P M creep stresses of rectangular section under P-M after creep and shrinkage

Setup for Units is the default to SI.

Initialization

$ORIGIN \equiv 1$ Count with fingers $TOL := 0.01$ $CTOL := 0.01$

$ton := 1000 \cdot kgf$ $ksi := 70.307 \cdot \frac{kgf}{cm^2}$ $psi := \frac{ksi}{1000}$ $kip := 453.592 \cdot kgf$ $MPa := 10.197 \cdot \frac{kgf}{cm^2}$ $°C := 1T$

$$AND2(a,b) := \begin{cases} \text{if } a = 1 \\ \quad \begin{cases} 1 & \text{if } b = 1 \\ 0 & \text{otherwise} \end{cases} \\ 0 \quad \text{otherwise} \end{cases}$$

$$OR2(a,b) := \begin{cases} 1 \quad \text{if } a = 1 \\ \text{otherwise} \\ \quad \begin{cases} 1 & \text{if } b = 1 \\ 0 & \text{otherwise} \end{cases} \end{cases}$$

$day := 86400 \cdot sec$
$month := 30 \cdot day$
$year := 365 \cdot day$

Ultimate Unrestrained Shrinkage Strain Data (CEB-FIP)

$RH := 70$ Relative humidity, enter in percent, i,e, 0 to 100 and without % symbol (permitted 40 and above)

$T_{mean\_while\_shrinking} := 17 \cdot °C$ in whole shrinkage period

$A_c := 900 \cdot cm^2$ section of the structural member

$u := 1.2m$ exposed perimeter

$\beta_{sc} := 8$
8 for rapid hardening high strength cement
5 normal or rapid hardening cement
4 slow hardening cement

$t_s := 1 \cdot day$ age at which shrinkage starts

Creep Factor (or Coefficient) Data (CEB-FIP)

measures times the elastic deformation due to creep

$t_{0L} := 30 \cdot day$ age of concrete at loading

$s := 0.2$
0.20 for rapid hardening high strength cement
0.25 normal or rapid hardening cement
0.38 slow hardening cement

$\alpha := 1$
1 for rapid hardening high strength cement
0 normal or rapid hardening cement
-1 slow hardening cement

must be paired

$aggr\_coef := 0.9$
1.0 for quartzitic
1.2 for basalt
1.2 for dense limestone
0.9 for limestone
0.7 for sandstone

$T_{mean\_while\_curing} := 16 \cdot °C$

$T_{lifespan} := 22 \cdot °C$ Enter here the mean temperature at lifespan (valid from 5 to 30 ℃ range)

Modulus of Rupture Data

$\alpha r := 4$

αr=**4.00** for cement Type I and moist curing
αr=**2.30** for cement Type III and moist curing
αr=**1.00** for cement Type I and steam curing
αr=**0.70** for cement Type III and steam curing

$\beta r := 0.85$

βr=**0.85** for cement Type I and moist curing
βr=**0.92** for cement Type III and moist curing
βr=**0.95** for cement Type I and steam curing
βr=**0.98** for cement Type III and steam curing

$Correction_{Factor} := 1$

1.00 for Normal weight
0.75 fo all-lightweight
0.85 for sand-lightweight

Concrete Data

$f_{ck} := 25 \cdot MPa$ specified strength $\quad b := 40 \cdot cm \quad h := 40 \cdot cm$

Steel Data

$f_y := 400 \cdot MPa$ yield strength $\quad E_s := 200000 \cdot MPa \quad d_1 := 5 \cdot cm$ common cover to axis of bars

bottom $\quad n_1 := 3 \quad \phi_1 := 20 \cdot mm$

sorry, limited to 2 layers

top $\quad n_2 := 3 \quad \phi_2 := 20 \cdot mm$

Load Data

$P := 125 \cdot ton$ purported real axial load $\quad M := 5 \cdot m \cdot ton$ purported real acting moment

