

Punch through 1



Areas subject to punch in decks will be assumed far from the ends, and supported by surrounding concrete.

Reinforcement := 1 1 if simple (opposed side to load)
2 if both sides

Area := 1 1 if rectangle
2 if circle

$$\phi_1 := \frac{4}{8} \cdot \text{in}$$

$$s := 12 \cdot \text{in}$$

$$t := 8 \cdot \text{in}$$

$$c_1 := 3 \cdot \text{in}$$

$$d := t - c_1$$

dimensions immediately below

$$b_L := 8 \cdot \text{in}$$

$$h_L := 8 \cdot \text{in}$$

dimensions of
rectangular loaded area
causing punching

square reinforcement mesh

in each reinforced face (1 in simply
supported and 2 in fixed) and both directions
geometrical reinforcement ratio is

$$\rho := \frac{\pi \cdot \frac{\phi_1^2}{4}}{s \cdot t}$$

$$\rho = 0.002$$

$$r_L := 9 \cdot \text{in}$$

radius of circular loaded
area causing punching

$f_{c28} := 4000 \cdot \text{psi}$ specified strength

$\beta_0 := 45 \cdot \text{deg}$ must be bigger than or equal to 30 deg

$$f_y := 60 \cdot \text{ksi}$$

$$\phi_b := 0.9$$

moment strength
reduction factor

$$E_s := 200000 \text{ MPa}$$

$$\beta_0 = 45 \text{ deg}$$

angle (from surface) at which
punching failure is assumed to
happen

- angles bigger than 30 deg will give less conservative assumptions of punching failure
- For 30 deg, failure load will be that of pure shear punching failure at reference cone 30 deg



$$r := \begin{cases} \sqrt{b_L \cdot h_L} & \text{if Area} = 1 \\ r_L & \text{otherwise} \end{cases} \quad \text{should be conservative}$$

$$r := \sqrt{1.6r^2 + t^2} - 0.675t$$

$$r = 19.05 \text{ cm}$$

for calculus

$$n_1 := \frac{1 \cdot \text{m}}{s}$$

$$n_2 := n_1$$

$$\phi_2 := \phi_1$$

$$c_2 := c_1 - \phi_2$$

$$b := 1 \cdot \text{m}$$

$$h := t$$

$$f_c := 0.85 \cdot f_{c28}$$

for long term loads and combinations of live and dead load a reduction factor is
needed since concrete won't stand at f_c under long term loads

Punching Flexural Failure

$$\varepsilon_y := \frac{f_y}{E_s}$$

$$f_s(\varepsilon) := \left| \begin{array}{l} (-f_y) \text{ if } \varepsilon \leq -\varepsilon_y \\ \text{otherwise} \\ \left| \begin{array}{l} E_s \cdot \varepsilon \text{ if } \varepsilon \leq \varepsilon_y \\ f_y \text{ otherwise} \end{array} \right. \end{array} \right|$$

$$A_{s1} := n_1 \cdot \pi \cdot \frac{\phi_1^2}{4} \qquad A_{s2} := n_2 \cdot \pi \cdot \frac{\phi_2^2}{4}$$

$$\varepsilon_2 := 0.01 \qquad \varepsilon_1 := -0.02 \qquad \text{unwarranted guesses of final status}$$

$$c(\varepsilon_1,\varepsilon_2) := \frac{h \cdot \varepsilon_2}{\varepsilon_2 - \varepsilon_1}$$

Below Attard in ACI SJ Vol 95-3

$$\beta_1 := \left| \begin{array}{l} 1.0276 \cdot \left(\frac{f_c}{\text{MPa}}\right)^{-0.0785} \text{ if } 1.0276 \cdot \left(\frac{f_c}{\text{MPa}}\right)^{-0.0785} \geq 0.67 \\ 0.67 \text{ otherwise} \end{array} \right|$$

$$\alpha_1 := \left| \begin{array}{l} 1.4037 \cdot \left(\frac{f_c}{\text{MPa}}\right)^{-0.116} \text{ if } 1.4037 \cdot \left(\frac{f_c}{\text{MPa}}\right)^{-0.116} \geq 0.71 \\ 0.71 \text{ otherwise} \end{array} \right|$$

$$E_c := 4370.3 \cdot \left(\frac{f_c}{\text{MPa}}\right)^{0.5164} \cdot \text{MPa}$$

$$\varepsilon_{fc} := \frac{\frac{f_c}{\text{MPa}} \cdot 4.11}{\frac{E_c}{\text{MPa}} \cdot \sqrt[4]{\frac{f_c}{\text{MPa}}}}$$

