3.0 – Joists

Analysis Criteria

Bending – The joist with experience bending forces
Lateral Buckling – The floor joist will unlikely experience any lateral force causing buckling
Shear – The floor joists will experience shear forces
Torsion – The floor joist will unlikely experience any torsional forces
Deflection – The floor joists will experience bending force that will cause deflection

3.1 - Design Strengths

$$X_d = k_{mod}\left[\frac{X_k}{\gamma M}\right]$$

3.1.1 - Design Shear Strength ($f_{v,d}$)

$$f_{v,d} = k_{mod}\left[\frac{f_{v,k}}{\gamma M}\right]$$

$f_{v,k}$ = 2.5 (based on appendix A)
K_{mod} = 0.7 (based on appendix B)
γM = 1.3 (based on appendix C)

$$f_{v,d} = 0.7\left[\frac{2.5}{1.3}\right]$$

$$f_{v,d} = 1.35 \text{ N/mm}^2$$

3.1.2 - Design Bending Strength ($f_{m,d}$)

$$f_{m,d} = k_{mod}\left[\frac{f_{m,k}}{\gamma M}\right]$$

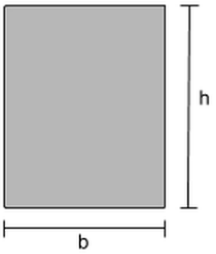
$f_{m,k}$ = 24 (based on appendix A)
K_{mod} = 0.7 (based on appendix B)
γM = 1.3 (based on appendix C)

$$f_{m,d} = 0.7\left[\frac{24}{1.3}\right]$$

$$f_{m,d} = 12.92 \text{ N/mm}^2$$

3.2 - Span 1

Width: 50mm
Height: 225mm
Span: 4.2m
Timber: C24 Grade
Modulus of Elasticity E = 11,000 MPa



Result	Notation	Value	Unit
Section Area	A	0.01125	m^2
Moment of Inertia about the x-axis	I_{xx}	0.000047461	m^4
Moment of Inertia about the y-axis	I_{yy}	0.0000023438	m^4
Distance from bottom of beam section to the centroid	C_y	0.1125	m
Distance from furthest left point of beam section to the centroid	C_x	0.025	m
Statical Moment of Area about the x-axis	Q_x	0.00031641	m^3
Statical Moment of Area about the y-axis	Q_y	0.000070313	m^3
Section Modulus about the x-axis	Z_x	0.00042188	m^3
Section Modulus about the y-axis	Z_y	0.00009375	m^3