Chart Data

$End := 10$ will be tracked for such number of months

$$A_{s1} := n_1 \cdot \pi \cdot \frac{{\phi_1}^2}{4} \qquad A_{s2} := n_2 \cdot \pi \cdot \frac{{\phi_2}^2}{4}$$

$A_s := A_{s1} + A_{s2}$ $A_c := b \cdot h - A_s$ $\rho_1 := \frac{A_{s1}}{A_c}$ $\rho_2 := \frac{A_{s2}}{A_c}$ $\rho := \frac{A_s}{A_c}$

Ec related calculus

$f_{cm} := f_{ck} + 8 \cdot MPa$ accepted mean value at 28 days age valid up to fcm=90 MPa

$E_{c0} := 21500 \cdot MPa$ $f_{cm0} := 10 \cdot MPa$ $RH_0 := 100$ $h_0 := 100 \cdot mm$ $t_1 := 1 \cdot day$ reference values

$temp\_coeff := 1.06 - 0.003 \cdot \frac{T_{mean\_while\_curing}}{°C}$ $temp\_coeff = 1.01$

$E_{c\_tangent} := temp\_coeff \cdot aggr\_coef \cdot E_{c0} \cdot \left(\frac{f_{cm}}{f_{cm0}}\right)^{\frac{1}{3}}$ without correction would be at 28 days and 20 degrees centigrade

$E_{c\_tangent} = 29154.09 MPa$ tangent modulus of elasticity

$$E_c(t) := \begin{cases} E_{c\_tangent} \cdot e^{\left[\frac{s}{2} \cdot \left(1 - \sqrt{\frac{28}{\frac{t}{day}}}\right)\right]} & \text{if } t \neq 28 \cdot day \\ \dfrac{E_{c\_tangent} \cdot e^{\left[\frac{s}{2} \cdot \left(1 - \sqrt{\frac{28}{\frac{27 \cdot day}{day}}}\right)\right]} + E_{c\_tangent} \cdot e^{\left[\frac{s}{2} \cdot \left(1 - \sqrt{\frac{28}{\frac{29 \cdot day}{day}}}\right)\right]}}{2} & \text{otherwise} \end{cases}$$

$E_c(28 \cdot day) = 29152.74 MPa$

$n := \frac{E_s}{E_c(28 \cdot day)}$ $n = 6.86$

Shrinkage related calculus

$$\beta_{sRH} := 1 - \left(\frac{RH}{RH_0}\right)^3$$

$$\beta_{RH} := \begin{cases} -1.55\cdot\beta_{sRH} & \text{if } 1 = \mathrm{AND2}(40 \le RH, RH \le 99) \\ 0.25 & \text{otherwise} \end{cases}$$

$$\beta_{RH\_corr} := \beta_{RH}\cdot\left[1 + \left(\frac{8}{103 - 100\cdot\frac{RH}{RH_0}}\right)\cdot\left(\frac{\frac{T_{mean\_while\_shrinking}}{day} - 20}{40}\right)\right]$$

$$\varepsilon_{s\_fcm} := \left[160 + 10\cdot\beta_{sc}\cdot\left(9 - \frac{f_{cm}}{f_{cm0}}\right)\right]\cdot 10^{-6}$$

$$\varepsilon_{cs0\_corr} := \varepsilon_{s\_fcm}\cdot\beta_{RH\_corr}$$

$h := 2\cdot\frac{A_c}{u}$ $\qquad h = 263.53\,mm$ $\qquad$ mean thickess for calculus

$$\beta_{SH} := 350\cdot\left(\frac{h}{h_0}\right)^2$$

$$\beta_{SH\_corr} := \beta_{SH}\cdot e^{-0.06\cdot\left(\frac{T_{mean\_while\_shrinking}}{°C} - 20\right)}$$

$$\beta_{s\_corr}(t) := \sqrt{\frac{\frac{t - t_s}{day}}{\beta_{SH\_corr} + \frac{t - t_s}{day}}}$$

