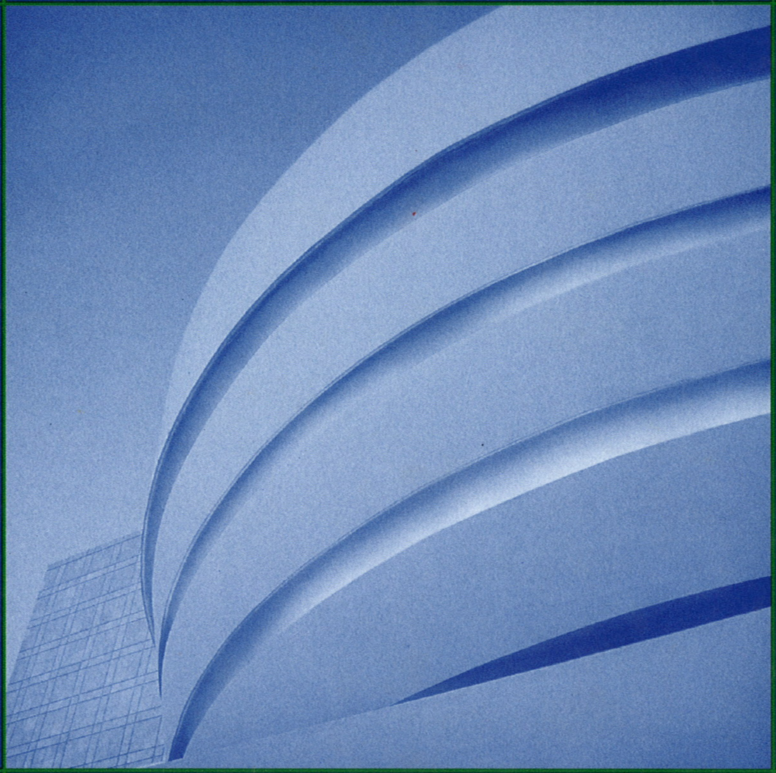
REINFORCED CONCRETE DESIGN

SIXTH EDITION



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3.7 Minimum Reinforcement

When steel reinforcement in a flexural member is only a small amount because the factored moment M_u is low, the beam may perform uncracked at service load. The computation of nominal moment strength M_n, in accordance with Sec. 3.4, assumes the tension concrete to be cracked. Thus, the computed nominal strength M_n for a section having a small amount of reinforcement could be less than the strength M_n (called M_{cr}) of the same section of plain concrete (i.e., no reinforcement). Since a ductile failure mode is desired, the lowest amount of steel permitted should be the amount that would equal the strength of an unreinforced section. The desired relationship then becomes

$$\begin{bmatrix}\text{strength of reinforced} \\ \text{concrete beam, } \phi M_n\end{bmatrix} \geq \begin{bmatrix}\text{strength of plain} \\ \text{concrete beam, } M_{cr}\end{bmatrix} \tag{3.7.1}$$

The strength of a plain concrete beam, known as the cracking moment M_{cr}, is achieved when the extreme fiber in tension reaches the modulus of rupture f_r (see Sec. 1.8). For normal-weight concrete, ACI-9.5.2.3 uses

$$f_r = 7.5\sqrt{f'_c} \tag{3.7.2)*}$$

Assuming plain (nonreinforced) concrete as an elastic homogeneous material, the flexure formula gives M_{cr} as

$$M_{cr} = f_r \frac{I_g}{y_t} \tag{3.7.3}$$

where I_g = moment of inertia of the gross concrete cross-section; $Cb_w h^3/12$
y_t = distance from the neutral axis to the extreme fiber in tension; $h/2$
C = coefficient to account for flanges of T-sections; $C = 1.0$ for rectangular section
b_w = width of section; width of web for T-section
h = overall depth of the section

Expanding Eq. (3.7.3), it becomes

$$M_{cr} = 7.5\sqrt{f'_c}\,\frac{Cb_w h^3/12}{h/2} = \frac{7.5\sqrt{f'_c}\,Cb_w h^2}{6} \tag{3.7.4}$$