$$\varepsilon_{cu} := \varepsilon_{fc} \cdot 2.8133 \cdot \left(\frac{f_c}{\text{MPa}}\right)^{-0.2093}$$

$$a(\varepsilon_1,\varepsilon_2) := \beta_1 \cdot c(\varepsilon_1,\varepsilon_2)$$

$$Fc(\varepsilon_1,\varepsilon_2) := \left\{\begin{array}{ll} \alpha_1 \cdot f_c & \text{if } c_2 \leq a(\varepsilon_1,\varepsilon_2) \\ 0 \cdot \frac{\text{kgf}}{\text{cm}^2} & \text{otherwise} \end{array}\right.$$

$$Fs2(\varepsilon_1,\varepsilon_2) := f_s \left[\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot (h - c_2) \right] \qquad \qquad Fs1(\varepsilon_1,\varepsilon_2) := f_s \left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot c_1 \right)$$

$$Steel_{Force}(\varepsilon_1,\varepsilon_2) := A_{s1} \cdot Fs1(\varepsilon_1,\varepsilon_2) + A_{s2} \cdot (Fs2(\varepsilon_1,\varepsilon_2) - Fc(\varepsilon_1,\varepsilon_2))$$

$$Concrete_{Force}(\varepsilon_1,\varepsilon_2) := b \cdot a(\varepsilon_1,\varepsilon_2) \cdot \alpha_1 \cdot f_c$$

$$Steel_{moment}(\varepsilon_1,\varepsilon_2) := c_1 \cdot A_{s1} \cdot Fs1(\varepsilon_1,\varepsilon_2) + (h - c_2) \cdot A_{s2} \cdot (Fs2(\varepsilon_1,\varepsilon_2) - Fc(\varepsilon_1,\varepsilon_2))$$

$$Concrete_{moment}(\varepsilon_1,\varepsilon_2) := \left(h - \frac{a(\varepsilon_1,\varepsilon_2)}{2}\right) \cdot b \cdot a(\varepsilon_1,\varepsilon_2) \cdot \alpha_1 \cdot f_c$$

$$Total_{Force}(\varepsilon_1,\varepsilon_2) := Steel_{Force}(\varepsilon_1,\varepsilon_2) + Concrete_{Force}(\varepsilon_1,\varepsilon_2)$$

$$Total_{Moment}(\varepsilon_1,\varepsilon_2) := Steel_{moment}(\varepsilon_1,\varepsilon_2) + Concrete_{moment}(\varepsilon_1,\varepsilon_2)$$

Given

$$Total_{Force}(\varepsilon_1,\varepsilon_2) = 0 \cdot \text{ton} \qquad \qquad \varepsilon_2 \leq \varepsilon_{cu}$$

$$P := \text{Maximize}(Total_{Moment},\varepsilon_1,\varepsilon_2)$$

$$P = \begin{pmatrix} -0.02661655 \\ 0.00271083 \end{pmatrix} \qquad \qquad \varepsilon_1 := P_1 \qquad \qquad \varepsilon_2 := P_2$$

$$Max_{Moment_bh} := 0.375 \cdot \left[0.85 \cdot f_c \cdot b \cdot (h - c_2) \right] \cdot (h - c_2) + A_{s2} \cdot (h - c_1 - c_2) \cdot f_y$$

$$Max_{Moment_bh} = 15.98 \text{ m}\cdot\text{ton}$$

the maximum moment that is accepted for any section b·h having As2 compression steel, whatever tensile steel area As1 is chosen (as long as the section is not over-reinforced)

Momento Tope

AS1, as a measure of protection against fragile
(compressive side) failure

$$M_n := \min \left(\left(\frac{\text{TotalMoment}(\varepsilon_1, \varepsilon_2)}{\text{MaxMoment_bh}} \right) \right)$$

here the control is set

We additionally investigate fragile rupture by

Failure :=

"Ductile even without moment cutoff, O.K." if $\frac{c(\varepsilon_1, \varepsilon_2)}{h} \leq 120 \cdot 0.0035$

"Would be No O.K., due to crushing of concrete and so fragile, except that moment strength cutoff has been applied " otherwise

Criteria is one as per Skogman, Tadros and Grasmic in PCI Journal separata JR 352.

