

V1 creep+shrinkage

singly and uniformly reinforced simple span beam subject to uniform load



$$L := 5 \cdot \text{m}$$

$$b := 1 \cdot \text{m}$$

$$h := 18 \cdot \text{cm}$$

$$n_1 := 6 \text{ bars of}$$

$$\phi_1 := 12 \cdot \text{mm}$$

diameter all over the span

$$f_c := 31 \cdot \text{MPa}$$

$$E_s := 29000 \cdot \text{ksi}$$

$$q := 0.66 \cdot \frac{\text{ton}}{\text{m}}$$

$$c_1 := 3 \cdot \text{cm}$$

cover to axis of bottom steel

$$d := h - c_1$$

$$\phi_t := 2.60$$

creep factor, final, ratio from creep deflection to elastic reference deflection

$$\epsilon_{SH} := 660 \cdot 10^{-6}$$

final unrestrained shrinkage strain expected



- A simple span rectangular beam of concrete is subject to uniform load.
- This sheet determines the long term status of such beam
- Shrinkage is not taken into account and its effects must be calculated and added apart.
- A triangular stress block is assumed in all calculations, so quite low stresses in concrete are assumed anyway, It could be extended to any sectional status by deriving closed form solutions similar to those involved in the process.
- Reinforcement and load extend uniformly all over the span
- See Samra in ACI SJ vol 94 N° 6, p. 748

$$E_c := 57000 \cdot \text{psi} \cdot \sqrt{\frac{f_c}{\text{psi}}}$$

$$E_c = 26352.32 \text{ MPa}$$

$$n := \frac{E_s}{E_c}$$

$$n = 7.59$$

$$f_r := 7.5 \cdot \text{psi} \cdot \sqrt{\frac{f_c}{\text{psi}}}$$

$$A_s := n_1 \cdot \pi \cdot \frac{\phi_1^2}{4}$$

$$\rho := \frac{A_s}{b \cdot d}$$

$$I_g := \frac{b \cdot h^3}{12}$$

$$I_g = 48600 \text{ cm}^4$$

gross section inertia

$$y_t := \frac{h}{2}$$

$$M_{cr} := f_r \cdot \frac{I_g}{y_t}$$

$$M_{cr} = 1.91 \text{ m} \cdot \text{ton}$$

cracking moment

$$c := 10$$

unwarranted guess, assumed cm

Given

$$\frac{\frac{b}{cm} \cdot c^2}{2} + n \cdot \frac{A_s}{cm^2} \cdot c - n \cdot \frac{A_s}{cm^2} \cdot \frac{d}{cm} = 0$$

$$c > 0$$

$c := cm \cdot \text{Find}(c)$

c = 3.45 cm

a triangular stress block is being assumed

$k_i := \sqrt{(\rho \cdot n)^2 + 2 \cdot \rho \cdot n} - \rho \cdot n$
 $k_i \cdot d = 3.45 \text{ cm}$

depth of triangular stress block, initial state

$j_i := 1 - \frac{k_i}{3}$
 $j_i \cdot d = 13.85 \text{ cm}$

initial mechanical arm

We will do the calculation at center and quarter points, so resuming in 2 different points from symmetry

At center

$M_c := q \cdot \frac{L^2}{8}$
 $M_c = 2.06 \text{ m} \cdot \text{ton}$

At quarter

$M_q := \frac{q \cdot \frac{L}{4}}{2} \cdot \left(L - \frac{L}{4} \right)$
 $M_q = 1.55 \text{ m} \cdot \text{ton}$

Center Point

$M := M_c$

Initial status

$f_{ci} := \frac{2 \cdot M}{k_i \cdot j_i \cdot b \cdot d^2}$
 $f_{ci} = 8.47 \text{ MPa}$
 $f_{si} := \frac{M}{A_s \cdot j_i \cdot d}$
 $f_{si} = 215.21 \text{ MPa}$

$\epsilon_{ci} := \frac{f_{ci}}{E_c}$
 $\epsilon_{ci} = 0.00032$
 $\epsilon_{si} := \frac{f_{si}}{E_s}$
 $\epsilon_{si} = 0.00108$

$\kappa_i := \frac{\epsilon_{ci} + \epsilon_{si}}{d}$
 $\kappa_i = 0.009318 \frac{\text{rad}}{\text{m}}$
 $R_{\kappa i} := \frac{1}{\kappa_i}$
 $R_{\kappa i} = 107.32 \text{ m}$