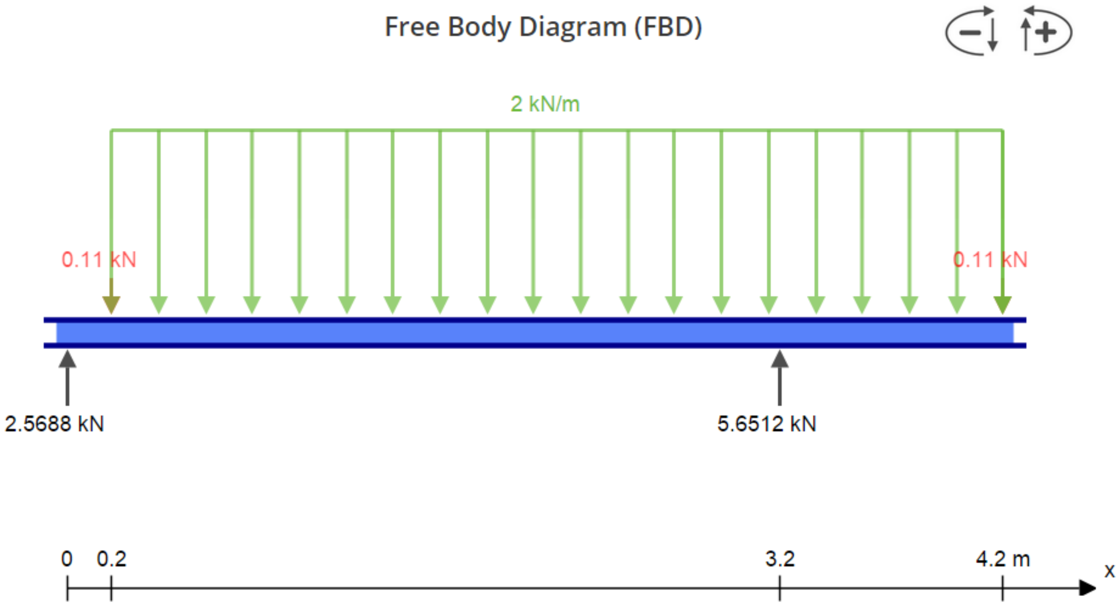
3.2.2 – Loads & Free Body Diagram

Dead Loads
UDL = 0.07 KN/m*
Point = 0.11 KN**

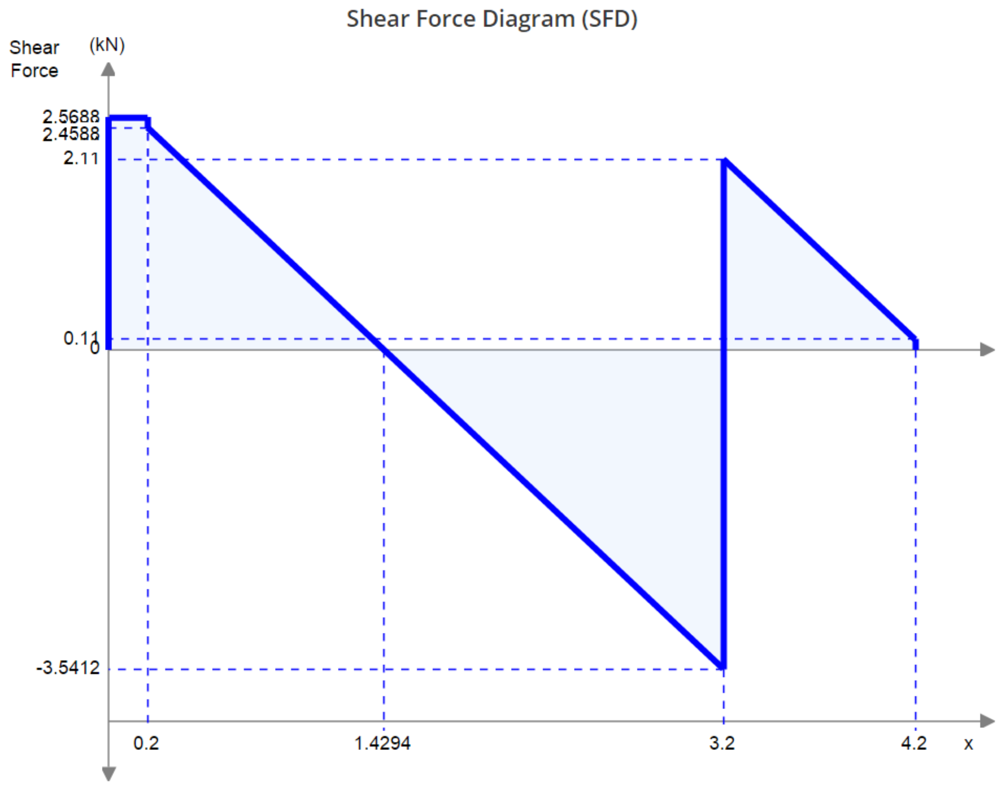
*Based on 400mm (0.4m) joist centres (includes Flooring Dead Loads)
Timber Joists
C24 Timber = 420 Kg/m^3
Joists A = 0.225m x 0.05m x 4.2m = 0.04725 m^3 = 19.85 Kg / Joist
UDL (Dead Load) = 4.73 Kg/m = 0.0473 KN/m