$$\varepsilon_{cs}(t) := \varepsilon_{cs0\_corr} \cdot \beta_{s\_corr}(t)$$

Creep related calculus

$$stress_{ratio} := \frac{\dfrac{\dfrac{P}{1+\rho \cdot n}}{A_C}}{f_{cm}} \qquad stress_{ratio} = 0.22$$

prior to shrinkage and creep,
on **mean fcm at the age of loading**,
up to a maximum 0.6 of such **fcm**

$$corr\_tOL\_T := e^{13.65 - \frac{4000}{273 + \frac{T_{lifespan}}{°C}}} \qquad corr\_tOL\_T = 1.09$$

$$t_{0L\_corr} := \begin{cases} t_{0L} \cdot \left[\dfrac{9}{2 + \left(\dfrac{t_{0L} \cdot corr\_tOL\_T}{t_1} \right)^{1.2}} + 1 \right]^{\alpha} & \text{if } t_{0L} \cdot \left[\dfrac{9}{2 + \left(\dfrac{t_{0L} \cdot corr\_tOL\_T}{t_1} \right)^{1.2}} + 1 \right]^{\alpha} \geq 0.5 \cdot day \\ 0.5 \cdot day & \text{otherwise} \end{cases}$$

$$\phi_{RH} := 1 + \frac{1 - \dfrac{RH}{RH_0}}{0.46 \cdot \sqrt[3]{\dfrac{h}{h_0}}} \qquad \phi_{RH} = 1.47$$

$$\beta_{fcm} := \frac{5.3}{\sqrt{\dfrac{f_{cm}}{f_{cm0}}}} \qquad \beta_{fcm} = 2.92$$

$$\beta_{t0L\_corr} := \frac{1}{0.1 + \left(\dfrac{t_{0L\_corr}}{t_1} \right)^{0.2}}$$

$$\beta_H := \begin{cases} 150\cdot\left[1+\left(1.2\cdot\frac{RH}{RH_0}\right)^{18}\right]\cdot\frac{h}{h_0}+250 & \text{if } 150\cdot\left[1+\left(1.2\cdot\frac{RH}{RH_0}\right)^{18}\right]\cdot\frac{h}{h_0}+250 \le 1500 \\ 1500 & \text{otherwise} \end{cases} \qquad \beta_H = 662.42$$

$$\beta_{H\_corr} := \beta_H\cdot e^{\frac{1500}{273+\frac{T_{lifespan}}{°C}}-5.12} \qquad \beta_{H\_corr} = 639.48$$

$$\beta_{c\_corr}(t) := \left(\frac{\frac{t-t_{0L}}{t_1}}{\beta_{H\_corr}+\frac{t-t_{0L}}{t_1}}\right)^{0.3}$$

$$\phi_{RH\_corr} := e^{\left[0.015\cdot\left(\frac{T_{lifespan}}{°C}-20\right)\right]}+\left(\phi_{RH}-1\right)\cdot\sqrt{e^{\left[0.015\cdot\left(\frac{T_{lifespan}}{°C}-20\right)\right]}} \qquad \phi_{RH\_corr} = 1.51$$

$$stress_{corr} := \begin{cases} e^{1.5\left(stress_{ratio}-0.4\right)} & \text{if } stress_{ratio} > 0.4 \\ 1 & \text{otherwise} \end{cases} \qquad stress_{corr} = 1$$

$$\phi(t) := stress_{corr}\cdot\phi_{RH\_corr}\cdot\beta_{fcm}\cdot\beta_{t0L\_corr}\cdot\beta_{c\_corr}(t)$$

