

Torsion analysis for concentrated torsion (Case 3):

Concentrated Torsion: $T_C := 200 \text{ lbs}$

Position of the load: $\alpha := 0.5$

Eccentricity: $\text{Off_QD} := e_o + 0.5b_f$ $\text{Off_QD} = 1.91 \text{ in}$

Eccentricity: $\text{Off_QL} := 36 \text{ in} + 2 \text{ in} + 0.5d$ $\text{Off_QL} = 42.50 \text{ in}$

$$a := \sqrt{\frac{E_s}{G_s} \cdot \frac{C_w}{J}} \quad a = 16.7 \text{ in} \quad K := \frac{L}{a} \quad K = 8.16$$

$$h := (d - t_f) \quad h = 8.59 \text{ in}$$

Normalized Warping Constant: $W_{n0} := .5 \cdot u \cdot h$ $W_{n0} = 6.97 \text{ in}^2$

$$W_{n2} := .5 \cdot E_o \cdot h \quad W_{n2} = 3.45 \text{ in}^2$$

Warping Statical Moment: $S_{w1} := .25 \cdot u^2 \cdot h \cdot t_f$ $S_{w1} = 2.31 \text{ in}^4$

$$S_{w2} := .25 \cdot h \cdot b' \cdot t_f \cdot (b' - 2E_o) \quad S_{w2} = 1.74 \text{ in}^4$$

$$S_{w3} := |S_{w2} - .125 \cdot E_o \cdot h^2 \cdot t_w| \quad S_{w3} = 1.58 \text{ in}^4$$

Statical moment for a point in the flange directly above the web: $Q_f := .5 \cdot d \cdot t_f \cdot (b_f - t_w)$ $Q_f = 4.1 \text{ in}^3$

Statical moment at mid depth of the section: $Q_w := .5 \cdot (h \cdot b_f \cdot t_f) + .125 \cdot (h - t_f)^2 \cdot t_w$ $Q_w = 8.40 \text{ in}^3$

Total Torsional load: $T := \text{Off_QL} \cdot T_C$ $T = 0.71 \text{ k}$

At the support: $z := 0 \text{ ft}$

For Case 3, from Appendix C for $0 < z < \alpha L$:

$$\theta(z) := \frac{T \cdot L}{G_s \cdot J} \cdot \left[(1 - \alpha) \cdot \frac{z}{L} + \frac{a}{L} \cdot \left(\sinh\left(\frac{\alpha \cdot L}{a}\right) + \tanh\left(\frac{L}{a}\right) - \cosh\left(\frac{\alpha \cdot L}{a}\right) \right) \cdot \sinh\left(\frac{z}{a}\right) \right]$$

$$\theta'(z) := \frac{T \cdot L}{(G_s \cdot J)} \cdot \left[\frac{(1 - \alpha)}{L} + \frac{1}{L} \cdot \left(\sinh\left(\frac{\alpha \cdot L}{a}\right) + \tanh\left(\frac{L}{a}\right) - \cosh\left(\frac{\alpha \cdot L}{a}\right) \right) \cdot \cosh\left(\frac{z}{a}\right) \right]$$

$$\theta''(z) := \frac{T}{(G_s \cdot J)} \cdot \left(\sinh\left(\frac{\alpha \cdot L}{a}\right) + \tanh\left(\frac{L}{a}\right) - \cosh\left(\frac{\alpha \cdot L}{a}\right) \right) \cdot \frac{1}{a} \cdot \sinh\left(\frac{z}{a}\right)$$

$$\theta'''(z) := \frac{T}{(G_s \cdot J)} \cdot \left(\sinh\left(\frac{\alpha \cdot L}{a}\right) + \tanh\left(\frac{L}{a}\right) - \cosh\left(\frac{\alpha \cdot L}{a}\right) \right) \cdot \frac{1}{a^2} \cdot \cosh\left(\frac{z}{a}\right)$$

All normal stresses are zero at the pinned ended support.

$$V_{sup} := R \quad V_{sup} = 0.83 \text{ kips}$$

$$\text{Flange stress, due to beam action: } \tau_{bf} := \frac{V_{sup} \cdot Q_f}{I_x \cdot t_f} \quad \tau_{bf} = 0.14 \text{ ksi}$$

$$\text{Web stress, due to beam action: } \tau_{bw} := \frac{V_{sup} \cdot Q_w}{I_x \cdot t_w} \quad \tau_{bw} = 0.26 \text{ ksi}$$

$$\text{Flange stress, due to St. Venant torsion: } \tau_{tf} := G_s \cdot t_f \cdot \theta'(z) \quad \tau_{tf} = 4.53 \text{ ksi}$$

$$\text{Web stress, due to St. Venant torsion: } \tau_{tw} := G_s \cdot t_w \cdot \theta'(z) \quad \tau_{tw} = 4.97 \text{ ksi}$$

$$\text{Flange stress, due to warping torsion: } \tau_{w1} := -\frac{E_s \cdot S_{w1} \cdot \theta'''(z)}{t_f} \quad \tau_{w1} = 0.02 \text{ ksi}$$

$$\text{Flange stress, due to warping torsion: } \tau_{w2} := -\frac{E_s \cdot S_{w2} \cdot \theta'''(z)}{t_f} \quad \tau_{w2} = 0.02 \text{ ksi}$$

$$\text{Web stress, due to warping torsion: } \tau_{w3} := -\frac{E_s \cdot S_{w3} \cdot \theta'''(z)}{t_w} \quad \tau_{w3} = 0.01 \text{ ksi}$$