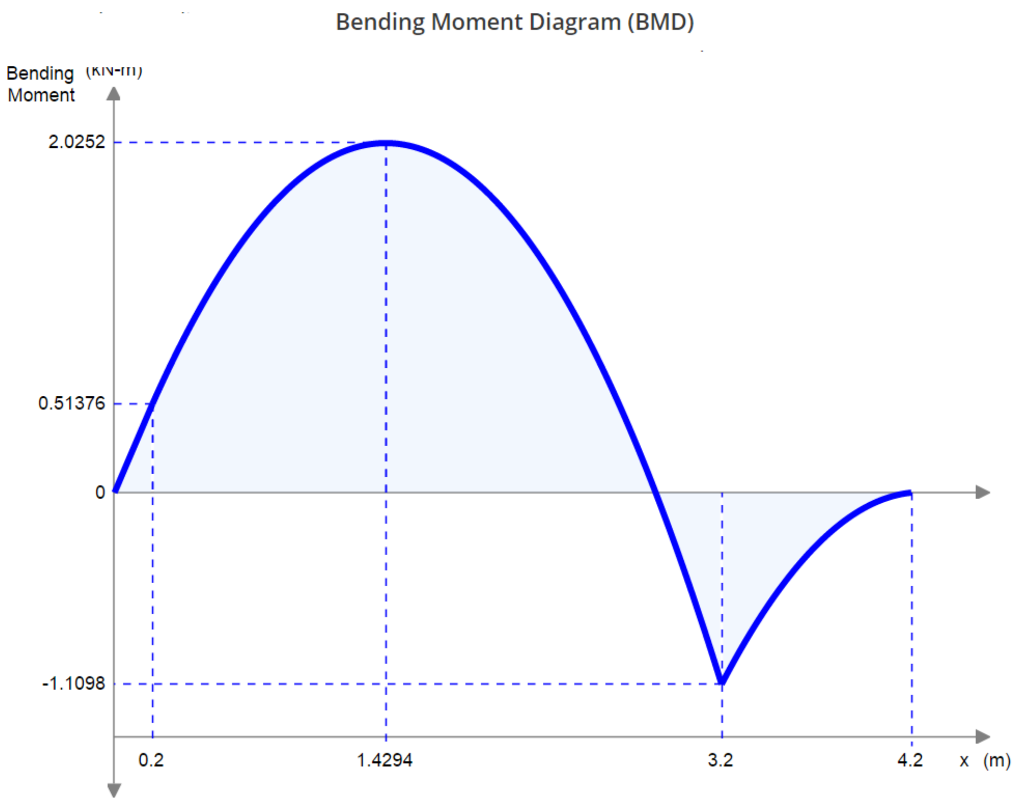
**Wall Dead weight, see force diagram for locations

Live Loads
UDL = 1.92 KN/m*
*Based on 400mm (0.4m) joist centres



3.2.3 - Shear Force and Bending Moment Diagrams





3.2.4 - Bending Stress

$$\sigma_{m,y,d} = \frac{M_{y,d}}{W_y} = \frac{M_{y,d}}{\frac{b.h^2}{6}}$$

Include Factor of Safety

$$\sigma_{m,y,d} = \frac{M_{y,d}}{\frac{b.h^2}{6}} \ . \ \mathrm{FoS}$$

$$\sigma_{m,y,d} = \frac{2.0252}{\frac{0.05 \ x \ 0.225^2}{6}} \times 1.4$$

$$\sigma_{m,y,d} = 6721 \ KN/m^2 = 6.721 \ N/mm^2$$

Design Bending Strength $f_{m,d}$ = 12.92 N/mm^2
Applied Bending Stress $\sigma_{m,d}$ = 6.72 N/mm^2
Usage Factor = 0.52

PASS

3.2.5 - Shear Stress

For a rectangular section……

$$\tau = \frac{3V}{2A}$$

Include Factor of Safety

$$\tau = \frac{3V}{2A} \ . FoS$$

$$\tau = \frac{3 \ x \ 3.5412}{2 \ x \ 0.01125} \ x \ 1.4$$

$$\tau = 661 \ KN/m^2 = 0.661 \ N/mm^2$$

Design Shear Strength $f_{v,d}$ = 1.35 N/mm^2
Applied Shear Stress $\tau_{v,d}$ = 0.661 N/mm^2
Usage Factor = 0.49

PASS

3.2.6 - Deflection

Deflection is calculated by a series of standard beam formulas for each loading type, and then combined by superposition.

The Modulus of Elasticity (E) is modified per Eurocode 5 to consider load duration and moisture influences on deformations having time dependent properties (I.E, Creep)

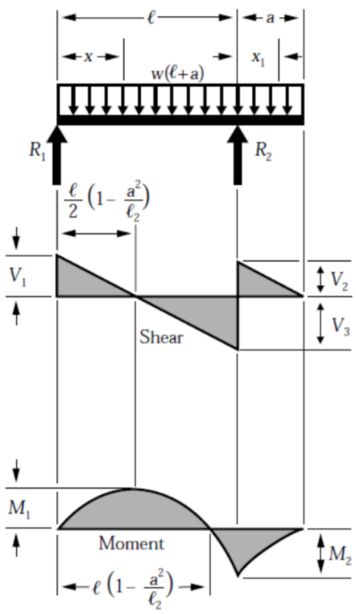
Therefore, Serviceability Limit State Design (SLS),