Modulus of rupture at the age

$$fc(t) := \frac{\frac{t}{day}}{\alpha r+\beta r\cdot\frac{t}{day}}\cdot f_{cm}$$ Branson's growth

$$f_r(t) := Correction_{Factor}\cdot psi\cdot 11.7\cdot\sqrt{\frac{fc(t)}{psi}}$$ Nawy's probabilistic **modulus of rupture** in RC p. 57

$$\varepsilon_{cr}(t) := -\left(\frac{f_r(t)}{E_c(t)}\right)$$

Stresses evolution calculus

solves the case similarly to Nawy approach in p. 314... in ACI SP-76

$$C_1 := E_s \cdot \left(A_{s1} + A_{s2}\right)$$

$$C_2 := \frac{E_s}{h} \cdot \left[d_1 \cdot A_{s1} + \left(h - d_1\right) \cdot A_{s2}\right]$$

$$C_3(j) := \frac{E_c(28 \cdot day) \cdot b \cdot h}{2 \cdot (1 + \phi(j \cdot month))}$$

$$C_4(j) := 2 \cdot C_3(j) \cdot \varepsilon_{cs}(j \cdot month)$$

$$C_5(j) := C_2 + C_3(j)$$

$$C_6(j) := C_1 - C_2 + C_3(j)$$

$$C_7 := E_s \cdot \left[A_{s1} \cdot d_1 + A_{s2} \cdot \left(h - d_1\right)\right]$$

$$C_8 := \frac{E_s}{h} \cdot \left[A_{s1} \cdot d_1^{\,2} + A_{s2} \cdot \left(h - d_1\right)^2\right]$$

$\varepsilon_1 := 0.001$ $\quad\quad$ $\varepsilon_2 := 0.003$ $\quad\quad$ this first block as if non-cracked

Given

$$\left(\frac{2 \cdot h}{3} \cdot C_3(j) + C_8\right) \cdot \varepsilon_2 + \left(\frac{h}{3} \cdot C_3(j) + C_7 - C_8\right) \cdot \varepsilon_1 - C_4(j) \cdot \frac{h}{3} - \frac{C_4(j) \cdot h \cdot \varepsilon_2}{3 \cdot \left(\varepsilon_2 + \varepsilon_1\right)} = M + P \cdot \frac{h}{2}$$ moments respect bottom

$$C_6(j) \cdot \varepsilon_1 + C_5(j) \cdot \varepsilon_2 - C_4(j) = P$$ equilibrium of forces

$PP(j) := \mathrm{Find}(\varepsilon_1, \varepsilon_2)$

$$\varepsilon\varepsilon_1 := \left| \begin{array}{l} j \leftarrow 1 \\ \text{while } j \le End \\ \quad \left| \begin{array}{l} t \leftarrow j \cdot month \\ a_{j,1} \leftarrow PP(j)_1 \\ a_{j,2} \leftarrow PP(j)_2 \\ j \leftarrow j + 1 \end{array} \right. \\ \text{return } a \end{array} \right.$$

$\varepsilon_1 := 0.001$ $\varepsilon_2 := 0.003$ c is substituted as function of epsilons this second block as if cracked

Given

$$\frac{E_s \cdot \varepsilon_2}{h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1}} \cdot \left[A_{s1} \cdot \left(d_1 + h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1} - h \right) + A_{s2} \cdot \left(h - d_1 + h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1} - h \right) \right] + \frac{b \cdot \left(h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1} \right) \cdot E_c(28 \cdot day) \cdot \left(\varepsilon_2 - \varepsilon_{cs}(j \cdot month) \right)}{2 \cdot (1 + \phi(j \cdot month))} = P$$

$$\frac{E_s \cdot \varepsilon_2}{h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1}} \cdot \left[A_{s2} \cdot \left[h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1} + \left(h - d_1 \right) - h \right] \cdot \left(h - d_1 \right) + A_{s1} \cdot \left(d_1 + h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1} - h \right) \cdot d_1 \right] + \frac{b \cdot \left(h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1} \right) \cdot E_c(28 \cdot day) \cdot \left(\varepsilon_2 - \varepsilon_{cs}(j \cdot month) \right) \cdot \left(h - \frac{h \cdot \frac{\varepsilon_2}{\varepsilon_2 - \varepsilon_1}}{3} \right)}{2 \cdot (1 + \phi(j \cdot month))} = M + P \cdot \frac{h}{2}$$