Now we solve the final status

$f_{ct} := \left \begin{array}{l} f_{ct} \leftarrow 0.6 \cdot f_{ci} \\ f_{ctp} \leftarrow 0.1 \cdot f_c \\ \text{while } f_{ct} - f_{ctp} \geq \text{CTOL} \cdot \text{MPa} \\ \quad \left \begin{array}{l} f_{ctp} \leftarrow f_{ct} \\ \alpha \leftarrow \frac{2 \cdot \rho}{f_{ci}} \cdot [f_{ci} \cdot (1 - 0.8) \cdot \phi_t + f_{ct} \cdot (1 + 0.8 \cdot \phi_t)] \\ k_t \leftarrow \frac{-\alpha + \sqrt{(\alpha \cdot n)^2 + 4 \cdot \alpha \cdot n}}{2} \\ f_{ct} \leftarrow \frac{2 \cdot M}{b \cdot d^2 \cdot \left(1 - \frac{k_t}{2}\right) \cdot k_t} \end{array} \right. \\ \text{return } f_{ct} \end{array} \right.$	$k_t := \left \begin{array}{l} f_{ct} \leftarrow 0.6 \cdot f_{ci} \\ f_{ctp} \leftarrow 0.1 \cdot f_c \\ \text{while } f_{ct} - f_{ctp} \geq \text{CTOL} \cdot \text{MPa} \\ \quad \left \begin{array}{l} f_{ctp} \leftarrow f_{ct} \\ \alpha \leftarrow \frac{2 \cdot \rho}{f_{ci}} \cdot [f_{ci} \cdot (1 - 0.8) \cdot \phi_t + f_{ct} \cdot (1 + 0.8 \cdot \phi_t)] \\ k_t \leftarrow \frac{-\alpha + \sqrt{(\alpha \cdot n)^2 + 4 \cdot \alpha \cdot n}}{2} \\ f_{ct} \leftarrow \frac{2 \cdot M}{b \cdot d^2 \cdot \left(1 - \frac{k_t}{2}\right) \cdot k_t} \end{array} \right. \\ \text{return } k_t \end{array} \right.$
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$f_{ct} = 5.49 \text{ MPa}$ see how concrete has relaxed under creep

$f_{st} := \frac{f_{ct} \cdot k_t}{2 \cdot \rho}$ $f_{st} = 250.42 \text{ MPa}$ contrarily, steel stress has grown due to loss of mechanical arm

$\epsilon_{ct} := \frac{f_{ct}}{E_c}$ $\epsilon_{ct} = 0.00021$ $\epsilon_{st} := \frac{f_{st}}{E_s}$ $\epsilon_{st} = 0.00125$

$\kappa_t := \frac{\epsilon_{ct} + \epsilon_{st}}{d}$ $\kappa_t = 0.009737 \frac{\text{rad}}{\text{m}}$ $R_{\kappa t} := \frac{1}{\kappa_t}$ $R_{\kappa t} = 102.7 \text{ m}$

curvature has grown (and consequently radius of curvature has diminished)

The steel stress and corresponding curvature just obtained correspond to the exact location of one crack. However, for most of the length, the beam remain (between cracks) uncracked, and this can reduce somewhat the mean stress in the steel (specially for situations where the moment is barely above the cracking moment). The mean stress in the zone will give a better assessment of curvature, and since there is a closed form solution for the mean stress, we will use it to make the correction.

$$\epsilon_{sm} := \begin{cases} \frac{f_{st}}{E_s} \cdot \left[1 - 0.5 \cdot \left[\frac{\frac{M_{cr}}{A_s \cdot \left(h - \frac{c}{3} - c_1 \right)}}{f_{st}} \right]^2 \right] & \text{if } \frac{f_{st}}{E_s} \cdot \left[1 - 0.5 \cdot \left[\frac{\frac{M_{cr}}{A_s \cdot \left(h - \frac{c}{3} - c_1 \right)}}{f_{st}} \right]^2 \right] > 0.4 \cdot \frac{f_{st}}{E_s} \\ 0.4 \cdot \frac{f_{st}}{E_s} & \text{otherwise} \end{cases}$$

we adopt 0.5 instead of 1 since assumed repeated opening of cracks

the closed form formula is taken from the spanish code EHE and since it is offered as a mean to evaluate the average stress in the steel, (lacking its derivation) it will be assumed here also represents the tension stiffening effects (tensile strength of concrete) at the location of the cracks themselves (tensile strength which is here not accounted for in the derivation of curvature)

$$f_{sm} := \epsilon_{sm} \cdot E_s \quad f_{sm} = 171.17 \text{ MPa} \quad \text{an slight reduction on } f_{st} \text{ for frankly cracked sections}$$

so reduction in curvature will be proportional, hence

$$K_c := \kappa_t \cdot \frac{f_{sm}}{f_{st}} \quad \text{we store terminal curvature at center for further use}$$

