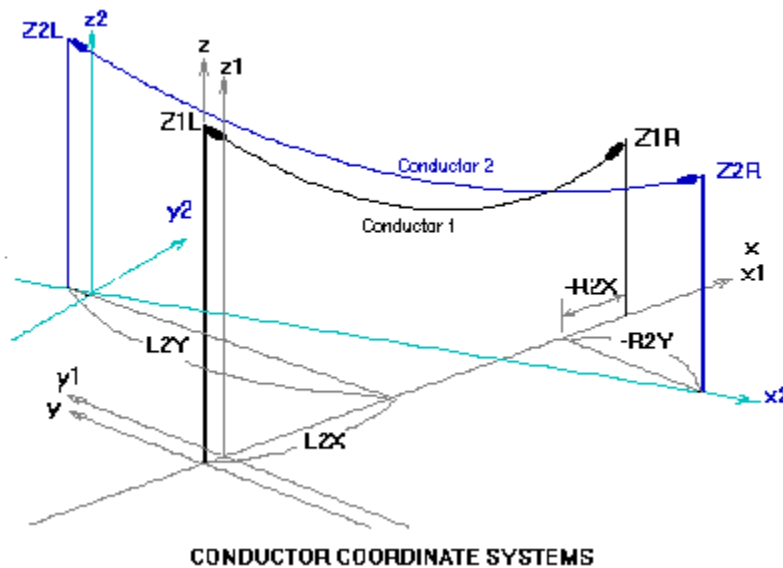


Client: _____ Computed by: _____
 Project: _____ Date: _____
 Project No.: _____ File No.: _____ Checked by: _____
 Title: Calculation of Distance Between Conductors Date: _____

These calculations determine the distance between two conductors at any point on each conductor. By varying the point locations, defined as the distance from one support of each conductor, the minimum distance can be found. Strain insulator droop (sag from horizontal) is considered separately from conductor sag to provide greater accuracy, particularly for slack spans. Distance from one conductor to a guy wire or other straight line structure, e.g. a pole, can be found by treating the guy or structure as a second conductor with a very high tension and low unit weight.

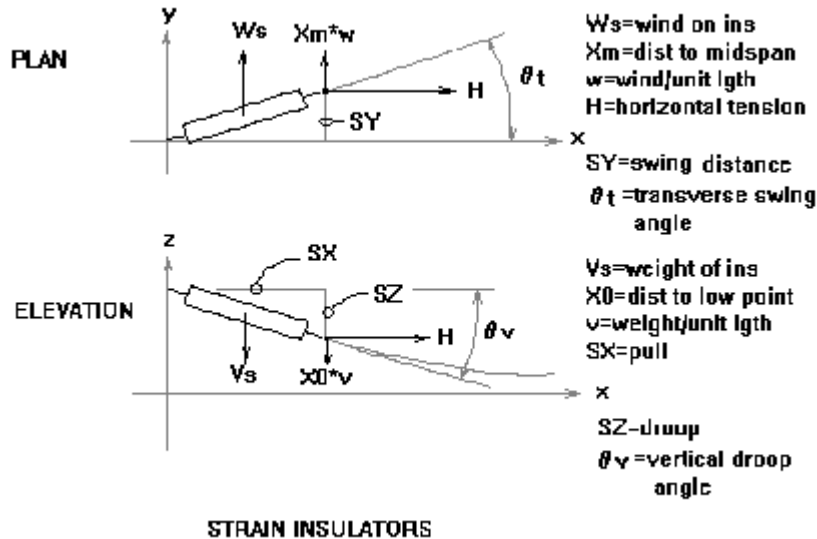
Catenary equations are based on equations given in "Sag-Tension Computations and Field Measurements of Bonneville Power Administration", Paul F. Winkelman, AIEE Paper 59-900, February 1960.

Calculations are based on a coordinate system where the left strain insulator attachment point of conductor 1 is at $x = 0$, $y = 0$, $z = \text{height}$ and the right attachment point is at $x = A1$ (span), $y = 0$, $z = \text{height}$. The coordinate system, span length, and attachment heights are adjusted to account for the effect of strain insulators. The following figure shows the definition of the coordinate systems and some key distances. Conductor positions are first calculated on two separate coordinate systems: $x1, y1, z1$ and $x2, y2, z2$. These coordinate systems are defined by the ends of the conductors. The left end of conductor 1 is at $x1 = 0$, $y1 = 0$, $z1 = \text{height}$. After calculating the conductor positions in terms of $x1, y1, z1$ and $x2, y2, z2$, the coordinates are translated and rotated to convert them to the x, y, z system.