$$E_{mean,fin} = \frac{E_{mean}}{(1 + K_{def})}$$

Where K_{def} = 0.6 based on the table in appendix D

$$E_{mean,fin} = \frac{11000}{(1 + 0.6)} = 6875\ MPa$$

Deflection from UDL



$R_1 = V_1 \dots = \frac{w}{2\ell}(\ell^2 - a^2)$

$R_2 = V_2 + V_3 \dots = \frac{w}{2\ell}(\ell + a)^2$

$V_2 \dots = wa$

$V_3 \dots = \frac{w}{2\ell}(\ell^2 + a^2)$

V_x (between supports) $\dots = R_1 - wx$

V_{x_1} (for overhang) $\dots = w(a - x_1)$

$M_1 \left(\text{at } x = \frac{\ell}{2}\left[1 - \frac{a^2}{\ell^2}\right]\right) \dots = \frac{w}{8\ell^2}(\ell + a)^2(\ell - a)^2$

M_2 (at R_2) $\dots = \frac{wa^2}{2}$

M_x (between supports) $\dots = \frac{wx}{2\ell}(\ell^2 - a^2 - x\ell)$

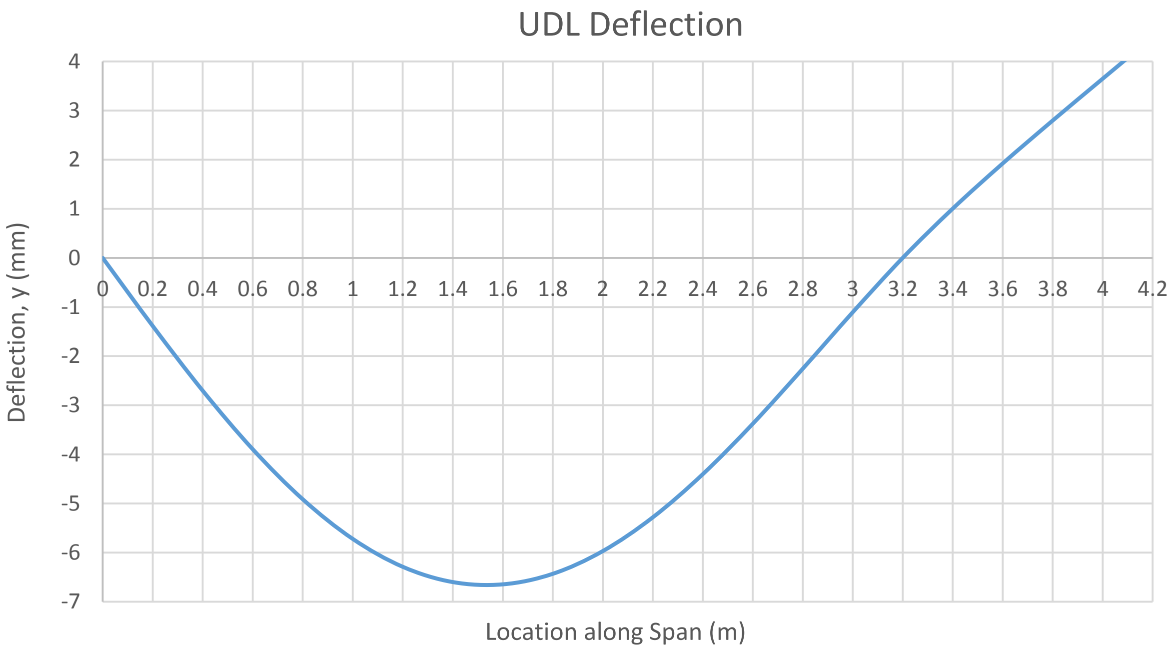
M_{x_1} (for overhang) $\dots = \frac{w}{2}(a - x_1)^2$

Δ_x (between supports) $\dots = \frac{wx}{24EI\ell}(\ell^4 - 2\ell^2x^2 + \ell x^3 - 2a^2\ell^2 + 2a^2x^2)$

Δ_{x_1} (for overhang) $\dots = \frac{wx_1}{24EI}(4a^2\ell - \ell^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$

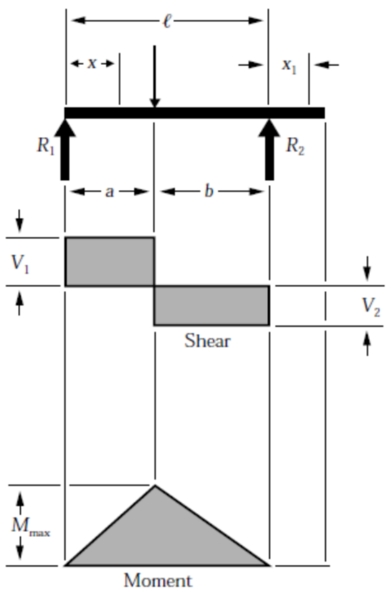
Take from standard beam formulas

Note: For simplification, the 0.2m section where the UDL does not act on the beam design is considered to have a negligible impact on the deflection curve, and therefore is omitted for these calculations.



