

Shrinkage 10c

Axis-symmetrically built and loaded reinforced concrete column



Initialization

ORIGIN \equiv 1 Count with fingers TOL := 0.01 CTOL := 0.01

ton := 1000·kgf ksi := 70.307· $\frac{\text{kgf}}{\text{cm}^2}$ psi := $\frac{\text{ksi}}{1000}$ kip := 453.592·kgf MPa := 10.197· $\frac{\text{kgf}}{\text{cm}^2}$ day := 86400·sec
month := 30·day
year := 365·day

AND2(a,b) := $\left\{ \begin{array}{l} \text{if } a = 1 \\ \quad \left\{ \begin{array}{l} 1 \text{ if } b = 1 \\ 0 \text{ otherwise} \end{array} \right. \\ 0 \text{ otherwise} \end{array} \right.$

OR2(a,b) := $\left\{ \begin{array}{l} 1 \text{ if } a = 1 \\ \text{otherwise} \\ \quad \left\{ \begin{array}{l} 1 \text{ if } b = 1 \\ 0 \text{ otherwise} \end{array} \right. \end{array} \right.$



Modulus of Rupture and Tensile Strength evaluation

f_{c28} := 35MPa α := 4 α=4.00 for cement Type I and moist curing
α=2.30 for cement Type III and moist curing
α=1.00 for cement Type I and steam curing
α=0.70 for cement Type III and steam curing β := 0.85 β=0.85 for cement Type I and moist curing
β=0.92 for cement Type III and moist curing
β=0.95 for cement Type I and steam curing
β=0.98 for cement Type III and steam curing

specified strength
as per ACI at 28
days

λ := 1 1.00 for Normal weight
0.75 fo all-lightweight
0.85 for sand-lightweight w_c := 2400· $\frac{\text{kgf}}{\text{m}^3}$ specific weight of
concrete

Parts := 400

$$f_c(t) := \frac{\frac{t}{\text{day}}}{\alpha + \beta \cdot \frac{t}{\text{day}}} \cdot f_{c28}$$

compressive strength at the age

according to Branson

probabilistic or expected, that of

$$f_r(t) := \text{psi} \cdot 11.7 \cdot \lambda \cdot \sqrt{\frac{f_c(t)}{\text{psi}}}$$

modulus of rupture, that we accept as good for wherever non cracked adjacent concrete provides some support

modulus of rupture (tensile in bending)

$$f_{ct}(t) := \text{psi} \cdot 7.5 \cdot \lambda \cdot \sqrt{\frac{f_c(t)}{\text{psi}}}$$

Modulus of Deformation evaluation

$$E_c(t) := \left\{ \begin{array}{l} \text{psi} \cdot 33 \cdot \left(\frac{w_c}{\frac{\text{lbf}}{\text{ft}^3}} \right)^{1.5} \cdot \sqrt{\frac{f_c(t)}{\text{psi}}} \quad \text{if } f_c(t) \leq 6000 \cdot \text{psi} \\ \text{psi} \cdot \left[\left(10^6 + 40000 \cdot \sqrt{\frac{f_c(t)}{\text{psi}}} \right) \cdot \left(\frac{w_c}{145 \cdot \frac{\text{lbf}}{\text{ft}^3}} \right)^{1.5} \right] \quad \text{otherwise} \end{array} \right.$$

Unrestrained Shrinkage

- Due mainly to material loss (water, vapour, CO₂ gas) it is assumed to occur in non reinforced concrete such a non reinforced prismatic vertically set small column left to itself freely shrinking in a non-restraining surface.
- It may or may not be directly employed for engineering purposes. For example, a fully compressed column or longitudinally non restrained prestressed beam will show axially the full unrestrained shrinkage shortening.