Wind is assumed to be normal to conductor 1 (direction $y1$). The wind force on conductor 2 is reduced by the cosine of the angle between the conductors. No in-line movement of conductor 2 by the wind is considered.

Strain insulators are assumed to be rigid and of uniform weight per unit length. The wind force on the insulators is assumed to be the same as on a solid cylinder of the same diameter. Any swing of suspension insulators must be accounted for separately in the coordinates for the attachment points; this is not considered in these calculations. Strain insulator length, droop, pull out, and sideways swing (as shown on the figure below) change the effective conductor span from between the conductor supports to between the ends of the insulators. If there are no strain insulators on one end of a conductor, enter a value of zero for the corresponding strain insulator length. This is also the case where the conductor is suspended from the structure on suspension insulators - the support locations are at the connections of the conductors to the insulators.



DERIVED UNITS:

$$\text{psf} \equiv \frac{\text{psi}}{144}$$

INPUT DATA:

Wind pressure $P_w := 4 \cdot \text{psf}$

Conductor 1:

Span length	$A1 := 250 \cdot \text{ft}$	Conductor unit weight	$v1 := 1.019 \cdot \frac{\text{lbf}}{\text{ft}}$
Conductor diameter	$Dc1 := 0.99 \cdot \text{in}$	Horizontal tension	$H1 := 1000 \cdot \text{lbf}$
Left support height	$Z1L := 50 \cdot \text{ft}$	Right support height	$Z1R := 55 \cdot \text{ft}$
Strain insulators:			
weight	$Vs1 := 84 \cdot \text{lbf}$	length (left str)	$Ls1L := 4.35 \cdot \text{ft}$
diameter	$Ds1 := 10 \cdot \text{in}$	length (right str)	$Ls1R := 4.35 \cdot \text{ft}$

Conductor 2:

		Conductor unit weight	$v2 := 0.875 \cdot \frac{\text{lbf}}{\text{ft}}$
Conductor diameter	$Dc2 := 0.99 \cdot \text{in}$	Horizontal tension	$H2 := 3000 \cdot \text{lbf}$
Left support height	$Z2L := 40 \cdot \text{ft}$	Right support height	$Z2R := 74 \cdot \text{ft}$
Strain insulators:			
weight	$Vs2 := 84 \cdot \text{lbf}$	length (left str)	$Ls2L := 4.35 \cdot \text{ft}$
diameter	$Ds2 := 10 \cdot \text{in}$	length (right str)	$Ls2R := 4.35 \cdot \text{ft}$

X,Y coordinates of supports in relation to the corresponding support of conductor 1:

left support x	$L2X := 50 \cdot \text{ft}$	right support x	$R2X := -50 \cdot \text{ft}$
left support y	$L2Y := -50 \cdot \text{ft}$	right support y	$R2Y := 250 \cdot \text{ft}$

VARIABLES:

Distance from left structure to measurement point:

Conductor 1	$xc1 := 70.7 \cdot \text{ft}$
Conductor 2	$xc2 := 52.85 \cdot \text{ft}$

CALCULATIONS:

Conductor 1:

Wind on conductor	$w1 := Pw \cdot Dc1$	$w1 = 0.33 \frac{\text{lbf}}{\text{ft}}$
Wind on left insulator	$Ws1L := Pw \cdot Ds1 \cdot Ls1L$	$Ws1L = 14.5 \text{ lbf}$
Wind on right insulator	$Ws1R := Pw \cdot Ds1 \cdot Ls1R$	$Ws1R = 14.5 \text{ lbf}$

Unadjusted support elevation difference: $Bu1 := Z1L - Z1R$ $Bu1 = -5 \text{ ft}$

Next calculate the strain insulator positions: droop down, pull out, and swing sideways. The conductor tension, wind load and weight used are unadjusted for the insulator droop and swing.