$PPP(j) := \mathrm{Find}(\varepsilon_1, \varepsilon_2)$

$$
\varepsilon\varepsilon_2 := \left|\begin{array}{l}
j \leftarrow 1 \\
\text{while } j \le End \\
\quad \left|\begin{array}{l}
t \leftarrow j \cdot month \\
a_{j,1} \leftarrow PPP(j)_1 \\
a_{j,2} \leftarrow PPP(j)_2 \\
j \leftarrow j + 1
\end{array}\right. \\
\text{return } a
\end{array}\right.
$$

Selection of appropriate solution

$$
\varepsilon\varepsilon := \left|\begin{array}{l}
j \leftarrow 1 \\
\text{while } j \le End \\
\quad \left|\begin{array}{l}
\text{if } \varepsilon\varepsilon1_{j,1} < \varepsilon_{cr}(j \cdot month) \\
\quad \left|\begin{array}{l}
a_{j,1} \leftarrow \varepsilon\varepsilon2_{j,1} \\
a_{j,2} \leftarrow \varepsilon\varepsilon2_{j,2}
\end{array}\right. \\
\text{otherwise} \\
\quad \left|\begin{array}{l}
a_{j,1} \leftarrow \varepsilon\varepsilon1_{j,1} \\
a_{j,2} \leftarrow \varepsilon\varepsilon1_{j,2}
\end{array}\right. \\
j \leftarrow j + 1
\end{array}\right. \\
\text{return } a
\end{array}\right.
$$

$j := 1 .. End$ $\qquad \kappa_j := \dfrac{\varepsilon\varepsilon_{j,2} - \varepsilon\varepsilon_{j,1}}{h}$ builds the evolution of section curvatures $\qquad R_j := \dfrac{1}{\kappa_j}$ and radiuses

$f_{s1_j} := \left(\varepsilon\varepsilon_{j,1} + \kappa_j \cdot d_1\right) \cdot E_s$

$f_{s2_j} := \left[\varepsilon\varepsilon_{j,1} + \kappa_j \cdot \left(h - d_1\right)\right] \cdot E_s$

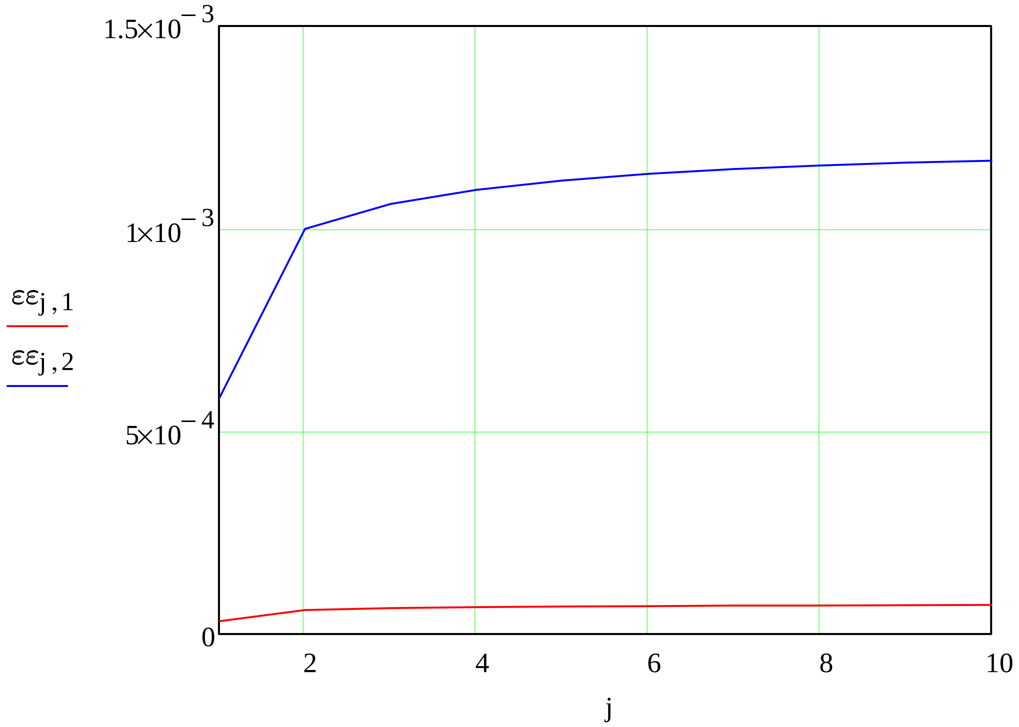
- steel stresses, where no yield is being assumed
- if yield is attained in tensile side, the section would be collapsing.
- Contrarily, the sectin might not collapse if only the compression side steel has attained fy.