Now we repeat the process for quarter point

Quarter Point $\underline{M} := M_q$

Initial status

$$\begin{aligned} \underline{f_{ci}} &:= \frac{2 \cdot M}{k_i \cdot j_i \cdot b \cdot d^2} & f_{ci} &= 6.35 \text{ MPa} & \underline{f_{si}} &:= \frac{M}{A_s \cdot j_i \cdot d} & f_{si} &= 161.41 \text{ MPa} \\ \underline{\epsilon_{ci}} &:= \frac{f_{ci}}{E_c} & \epsilon_{ci} &= 0.00024 & \underline{\epsilon_{si}} &:= \frac{f_{si}}{E_s} & \epsilon_{si} &= 0.00081 \\ \underline{\kappa_i} &:= \frac{\epsilon_{ci} + \epsilon_{si}}{d} & \kappa_i &= 0.006988 \frac{\text{rad}}{\text{m}} & \underline{R_{\kappa i}} &:= \frac{1}{\kappa_i} & R_{\kappa i} &= 143.1 \text{ m} \end{aligned}$$

Now we solve the final status

$\begin{aligned} \text{\textcolor{green}{\text{\textit{f}}}_{ct}} &:= \left \begin{array}{l} f_{ct} \leftarrow 0.6 \cdot f_{ci} \\ f_{ctp} \leftarrow 0.1 \cdot f_c \\ \text{while } f_{ct} - f_{ctp} \geq \text{CTOL} \cdot \text{MPa} \\ \quad \left \begin{array}{l} f_{ctp} \leftarrow f_{ct} \\ \alpha \leftarrow \frac{2 \cdot \rho}{f_{ci}} \cdot [f_{ci} \cdot (1 - 0.8) \cdot \phi_t + f_{ct} \cdot (1 + 0.8 \cdot \phi_t)] \\ k_t \leftarrow \frac{-\alpha + \sqrt{(\alpha \cdot n)^2 + 4 \cdot \alpha \cdot n}}{2} \\ f_{ct} \leftarrow \frac{2 \cdot M}{b \cdot d^2 \cdot \left(1 - \frac{k_t}{2}\right) \cdot k_t} \end{array} \right. \\ \text{return } f_{ct} \end{array} \right. \end{aligned}$	$\begin{aligned} \text{\textcolor{green}{\text{\textit{k}}}_{t}} &:= \left \begin{array}{l} f_{ct} \leftarrow 0.6 \cdot f_{ci} \\ f_{ctp} \leftarrow 0.1 \cdot f_c \\ \text{while } f_{ct} - f_{ctp} \geq \text{CTOL} \cdot \text{MPa} \\ \quad \left \begin{array}{l} f_{ctp} \leftarrow f_{ct} \\ \alpha \leftarrow \frac{2 \cdot \rho}{f_{ci}} \cdot [f_{ci} \cdot (1 - 0.8) \cdot \phi_t + f_{ct} \cdot (1 + 0.8 \cdot \phi_t)] \\ k_t \leftarrow \frac{-\alpha + \sqrt{(\alpha \cdot n)^2 + 4 \cdot \alpha \cdot n}}{2} \\ f_{ct} \leftarrow \frac{2 \cdot M}{b \cdot d^2 \cdot \left(1 - \frac{k_t}{2}\right) \cdot k_t} \end{array} \right. \\ \text{return } k_t \end{array} \right. \end{aligned}$
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$f_{ct} = 4.12 \text{ MPa}$ see how concrete has relaxed under creep

$\text{\textcolor{green}{\text{\textit{f}}}_{st}} := \frac{f_{ct} \cdot k_t}{2 \cdot \rho}$ $f_{st} = 187.81 \text{ MPa}$ contrarily, steel stress has grown due to loss of mechanical arm

$\text{\textcolor{green}{\text{\textit{\epsilon}}}_{ct}} := \frac{f_{ct}}{E_c}$ $\epsilon_{ct} = 0.00016$ $\text{\textcolor{green}{\text{\textit{\epsilon}}}_{st}} := \frac{f_{st}}{E_s}$ $\epsilon_{st} = 0.00094$