Distance to low point:

$$Au1 := A1 - Ls1L - Ls1R \quad Au1 = 241.3 \text{ ft}$$

$$X0u1 := \frac{Au1}{2} + \frac{H1}{v1} \cdot \text{asinh} \left[\frac{\frac{Bu1}{2}}{\left[\frac{H1}{v1} \cdot \sinh \left[\frac{\frac{Au1}{2}}{\left(\frac{H1}{v1} \right)} \right]} \right]} \right] \quad X0u1 = 100.368 \text{ ft}$$

Left insulator:

$$\text{droop angle} \quad \theta_{v1L} := \text{atan}\left(\frac{v1 \cdot X0u1 + \frac{Vs1}{2}}{H1}\right) \quad \theta_{v1L} = 8.21 \text{ deg}$$

$$\text{droop} \quad SZ1L := Ls1L \cdot \sin(\theta_{v1L}) \quad SZ1L = 0.621 \text{ ft}$$

$$\text{swing angle} \quad \theta_{t1L} := \text{atan}\left(\frac{w1 \cdot \frac{Au1}{2} + \frac{Ws1L}{2}}{H1}\right) \quad \theta_{t1L} = 2.695 \text{ deg}$$

$$\text{swing} \quad SY1L := Ls1L \cdot \sin(\theta_{t1L}) \quad SY1L = 0.205 \text{ ft}$$

$$\text{pull} \quad SX1L := Ls1L \cdot \cos(\theta_{v1L}) \cdot \cos(\theta_{t1L}) \quad SX1L = 4.301 \text{ ft}$$

Right insulator:

$$\text{droop angle} \quad \theta_{v1R} := \text{atan}\left[\frac{v1 \cdot (Au1 - X0u1) + \frac{Vs1}{2}}{H1}\right] \quad \theta_{v1R} = 10.515 \text{ deg}$$

$$\text{droop} \quad SZ1R := Ls1R \cdot \sin(\theta_{v1R}) \quad SZ1R = 0.794 \text{ ft}$$

$$\text{swing angle} \quad \theta_{t1R} := \text{atan}\left(\frac{w1 \cdot \frac{Au1}{2} + \frac{Ws1R}{2}}{H1}\right) \quad \theta_{t1R} = 2.695 \text{ deg}$$

$$\text{swing} \quad SY1R := Ls1R \cdot \sin(\theta_{t1R}) \quad SY1R = 0.205 \text{ ft}$$

$$\text{pull} \quad SX1R := Ls1R \cdot \cos(\theta_{v1R}) \cdot \cos(\theta_{t1R}) \quad SX1R = 4.272 \text{ ft}$$

Now make adjustments for insulator droop, pull and swing.

$$\text{Span length} \quad Aa1 := A1 - SX1L - SX1R \quad Aa1 = 241.427 \text{ ft}$$

$$\text{Left support height} \quad Za1L := Z1L - SZ1L \quad Za1L = 49.379 \text{ ft}$$

$$\text{Right support height} \quad Za1R := Z1R - SZ1R \quad Za1R = 54.206 \text{ ft}$$

$$\text{Support height difference} \quad B1 := Za1L - Za1R \quad B1 = -4.827 \text{ ft}$$

$$\text{Slope distance} \quad C1 := \sqrt{Aa1^2 + B1^2} \quad C1 = 241.475 \text{ ft}$$

$$\text{Distance to low point} \quad X01 := \frac{Aa1}{2} + \frac{H1}{v1} \cdot \text{asinh}\left[\frac{\frac{B1}{2}}{\frac{H1}{v1} \cdot \sinh\left[\frac{\frac{Aa1}{2}}{\left(\frac{H1}{v1}\right)}\right]}\right] \quad X01 = 101.142 \text{ ft}$$

$$\text{Height at low point} \quad Z01 := Za1L - \frac{H1}{v1} \cdot \left[\cosh\left[\frac{X01}{\left(\frac{H1}{v1}\right)}\right] - 1\right] \quad Z01 = 44.162 \text{ ft}$$

$$\text{Distance to wide point} \quad Xw1 := \frac{Aa1}{2} \quad Xw1 = 120.714 \text{ ft}$$

Maximum conductor blowout
(not including insulators)

$$Y_{w1} := \frac{H1}{w1} \cdot \left[\cosh \left[\frac{X_{w1}}{\left(\frac{H1}{w1} \right)} \right] - 1 \right] \quad Y_{w1} = 2.405 \text{ ft}$$

Height at measurement point

$$Z1(x1) := Z01 + \frac{H1}{v1} \cdot \left[\cosh \left[\frac{x1 - X01}{\left(\frac{H1}{v1} \right)} \right] - 1 \right]$$

Blowout at measurement point

$$Y1(x1) := Y_{w1} - \frac{H1}{w1} \cdot \left[\cosh \left[\frac{(x1 - X_{w1}) \cdot \frac{C1}{Aa1}}{\left(\frac{H1}{w1} \right)} \right] - 1 \right] + SY1L - (SY1L - SY1R) \cdot \frac{x1}{Aa1}$$

