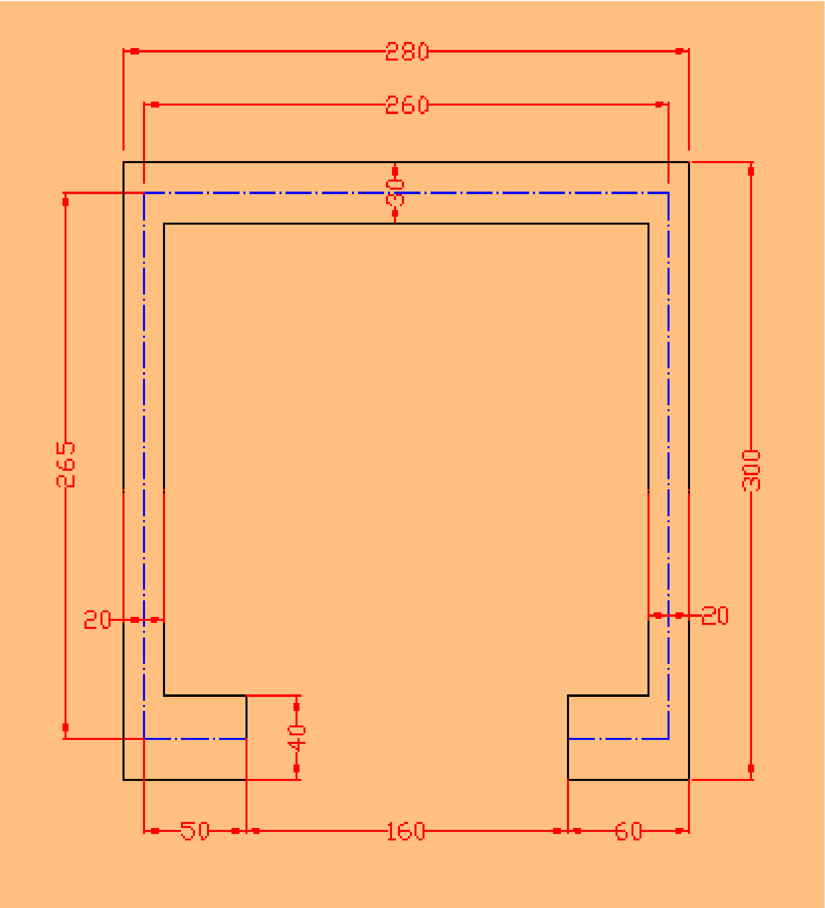
Computation of Mechanical Properties of an U Lipped Open Section

Due to vagaries in the tolerances this sheet solves more quickly for members with dimensions in the range of thin gage members than for those in the lift core range. We exploit this effect in this sheet to calculate the properties of a lift core section at 1/10 scale, hence where we read in the sheet 1 mm, the actual properties are in cm.



$$x0 := \begin{pmatrix} 50 \\ 0 \\ 0 \\ 260 \\ 260 \\ 210 \end{pmatrix} \cdot mm \qquad y0 := \begin{pmatrix} 0 \\ 0 \\ 265 \\ 265 \\ 0 \\ 0 \end{pmatrix} \cdot mm \qquad thickness := \begin{pmatrix} 40 \\ 20 \\ 30 \\ 20 \\ 40 \end{pmatrix} \cdot mm$$

$N_{segs} := \mathrm{length}(x0) - 1$

$N_{segs} = 5$

$i := 1 .. N_{segs}$ $\qquad x0_{med_i} := \frac{x0_i + x0_{i+1}}{2} \quad y0_{med_i} := \frac{y0_i + y0_{i+1}}{2}$

$L_i := \sqrt{(x0_i - x0_{i+1})^2 + (y0_i - y0_{i+1})^2}$

$$\mathrm{ang}(x_1, y_1, x_2, y_2) := \begin{cases} \text{if } x_2 = x_1 \\ \quad \begin{cases} 90\cdot\mathrm{deg} & \text{if } y_2 > y_1 \\ -90\cdot\mathrm{deg} & \text{otherwise} \end{cases} \\ \text{otherwise} \\ \quad \begin{cases} \text{if } y_2 = y_1 \\ \quad \begin{cases} 180\cdot\mathrm{deg} & \text{if } x_2 < x_1 \\ 0\cdot\mathrm{deg} & \text{otherwise} \end{cases} \\ \text{otherwise} \\ \quad \begin{cases} \mathrm{atan}\left(\frac{y_2 - y_1}{x_2 - x_1}\right) & \text{if } x_2 > x_1 \\ \pi + \mathrm{atan}\left(\frac{y_2 - y_1}{x_2 - x_1}\right) & \text{otherwise} \end{cases} \end{cases} \end{cases}$$

