

Effective and pseudo-tangent Young modulus inferred from LRFD F_{cr} formulation



We obtain the effective Young modulus as that producing the same critical strength than LRFD but in a pure elastic formulation. We define parametrically both stress and effective Young modulus on slenderness to chart the relationship.

$$F_y := 260 \cdot \text{MPa} \quad E := 200000 \cdot \text{MPa}$$

$$\sigma(\lambda) := F_y \cdot \begin{cases} 0.658 \left[\left(\frac{\lambda}{\pi} \sqrt{\frac{F_y}{E}} \right)^2 \right] & \text{if } 0 \leq \frac{\lambda}{\pi} \sqrt{\frac{F_y}{E}} \leq 1.5 \\ \frac{0.877}{\left(\frac{\lambda}{\pi} \sqrt{\frac{F_y}{E}} \right)^2} & \text{otherwise} \end{cases} \quad \text{the LRFD critical (limit strength) stress}$$

$$E_{\text{eff}}(\lambda) := \sigma(\lambda) \cdot \frac{\lambda^2}{\pi^2}$$

- inferred effective modulus at the stress, is a SECANT modulus to the point of σ stress, linear as E itself
- the reduced modulus is secant and should not be denoted E_t implying tangent as sometimes is

$$E_{\text{effe}}(\sigma) := E \cdot \begin{cases} 0.877 & \text{if } \frac{\sigma}{F_y} \leq 0.39 \\ \text{otherwise} \\ \left| \begin{array}{l} 2.39 \cdot \frac{\sigma}{F_y} \cdot \ln \left(\frac{1}{\frac{\sigma}{F_y}} \right) & \text{if } \text{AND2} \left(\frac{\sigma}{F_y} > 0.39, \frac{\sigma}{F_y} \leq 1 \right) \\ 0 & \text{otherwise} \end{array} \right. & \end{cases}$$

a closed form formulation of the same effective modulus by Baker

$$E_{\text{effeCRC}}(\sigma) := 0.877E \cdot \begin{cases} 4 \cdot \frac{\sigma}{F_y} \cdot \left(1 - \frac{\sigma}{F_y} \right) & \text{if } \frac{\sigma}{F_y} \geq 0.5 \\ 1 & \text{otherwise} \end{cases}$$

CRC based (CRC's multiplied by 0.877)
closed form formulation of effective modulus

E_{eff} substitutes E in the critical elastic stresses closed form formulations to give elastic or inelastic critical stress

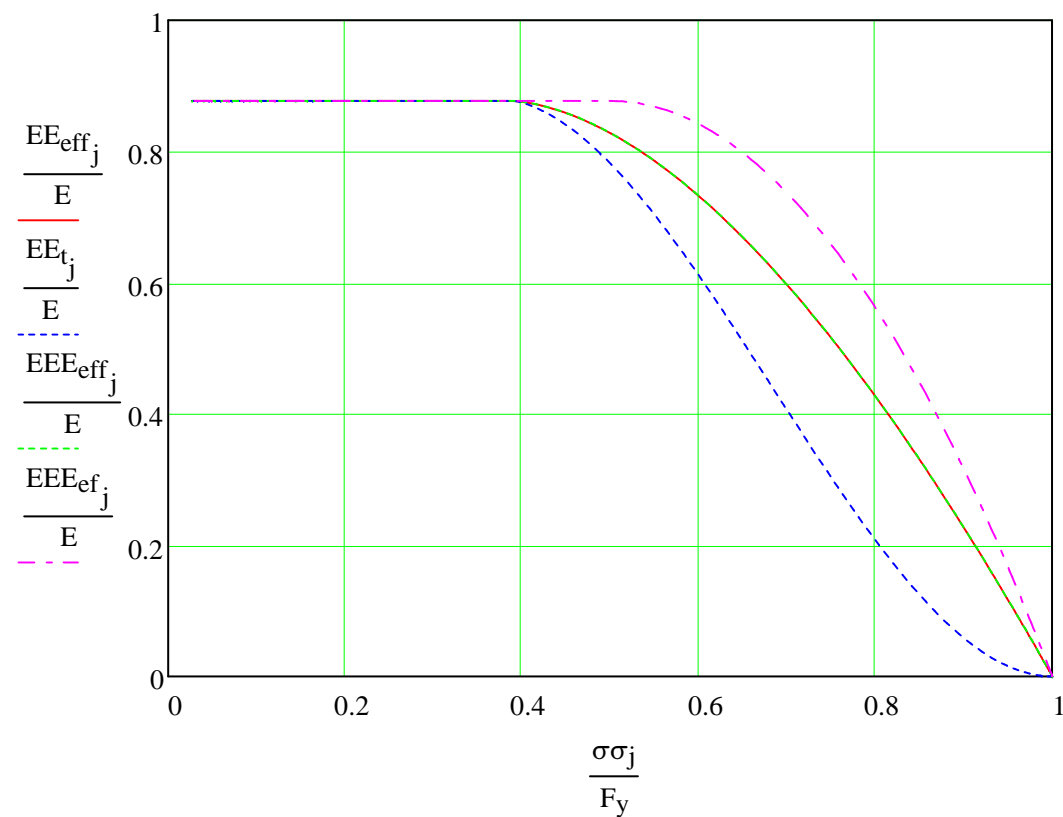
$$E_t(\lambda) := \frac{\left(\sigma(\lambda) \cdot \frac{\lambda^2}{\pi^2}\right)^2}{0.877 \cdot E}$$

one formulation of inferred TANGENT modulus at the stress
the 0.877 in denominator required for consistency with effective modulus in the proportionality region
 $\sqrt{E \cdot E_t}$ substitutes E in the critical elastic stresses closed form formulations to give elastic or inelastic critical stress

Chart Young's reduced or effective modulus as a function of the stress

Chart Young's inferred pseudo-tangent modulus as a function of the stress

$$j := 1..500 \quad \lambda\lambda_j := j \quad \sigma\sigma_j := \sigma(\lambda\lambda_j) \quad EE_{eff_j} := E_{eff}(\lambda\lambda_j) \quad EE_{t_j} := E_t(\lambda\lambda_j) \quad EEE_{eff_j} := E_{effe}(\sigma\sigma_j) \quad EEE_{ef_j} := E_{effeCRC}(\sigma\sigma_j)$$



- Note that even at the first stages of axial loading there is a surmised reduction on stiffness with rapport to E of material.
- Note also that this reduction in axial stiffness is very alike if not the same that charted for BENDING stiffness reduction in p. 160 of *Is your structure suitably braced?* SSRC quoting Baker 1991.
- In our chart the Baker's closed form formulation is seen to directly lie on the LRFD deduced effective (secant) modulus
- Hence may be this formulation could be used for simplicity for both bending and axial stiffness reduction instead of 2 formulations.
- However doing so causes some common permissible slenderness of plates be reduced; this could however with the plates in seismic members requiring to be thicker.
- In magenta CRC derived effective modulus

- Then (with some risk) we can use the closed form formulation of the secant modulus directly in the formulas of elastic buckling stress (such as Bleich's) to get elastic and inelastic critical stress.
- If you use reduced stiffnesses in analysis, such as per secant modulus, you can use such stiffness directly to get the proper K factors to check the sections by formulations in LRFD.
- Otherwise (if you don't use reduced stiffnesses in analysis), a correction due to inelastic buckling is warranted when calculating the K factors. Stiffness of the columns must be in such case multiplied by F_{cr}/F_e