$\text{\textcolor{green}{\text{\textit{\kappa}}}_{t}} := \frac{\epsilon_{ct} + \epsilon_{st}}{d}$ $\kappa_t = 0.007303 \frac{\text{rad}}{\text{m}}$ $\text{\textcolor{green}{\text{\textit{R}}}_{\kappa t}} := \frac{1}{\kappa_t}$ $R_{\kappa t} = 136.93 \text{ m}$

curvature has grown (and consequently radius of curvature has diminished)

The steel stress and corresponding curvature just obtained correspond to the exact location of one crack. However, for most of the length, the beam remain (between cracks) uncracked, and this can reduce somewhat the mean stress in the steel (specially for situations where the moment is barely above the cracking moment). The mean stress in the zone will give a better assessment of curvature, and since there is a closed form solution for the mean stress, we will use it to make the correction.

$$\varepsilon_{sm} := \begin{cases} \frac{f_{st}}{E_s} \cdot \left[1 - 0.5 \cdot \left[\frac{A_s \cdot \left(h - \frac{c}{3} - c_1 \right)}{f_{st}} \right]^2 \right] & \text{if } \frac{f_{st}}{E_s} \cdot \left[1 - 0.5 \cdot \left[\frac{A_s \cdot \left(h - \frac{c}{3} - c_1 \right)}{f_{st}} \right]^2 \right] > 0.4 \cdot \frac{f_{st}}{E_s} \\ 0.4 \cdot \frac{f_{st}}{E_s} & \text{otherwise} \end{cases}$$

we adopt 0.5 instead of 1 since assumed repeated opening of cracks

$$f_{sm} := \varepsilon_{sm} \cdot E_s \quad f_{sm} = 82.15 \text{ MPa} \quad \text{an slight redcuton on fst for frankly cracked sections}$$

so reduction in curvature will be proportional, hence

$$K_q := \kappa_t \cdot \frac{f_{sm}}{f_{st}} \quad \text{store for deflection evaluation}$$

Now we create a function describing curvature along the beam by one interpolation function

Shrinkage

We will determine curvature per a triangular concrete stress block and a reduced final modulus as per creep, which is coherent with the previous procedure, and common in creep-shrinkage studies, many times to ascertain service level or real weight cases, normally still quite far from limit strength

$$A_c := b \cdot h - A_s$$

$$y_{gc} := \frac{b \cdot h \cdot \frac{h}{2} - A_s \cdot c_1}{A_c} \quad y_{gc} = 9.02 \text{ cm} \quad \text{shrinkage will exert its force centered here}$$

$$\varepsilon_1 := 0.003 \quad \varepsilon_2 := 0.001 \quad \text{unwarranted guesses, the rebound strains at bottom and atop suffice to describe and solve the problem}$$

$$\varepsilon_{\text{ygc}}(\varepsilon_1,\varepsilon_2) := \varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot y_{\text{gc}}$$

$$\text{Steel}_{\text{force}}(\varepsilon_1,\varepsilon_2) := A_s \cdot \left[\varepsilon_{\text{SH}} - \left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot c_1 \right) \right] \cdot E_s$$

$$\text{Concrete}_{\text{force}}(\varepsilon_1,\varepsilon_2) := \int\limits_{0 \cdot \text{m}}^h \left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot z \right) \cdot \frac{E_c}{1 + \phi_t} \cdot b \, \text{d}z - A_s \cdot \left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot c_1 \right) \cdot \frac{E_c}{1 + \phi_t}$$

$$\text{Steel}_{\text{moment}}(\varepsilon_1,\varepsilon_2) := A_s \cdot \left[\varepsilon_{\text{SH}} - \left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot c_1 \right) \right] \cdot E_s \cdot c_1 \qquad \text{respect bottom}$$

$$\text{Concrete}_{\text{moment}}(\varepsilon_1,\varepsilon_2) := \int\limits_{0 \cdot \text{m}}^h z \cdot \left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot z \right) \cdot \frac{E_c}{1 + \phi_t} \cdot b \, \text{d}z - A_s \cdot \left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{h} \cdot c_1 \right) \cdot \frac{E_c}{1 + \phi_t} \cdot c_1$$