$\alpha_i := \mathrm{ang}(x0_i, y0_i, x0_{i+1}, y0_{i+1})$

$\alpha =$ (deg)

	1
1	180
2	90
3	0
4	-90
5	180

$i := 2 .. N_{segs} + 1$

$$LA_i := \sum_{j=1}^{i-1} L_j$$

accumulated length to the points, marks dividers of the definidn equations alond length

lengths of segments

accumulated length to point

$L =$

	1
1	50
2	265
3	260
4	265
5	50

mm

$$LA = \begin{pmatrix} 0 \\ 50 \\ 315 \\ 575 \\ 840 \\ 890 \end{pmatrix} mm$$

$$L_{total} := \sum L$$

$L_{total} = 890\,mm$

$$Area := \sum_{k=1}^{N_{segs}} L_k \cdot thickness_k$$

$Area = 22400\,mm^2$

$$I_t := \frac{1}{3} \cdot \sum_{k=1}^{N_{segs}} L_k \cdot \left(thickness_k\right)^3$$

$I_t = 5886666.67\,mm^4$

$$x0_g := \frac{\sum_{k=1}^{N_{segs}} L_k \cdot thickness_k \cdot x0_{med_k}}{Area}$$

$$y0_g := \frac{\sum_{k=1}^{N_{segs}} L_k \cdot thickness_k \cdot y0_{med_k}}{Area}$$

$x0_g = 130\,mm$ $y0_g = 154.98\,mm$ coordinates of the center of gravity in the original system of coordinates

$xp := x0 - x0_g$ $yp := y0 - y0_g$

$$xp = \begin{pmatrix} -80 \\ -130 \\ -130 \\ 130 \\ 130 \\ 80 \end{pmatrix} mm \qquad yp = \begin{pmatrix} -154.98 \\ -154.98 \\ 110.02 \\ 110.02 \\ -154.98 \\ -154.98 \end{pmatrix} mm$$

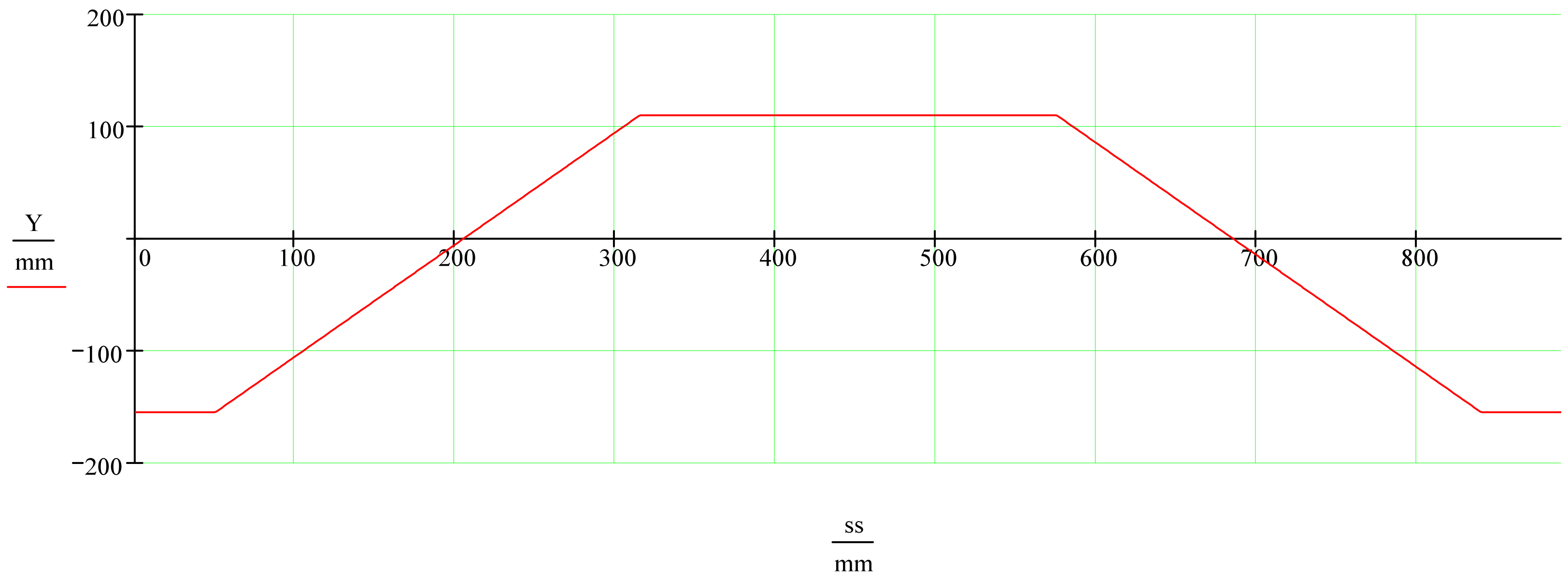
coordinates referred to center of gravity of the points defining the segments