For a reinforced concrete section, using Eq. (3.3.4),

$$\phi M_n = \phi A_s f_y (d - a/2) \qquad [3.3.4]$$

Substituting Eqs. (3.7.4) and (3.3.4) into Eq. (3.7.1) gives

$$\phi A_s f_y \left(d - \frac{a}{2}\right) \geq M_{cr} = \frac{7.5\sqrt{f'_c}\,Cb_w h^2}{6} \tag{3.7.5}$$

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*For SI, with f_r and f'_c in MPa, $f_r = 0.62\sqrt{f'_c}$ (3.7.2)

Estimating $a/2$ as $0.05d$, and using $\phi = 0.9$ for flexure, Eq. (3.7.5) gives

$$A_{s,\min} = \left[\frac{7.5\sqrt{f'_c}}{f_y}\right]\left[\frac{h}{d}\right]^2\left[\frac{C}{5.13}\right]b_w d \tag{3.7.6}$$

Rectangular Sections. For rectangular beams, slabs, and footings, $C = 1.0$ and h/d varies from about 1.05 to 1.2. For such sections, Eq. (3.7.6) becomes

$$A_{s,\min} = \frac{1.6\sqrt{f'_c}}{f_y}b_w d \quad \text{to} \quad A_{s,\min} = \frac{2.1\sqrt{f'_c}}{f_y}b_w d \tag{3.7.7}$$

T-sections Having Slab in Compression. For this case, C will vary from about 1.3 to 1.6 for a practical range of variables in flange thickness to overall depth and flange width to web width. Taking $C = 1.5$ along with h/d from 1.05 to 1.2, Eq. (3.7.6) becomes

$$A_{s,\min} = \frac{2.4\sqrt{f'_c}}{f_y}b_w d \quad \text{to} \quad A_{s,\min} = \frac{3.2\sqrt{f'_c}}{f_y}b_w d \tag{3.7.8}$$

New in 1995, ACI-10.5.1 gives as Formula (10-3) for the minimum reinforcement "At every section of a flexural member, where tensile reinforcement is required by analysis . . ."

$$A_{s,\min} = \frac{3\sqrt{f'_c}}{f_y}b_w d \tag{3.7.9}^*$$

but not less than $200b_w d/f_y$. This latter limit was a last minute addition to these Code changes to satisfy negative voters. In the authors' view there is no scientific basis for keeping the 1989 ACI Code limit as a lower bound.

T-sections Having Slab in Tension. For this case, C will vary from about 3.0 to 4.0 for a practical range of variables flange thickness to overall depth and flange width to web width. Taking $C = 3.5$ along with h/d from 1.05 to 1.2, Eq. (3.7.6) becomes

$$A_{s,\min} = \frac{5.6\sqrt{f'_c}}{f_y}b_w d \quad \text{to} \quad A_{s,\min} = \frac{7.4\sqrt{f'_c}}{f_y}b_w d \tag{3.7.10}$$

New in 1995, ACI-10.5.2 gives as Formula (10-4) for the minimum reinforcement "For a statically determinate T-section with flange in tension, . . ."

$$A_{s,\min} = \frac{6\sqrt{f'_c}}{f_y}b_w d \tag{3.7.11}^\dagger$$

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*For SI, with f_r and f'_r in MPa, and b_w and d in mm,

$$A_{s,\min} = \frac{\sqrt{f'_c}}{4f_y}b_w d \geq \frac{1.4b_w d}{f_y} \tag{3.7.9}$$

†For SI, with f_r and f'_c in MPa,

$$A_{s,\min} = \frac{\sqrt{f'_c}}{2f_y}b_w d \tag{3.7.11}$$

but *not more than* ACI Formula (10-3), Eq. (3.7.9), with b_w set equal to the width of the flange. This modifier of Eq. (3.7.11) was an addition to address concerns of Code committee members that Eq. (3.7.11) would require too much minimum reinforcement.

Note that the 1995 ACI Code refers in ACI-10.5.2 to a "statically determinate" T-section having the "flange in tension," which means a cantilever T-section beam. Prior codes did not have such a reference; instead required minimum reinforcement where "positive reinforcement is required by analysis." "Positive" reinforcement can be interpreted as "positive moment" reinforcement or as "tension" reinforcement. Obviously, there is not agreement on when minimum reinforcement is required, particularly on continuous beams. The authors believe that minimum reinforcement should be used in both positive and negative moment regions of continuous beams as well, because sudden cracking in flexure should be avoided.

