

Modeling of Material Damping Properties in ANSYS

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Abstract

A comprehensive review of vibration damping in vibration and acoustics analysis is presented. The treatment of damping material is an important measure for vibration and acoustics control in engineering. The simulation-based results on vibration and acoustics analysis are very sensitive to the description and input methods of damping properties. In this paper, the consideration of vibration damping using software ANSYS for harmonic and modal analysis is addressed. Several key points are summarized.

Introduction

When an unacceptable vibration and acoustics problem needs to be controlled, it is firstly desirable and often necessary to understand its whole nature, such as its originating source, the nature and direction of the vibration and acoustics at the problem location, transmission path and frequency content. Then, it must be decided whether the problem would be best solved by passive or active control methods. The passive control involves modification of the stiffness, mass and damping of the vibration system to make the system less responsive to its vibratory environment. This paper is concerned only with the damping modification in passive control methods. If the undesirable vibration and acoustics is dominated by one or more resonance of the structure, it can be often adequately controlled by increasing the damping of the system. Most non-resonant vibration and acoustics problems cannot be solved by the damping treatment. If an added damping system is to be effective, the increased damping must be significantly larger than the initial damping. The most commonly used method of increasing the damping is to include highly damped polymeric material at strategic locations onto the structure. The structure and polymer must interact with one another in such a way as to cause the polymer to dissipate as much energy as possible. In practice, there are two kinds of damping treatments for vibration and acoustics control.

The first is called extensional damping treatment. This treatment is also referred to as the unconstrained- or free-layer damping treatment. The treatment is coated on one or both sides of a structure, so that whenever the structure is subjected to flex, the damping material will be subjected to tension-compression deformation. The second one is named as shear type of damping treatment. For a given weight, the shear type of damping treatment is more efficient than the extensional damping treatment. However, this efficiency is balanced by greater complication in analysis and application. The treatment is similar to the unconstrained-layer type, except the viscoelastic material is constrained by another layer. Therefore, whenever the structure is subjected to flex, the extra layer will constrain the viscoelastic material and force it to deform in shear. The maximum shear deformation in the middle layer is a function of the modulus and the thickness of the constraining layer, the thickness and the damping material and the wavelength of vibration in addition to the properties of the damping material.

The actual description of the damping force associated with the dissipation of energy is difficult. It may be a function of the displacement, velocity, stress or other factors. Most of the mechanisms which dissipate energy with a vibrating system are non-linear and conform neither to the linear viscous nor to the linear hysteretic damping ^[1]. However, ideal damping models can be conceived which will often permit a satisfactory approximation.

In this paper, the categories of common damping materials in engineering are reviewed. After that, the model for description of structural damping is introduced. Thirdly, it is elaborated how ANSYS considers the damping properties for engineering purpose. Several case studies are carried out to explain the difference of various consideration methods of material damping effects. Finally, some key points are drawn for correct application of damping effects for harmonic and modal analysis in ANAYS.

Categories of Damping Materials

There are several types of damping. Viscous damping is the form of damping that we are familiar with. It is caused by energy loss that results from fluid flow. An example would be the damping used in vehicle's shock absorbers. Frictional damping occurs when two objects rub against each other. That is why our hands get warm when we rub them together. Most of the damping materials in the market provided by various manufacturers belong to hysteretic damping.

Viscous Damping [2,3]

When mechanical systems vibrate in a fluid medium such as air, gas, water and oil, the resistance offered by the fluid to the moving body causes energy to be dissipated. The amount of dissipated energy depends on many factors, such as the size and shape of the vibrating body, the viscosity of the fluid, the frequency of vibration, and the velocity of the vibrating body. In viscous damping, the damping force is proportional to the velocity of the vibrating body.

Viscous damping force can be expressed by the equation

$$F = -c\dot{x} \quad (1)$$

where c is a constant of proportionality and \dot{x} the velocity of the mass shown in Figure 1.

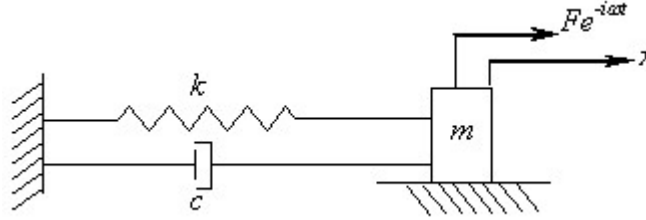


Figure 1 - A forced damped vibration of single DOF
[$m=0.5$ (kg), $k=200$ (N/m), $c=6$ (N•s/m), $F=10$ (N)]

When the single spring mass system undergoes free vibration, the equation of motion becomes

$$m\ddot{x} + c\dot{x} + kx = 0 \quad (2)$$

Assuming a solution of the form $x = e^{st}$, we have the eigen or the characteristic equation of the system as

$$ms^2 + cs + k = 0 \quad (3)$$

The solution of equation 3 is

$$x = e^{-\frac{c}{2m}t} \left(Ae^{\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}t} + Be^{-\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}t} \right) \quad (4)$$

where A and B are arbitrary constants depending on how the motion is started.