$$f_{c1_j} := \begin{cases} \varepsilon\varepsilon_{j,1} \cdot \dfrac{E_c(28 \cdot day)}{1 + \phi(j \cdot month)} & \text{if } \varepsilon\varepsilon_{1_{j,1}} \ge \varepsilon_{cr}(j \cdot month) \\ 0 \cdot MPa & \text{otherwise} \end{cases}$$

$$f_{c2_j} := \varepsilon\varepsilon_{j,2} \cdot \frac{E_c(28 \cdot day)}{1 + \phi(j \cdot month)}$$

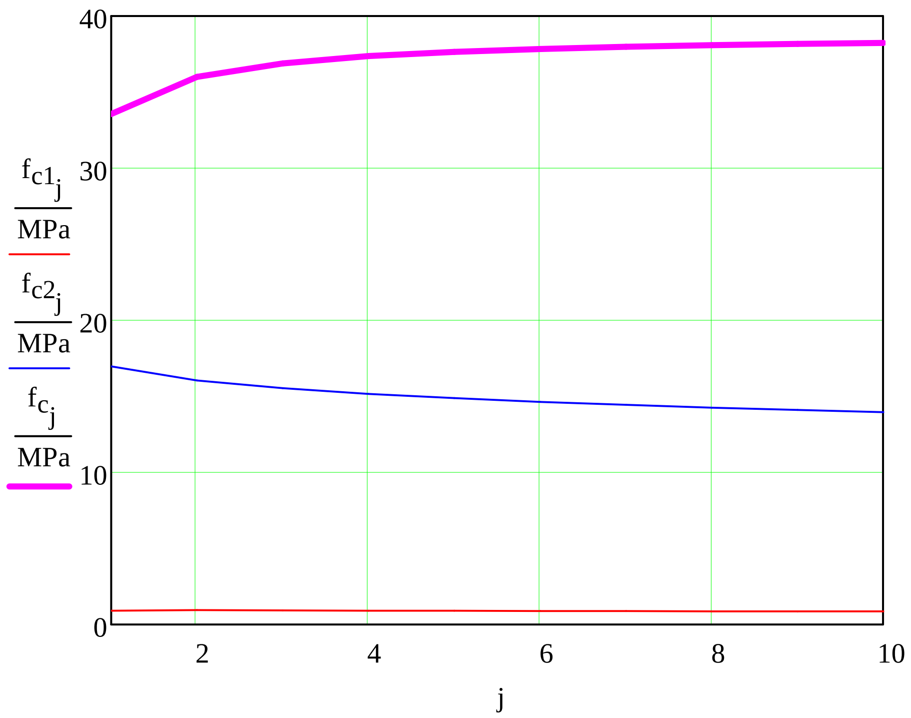
$$f_{c_j} := fc(j \cdot month)$$

ε_1 and ε_2 changing with time



- Once taken into account the creep, total strain need not (always) be less than, say, 0.003.
- See in the stresses chart whether the stresses are met by available strength