RH := 70·%	arithmetic mean value of Relative Humidity along the whole (analyzed) period of life of the structure	$\epsilon_{SHU} := 780 \cdot 10^{-6}$ <div> Ultimate unrestrained shrinkage. Don't change if you don't have data </div>	Concrete _{Type} := 1 <div> 1 for moist cured after 7 days (~ in situ) 2 for steam cured after 1 or 3 days (~ prefab) </div>
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In the formulation of Branson...

$$\epsilon_{SH}(t) := \left\{ \begin{array}{ll} 1.4 - 0.01 \cdot RH \cdot 100 & \text{if } AND2(40\% \leq RH, RH \leq 80\%) \\ 3 - 0.03 \cdot RH \cdot 100 & \text{otherwise} \end{array} \right. \cdot \left\{ \begin{array}{ll} \frac{\frac{t}{day}}{55 + \frac{t}{day}} \cdot \epsilon_{SHU} & \text{if } Concrete_{Type} = 2 \\ \frac{\frac{t}{day}}{35 + \frac{t}{day}} \cdot \epsilon_{SHU} & \text{otherwise} \end{array} \right.$$

So only valid for mean humidities 40% and above

it could be argued to put $t - t_0$ where t but it is of scarce meaning for the sought use

Creep

By one simple formulation (EHE)

$f_{ck} := 0.9 \cdot f_{c28}$

surmised equivalence between european and american specified strengths, from statistical estimators

$HR := 100 \cdot RH$

$HR = 70$

Relative humidity, enter in percent, i,e, 0 to 100 and without % symbol

$VS := 150 \cdot mm$

volume to surface ratio

$t_0 := 14 \cdot day$

age of loading

$$\phi_{HR} := 1 + \frac{100 - HR}{9.9 \cdot \left(\frac{VS}{mm}\right)^{\frac{1}{3}}}$$

$$\beta(t_0) := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}}$$

$$\beta_1 := \frac{16.8}{\sqrt{\frac{f_{ck}}{MPa}} + 8}$$

a function but of mean strength at 28 days, so a constant

$$\phi_0 := \phi_{HR} \cdot \beta_1 \cdot \beta(t_0)$$

$$\beta_H := \begin{cases} 1.5 \cdot \frac{VS}{mm} \cdot \left[\left[1 + (0.012 \cdot HR)^{18} \right] + 250 \right] & \text{if } 1.5 \cdot \frac{VS}{mm} \cdot \left[\left[1 + (0.012 \cdot HR)^{18} \right] + 250 \right] < 1500 \\ 1500 & \text{otherwise} \end{cases}$$

$$\beta_c(t) := \left(\frac{\frac{t-t_0}{\text{day}}}{\beta_H + \frac{t-t_0}{\text{day}}} \right)^{0.3} \quad \phi(t) := \phi_0 \cdot \beta_c(t)$$

Some considerations regarding shortening axially loaded columns

Elastic shortening (central symmetric reinforcements)

$$\Delta \epsilon \cdot E_c \cdot A_c + \Delta \epsilon \cdot E_s \cdot A_s = P$$

hence

$$\Delta \epsilon := \frac{P}{E_c \cdot A_c + E_s \cdot A_s} \quad \Delta \epsilon := \frac{P}{E_c \cdot A_c \cdot (1 + \rho n)}$$

Axis-symmetrically built and loaded reinforced concrete column

- Note that we will be analyzing a member loaded early (14 days age) at quite high service load (1/2 of nominal) and keep it constant for 100 years. This is quite exacting but I would not say unseen in concrete columns mainly due to bad accounting of loads.
- in the procedure that follows to trace the time dependent behaviour of concrete, we will be considering the stepwise change of forces, and since the process will be linearized in the step, at the current modulus of deformation for the age, the opposition to compression will be so in reason of the added stiffnesses of concrete and passive rebar.
- due to considering the current (aged) properties of materials at every step, no aging factor needs to be included
- A particularity is that in the process of solving for stress and deformation at any time, a rebound on the passive rebar stiffness needs to be included.

$$f_y := 400 \cdot \text{MPa}$$

$A_s := 8 \cdot \text{cm}^2$ centrally symmetric area of passive steel $E_s := 200000 \cdot \text{MPa}$ $A_c := 900 \cdot \text{cm}^2 - A_s$ net area of concrete. $A_c = 892 \text{ cm}^2$

$\rho_s := \frac{A_s}{A_c}$ $\rho_s = 0.009$ $E_c(14 \cdot \text{day}) = 27897.29 \text{ MPa}$

$P := 150 \cdot \text{ton}$ held constant from 14 days to 100 years

$\Delta \epsilon := \frac{P}{E_c(14 \cdot \text{day}) \cdot A_c + E_s \cdot A_s}$ $\Delta \epsilon = 0.00056$ elastic shortening