Note: The stresses are not necessarily additive, but for simplicity and conservatism, I will add all the stresses.

$$\text{Flange stress: } f_{vf} := \tau_{tf} + \tau_{w1} + \tau_{bf} \quad f_{vf} = 4.7 \text{ ksi} \quad F_v(h, t_w) = 12.0 \text{ ksi}$$

$$\text{Web stress: } f_{vw} := \tau_{tw} + \tau_{bw} + \tau_{w3} \quad f_{vw} = 5.2 \text{ ksi} \quad \text{if}(\max(f_{vf}, f_{vw}) \leq F_v(h, t_w), \text{OK}, \text{NG}) = \text{"O.K."}$$

At the midspan:

$$z := 0.5L$$

For Case 3, from Appendix C for $\alpha L < z < L$:

$$\theta(z) := \frac{T \cdot L}{G_s \cdot J} \cdot \left[(L - z) \cdot \frac{\alpha}{L} + \frac{a}{L} \cdot \left[\left(\sinh\left(\frac{\alpha \cdot L}{a}\right) + \tanh\left(\frac{L}{a}\right) \right) \cdot \sinh\left(\frac{z}{a}\right) - \sinh\left(\frac{\alpha \cdot L}{a}\right) \cdot \cosh\left(\frac{z}{a}\right) \right] \right]$$

$$\theta'(z) := \frac{T \cdot L}{G_s \cdot J} \cdot \left[\frac{-\alpha}{L} + \frac{a}{L} \cdot \left[\left(\sinh\left(\frac{\alpha \cdot L}{a}\right) + \tanh\left(\frac{L}{a}\right) \right) \cdot \frac{1}{a} \cdot \cosh\left(\frac{z}{a}\right) - \sinh\left(\frac{\alpha \cdot L}{a}\right) \cdot \frac{1}{a} \cdot \sinh\left(\frac{z}{a}\right) \right] \right]$$

$$\theta''(z) := \frac{T \cdot a}{G_s \cdot J} \cdot \left[\left(\sinh\left(\frac{\alpha \cdot L}{a}\right) + \tanh\left(\frac{L}{a}\right) \right) \cdot \frac{1}{a^2} \cdot \sinh\left(\frac{z}{a}\right) - \sinh\left(\frac{\alpha \cdot L}{a}\right) \cdot \frac{1}{a^2} \cdot \cosh\left(\frac{z}{a}\right) \right]$$

$$\theta'''(z) := \frac{T \cdot a}{G_s \cdot J} \cdot \left[\left(\sinh\left(\frac{\alpha \cdot L}{a}\right) + \tanh\left(\frac{L}{a}\right) \right) \cdot \frac{1}{a^3} \cdot \cosh\left(\frac{z}{a}\right) - \sinh\left(\frac{\alpha \cdot L}{a}\right) \cdot \frac{1}{a^3} \cdot \sinh\left(\frac{z}{a}\right) \right]$$

Design Moment: $M_{mid} := M$

$$M_{mid} = 2.28 \text{ k'}$$

Flange stress, due to bending: $\sigma_{bf} := \frac{M_{mid}}{-S_x}$

$$\sigma_{bf} = -2.03 \text{ ksi}$$

Flange stress, due to warping torsion: $\sigma_{w0} := E_s \cdot W_{n0} \cdot \theta''(z)$

$$\sigma_{w0} = -12.4 \text{ ksi}$$

Flange stress, due to warping torsion: $\sigma_{w2} := E_s \cdot W_{n2} \cdot \theta''(z)$

$$\sigma_{w2} = -6.1 \text{ ksi}$$

Flange normal stress: $f_n := |\sigma_{bf}| + \max(|\sigma_{w0}|, |\sigma_{w2}|)$

$$f_n = 14.4 \text{ ksi}$$

$$\text{if}(f_n \leq F_{a1}, \text{OK}, \text{NG}) = \text{"O.K."}$$

Design Shear: $V_{mid} := R$

$$V_{mid} = 0.83 \text{ kips}$$

Flange stress, due to beam action: $\tau_{bf} := \frac{V_{mid} \cdot Q_f}{I_x \cdot t_f}$