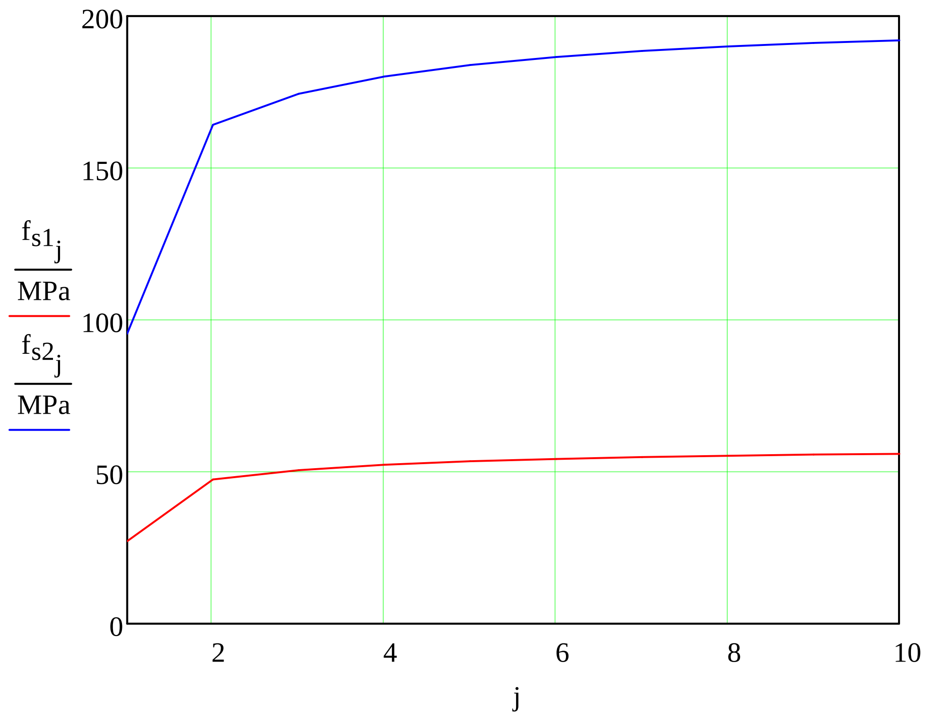
f_{c1} and f_{c2} changing with time



- The thicker magenta line traces attained strength and compressive stresses must be under it to avoid collapse
- See that in all the process a concrete triangular stress block has been assumed, so significant degree of accuracy can be expected only for low (initial-at loading) stresses (under 0,45 fcm or so)