Max y_{UDL} = -6.65mm

Deflection from Point Load 1



$$R_1 = V_1 \text{ (max when } a < b) \; \ldots\ldots = \frac{Pb}{\ell}$$

$$R_2 = V_2 \text{ (max when } a > b) \; \ldots\ldots = \frac{Pa}{\ell}$$

$$M_{max} \text{ (at point of load)} \; \ldots\ldots = \frac{Pab}{\ell}$$

$$M_x \text{ (when } x < a) \; \ldots\ldots = \frac{Pbx}{\ell}$$

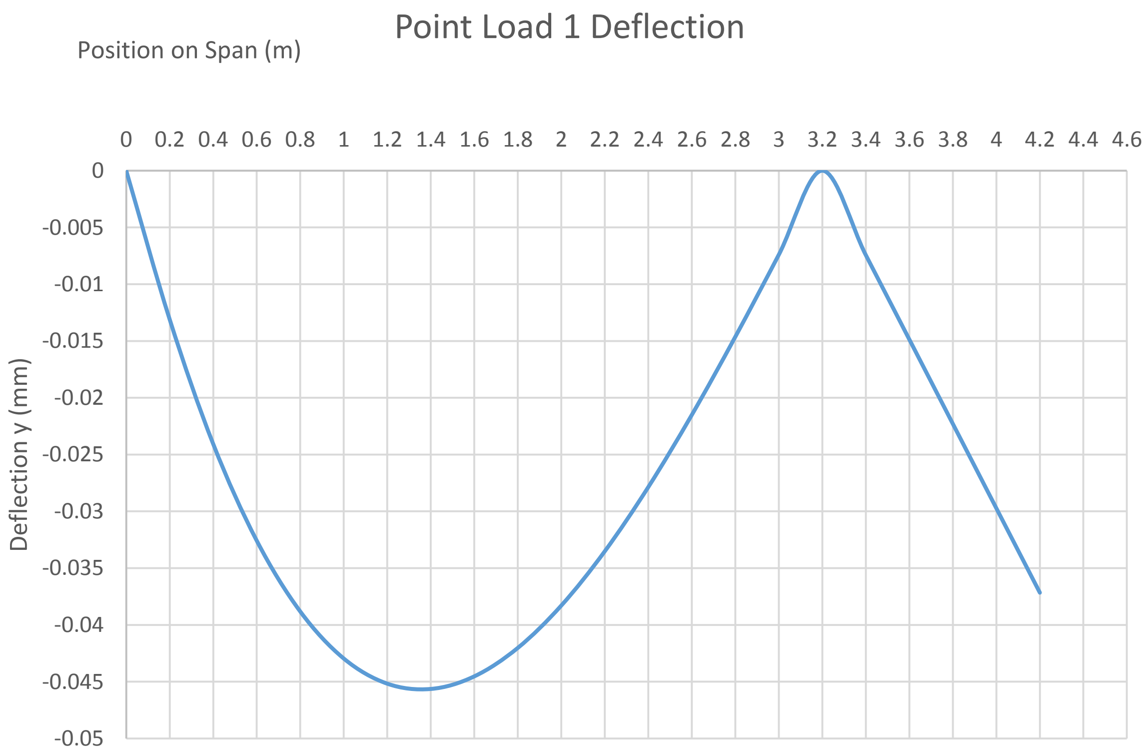
$$\Delta_{max} \left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b \right) \; . \; = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell}$$

$$\Delta_a \text{ (at point of load)} \; \ldots\ldots = \frac{Pa^2b^2}{3EI\ell}$$

$$\Delta_x \text{ (when } x < a) \; \ldots\ldots = \frac{Pbx}{6EI\ell}(\ell^2 - b^2 - x^2)$$