$$\tau_{bf} = 0.14 \text{ ksi}$$

Web stress, due to beam action: $\tau_{bw} := \frac{V_{mid} \cdot Q_w}{I_x \cdot t_w}$

$$\tau_{bw} = 0.26 \text{ ksi}$$

Flange stress, due to St. Venant torsion: $\tau_{tf} := 0 \text{ ksi}$

Flange stress, due to warping torsion: $\tau_{w1} := \frac{E_s \cdot S_{w1} \cdot \theta'''(z)}{t_f}$

$$\tau_{w1} = 0.6 \text{ ksi}$$

Flange stress, due to warping torsion: $\tau_{w2} := -\frac{E_s \cdot S_{w2} \cdot \theta'''(z)}{t_f}$

$$\tau_{w2} = -0.46 \text{ ksi}$$

Web stress, due to warping torsion: $\tau_{w3} := -\frac{E_s \cdot S_{w3} \cdot \theta'''(z)}{t_w}$

$$\tau_{w3} = -0.38 \text{ ksi}$$

Flange stress: $f_{vf} := |\tau_{tf}| + |\tau_{w1}| + |\tau_{bf}|$

$$f_{vf} = 0.74 \text{ ksi}$$

Web stress: $f_{vw} := |\tau_{tw}| + |\tau_{bw}| + |\tau_{w3}|$

$$f_{vw} = 5.6 \text{ ksi}$$

$$\text{if}(\max(f_{vf}, f_{vw}) \leq F_v(h, t_w), \text{OK}, \text{NG}) = \text{"O.K."}$$

At $z := \alpha L$

For Case 3, from Appendix C for $\alpha L < z < L$:

$$M_{\alpha} := (w_b + w_{sl} + w_{ad} + w_l) \cdot (5 \cdot \alpha \cdot L) \cdot (L - \alpha \cdot L)$$

$$M_{\alpha} = 2.3 \text{ k'}$$

$$\text{Flange stress, due to bending: } \sigma_{bf} := \frac{M_{\alpha}}{-S_x}$$

$$\sigma_{bf} = -2.03 \text{ ksi}$$

$$\text{Flange stress, due to warping torsion: } \sigma_{w0} := E_s \cdot W_{n0} \cdot \theta''(z)$$

$$\sigma_{w0} = -12.4 \text{ ksi}$$

$$\text{Flange stress, due to warping torsion: } \sigma_{w2} := E_s \cdot W_{n2} \cdot \theta''(z)$$

$$\sigma_{w2} = -6.1 \text{ ksi}$$

$$\text{Flange normal stress: } f_n := |\sigma_{bf}| + \max(|\sigma_{w0}|, |\sigma_{w2}|)$$

$$f_n = 14.4 \text{ ksi}$$

$$\text{if}(f_n \leq F_{a1}, \text{OK}, \text{NG}) = \text{"O.K."}$$

$$\text{Design Shear: } V_{\alpha} := (w_b + w_{sl} + w_{ad} + w_l) \cdot (L - \alpha \cdot L)$$

$$V_{\alpha} = 0.80 \text{ kips}$$

$$\text{Flange stress, due to beam action: } \tau_{bf} := \frac{V_{\alpha} \cdot Q_f}{I_x \cdot t_f}$$

$$\tau_{bf} = 0.13 \text{ ksi}$$

$$\text{Web stress, due to beam action: } \tau_{bw} := \frac{V_{\alpha} \cdot Q_w}{I_x \cdot t_w}$$

$$\tau_{bw} = 0.25 \text{ ksi}$$

$$\text{Flange stress, due to St. Venant torsion: } \tau_{tf} := 0 \text{ ksi}$$

$$\text{Flange stress, due to warping torsion: } \tau_{w1} := \frac{E_s \cdot S_{w1} \cdot \theta'''(z)}{t_f}$$

$$\tau_{w1} = 0.6 \text{ ksi}$$

$$\text{Flange stress, due to warping torsion: } \tau_{w2} := -\frac{E_s \cdot S_{w2} \cdot \theta'''(z)}{t_f}$$

$$\tau_{w2} = -0.46 \text{ ksi}$$

$$\text{Web stress, due to warping torsion: } \tau_{w3} := -\frac{E_s \cdot S_{w3} \cdot \theta'''(z)}{t_w}$$

$$\tau_{w3} = -0.38 \text{ ksi}$$

$$\text{Flange stress: } f_{vf} := |\tau_{tf}| + |\tau_{w1}| + |\tau_{bf}| \quad f_{vf} = 0.73 \text{ ksi}$$

$$\text{Web stress: } f_{vw} := |\tau_{tw}| + |\tau_{bw}| + |\tau_{w3}| \quad f_{vw} = 5.59 \text{ ksi}$$

$$\text{if}(\max(f_{vf}, f_{vw}) \leq F_v(h, t_w), \text{OK}, \text{NG}) = \text{"O.K."}$$