Escape Clause. For situations where the reinforcement required for strength is far below the minimum required by ACI Formulas (10-3) or (10-4), ACI-10.5.3 permits the use of a lesser minimum, as long as the amount is "at least one-third greater than that required by analysis."

Structural Slabs and Footings of Uniform Thickness. Note that the derived requirement for rectangular sections, Eqs. (3.7.7), is roughly two-thirds of Eqs. (3.7.8) and is only one-half to two-thirds of ACI Formula (10-3), Eq. (3.7.9). There could have been another category for rectangular sections. Structural slabs and footings of uniform thickness are the most common *actual* rectangular sections; thus the special category. Converting Eqs. (3.7.7) for rectangular sections to be in terms of overall thickness h by substituting $d = h/1.05$ and $d = h/1.2$, respectively, gives

$$A_{s,\min} = \frac{1.5\sqrt{f'_c}}{f_y} b_w h \quad \text{to} \quad A_{s,\min} = \frac{1.8\sqrt{f'_c}}{f_y} b_w h \tag{3.7.12}$$

Equation (3.7.12) converted to reinforcement ratio $A_s/b_w h$ gives the following range of values for the ratio of reinforcement area to gross concrete area:

	f'_c	3000	4000	5000	6000
f_y	40,000	0.0020–0.0024	0.0024–0.0029	0.0026–0.0032	0.0029–0.0035
	50,000	0.0017–0.0019	0.0019–0.0023	0.0021–0.0025	0.0023–0.0028
	60,000	0.0014–0.0017	0.0016–0.0019	0.0018–0.0021	0.0019–0.0023

ACI-10.5.4 requires $A_s/b_w h$ to be not less than 0.0020 for Grades 40 and 50 deformed bars, and not less than 0.0018 for Grade 60. These amounts agree with the lower end of the ranges indicated by Eqs. (3.7.12) for rectangular sections. Opinions differ on how much minimum reinforcement is needed for slabs and footings; generally designers believe less is needed than for important beams. ACI Code Committee 318 settled on the lower minimum for these rectangular sections to be the amount specified in ACI-7.12 for temperature and shrinkage reinforcement. The amount prescribed by ACI-10.5.4 is *not* temperature and shrinkage reinforcement, but rather the amount that generally agrees with the above table and Eqs. (3.7.12).

3.8 Design of Rectangular Sections in Bending Having Tension Reinforcement Only Under ACI-10.3 and 10.5

In the design of rectangular sections in bending with tension reinforcement only, the problem is to determine b, d, and A_s from the required value of $M_n = M_u/\phi$, and the given material properties f_c' and f_y.

The two conditions of equilibrium are

$$C = T \tag{3.8.1a}$$

and

$$M_n = (C \text{ or } T)\left(d - \frac{a}{2}\right) \tag{3.8.1b}$$

Since there are three unknowns but only two conditions, there are many possible solutions. If the reinforcement ratio ρ is preset, then, from Eq. (3.8.1a),

$$0.85 f_c' ba = \rho bd f_y$$

$$a = \rho\left(\frac{f_y}{0.85 f_c'}\right)d \tag{3.8.2}$$

Substituting Eq. (3.8.2) into Eq. (3.8.1b),

$$M_n = \rho bd f_y\left[d - \frac{\rho}{2}\left(\frac{f_y}{0.85\, f_c'}\right)d\right] \tag{3.8.3}$$

A strength *coefficient of resistance* R_n is obtained by dividing Eq. (3.8.3) by bd^2 and letting

$$m = \frac{f_y}{0.85 f_c'} \tag{3.8.4a}$$

Thus

$$R_n = \frac{M_n}{bd^2} = \rho f_y\left(1 - \tfrac{1}{2}\rho m\right) \tag{3.8.4b}$$

The relationship between ρ and R_n for various values of f_c' and f_y is shown in Fig. 3.8.1.

In some situations the values of b and d may be preset, which is equivalent to having R_n preset; then ρ may be determined by solving the quadratic equation (3.8.4). Thus

$$R_n = \rho f_y\left(1 - \tfrac{1}{2}\rho m\right)$$

from which

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) \tag{3.8.5}$$