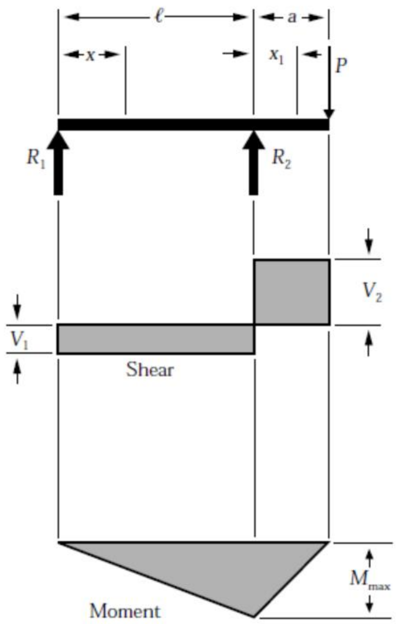
$$\Delta_x \text{ (when } x > a) \; \ldots\ldots = \frac{Pa(\ell - x)}{6EI\ell}(2\ell x - x^2 - a^2)$$

$$\Delta_{x_1} \; \ldots\ldots = \frac{Pabx_1}{6EI\ell}(\ell + a)$$



<u>Max y_{p2} = -0.04564 mm</u>

Deflection from Point Load 2



$$R_1 = V_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad = \frac{Pa}{\ell}$$

$$R_2 = V_1 + V_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad = \frac{P}{\ell}(\ell + a)$$

$$V_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad = P$$

$$M_{max} \text{ (at } R_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad = Pa$$

$$M_x \text{ (between supports)} \quad . \quad . \quad . \quad . \quad . \quad . \quad = \frac{Pax}{\ell}$$

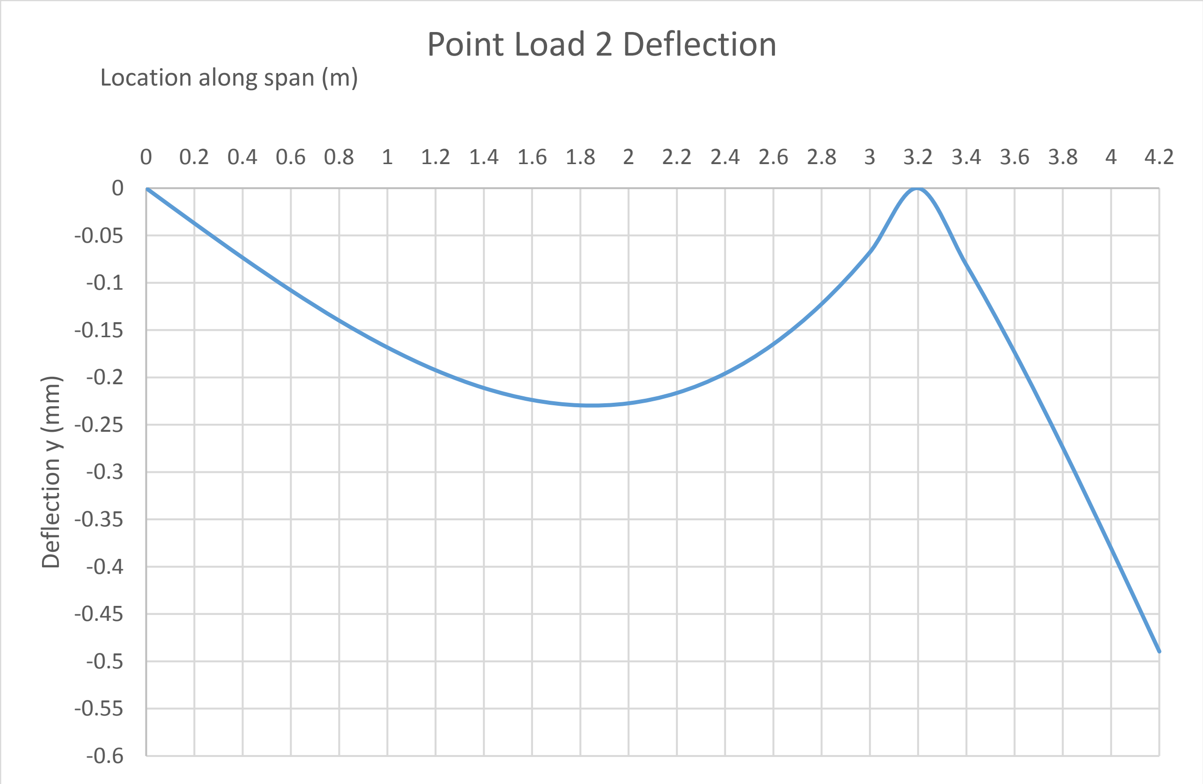
$$M_{x_1} \text{ (for overhang)} \quad . \quad . \quad . \quad . \quad . \quad . \quad = P(a - x_1)$$