Determination of ordinate on cog from input s along the shape

$$yy(i,xx) := yp_i + (xx - xp_i) \cdot \tan(\alpha_i)$$

$$y(s) := \sum_{i=1}^{N_{segs}} \left| \begin{array}{l} yy\left[i, (s - LA_i) \cdot \cos(\alpha_i)\right] \text{ if } \alpha_i \neq 90 \cdot deg \wedge \alpha_i \neq -90 \cdot deg \\ \text{otherwise} \\ \quad \left| \begin{array}{ll} yp_i + (s - LA_i) & \text{if } \alpha_i = 90 \cdot deg \\ yp_i - (s - LA_i) & \text{otherwise} \end{array} \right. \end{array} \right. \cdot \left| \begin{array}{ll} LA_i \le s \le LA_{i+1} & \text{if } i = 1 \\ LA_i < s \le LA_{i+1} & \text{otherwise} \end{array} \right.$$

$N_{parts} := 790$ $k := 1 .. N_{parts} + 1$ $ss_k := \frac{L_{total}}{N_{parts}} \cdot (k-1)$ $Y_k := y(ss_k)$



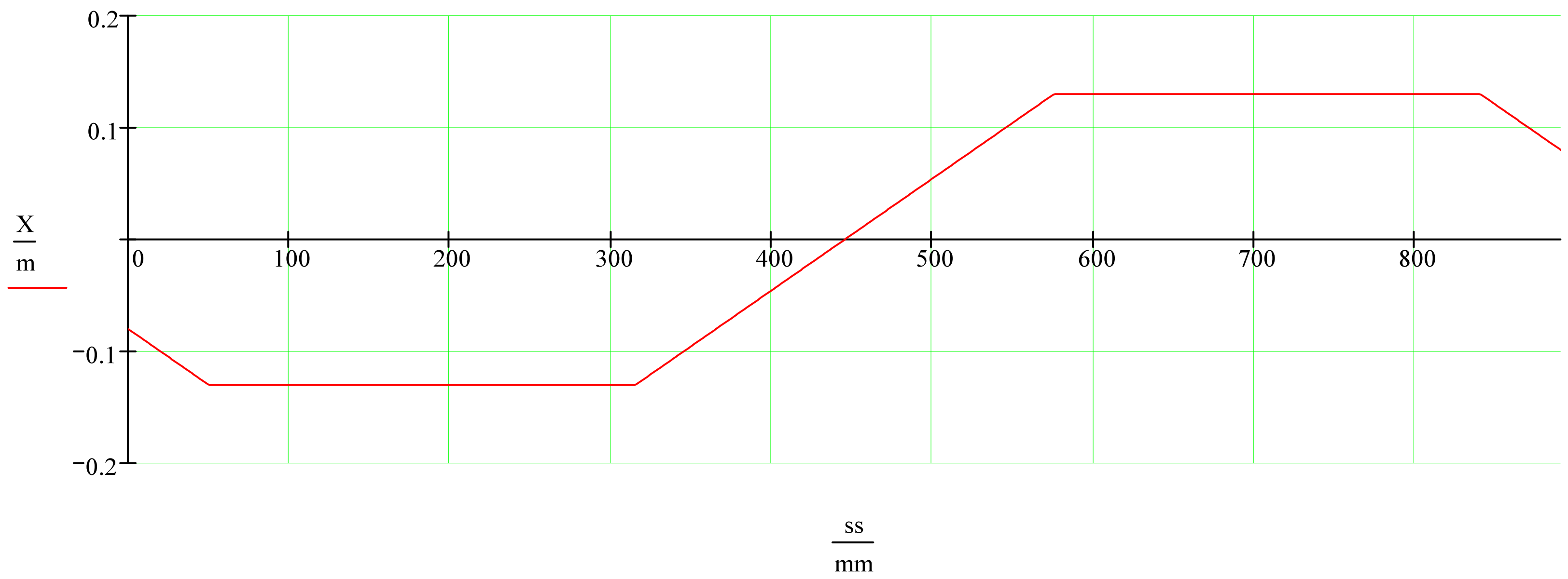
The definition of the ordinate respect cog on s portraits well in the chart