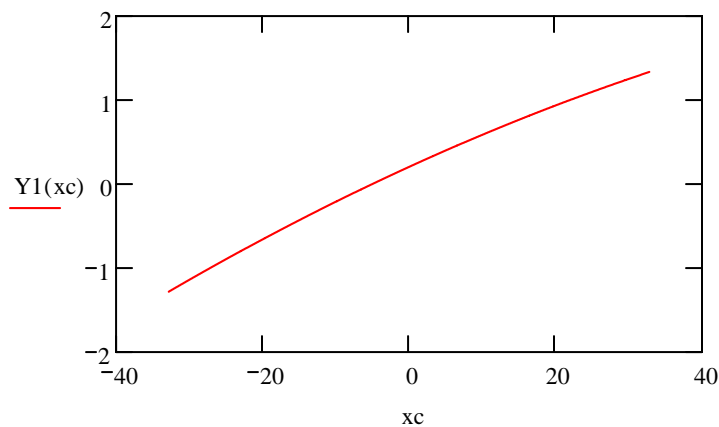
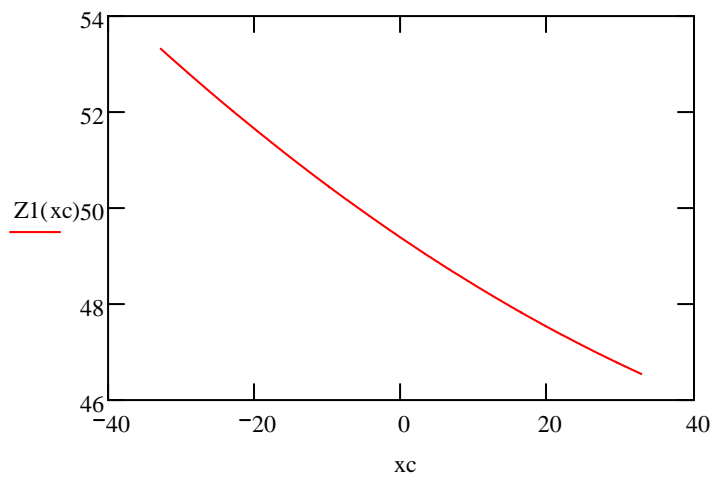
X-coordinate

$$X1(x1) := x1 + SX1L$$

Position vector

$$\mathbf{cond1}(x1) := \begin{pmatrix} X1(x1) \\ Y1(x1) \\ Z1(x1) \end{pmatrix} \quad \mathbf{cond1}(xc1) = \begin{pmatrix} 75.001 \\ 2.196 \text{ ft} \\ 44.634 \end{pmatrix}$$

Graph the conductor location: $xc := 0 \cdot \text{ft} \dots \mathbf{Aa1}$



Conductor 2:

$$\begin{aligned}
 \text{Span length} \quad A2 &:= \sqrt{(A1 + R2X - L2X)^2 + (R2Y - L2Y)^2} & A2 &= 335.41 \text{ ft} \\
 \text{Angle from conductor 1} \quad \theta_{12} &:= \text{atan}\left(\frac{R2Y - L2Y}{A1 + R2X - L2X}\right) & \theta_{12} &= 63.435 \text{ deg} \\
 \text{Wind on conductor} \quad w2 &:= Pw \cdot Dc2 \cdot \cos(\theta_{12}) & w2 &= 0.148 \frac{\text{lbf}}{\text{ft}} \\
 \text{Wind on left insulator} \quad Ws2L &:= Pw \cdot Ds2 \cdot Ls2L & Ws2L &= 14.5 \text{ lbf} \\
 \text{Wind on right insulator} \quad Ws2R &:= Pw \cdot Ds2 \cdot Ls2R & Ws2R &= 14.5 \text{ lbf} \\
 \text{Unadjusted support elevation difference:} \quad Bu2 &:= Z2L - Z2R & Bu2 &= -34 \text{ ft}
 \end{aligned}$$

Next calculate the strain insulator positions: droop down, pull out, and swing sideways. The conductor tension, wind load and weight used are unadjusted for the insulator droop and swing.