So, just after elastic shortening

$\sigma_0 := E_c(14 \cdot \text{day}) \cdot \Delta \epsilon$ $\sigma_0 = 15.49 \text{ MPa}$ (about half of f_c) (note that it is about equal than in the prior case, but now we have different prestress force and initial conditions)

$f_{s0} := \Delta \epsilon \cdot E_s$ $f_{s0} = 111.09 \text{ MPa}$

We denote with subscript 0 our particularly after elastic shortening defined state

$t_0 = 14 \text{ day}$ a quite early age of loading

also $E_{c0} := E_c(14 \cdot \text{day})$

The integration of stresses is made by sampling the situation at succesive time intervals. At each interval-step...

- Shrinkage is applied: concrete shortens and so the applied stress by the prestressed steel diminish
- Creep is applied: concrete shortens, its value determined using the compression found in the previous step, and consequently to shortening the applied stress by the prestressed steel diminish
- Relaxation of steel acts: the applied stress diminishes further, while the shortenings from shrinkage and creep are a bit counteracted by the elongation in the forcing prestressing steel
- Rebound force in steel corresponding to the unshared shortening is then weighed according to axial stiffness

$N := 100$ • moments (ages) at which we will sample shrinkage, creep and relaxation values.

Time Varying Stresses in Concrete

last

$$\sigma_c(T) := \left| \begin{array}{l} j \leftarrow 1 \\ \sigma \leftarrow \sigma_0 \\ \text{while } j \leq N \\ \left| \begin{array}{l} \Delta\sigma \leftarrow \left| \begin{array}{l} \frac{1}{A_c + \frac{E_s}{E_{c0}} \cdot A_s} \cdot A_s \cdot E_s \cdot \frac{A_c}{A_c + \frac{E_s}{E_{c0}} \cdot A_s} \cdot \left[\varepsilon_{SH}\left(t_0 + j \cdot \frac{T}{N}\right) - \varepsilon_{SH}(t_0) + \frac{\sigma}{E_{c0}} \cdot \left(\phi\left(t_0 + j \cdot \frac{T}{N}\right) \right) \right] \text{ if } j = 1 \\ \frac{1}{A_c + \frac{E_s}{E_c\left(t_0 + j \cdot \frac{T}{N}\right)} \cdot A_s} \cdot A_s \cdot E_s \cdot \frac{A_c}{A_c + \frac{E_s}{E_c\left(t_0 + j \cdot \frac{T}{N}\right)} \cdot A_s} \cdot \left[\varepsilon_{SH}\left(t_0 + j \cdot \frac{T}{N}\right) - \varepsilon_{SH}\left[t_0 + (j-1) \cdot \frac{T}{N}\right] + \frac{\sigma}{E_c\left(t_0 + j \cdot \frac{T}{N}\right)} \cdot \left[\phi\left(t_0 + j \cdot \frac{T}{N}\right) - \phi\left[t_0 + (j-1) \cdot \frac{T}{N}\right] \right] \right] \text{ otherwise} \end{array} \right. \\ \sigma \leftarrow \sigma - \Delta\sigma \\ j \leftarrow j + 1 \end{array} \right. \\ \sigma \end{array} \right.$$

- creep factor gives creep between two time tick marks (times the elastic shortening at the first tick mark)
- when computing in the stepwise linearized process the change in strain due to creep between beginning and end of step, it is proper to use modulus of

deformation at the age, as a result of $\frac{E_{c0}}{E_c\left(t_0 + j \cdot \frac{T}{N}\right)} \cdot \frac{\sigma}{E_{c0}}$

- any age effect must be being captured by the time-stepwise approach, son I don't see any need of aging factor to be included in the formulation; if results don't correlate, it is creep or shrinkage formulation what would need correction, never aging factor introduction
- formulation captures, from left to right, shrinkage and creep