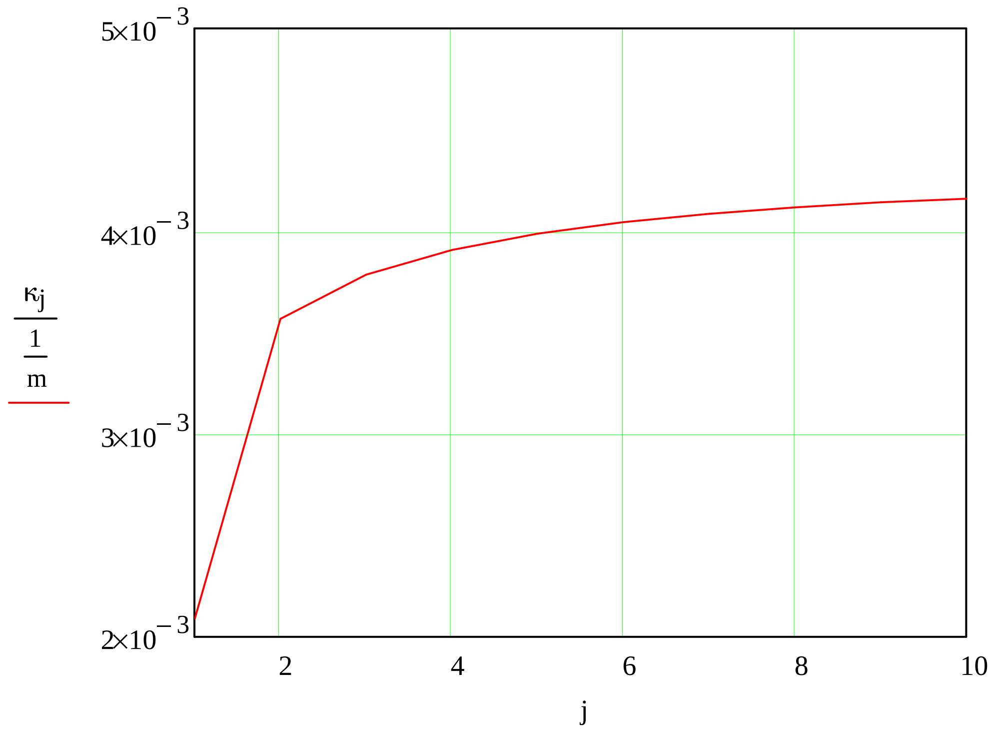
$0.45 \cdot f_{cm} = 14.85\,MPa$

f_{s1} and f_{s2} changing with time

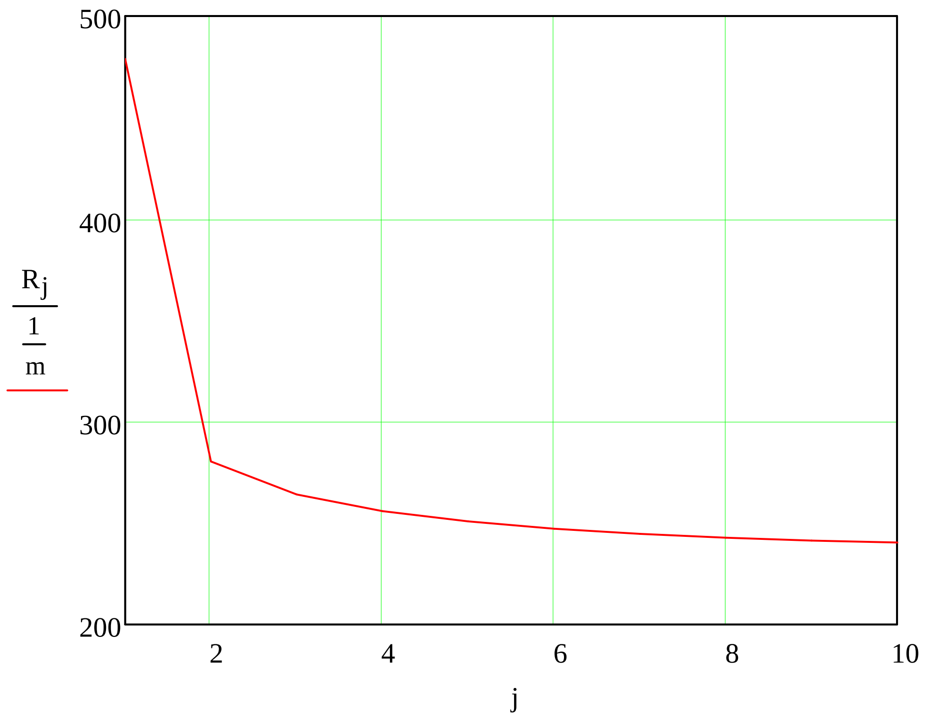


- Note that if the steel enters the over fy range, the solution is not valid since there is no provision in the calculations for yielded steel

Curvature at the section changing with time



Radius of Curvature at the section changing with time



- Limit the chart at the left abscissa (and corresponding calculations) to a age greater than the age of loading.
- A single step of loading taking reference age 28 days is the default built situation; anyway it can be easily changed to another reference time for modulus of deformation and by whatever suitable method (differences, or creep factor for limited steps) can help to trace the changes in the column.
- You can use the sheet to evaluate flexural cases using the artifice of quite little axial load.