Buckling of symmetric antifunicular T section arches

$L := 50 \cdot m$ span $f := 20 \cdot m$ sagitta $f_c := 35 \cdot MPa$ $\gamma_c := 2400 \cdot \frac{kgf}{m^3}$

$b(y) := \begin{cases} 30 \cdot cm & \text{if } y \le 58 \cdot cm \\ 222 \cdot cm & \text{otherwise} \end{cases}$ $d := 70 \cdot cm$ total depth

$w := 3 \cdot \frac{ton}{m}$ $Arch_{Type} := 3$

1 for 3 hinges
2 for 2 hinges
3 for fixed

out of plane fixity is assumed at supports

Section properties

We will be considering the T beam defined above. In our case the section remains constant along the arch length.

$A_c := \int_{0 \cdot m}^{d} b(y)\, dy$ $A_c = 4405.35\,cm^2$ $own\_weight := A_c \cdot \gamma_c$ $own\_weight = 1.06 \frac{ton}{m}$ merely information

$y_g := \frac{\int_{0 \cdot m}^{d} y \cdot b(y)\, dy}{A_c}$ $y_g = 49.6\,cm$

$I_x := \int_{0 \cdot m - y_g}^{d - y_g} y^2 \cdot b(y)\, dy$ $I_x = 1304896.94\,cm^4$

$i_x := \sqrt{\frac{I_x}{A_c}}$ $i_x = 17.21\,cm$

Transversally

$$B(y) := \begin{cases} 12\cdot cm & \text{if } y \le -15\cdot cm \\ \text{otherwise} & \begin{cases} 12\cdot cm & \text{if } y \ge 15\cdot cm \\ 70\cdot cm & \text{otherwise} \end{cases} \end{cases}$$

$$I_y := \int_{-111\cdot cm}^{111\cdot cm} y^2\cdot B(y)\,dy$$

change limits of integration for your case

we have taken 8 times thickness at each side since we are thinking of arches solidary to equal shape shells, to avoid in such case y buckling controlling, which will not due to impossibility of the arches going out of the plane

$$I_y = 1.09\times 10^7\,cm^4$$

$$i_y := \sqrt{\frac{I_y}{A_c}} \qquad i_y = 49.84\,cm$$

Buckling

Source of buckling treatment is *Arcos y Pandeo*, Arangoá, ETSAM 1974
Of course the treatment is conventional and only approximate.

In plane buckling

$$ps := \begin{pmatrix} 0.05 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \end{pmatrix} \qquad \beta1s := \begin{pmatrix} 1.2 \\ 1.16 \\ 1.13 \\ 1.19 \\ 1.25 \end{pmatrix} \qquad \beta2s := \begin{pmatrix} 1 \\ 1.06 \\ 1.13 \\ 1.19 \\ 1.25 \end{pmatrix} \qquad \beta3s := \begin{pmatrix} 0.7 \\ 0.72 \\ 0.74 \\ 0.75 \\ 0.76 \end{pmatrix}$$

$$vs1 := pspline(ps, \beta1s) \qquad vs2 := pspline(ps, \beta2s) \qquad vs3 := pspline(ps, \beta3s)$$

$$\beta_1(p) := interp(vs1, ps, \beta1s, p) \quad \beta_2(p) := interp(vs2, ps, \beta2s, p) \quad \beta_3(p) := interp(vs3, ps, \beta3s, p)$$

$$\beta(p) := \begin{array}{|l} \beta_3(p) \ \text{ if } \ Arch_{Type} = 3 \\ \text{otherwise} \\ \quad \begin{array}{|l} \beta_2(p) \ \text{ if } \ Arch_{Type} = 2 \\ \beta_1(p) \ \text{ otherwise} \end{array} \end{array}$$

$$\lambda s := \begin{pmatrix} 50 \\ 70 \\ 85 \\ 105 \\ 120 \\ 140 \end{pmatrix} \qquad \omega s := \begin{pmatrix} 1 \\ 1.08 \\ 1.32 \\ 1.72 \\ 2.28 \\ 3 \end{pmatrix}$$

$$vs4 := \text{lspline}(\lambda s, \omega s) \qquad \omega(\lambda) := \begin{array}{|l} \text{interp}(vs4, \lambda s, \omega s, \lambda) \ \text{ if } \ \text{interp}(vs4, \lambda s, \omega s, \lambda) \geq 1 \\ 1 \ \text{ otherwise} \end{array}$$

Parabola arch equation (centered) is

$$y(x, f) := -4 \cdot \frac{f}{L^2} \cdot x^2 + f$$

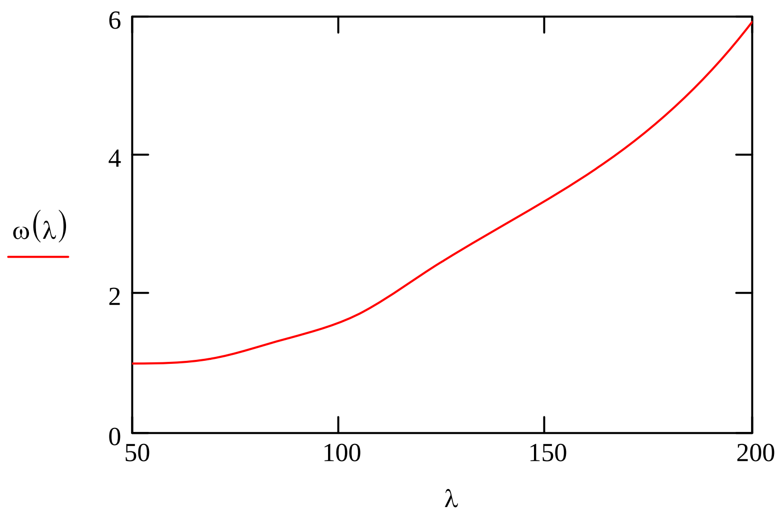
$$\text{slope}(x) := \frac{d}{dx} y(x, f) \qquad \alpha(x) := \text{atan}(\text{slope}(x))$$