Given

$$\text{Concrete}_{\text{force}}(\varepsilon_1,\varepsilon_2) = \text{Steel}_{\text{force}}(\varepsilon_1,\varepsilon_2) \qquad \text{tensile force in concrete (of average at ygc height equaled to forcing)}$$

$$\text{Concrete}_{\text{moment}}(\varepsilon_1,\varepsilon_2) = \text{Steel}_{\text{moment}}(\varepsilon_1,\varepsilon_2)$$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} := \text{Find}(\varepsilon_1,\varepsilon_2) \qquad \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \begin{pmatrix} 0.000166 \\ -0.000055 \end{pmatrix} \begin{matrix} \text{ound at bottom} \\ \text{rebound, er, no, deepening atop} \end{matrix}$$

$$\text{Steel}_{\text{force}}(\varepsilon_1,\varepsilon_2) = 7.35 \, \text{ton} \qquad \text{Concrete}_{\text{force}}(\varepsilon_1,\varepsilon_2) = 7.35 \, \text{ton}$$

$$\begin{pmatrix} \varepsilon_{SH} - \varepsilon_1 \\ \varepsilon_{SH} - \varepsilon_2 \end{pmatrix} = \begin{pmatrix} 0.000494 \\ 0.000715 \end{pmatrix} \begin{matrix} \text{shortening at bottom} \\ \text{shortening atop} \end{matrix} \quad \varepsilon_{SH} - \varepsilon_{ygc}(\varepsilon_1, \varepsilon_2) = 0.000605$$

mean shortening, less than free shrinkage shortening due to the presence of reinforcement

$$\varepsilon_{ygc}(\varepsilon_1, \varepsilon_2) = 0.000055 \quad \text{mean rebound strain due to the presence of steel}$$

$$\frac{E_c}{1 + \phi_t} \cdot \varepsilon_{ygc}(\varepsilon_1, \varepsilon_2) \cdot A_c = 7.35 \text{ ton} \quad \text{equal to total force, due to mean strain rebound}$$

You see appear compressive stresses atop: the urge to centered on ygc shorten is so effective that causes compression atop.
In no other way the non concentrical with ygc steel opposing force could by itself equilibrate the (mean) tensile (centered on ygc) stresses in concrete.

Will check tensile stresses, long term, maximum

$$\varepsilon_1 \cdot \frac{E_c}{1 + \phi_t} = 1.21 \text{ MPa}$$

Correction_{Factor} := 1

1.00 for Normal weight
0.75 fo all-lightweight
0.85 for sand-lightweight

$$f_r := \text{Correction}_{\text{Factor}} \cdot \text{psi} \cdot 11.7 \cdot \sqrt{\frac{f_c}{\text{psi}}}$$

while the probabilistic longterm modulus of rupture (tensile) strength will be more than $f_r = 5.41 \text{ MPa}$ even wehen we haven't taken into account gain in strength with time

It is not likely for this case the section breaking due to shrinkage alone.
Under the uniform section, curvature of shrinkage will be constant except for a shorl length from the ends of the beams needed to gain the 7.43 ton in bond (some cm). We will assume constant added curvature by shrinkage, of value...

$$\kappa_{SH} := \frac{\varepsilon_{SH} - \varepsilon_2 - (\varepsilon_{SH} - \varepsilon_1)}{h} \quad \kappa_{SH} = 0.001226 \frac{1}{\text{m}}$$

In this we are being a bit unconservative in that a more correct approach would have involved dumping the stresses and strains of shrinkage on an initial condition (at unreduced original E_c modulus) and then relax these with all other actions through creep. However it is expected that the introduced inaccuracy may be dismissed as one more of the many simplifications assumed to get an appraisal of long term behaviour of the beam, no worse than others. So adding shrinkage curvature to creep...

$$Xs := \begin{pmatrix} 0 \cdot m \\ \frac{L}{4} \\ \frac{L}{2} \\ 0.75 \cdot L \\ L \end{pmatrix}$$

$$\kappa s := \begin{pmatrix} \kappa_{SH} + 0 \cdot \frac{1}{m} \\ \kappa_{SH} + K_q \\ \kappa_{SH} + K_c \\ \kappa_{SH} + K_q \\ \kappa_{SH} + 0 \cdot \frac{1}{m} \end{pmatrix}$$