$$\Delta_{max} \left(\text{between supports at } x = \frac{\ell}{\sqrt{3}} \right) = \frac{Pa\ell^2}{9\sqrt{3}EI} = .06415 \frac{Pa\ell^2}{EI}$$

$$\Delta_{max} \text{ (for overhang at } x_1 = a) \quad . \quad . \quad . \quad . \quad = \frac{Pa^2}{3EI}(\ell + a)$$

$$\Delta_x \text{ (between supports)} \quad . \quad . \quad . \quad . \quad . \quad . \quad = \frac{Pax}{6EI\ell}(\ell^2 - x^2)$$

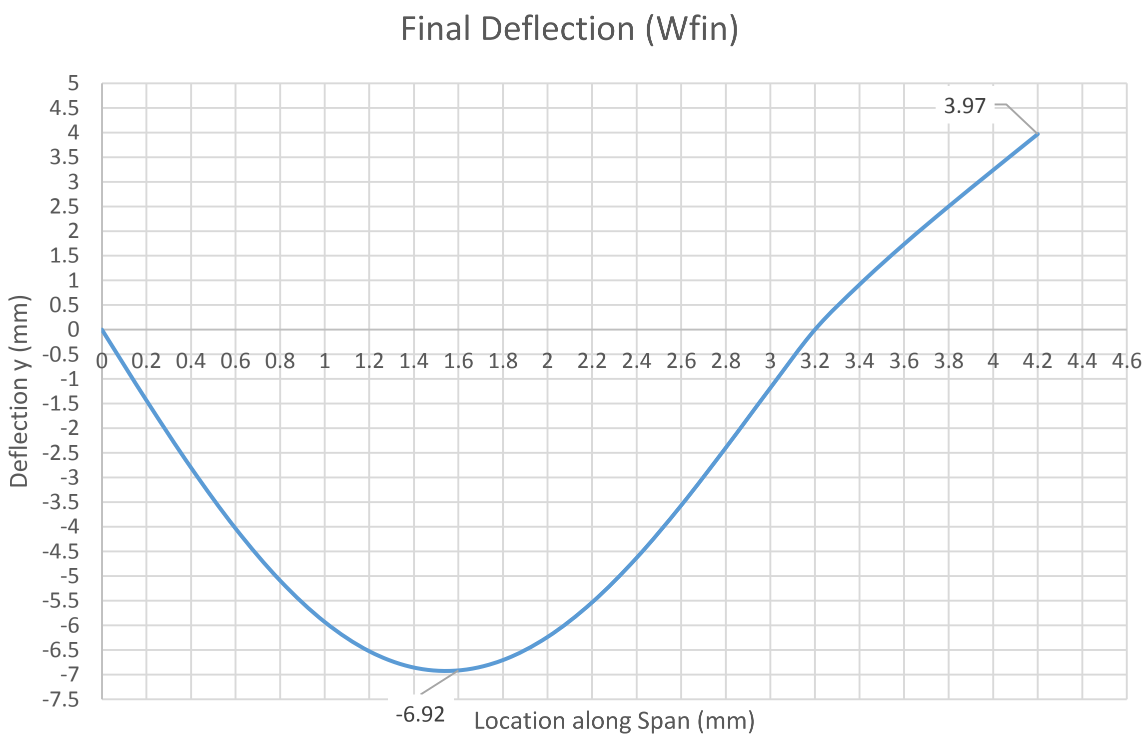
$$\Delta_{x_1} \text{ (for overhang)} \quad . \quad . \quad . \quad . \quad . \quad . \quad = \frac{Px_1}{6EI}(2a\ell + 3ax_1 - x_1^2)$$



Max y_{p2} = -0.22951 mm

Final Deflection of Beam

y_{max} = 6.92mm



Limiting Value for Deflection (based on Eurocode 5)

$W_{net, fin}$ = l / 350,   where l = span = 4200mm

$W_{net, fin}$ = 12mm
y_{max} = 6.92mm
Usage Factor = 0.58
PASS

Appendix A

Table of Timber Structural strength class according to BS EN 338 table 1

Important Note; This table is equivalent to the table found in BS 5628 Timber Design however the strength values in this table are characteristic strengths (fifth percentile values derived directly from laboratory tests of five minutes) whereas the equivalent values in the BS 5628 table are grade stresses which have been reduced for long-term duration and already include a safety factor.