Distance to low point:

$$\begin{aligned}
 Au2 &:= A2 - Ls2L - Ls2R & Au2 &= 326.71 \text{ ft} \\
 X0u2 &:= \frac{Au2}{2} + \frac{H2}{v2} \cdot \text{asinh}\left[\frac{\frac{Bu2}{2}}{\frac{H2}{v2} \cdot \sinh\left[\frac{\frac{Au2}{2}}{\left(\frac{H2}{v2}\right)}\right]}\right] & X0u2 &= -192.674 \text{ ft}
 \end{aligned}$$

Left insulator:

$$\begin{aligned}
 \text{droop angle} \quad \theta_{v2L} &:= \text{atan}\left(\frac{v2 \cdot X0u2 + \frac{Vs2}{2}}{H2}\right) & \theta_{v2L} &= -2.416 \text{ deg} \\
 \text{droop} \quad SZ2L &:= Ls2L \cdot \sin(\theta_{v2L}) & SZ2L &= -0.183 \text{ ft} \\
 \text{swing angle} \quad \theta_{t2L} &:= \text{atan}\left(\frac{w2 \cdot \frac{Au2}{2} + \frac{Ws2L}{2}}{H2}\right) & \theta_{t2L} &= 0.599 \text{ deg} \\
 \text{swing} \quad SY2L &:= Ls2L \cdot \sin(\theta_{t2L}) & SY2L &= 0.045 \text{ ft} \\
 \text{pull} \quad SX2L &:= Ls2L \cdot \cos(\theta_{v2L}) \cdot \cos(\theta_{t2L}) & SX2L &= 4.346 \text{ ft}
 \end{aligned}$$

Right insulator:

$$\begin{aligned}
 \text{droop angle} \quad \theta_{v2R} &:= \text{atan}\left[\frac{v2 \cdot (Au2 - X0u2) + \frac{Vs2}{2}}{H2}\right] & \theta_{v2R} &= 9.397 \text{ deg} \\
 \text{droop} \quad SZ2R &:= Ls2R \cdot \sin(\theta_{v2R}) & SZ2R &= 0.71 \text{ ft}
 \end{aligned}$$

$$\text{swing angle} \quad \theta_{t2R} := \text{atan}\left(\frac{w2 \cdot \frac{Au2}{2} + \frac{Ws2R}{2}}{H2}\right) \quad \theta_{t2R} = 0.599 \text{ deg}$$

$$\text{swing} \quad SY2R := Ls2R \cdot \sin(\theta_{t2R}) \quad SY2R = 0.045 \text{ ft}$$

$$\text{pull} \quad SX2R := Ls2R \cdot \cos(\theta_{v2R}) \cdot \cos(\theta_{t2R}) \quad SX2R = 4.291 \text{ ft}$$

Now make adjustments for insulator droop, pull and swing.

$$\text{Span length} \quad Aa2 := A2 - SX2L - SX2R \quad Aa2 = 326.773 \text{ ft}$$

$$\text{Left support height} \quad Za2L := Z2L - SZ2L \quad Za2L = 40.183 \text{ ft}$$

$$\text{Right support height} \quad Za2R := Z2R - SZ2R \quad Za2R = 73.29 \text{ ft}$$

$$\text{Support height difference} \quad B2 := Za2L - Za2R \quad B2 = -33.106 \text{ ft}$$

$$\text{Slope distance} \quad C2 := \sqrt{Aa2^2 + B2^2} \quad C2 = 328.446 \text{ ft}$$

$$\text{Distance to low point} \quad X02 := \frac{Aa2}{2} + \frac{H2}{v2} \cdot \text{asinh}\left[\frac{\frac{B2}{2}}{\frac{H2}{v2} \cdot \text{sinh}\left[\frac{\frac{Aa2}{2}}{\left(\frac{H2}{v2}\right)}\right]}\right] \quad X02 = -183.251 \text{ ft}$$

$$\text{Height at low point} \quad Z02 := Za2L - \frac{H2}{v2} \cdot \left[\cosh\left[\frac{X02}{\left(\frac{H2}{v2}\right)}\right] - 1 \right] \quad Z02 = 35.285 \text{ ft}$$

$$\text{Distance to wide point} \quad Xw2 := \frac{Aa2}{2} \quad Xw2 = 163.386 \text{ ft}$$

$$\text{Maximum conductor blowout (not including insulators)} \quad Yw2 := \frac{H2}{w2} \cdot \left[\cosh\left[\frac{Xw2}{\left(\frac{H2}{w2}\right)}\right] - 1 \right] \quad Yw2 = 0.657 \text{ ft}$$

$$\text{Height at measurement point} \quad Z2(x2) := Z02 + \frac{H2}{v2} \cdot \left[\cosh\left[\frac{x2 - X02}{\left(\frac{H2}{v2}\right)}\right] - 1 \right] \quad Z2(xc2) = 43.418 \text{ ft}$$