a simple span beam curvature at end
 ; assume zero since there it is so M/EI

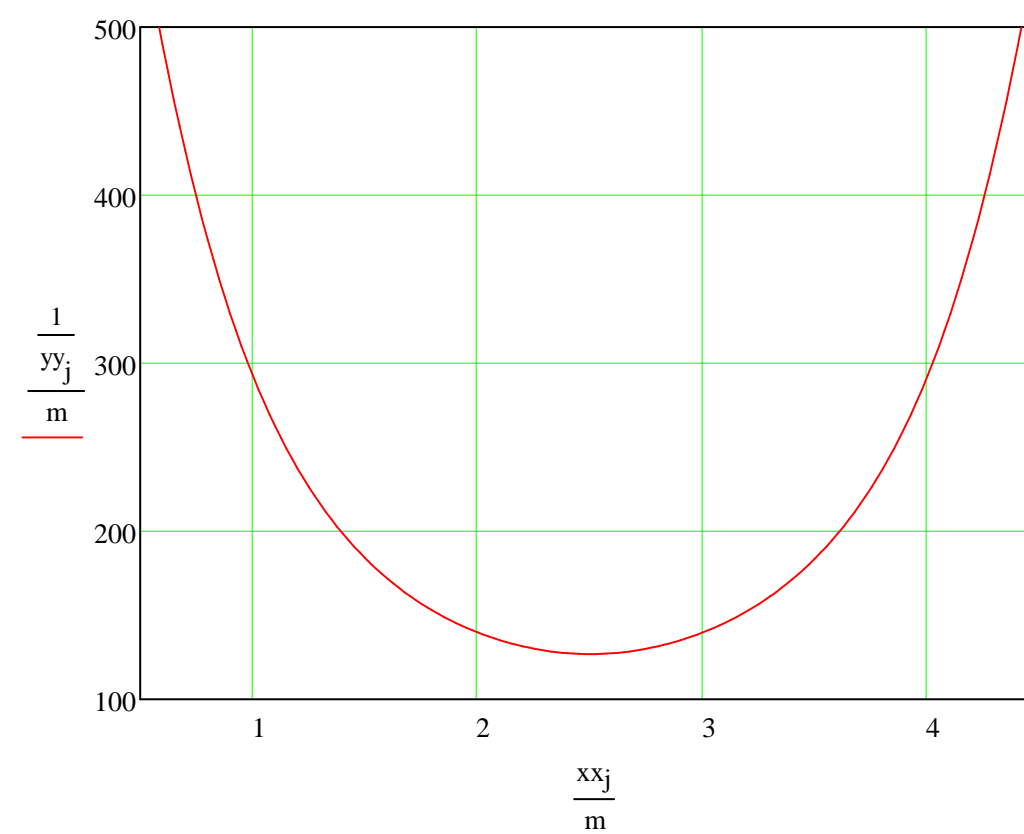
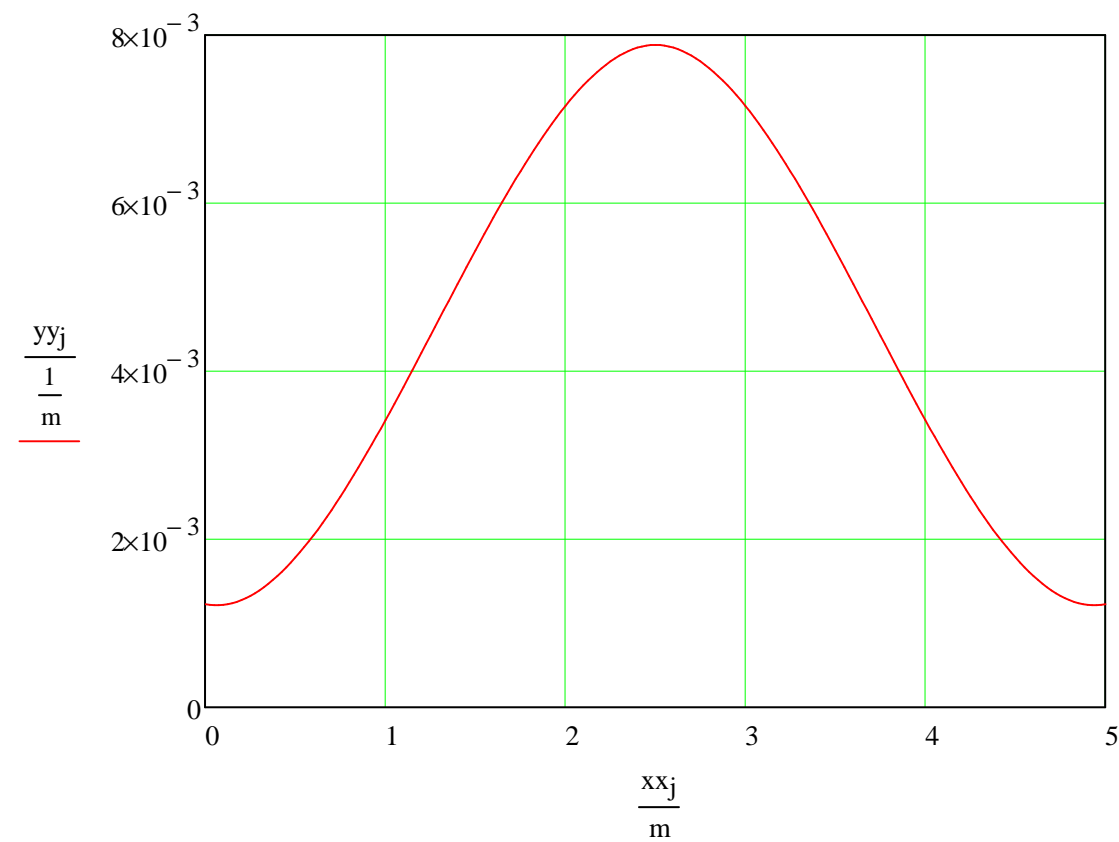
$vs := cspline(Xs, \kappa s)$
 $\kappa(x) := interp(vs, Xs, \kappa s, x)$