Species type	Poplar and conifer species							
Class	C14	C16	C18	C22	C24	C27	C30	C35
						Strength properties (N/mm^2)		
Bending $f_{m,k}$	14	16	18	22	24	27	30	35
Tension par'l. $f_{t,0,k}$	8	10	11	13	14	16	18	21
Tension perp. $f_{t,90,k}$	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4
Compression par'l. $f_{c,0,k}$	16	17	18	20	21	22	23	25
Compression perp. $fc_{,90,k}$	4.3	4.6	4.8	5.1	5.3	5.6	5.7	6.0
Shear $f_{v,k}$	1.7	1.8	2.0	2.4	2.5	2.8	3.0	3.4
						Stiffness properties (kN/mm^2)		
Mean modulus of elasticity parallel $E_{0,mean}$	7	8	9	10	11	12	12	13
5% modulus of elasticity parallel $E_{0,05}$	4.7	5.4	6.0	6.7	7.4	8.0	8.0	8.7
Mean modulus of elasticity Perpendicular $E_{90,mean}$	0.23	0.27	0.30	0.33	0.37	0.40	0.40	0.43
Mean shear modulus G_{mean}	0.44	0.50	0.56	0.63	0.69	0.75	0.75	0.81
						Density (kg/m^3)		
Density ρ_k	290	310	320	340	350	370	380	400
Average density ρ_{mean}	350	370	380	410	420	450	460	480

Appendix B

Table for partial factor γ_M

Design situation	Partial factor γ_M
Fundamental combinations ...	
Solid timber	1.3
Glued laminated timber	1.25
Laminated veneer lumber (LVL),plywood, OSB	1.2
Particle board	1.3
Fibreboard hard	1.3
Fibreboard medium	1.3
Fibreboard, MDF	1.3
Fibreboard , soft	1.3
Connections	1.3
Punched metal plate fasteners	1.25
Accidental combinations	1.0
Serviceability limit states	1.0

Appendix C

table for k_{mod}
This is applicable to solid timber , Glued laminated timber, LVL , and Plywood

Load duration class	Service class		
	1	2	3
Permanent (> 10 years)	0.60	0.60	0.50
Long-term (6 months - 10 years)	0.70	0.70	0.55
Medium-term (1 week - 6 months)	0.80	0.80	0.65
Short-term (< 1 week)	0.90	0.90	0.70
Instantaneous	1.10	1.10	0.90

Appendix D

table for k_{def}

Load duration class	Service class		
	1	2	3
Solid Timber. EN 14081-1	0.60	0.80	2,00
Glued laminated timber./ EN 14080	0.60	0.80	2,00
LVL. EN 14374, EN 14279	0.60	0.80	2,00
Plywood type EN636-1	0.80	-	-
OSB. EN 312 OSB/2	2,25	-	-
Particleboard EN312 Type P4	2,25	-	-
Fibreboard hard. EN 622-2 HB0LA	2,25	-	-
Fibreboard med. EN 622-3 MBH-LA1, MBH.LA2	3,00	-	-
Fibreboard MDF EN 622-5 MDF-LA	2,25	-	-

Table 2.10. *Recommended values of ψ factors for buildings [Table (A1.1)-EN1990]*

Imposed loads in buildings, category (see EN 1991-1-1)	ψ_0	ψ_1	ψ_2
Category A: domestic, residential areas	0,7	0,5	0,3
Category B: office areas	0,7	0,5	0,3
Category C: congregation areas	0,7	0,7	0,6
Category D: shopping areas	0,7	0,7	0,6
Category E: storage areas	1,0	0,9	0,8
Category F: traffic area, vehicle weight ≤ 30 kN	0,7	0,7	0,6
Category G: traffic area, 30 kN < vehicle weight ≤ 160 kN	0,7	0,5	0,3
Category H: roofs	0	0	0
Snow loads on buildings (see EN 1991-1-3)* in Finland, Iceland, Norway, Sweden and other CEN Member, for sites located at altitude H > 1000 m a.s.l.	0,7	0,5	0,2
Other CEN Member States, for sites located at altitude H < 1000 m a.s.l.	0,5	0,2	0
Wind load on buildings (see EN 1991-1-4)	0,6	0,2	0
Temperature (non-fire) in buildings (see EN 1991-1-5)	0,6	0,5	0

NOTE The ψ values may be set by the National annex.
* For countries not mentioned below, see relevant local conditions.