Determination of abscissa on cog from input s along the shape

$$yy(i, xx) := yp_i + (xx - xp_i) \cdot \tan(\alpha_i)$$

$$x(s) := \sum_{i=1}^{N_{segs}} \left| \begin{array}{ll} xp_i + (s - LA_i) \cdot \cos(\alpha_i) & \text{if } \alpha_i \neq 90 \cdot deg \wedge \alpha_i \neq -90 \cdot deg \\ xp_i & \text{otherwise} \end{array} \right. \cdot \left| \begin{array}{ll} LA_i \le s \le LA_{i+1} & \text{if } i = 1 \\ LA_i < s \le LA_{i+1} & \text{otherwise} \end{array} \right.$$

$X_k := x(ss_k)$

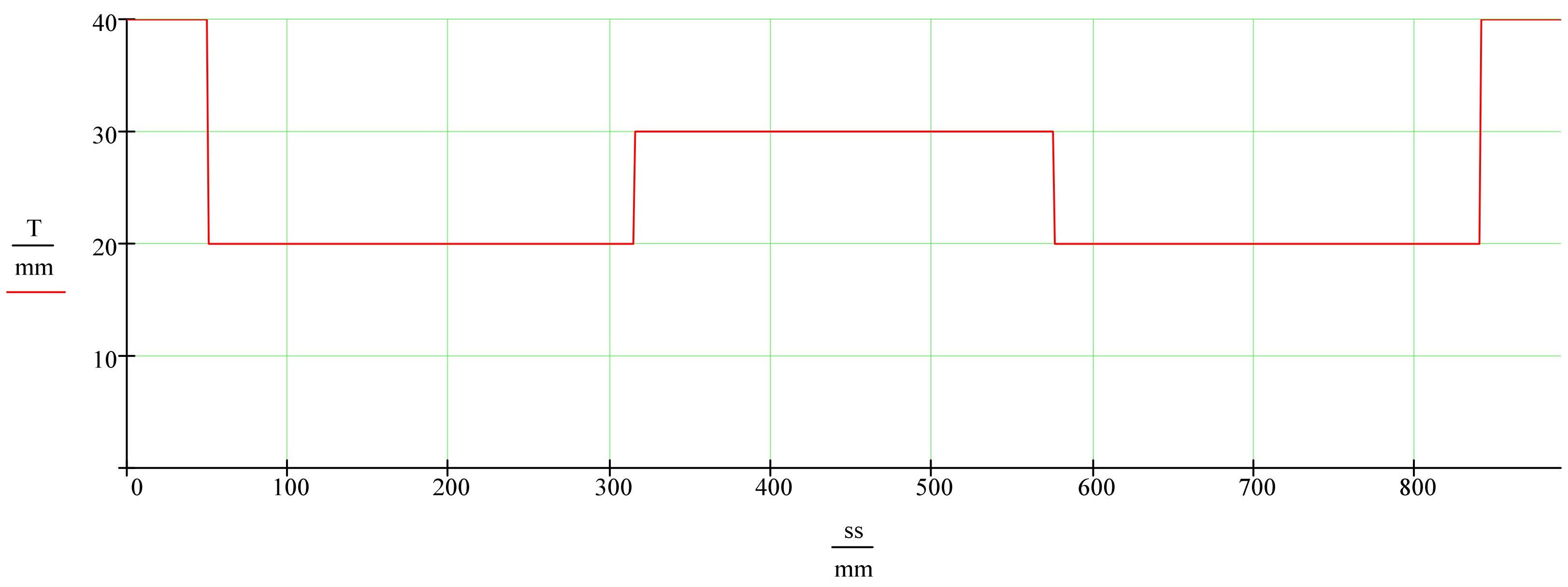


The definition of the abscissa respect cog on s portraits well in the chart

Determination of thickness from input s along the shape

$$t(s) := \sum_{i=1}^{N_{segs}} thickness_i \cdot \begin{vmatrix} LA_i \le s \le LA_{i+1} & \text{if } i = 1 \\ LA_i < s \le LA_{i+1} & \text{otherwise} \end{vmatrix}$$

$T_k := t(ss_k)$



The definition of the thickness along s portraits well in the chart

Moments of Inertia respect axis passing for cog

cog stands for center of gravity

$$I_x := \int_{0\cdot cm}^{L_{total}} y(s)^2 \cdot t(s)\, ds$$