$Parts := 200$
 $j := 1 .. Parts + 1$
 $xxj := \frac{L}{Parts} \cdot (j - 1)$
 $yyj := \kappa(xxj)$

(Inferred) CURVATURES Chart

not deflections

(Inferred) RADIUSES OF CURVATURE Chart



DCI no Directorate of Central Intelligence but **Direct CURVATURE Integrator** a courtesy of the mind of Mohr

Once we have determined curvature along the span, we are in the geometrical know-how of getting by integration the deflection, by the procedure that follows (of which the clearer exposition I've found in Merrit and Ricketts p. 5.41 of the 5th ed)

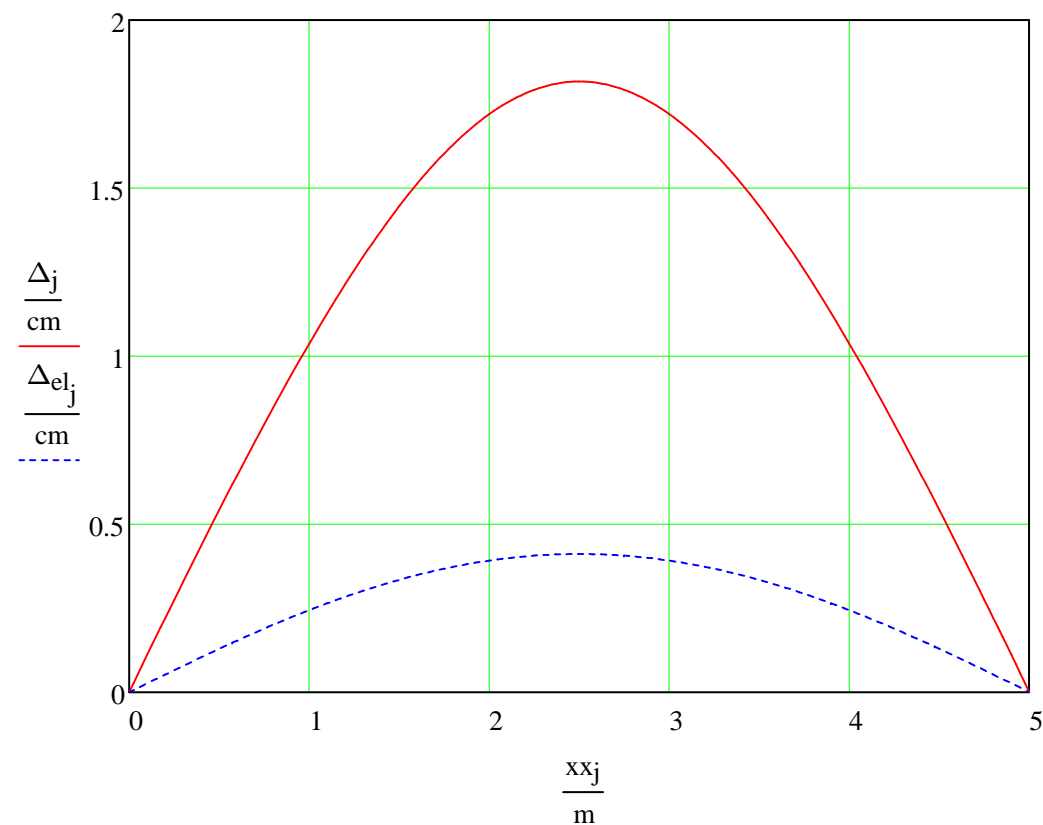
$$\delta_{\text{ww}}(X) := \frac{X}{L} \cdot \int_{0 \cdot \text{m}}^L (L - x) \cdot \kappa(x) \, dx - \int_{0 \cdot \text{m}}^X (X - x) \cdot \kappa(x) \, dx \qquad \Delta_j := \delta(\text{xxj}) \qquad \text{so we chart}$$

