $x = \frac{l_1}{W_1} (W_1 - R_1)$ <p>and is $\nu = \frac{2bl}{l + 2b}$</p> <p>Greatest stress in the second span is at</p> $u = \frac{l_2}{W_2} (W_2 - R_2)$ <p>and is, $-\frac{R_2^2 l_2}{2ZW_2}$</p>	<p>Between R_1 and R,</p> $y = \frac{x(l_1 - x)}{24EI} \left\{ (2l_1 - x)(4R_1 - W_1) - \frac{W_1(l_1 - x)^2}{l_1} \right\}$ <p>Between R_2 and R,</p> $y = \frac{u(l_2 - u)}{24EI} \left\{ (2l_2 - u)(4R_2 - W_2) - \frac{W_2(l_2 - u)^2}{l_2} \right\}$	<p>This case is so complicated that convenient general expressions for the critical deflections cannot be obtained.</p>

Stresses and Deflections in Beams (*Continued*)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 22. — Continuous Beam, with Two Equal Spans, Uniform Load				
 <p>TOTAL LOAD ON EACH SPAN, W</p>	$s = \frac{W(l-x)}{2Zl} \left(\frac{1}{4}l - x \right)$	<p>Maximum stress at point</p> $u = \frac{l_2}{W_2} (W_2 - R_2)$ <p>Stress is zero at $x = \frac{5}{8}l$</p> <p>Greatest negative stress is at $x = \frac{5}{8}l$ and is,</p> $-\frac{9}{128} \frac{Wl}{Z}$	$y = \frac{Wx^2(l-x)}{48EI} (3l - 2x)$	<p>Maximum deflection is at $x = 0.5785l$, and is $\frac{Wl^3}{185EI}$</p> <p>Deflection at center of span,</p> $\frac{Wl^3}{192EI}$ <p>Deflection at point of greatest negative stress, at $x = \frac{5}{8}l$ is</p> $\frac{Wl^3}{187EI}$
Case 23. — Continuous Beam, with Two Equal Spans, Equal Loads at Center of Each				
	<p>Between point A and load,</p> $s = \frac{W}{16Z} (3l - 11x)$ <p>Between point B and load,</p> $s = -\frac{5}{16} \frac{Wv}{Z}$	<p>Maximum stress at point A,</p> $\frac{3}{16} \frac{Wl}{Z}$ <p>Stress is zero at</p> $x = \frac{3}{11}l$ <p>Greatest negative stress at center of span,</p> $-\frac{5}{32} \frac{Wl}{Z}$	<p>Between point A and load,</p> $y = \frac{Wx^2}{96EI} (9l - 11x)$ <p>Between point B and load,</p> $y = \frac{Wv}{96EI} (3l^2 - 5v^2)$	<p>Maximum deflection is at $v = 0.4472l$, and is $\frac{Wl^3}{107.33EI}$</p> <p>Deflection at load, $\frac{7}{768} \frac{Wl^3}{EI}$</p>

Stresses and Deflections in Beams (*Continued*)

Type of Beam	Stresses		Deflections	
	General Formula for Stress at any Point	Stresses at Critical Points	General Formula for Deflection at any Point ^a	Deflections at Critical Points ^a
Case 24. — Continuous Beam, with Two Unequal Spans, Unequal Loads at any Point of Each				
 $m = \frac{I}{2(l_1 + l_2)} \left(\frac{W_1 a_1 b_1}{l_1} (l_1 + a_1) + \frac{W_2 a_2 b_2}{l_2} (l_2 + a_2) \right)$ $\frac{W_1 b_1 - m}{l_1} = \frac{\sqrt{W_1 a_1 + m}}{l_1} = \frac{W_2 b_2 - m}{l_2} = \frac{\sqrt{W_2 a_2 + m}}{l_2}$ $= r_1 \qquad = r \qquad = r_2$	<p>Between R_1 and W_1,</p> $s = -\frac{w r_1}{Z}$ <p>Between R and W_1, $s =$</p> $\frac{1}{l_1 Z} [m(l_1 - u) - W_1 a_1 u]$ <p>Between R and W_2, $s =$</p> $\frac{1}{l_2 Z} [m(l_2 - x) - W_2 a_2 x]$ <p>Between R_2 and W_2,</p> $s = -\frac{v r_2}{Z}$	<p>Stress at load W_1,</p> $-\frac{a_1 r_1}{Z}$ <p>Stress at support R,</p> $\frac{m}{Z}$ <p>Stress at load W_2,</p> $-\frac{a_2 r_2}{Z}$ <p>The greatest of these is the maximum stress.</p>	<p>Between R_1 and W_1,</p> $y = \frac{w}{6EI} \left\{ (l_1 - w)(l_1 + w)r_1 - \frac{W_1 b_1^3}{l_1} \right\}$ <p>Between R and W_1,</p> $y = \frac{u}{6EI l_1} [W_1 a_1 b_1 (l_1 + a_1) - W_1 a_1 u^2 - m(2l_1 - u)(l_1 - u)]$ <p>Between R and W_2,</p> $y = \frac{x}{6EI l_2} [W_2 a_2 b_2 (l_2 + a_2) - W_2 a_2 x^2 - m(2l_2 - x)(l_2 - x)]$ <p>Between R_2 and W_2,</p> $y = \frac{v}{6EI} \left\{ (l_2 - v)(l_2 + v)r_2 - \frac{W_2 b_2^3}{l_2} \right\}$	<p>Deflection at load W_1,</p> $\frac{a_1 b_1}{6EI l_1} [2a_1 b_1 W_1 - m(l_1 + a_1)]$ <p>Deflection at load W_2,</p> $\frac{a_2 b_2}{6EI l_2} [2a_2 b_2 W_2 - m(l_2 + a_2)]$ <p>This case is so complicated that convenient general expressions for the maximum deflections cannot be obtained.</p>

^aThe deflections apply only to cases where the cross section of the beam is constant for its entire length.

In the diagrammatical illustrations of the beams and their loading, the values indicated near, but below, the supports are the “reactions” or upward forces at the supports. For Cases 1 to 12, inclusive, the reactions, as well as the formulas for the stresses, are the same whether the beam is of constant or variable cross-section. For the other cases, the reactions and the stresses given are for constant cross-section beams only.

The bending moment at any point in inch-pounds is $s \times Z$ and can be found by omitting the divisor Z in the formula for the stress given in the tables. A positive value of the bending moment denotes tension in the upper fibers and compression in the lower ones. A negative value denotes the reverse. The value of W corresponding to a given stress is found by transposition of the formula. For example, in Case 1, the stress at the critical point is $s = -Wl \div 8Z$. From this formula we find $W = -8Zs \div l$. Of course, the negative sign of W may be ignored.