Transformer Regulation

Regulation is defined as the change in output (secondary) voltage which ocurs when the load is reduced from rated kVA to zero, with the applied (primary) voltage maintained constant and is expressed as a percentage of the full-load secondary voltage.

Example:

$Z := 7.94\%$

$R := \frac{51.192}{12000}$ $\quad R = 0.427\,\%$

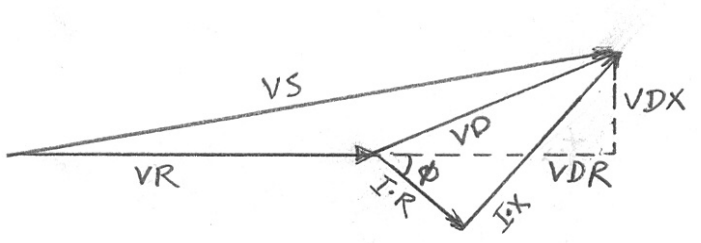
$X := \sqrt{Z^2 - R^2}$ $\quad X = 7.929\,\%$

$pf := 80\%$ $\quad \phi := \text{acos}(pf)$ $\quad \phi = 36.87\text{deg}$

Formula from *ABB Transformer Tests, Their Purpose and Importance*:

$$Reg := R\cdot\cos(\phi) + X\cdot\sin(\phi) + \frac{(X\cdot\cos(\phi) - R\cdot\sin(\phi))^2}{2} \qquad Reg = 5.283649\%$$

Derive Exact Formula:



$I := 1$ load = 1 pu

$1 = (VR + VDR)^2 + VDX^2$ sending end voltage = 1 pu

$VDR = R\cdot\cos(\phi) + X\cdot\sin(\phi)$

$VDX := X\cdot\cos(\phi) - R\cdot\sin(\phi)$

substituting:

$1 = (VR + R\cdot\cos(\phi) + X\cdot\sin(\phi))^2 + (X\cdot\cos(\phi) - R\cdot\sin(\phi))^2$

solving for VR:

$$\begin{bmatrix} (-R)\cdot\cos(\phi) - X\cdot\sin(\phi) + \left(R^2\cdot\cos(\phi)^2 + 2\cdot R\cdot\cos(\phi)\cdot X\cdot\sin(\phi) + X^2\cdot\sin(\phi)^2 + 1 - X^2 - R^2\right)^{\frac{1}{2}} \\ (-R)\cdot\cos(\phi) - X\cdot\sin(\phi) - \left(R^2\cdot\cos(\phi)^2 + 2\cdot R\cdot\cos(\phi)\cdot X\cdot\sin(\phi) + X^2\cdot\sin(\phi)^2 + 1 - X^2 - R^2\right)^{\frac{1}{2}} \end{bmatrix} = \begin{pmatrix} 0.947 \\ -1.049 \end{pmatrix}$$

$\text{Reg} = 1 - VR$

substitute the first solution from the above vector:

$$\text{Reg} = 1 + R\cdot\cos(\phi) + X\cdot\sin(\phi) - \left(R^2\cdot\cos(\phi)^2 + 2\cdot R\cdot\cos(\phi)\cdot X\cdot\sin(\phi) + X^2\cdot\sin(\phi)^2 + 1 - X^2 - R^2\right)^{\frac{1}{2}}$$

$(R\cdot\cos(\phi) + X\cdot\sin(\phi))^2$ expands to $R^2\cdot\cos(\phi)^2 + 2\cdot R\cdot\cos(\phi)\cdot X\cdot\sin(\phi) + X^2\cdot\sin(\phi)^2$

substituting:

$$\text{Reg} := 1 + R\cdot\cos(\phi) + X\cdot\sin(\phi) - \left[(R\cdot\cos(\phi) + X\cdot\sin(\phi))^2 + 1 - X^2 - R^2\right]^{\frac{1}{2}}$$

$\text{Reg} = 5.28382\ \%$

$$\frac{5.28382\ \%}{5.283649\ \%} = 100.00324\ \%$$