The elastic line for gross section Inertia is

$$\delta_{\text{el}}(X) := \frac{q \cdot X}{24 \cdot E_c \cdot b \cdot \frac{h^3}{12}} \cdot (X^3 - 2 \cdot L \cdot X^2 + L^3) \qquad \Delta_{\text{el},j} := \delta_{\text{el}}(\text{xxj})$$



Long Term Deflection of the singly uniformly reinforced beam under the assumed permanent input load, at the age implied by the data Creep Factor, with CREEP and SHRINKAGE taken into account



- red line plots the long term deflection
- in blue the elastic line at initial gross section inertia
- change the Creep Factor to see how the long term deflection proves to be in this formulation very insensitive to it

$\phi_t = 2.6$ assumed creep factor

$\epsilon_{SH} = 0.00066$ final unrestrained shrinkage strain

This deflection is short by about 35% of experimental in ACI SJ 96/6 in p. 1028
Data seem to be for the age, not final, but theoretically should provide agreement.

Branson's formulation for LONG TERM curvature and deflection
added by shrinkage in doubly reinforced rectangular sections



Curvature

Instead of going through the "exact" evaluation given above, Branson proposed to add curvature

$$\rho_1 := \rho \quad \rho_2 := 0$$

$$\kappa_{\text{add}} := \begin{cases} 0.7 \cdot \frac{\epsilon_{\text{SH}}}{h} \cdot [100 \cdot (\rho_1 - \rho_2)]^{\frac{1}{3}} \cdot \left(\frac{\rho_1 - \rho_2}{\rho_1} \right)^{\frac{1}{2}} & \text{if } \rho_1 - \rho_2 \leq 0.03 \\ 0.7 \cdot \frac{\epsilon_{\text{SH}}}{h} \cdot [100 \cdot (\rho_1 - \rho_2)]^{\frac{1}{3}} \cdot \left(\frac{\rho_1 - \rho_2}{\rho_1} \right)^{\frac{1}{2}} & \text{otherwise} \end{cases}$$

$$\kappa_{\text{add}} = 0.00197 \frac{1}{\text{m}} \quad \text{where we have with more effort obtained} \quad \kappa_{\text{SH}} = 0.001226 \frac{1}{\text{m}}$$

- Note that Branson's closed form can be even more accurate than the "exact" curvature from the calculation above, since this is based on a theoretical triangular stress block, while Branson's fits well to experimental tests.
- If you can accept these, you can directly proceed to creep relaxation of loads without concern for long term deflection from shrinkage
- Obviously, it must refer to a specific range of beam depths and may be your beam is not in such range

Deflection

$L_{\text{cantilever}} := 6 \cdot \text{m}$

$L_{\text{cantilever}} := 1.25 \cdot \text{m}$

Cantilevers	$\Delta\delta_{\text{SH_CV}} := 0.5 \cdot \kappa_{\text{add}} \cdot L_{\text{cantilever}}^2$	$\Delta\delta_{\text{SH_CV}} = 1.54 \text{ mm}$
Simply supported beams	$\Delta\delta_{\text{SH_SS}} := 0.125 \cdot \kappa_{\text{add}} \cdot L^2$	$\Delta\delta_{\text{SH_SS}} = 8.87 \text{ mm}$
Wholly fixed end beams	$\Delta\delta_{\text{SH_FE}} := 0.063 \cdot \kappa_{\text{add}} \cdot L^2$	$\Delta\delta_{\text{SH_FE}} = 4.47 \text{ mm}$