Blowout at measurement point

$$Y2(x2) := Yw2 - \frac{H2}{w2} \cdot \left[\cosh\left[\frac{(x2 - Xw2) \cdot \frac{C2}{Aa2}}{\left(\frac{H2}{w2}\right)}\right] - 1 \right] + SY2L - (SY2L - SY2R) \cdot \frac{x2}{Aa2}$$

$$Y2(xc2) = 0.398 \text{ ft}$$

X-coordinate

$$X2(x2) := x2 + SX2L$$

$$X2(xc2) = 57.196 \text{ ft}$$

Rotation of coordinates:

$$\text{Radius} \quad R(x2) := \sqrt{X2(x2)^2 + Y2(x2)^2}$$

$$R(xc2) = 57.197 \text{ ft}$$

$$\text{Angle} \quad \theta(x2) := \text{atan}\left(\frac{Y2(x2)}{X2(x2)}\right)$$

$$\theta(xc2) = 0.399 \text{ deg}$$

$$\text{Rotated angle} \quad \theta_r(x2) := \theta(x2) + \theta_{12}$$

$$\theta_r(xc2) = 63.834 \text{ deg}$$

$$\text{Rotated X-coordinate} \quad Xr2(x2) := R(x2) \cdot \cos(\theta_r(x2))$$

$$Xr2(xc2) = 25.222 \text{ ft}$$

$$\text{Rotated Y-coordinate} \quad Yr2(x2) := R(x2) \cdot \sin(\theta_r(x2))$$

$$Yr2(xc2) = 51.336 \text{ ft}$$

$$\text{Translated X-coordinate} \quad Xt2(x2) := Xr2(x2) + L2X$$

$$Xt2(xc2) = 75.222 \text{ ft}$$

$$\text{Translated Y-coordinate} \quad Yt2(x2) := Yr2(x2) + L2Y$$

$$Yt2(xc2) = 1.336 \text{ ft}$$

$$\text{Position vector} \quad \mathbf{cond2}(x2) := \begin{pmatrix} Xt2(x2) \\ Yt2(x2) \\ Z2(x2) \end{pmatrix} \quad \mathbf{cond2}(xc2) = \begin{pmatrix} 75.222 \\ 1.336 \\ 43.418 \end{pmatrix} \text{ ft}$$

$$\text{Distance between conductors} \quad \mathbf{dist12}(x1, x2) := \mathbf{cond2}(x2) - \mathbf{cond1}(x1)$$

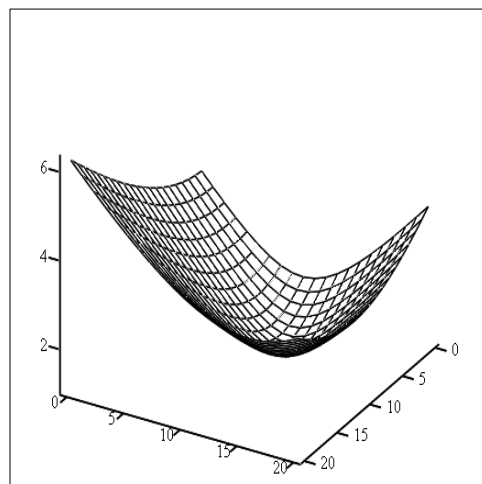
$$\mathbf{dist12}(xc1, xc2) = \begin{pmatrix} 0.222 \\ -0.861 \\ -1.217 \end{pmatrix} \text{ ft}$$

$$\text{magnitude} \quad |\mathbf{dist12}(xc1, xc2)| = 1.507 \text{ ft}$$

Plot the magnitude of the distance between conductors for a matrix of xc1 & xc2 distances.

$$N := 20 \quad i := 0..N \quad j := 0..N \quad x_i := .28 \cdot A1 + \frac{.02 \cdot A1}{N} \cdot i \quad y_j := .15 \cdot A2 + \frac{.02 \cdot A2}{N} \cdot j$$

$$\mathbf{M}_{(i,j)} := |\mathbf{dist12}(x_i, y_j)| \quad x_0 = 70 \text{ ft} \quad x_N = 75 \text{ ft} \quad y_0 = 50.312 \text{ ft} \quad y_N = 57.02 \text{ ft}$$



$$\min(\mathbf{M}) = 1.126 \text{ ft}$$

M