$$I_x = 270186495.65\,mm^4$$

$$I_y := \int_{0\cdot mm}^{L_{total}} x(s)^2 \cdot t(s)\, ds$$

$$I_y = 279009865.3\,mm^4$$

Product of Inertia respect cog

$$I_{xy} := \int_{0\cdot mm}^{L_{total}} x(s)\cdot y(s)\cdot t(s)\, ds$$

$$I_{xy} = -0\,mm^4$$

Radiuses of gyration respect axes passing by the cog

$$r_x := \sqrt{\frac{I_x}{Area}}$$ $r_x = 109.83\,mm$

$$r_y := \sqrt{\frac{I_y}{Area}}$$ $r_y = 111.61\,mm$

Polar Moment of Inertia respect cog

$$I_0 := I_x + I_y$$ $I_0 = 549196360.94\,mm^4$

$$r(s) := \sqrt{x(s)^2 + y(s)^2}$$

Polar Radius of Gyration respect cog

$$r_0 := \sqrt{\frac{I_0}{Area}}$$ $r_0 = 156.58\,mm$

Principal Moments of Inertia

Location from Ox

$$\theta_p := \frac{\operatorname{atan}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right)}{2}$$ $\theta_p = 0\,deg$

$$I_I := \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_y - I_x}{2}\right)^2 + I_{xy}^2}$$

$$I_I = 279009865.3\,mm^4$$

$$I_{II} := \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_y - I_x}{2}\right)^2 + I_{xy}^2}$$