$$f_s(T) := \frac{P - A_c \cdot \sigma_c(T)}{A_s}$$

from a continuous requirement of equilibrium

$T_f := 100\text{-year}$

when to end chart

$N_c := 10$

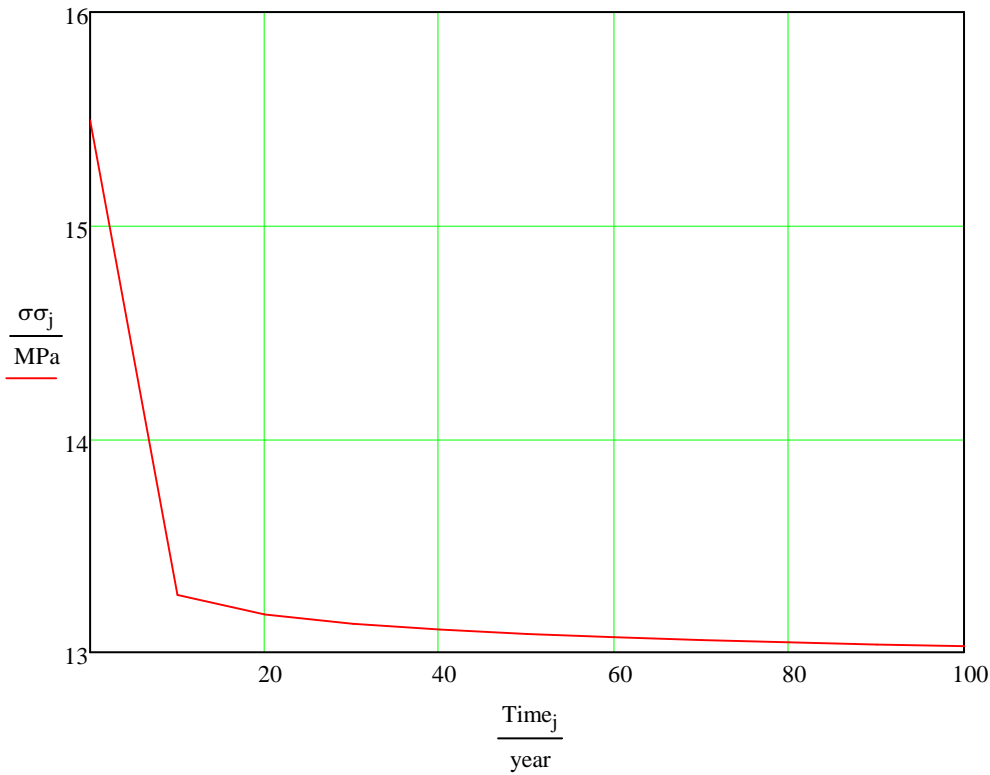
moments at which we will sample shrinkage and creep values for the chart

$j := 1..N_c + 1$

$\text{Time}_j := \frac{T_f}{N_c} \cdot (j - 1)$

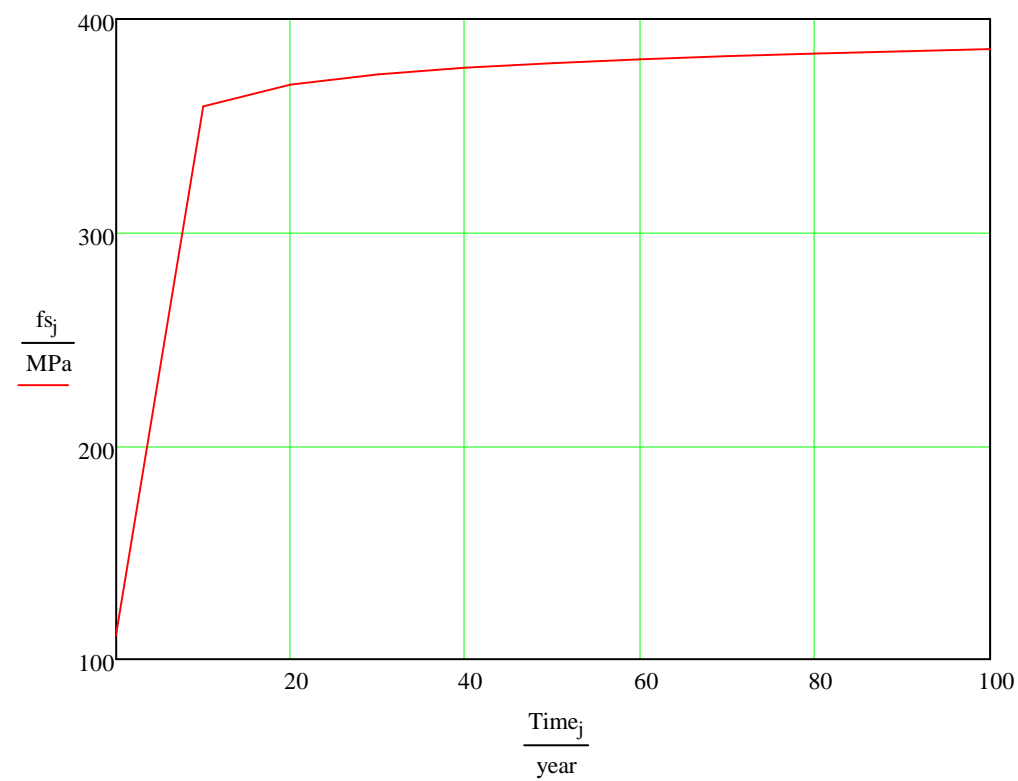
$$\sigma\sigma_j := \sigma_c(\text{Time}_j)$$