$$S_2 := \int_{0 \cdot m}^{\frac{L}{2}} \sqrt{1 + \left(\frac{d}{dx} y(x, f)\right)^2} \, dx \qquad S_2 = 33.34 \, m$$

semiarch length

$$p_0 := \frac{f}{L} \qquad p_0 = 0.4$$

sagitta to span ratio



$$\lambda_x := \beta(p_0)\cdot\frac{S_2}{i_x}$$

$\lambda_x = 145.3$

better don't exceed
slenderness 140

$\omega(\lambda_x) = 3.18$

Out of plane buckling

Out of Plane Buckling is of not concern IF bracing effectively prevents the arches going out of their plane, such when they are part of a shell of same shape.
For the other cases...

$Type_{arch2} := 1$

1 for Ix=constant inertia
2 for Ix·cosα=constant

$$p2s := \begin{pmatrix} 0.05 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{pmatrix} \qquad \beta2s1 := \begin{pmatrix} 0.5 \\ 0.54 \\ 0.65 \\ 0.82 \\ 1.07 \end{pmatrix} \qquad \beta2s2 := \begin{pmatrix} 0.5 \\ 0.52 \\ 0.59 \\ 0.71 \\ 0.86 \end{pmatrix}$$

$vs5 := pspline(p2s, \beta2s1)$ $\qquad$ $vs6 := pspline(p2s, \beta2s2)$

$\beta_{2\_1}(p) := \text{interp}(vs5, p2s, \beta2s1, p)$ $\beta_{2\_2}(p) := \text{interp}(vs6, p2s, \beta2s2, p)$

$$\beta_2(p) := \begin{vmatrix} \beta_{2\_1}(p) & \text{if } Type_{arch2} = 1 \\ \beta_{2\_2}(p) & \text{otherwise} \end{vmatrix}$$

$\lambda_y := \beta_2\left(\frac{f}{L}\right)\cdot\frac{L}{i_y}$ $\lambda_y = 107.35$ better don't exceed 140

$\omega(\lambda_y) = 1.8$

$\omega := \max\left(\begin{pmatrix} \omega(\lambda_x) \\ \omega(\lambda_y) \end{pmatrix}\right)$ $\omega = 3.18$

curvature $\kappa(x) := \left| \dfrac{\frac{d^2}{dx^2}y(x,f)}{\left[1+\left(\frac{d}{dx}y(x,f)\right)^2\right]^{\frac{3}{2}}} \right|$ $R(x) := \dfrac{1}{\kappa(x)}$ radius of curvature

These two equations make nothing but to keep information available

$\text{weight}(X) := 2\cdot X\cdot w$ vertical action of symmetric central cut, origin at center

$N_\phi(x) := \dfrac{\frac{\text{weight}(x)}{\cos(\text{atan}(\text{slope}(x)))}}{2}$ tangential force

At ends

$x := \frac{L}{2}$

$\sigma_{\phi}(x) := \frac{N_{\phi}(x)}{A_c}$

$N_{\phi}(x) = 141.51\,\text{ton}$

$\sigma_{\phi}(x) = 3.15\,\text{MPa}$

$\alpha(x) = -57.99\,\text{deg}$

At 1/4th span

$x := \frac{L}{4}$

$\sigma_{\phi}(x) := \frac{N_{\phi}(x)}{A_c}$

$N_{\phi}(x) = 48.02\,\text{ton}$

$\sigma_{\phi}(x) = 1.07\,\text{MPa}$

$\alpha(x) = -38.66\,\text{deg}$

Axial Stress that controls

$$\sigma_{\phi\_max} := \max\left(\begin{pmatrix} \sigma_{\phi}\left(\frac{L}{2}\right) \\ \omega \cdot \sigma_{\phi}\left(\frac{L}{4}\right) \end{pmatrix}\right)$$

$\sigma_{\phi\_max} = 3.4\,\text{MPa}$

$\frac{\sigma_{\phi\_max}}{f_c} = 9.72\,\%$

OK, buckling seems of not concern

I wouldn't let go it over 40% in non seismic zones and over 25% in seismic zones, without further consideration of concomitant bending

- The stress at 1/4 point would be combined with the strength of materials derived stresses from bending in x direction (and y, if significant).
- At supports a common strength of materials check (without amplification due to buckling) seems to be enough.

Reactions (if hinged)

$$\text{Reaction} := N_\phi\left(\frac{L}{2}\right) \qquad R_v := -N_\phi\left(\frac{L}{2}\right)\cdot\sin(\alpha(x)) \qquad R_h := N_\phi\left(\frac{L}{2}\right)\cdot\cos(\alpha(x))$$

$$\text{Reaction} = 141.51\,\text{ton} \qquad R_v = 88.4\,\text{ton} \qquad R_h = 110.5\,\text{ton}$$

Chart parabolic arch

$$\text{Parts} := 100 \qquad j := 1..\text{Parts}+1 \qquad x_j := -\frac{L}{2} + (j-1)\cdot\frac{L}{\text{Parts}} \qquad y_j := y(x_j, f)$$