$$\kappa := \frac{\varepsilon_2 - \varepsilon_1}{h}$$

$$R := \frac{1}{\kappa}$$

$$M_n = 3.08 \text{ m}\cdot\text{ton}$$

limit moment to be compared with the factored moment

all the data below corresponds to the non curtailed limit moment strength

$$c(\varepsilon_1, \varepsilon_2) = 1.88 \text{ cm}$$

depth of neutral axis from top

$$a(\varepsilon_1, \varepsilon_2) = 1.51 \text{ cm}$$

accepted depth of the compressed stress block

$$Fs1(\varepsilon_1, \varepsilon_2) = -4218.42 \frac{\text{kgf}}{\text{cm}^2}$$

$$Fs2(\varepsilon_1, \varepsilon_2) = -4218.42 \frac{\text{kgf}}{\text{cm}^2}$$

$$f_y = 4218.42 \frac{\text{kgf}}{\text{cm}^2}$$

$$f_y = 413.69 \text{ MPa}$$

at bottom, negative is tensile

atop, positive is compressive

$$\varepsilon_1 = -0.02662$$

$$\varepsilon_2 = 0.00271$$

$$\kappa = 0.14 \frac{\text{rad}}{\text{m}}$$

at non-curtailed limit strength

$$R = 6.93 \text{ m}$$

Failure = "Ductile even without moment cutoff, O.K."

$$\phi_b \cdot M_n = 2.77 \text{ m}\cdot\text{ton}$$

both for positives and negatives and following Massonnet and Save formula 6-11

$$M_p := \phi_b \cdot M_n$$

$$R := 50 \cdot \text{cm}$$

unwarranted guess for R that gives maximum

$$P(R) := \frac{2 \cdot \pi \cdot 2 \cdot \frac{M_p}{\text{m}}}{1 - \frac{2}{3} \cdot \frac{r}{R}}$$

$$R = 0.5 \text{ m}$$

Given $R = 50\text{ cm}$

$$R \geq r \qquad R \leq 10 \cdot t$$

$R := \text{Maximize}(P,R)$ $R = 19.05\text{ cm}$

$$P_F := \begin{cases} \frac{P(R)}{2} & \text{if Reinforcement} = 1 \\ P(R) & \text{otherwise} \end{cases}$$
 $P_F = 52.18\text{ ton}$

a whopping load in concentric punching flexure

Punching Shear Failure

Now we estimate failure per cones in pure shear born from radius r_L

$f_{ct} := 0.55 \cdot \sqrt{\frac{f_{c28}}{\text{MPa}}} \cdot \text{MPa}$ $f_{ct} = 2.89\text{ MPa}$

equal to strength in pure shear

$R := r_L + \frac{t}{\tan(\beta_0)}$ $R = 58.06\text{ cm}$

$H_R := \frac{R}{\cos(\beta_0)}$ $H_{rL} := \frac{r_L}{\cos(\beta_0)}$ $A_{\text{cone}} := \pi \cdot (H_R^2 - H_{rL}^2) \cdot \frac{2 \cdot \pi \cdot R}{2 \cdot \pi \cdot H_R}$ $A_{\text{cone}} = 10330.8\text{ cm}^2$

$$A_{\text{pyramid}} := 2 \cdot \left(\frac{b_L + b_L + 2 \cdot \frac{b_L}{\tan(\beta_0)}}{2} \cdot \frac{h}{\sin(\beta_0)} \right) + 2 \cdot \left(\frac{h_L + h_L + 2 \cdot \frac{h_L}{\tan(\beta_0)}}{2} \cdot \frac{h}{\sin(\beta_0)} \right)$$
 $A_{\text{pyramid}} = 9024.56\text{ cm}^2$

$$\text{Area}_{\text{in shear}} := \begin{cases} A_{\text{pyramid}} & \text{if Area} = 1 \\ A_{\text{cone}} & \text{otherwise} \end{cases}$$

$P_S := \text{Area}_{\text{in shear}} \cdot f_{ct} \cdot \sin(\beta_0)$ $P_S = 132.9\text{ ton}$

Punching Failure, inferred from Menétrey

$$P := P_S + (P_F - P_S) \cdot \sqrt{\sin\left(1.5 \cdot \beta_0 - \frac{\pi}{4}\right)}$$



$P = 152.96 \text{ kip}$

predicted punching failure load

$P_S = 214.49 \text{ kip}$

cone at 30 deg failing in pure shear at fct (till then perfectly undamaged state in shear)

$P_F = 115.03 \text{ kip}$

yield lines ultimate load failure, so quite damaged when attaining this load