$$I_{II} = 270186495.65\,mm^4$$

Torsional Constant

$$J := \frac{1}{3} \cdot \int_{0 \cdot mm}^{L_{total}} t(s)^3\, ds$$

$$J = 6381628.67\,mm^4$$

We had already calculated it above under its notation in Spain

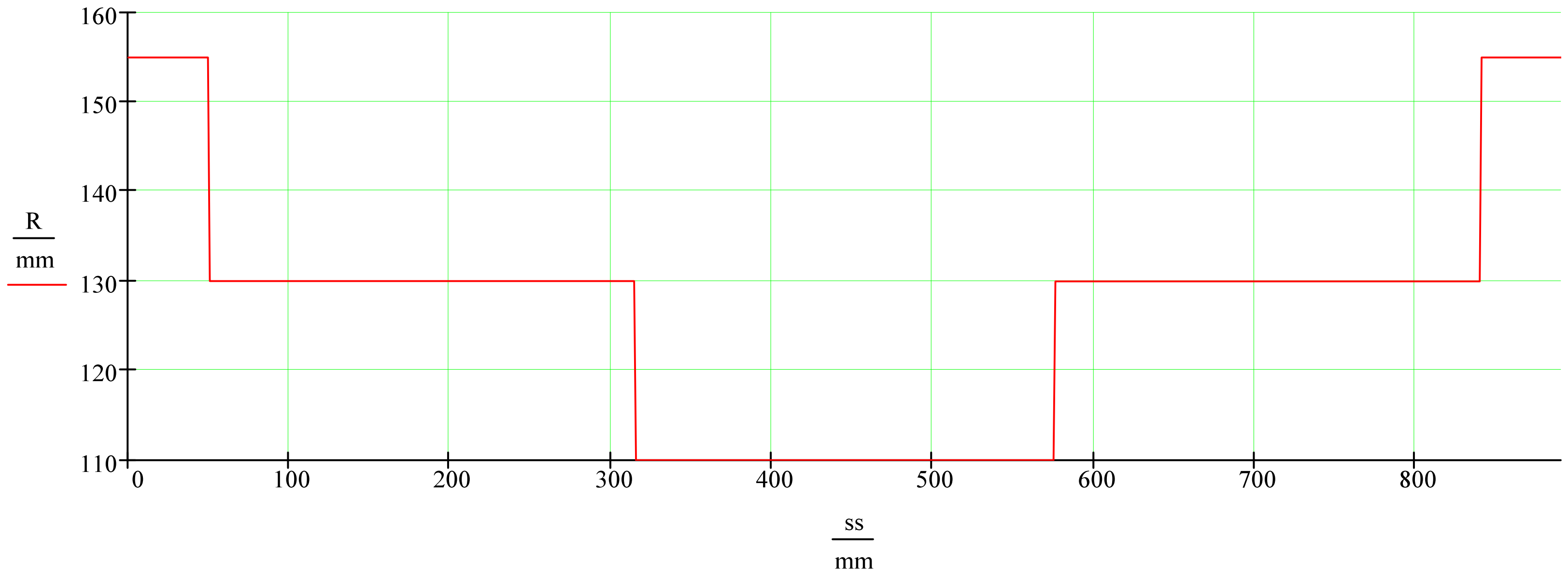
$$I_t = 5886666.67\,mm^4$$

Determination of Distance of cog to the segments from input s along the shape

$\theta(s) := ang(0 \cdot cm, 0 \cdot cm, x(s), y(s))$ angle with Ox of the vector from cog to point along the line

$$r_{0P}(s) := \sum_{i=1}^{N_{segs}} \sqrt{(xp_i)^2 + (yp_i)^2} \cdot \cos\left(\theta(LA_i) - \alpha_i - \frac{\pi}{2}\right) \cdot \begin{vmatrix} LA_i \le s \le LA_{i+1} & \text{if } i = 1 \\ LA_i < s \le LA_{i+1} & \text{otherwise} \end{vmatrix}$$

$N_{parts} := 790$ $\quad k := 1 .. N_{parts} + 1$ $\quad ss_k := \frac{L_{total}}{N_{parts}} \cdot (k - 1)$ $\quad R := \overrightarrow{r_{0P}(ss)}$



The definition of the distance respect cog on s portraits well in the chart

Coordinates of the Shear Center referred to cog

$D := I_x \cdot I_y - I_{xy}^2$ Constant of Inertia

$S_x(S) := \int_{0 \cdot mm}^{S} y(s) \cdot t(s)\, ds$ $\quad S_y(S) := \int_{0 \cdot mm}^{S} x(s) \cdot t(s)\, ds$

$$x_S := -\frac{1}{D} \cdot \left(I_{xy} \cdot \int_{0 \cdot mm}^{L_{total}} S_y(s) \cdot r_{0P}(s)\, ds - I_y \cdot \int_{0 \cdot mm}^{L_{total}} S_x(s) \cdot r_{0P}(s)\, ds \right)$$

$x_S = -0.03\,mm$

$$y_S := -\frac{1}{D} \cdot \left(I_x \cdot \int_{0 \cdot mm}^{L_{total}} S_y(s) \cdot r_{0P}(s)\, ds - I_{xy} \cdot \int_{0 \cdot mm}^{L_{total}} S_x(s) \cdot r_{0P}(s)\, ds \right)$$