(Compounded) (diminishing) actual compression in the reinforced section, due to shrinkage and creep



$$fs_j := f_s(\text{Time}_j)$$

(Compounded) (augmenting) actual compression in the passive steel, due to shrinkage and creep



- The analysis for this case shows a far bigger (and more normal) transfer of stresses towards passive steel, to a practically yielded state long term.
- Having the rebar yielded is highly inconvenient (what do you try, to reinforce with some kind of hardened butter?) in that the building is waiting the shock in a fragile state, won't show any further stiffness than initial till meeting the start of strain hardening, so to the risk of easily initiating structural and non-structural damage from the start.
- So for cases like this use $f_y=500$ MPa rather than 400 MPa; you will have a structure showing positive stiffness from the start, which must reduce damage.

(Compounded) (diminishing) actual compression in the concrete, due to shrinkage and creep of concrete

$$\sigma_0 = 15.49 \text{ MPa}$$

$$\sigma_c(6\text{-month}) = 14.05 \text{ MPa}$$

$$\sigma_0 - \sigma_c(6\text{-month}) = 1.44 \text{ MPa}$$

$$\frac{\sigma_c(6\text{-month})}{\sigma_0} = 90.69 \%$$

$$\sigma_0 = 15.49 \text{ MPa}$$

$$\sigma_c(10\text{-year}) = 13.27 \text{ MPa}$$

$$\sigma_0 - \sigma_c(10\text{-year}) = 2.22 \text{ MPa}$$

$$\frac{\sigma_c(10\text{-year})}{\sigma_0} = 85.64 \%$$

$$\sigma_0 = 15.49 \text{ MPa}$$

$$\sigma_c(100\text{-year}) = 13.03 \text{ MPa}$$

$$\sigma_0 - \sigma_c(100\text{-year}) = 2.47 \text{ MPa}$$

$$\frac{\sigma_c(100\text{-year})}{\sigma_0} = 84.09 \%$$

- The compounded effects of these high but not exorbitants values of shrinkage and creep have washed out about 10% percent of concrete stress in 6 months and 15% in 10 years.

(Compounded) (augmenting) actual compression in the passive steel, due to shrinkage and creep of concrete

$f_{s0} = 111.09 \text{ MPa}$	$f_s(6\cdot\text{month}) = 272.01 \text{ MPa}$	$f_s(6\cdot\text{month}) - f_{s0} = 160.93 \text{ MPa}$	$\frac{f_s(6\cdot\text{month})}{f_{s0}} = 244.87 \%$
$f_{s0} = 111.09 \text{ MPa}$	$f_s(10\cdot\text{year}) = 359.13 \text{ MPa}$	$f_s(10\cdot\text{year}) - f_{s0} = 248.04 \text{ MPa}$	$\frac{f_s(10\cdot\text{year})}{f_{s0}} = 323.29 \%$
$f_{s0} = 111.09 \text{ MPa}$	$f_s(100\cdot\text{year}) = 385.99 \text{ MPa}$	$f_s(100\cdot\text{year}) - f_{s0} = 274.9 \text{ MPa}$	$\frac{f_s(100\cdot\text{year})}{f_{s0}} = 347.47 \%$

- Once the rebar exceeds f_y the algorithm should be modified, in that it won't take stress in excess of f_y and no further stress transfer is possible.