$y_S = 250.76\,mm$

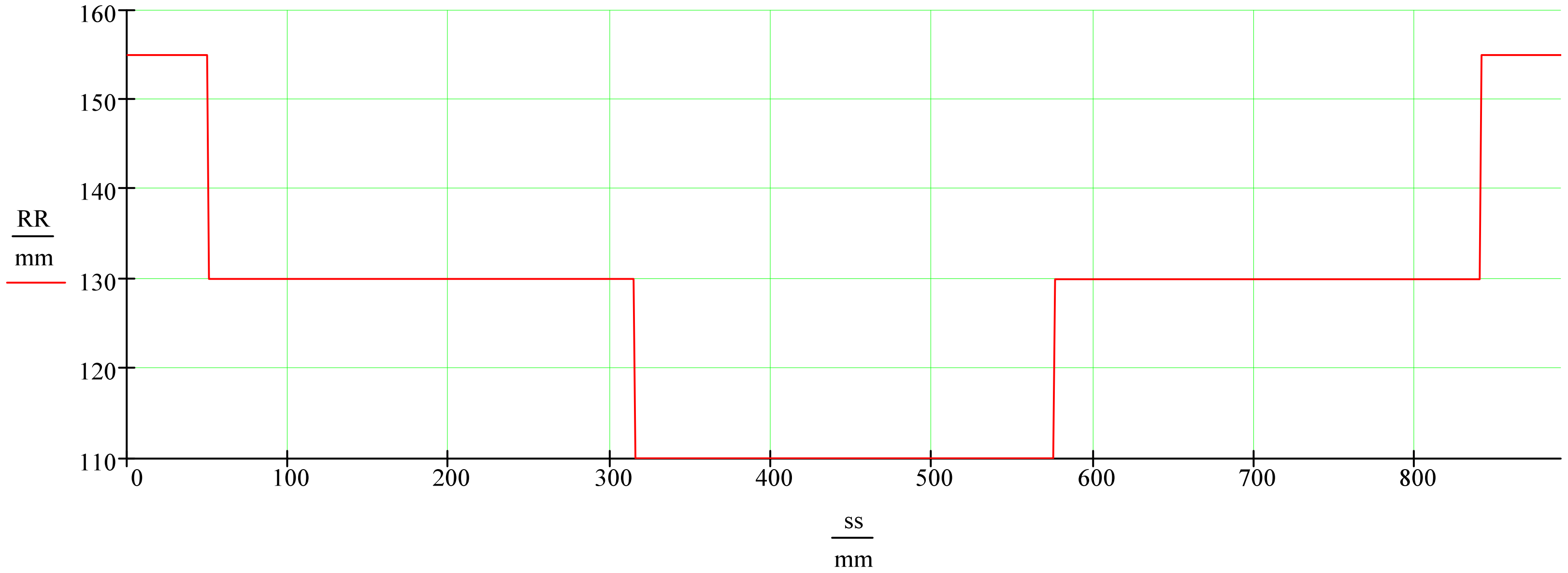
Sectorial Area referred to Shear Center

Determination of Distance of Shear Center to the segments from input s along the shape

$\theta S(s) := \text{ang}\left(x_S, y_S, x(s), y(s)\right)$ angle with Ox of the vector from Shear Center to point along the line

$$r_{SP}(s) := \sum_{i=1}^{N_{segs}} \sqrt{\left(xp_i - x_S\right)^2 + \left(yp_i - y_S\right)^2} \cdot \cos\left(\theta S\left(LA_i\right) - \alpha_i - \frac{\pi}{2}\right) \cdot \left| \begin{array}{ll} LA_i \le s \le LA_{i+1} & \text{if } i = 1 \\ LA_i < s \le LA_{i+1} & \text{otherwise} \end{array} \right.$$

$N_{parts} := 790$ $k := 1 .. N_{parts} + 1$ $ss_k := \frac{L_{total}}{N_{parts}} \cdot (k - 1)$ $RR := \overrightarrow{r_{0P}(ss)}$



The definition of the distance respect Shear Center on s portraits well in the chart

$\omega_S(S) := \int_{0 \cdot mm}^{S} r_{SP}(s)\, ds$ sectorial area referred to shear center

$$\omega(s) := \omega_S(s) - \frac{\int_{0 \cdot mm}^{L_{total}} \omega_S(s) \cdot t(s)\, ds}{Area}$$

exact formulation

Warping Constant

$$C_w := \int_{0 \cdot mm}^{L_{total}} \omega(s)^2 \cdot t(s)\, ds$$

$C_w = 5.5 \times 10^{12} mm^6$

$$I_a := C_w$$

$I_a = 5.5 \times 10^{12} mm^6$

in spanish notation

Polar radius of gyration respect the shear center

$$r_{0S} := \sqrt{r_x^2 + r_y^2 + x_S^2 + y_S^2}$$

$r_{0S} = 295.64\, mm$