It is observed that the behavior of the damped system depends on the numerical value of the radical in the exponential of equation 4. As a reference quality, a critical damping c_c is defined which reduces this radical to zero

$$\left(\frac{c_c}{2m} \right)^2 - \frac{k}{m} = 0 \quad \text{or} \quad c_c = 2\sqrt{km} = 2m\omega_n \quad (5, 6)$$

where ω_n is the natural circular frequency of the system. $\omega_n = \sqrt{k/m}$

An important parameter to describe the properties of the damping is damping ratio ζ , which is a non-dimensional ratio as

$$\zeta = \frac{c}{c_c} = \frac{c}{2m\omega_n} \quad (7)$$

Based on the value of damping ratio, the motion of the mass in Figure 1 can be divided into the following three cases: (1) Oscillatory motion when $\zeta < 1.0$; (2) Nonoscillatory motion when $\zeta > 1.0$ and (3) Critical damped motion when $\zeta = 1.0$. In last case, the general solution of the system is $x = (A + Bt)e^{-\omega_n t}$.

Viscous damping can be used whatever the form of the excitation. The most common form of viscous damping is the Rayleigh-type damping given by

$$c = \alpha m + \beta k \quad (8)$$

Coulomb or Frictional Damping

Coulomb damping results from the sliding of two dry surfaces. The damping force is equal to the product of the normal force and the coefficient of friction μ and is assumed to be independent of the velocity, once the motion is initiated.

Because the sign of the damping force is always opposite to that of the velocity, the differential equation of motion for each sign is valid only for half-cycle intervals.

$$m\ddot{x} + kx = -\mu N \quad (9)$$

This is a second order nonhomogeneous differential equation. The solution can be expressed as

$$x(t) = A \cos \sqrt{\frac{k}{m}}t + B \sin \sqrt{\frac{k}{m}}t - \frac{\mu N}{k} \quad (10)$$

Hysteretic or Structural Damping

In general, the damping materials are polymers (synthetic rubbers) which have been suitably formulated to yield high damping capacities in the frequency and temperature ranges of interest. When the materials are deformed, energy is absorbed and dissipated by the material itself. The effect is due to friction between the internal planes, which slip or slide as the deformations take place. When a structure having material damping is subjected to vibration, the stress-strain diagram shows a hysteresis loop. Therefore, the structural damping is also called hysteretic damping. The area of this loop denotes the energy lost per unit volume of the body per cycle due to the damping.

To explain the hysteretic damping, we first review the relationship between the response x and excitation force for viscous damping. For a harmonic motion, $x = Xe^{-i\omega t}$, the relationship between them behaves as

$$F(t) = (-i c \omega + k)x \quad (11)$$

It can be shown in Figure 2. Equation 12 gives the energy dissipated in one vibration cycle which is the area of the loop above

$$\Delta W = \oint \text{Im}(F) d \text{Im}(x) = \int_0^{2\pi/\omega} (cX\omega \cos \omega t + kX \sin \omega t)(X\omega \cos \omega t) dt = \pi \omega c X^2 \quad (12)$$

where “Im” is the imaginary symbol.

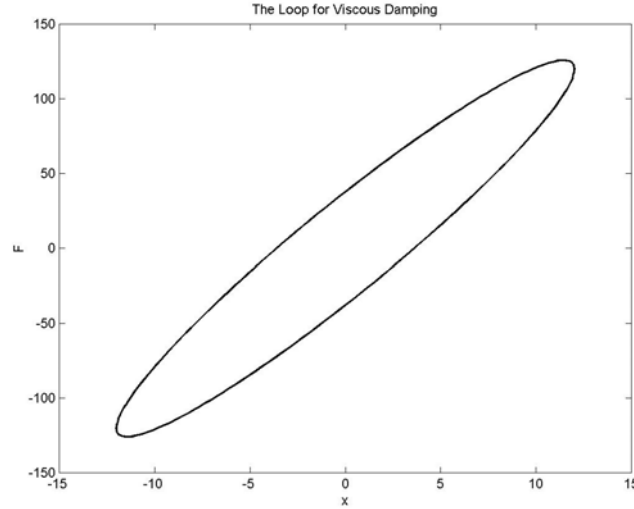


Figure 2 - The Loop for Viscous Damping

For the hysteretic damping, similarly, there is a hysteresis loop to be formed in the stress-strain or force-displacement curve in one loading and unloading cycle. It has been found experimentally that the energy loss per cycle due to internal friction is independent of the frequency for most structural metals, but approximately proportional to the square of the amplitude. In order to achieve the observed behavior from the equation above, the equivalent damping coefficient c_{eq} is assumed to be inversely proportional to the frequency as

$$c_{eq} = \frac{h}{\omega} \quad (13)$$

where h is a hysteretic damping coefficient.

Substitution of equation 13 into 12 results in the energy dissipated by the hysteretic damping in a cycle of motion.

$$\Delta W = \pi h X^2 \quad (14)$$

Model of Damping Properties[5]

Structural damping factor γ

Beside the viscous damping coefficient c , hysteretic damping coefficient h and the damping ratio ζ , there is another very important parameter, structural damping factor, to describe the property of the damping material.

The forced motion equation of a single spring mass system with a hysteretic damper is

$$m\ddot{x} + c_{eq}\dot{x} + kx = f(t) \quad (15)$$

For a harmonic problem, it becomes

$$-\omega^2 mx + k \left(1 - i 2 \frac{\omega}{\omega_n} \zeta_{eq} \right) x = f(t) \quad (16)$$

where $\zeta_{eq} = \frac{c_{eq}}{c_c} = \frac{h}{2m\omega_n\omega}$.

For the modal damping, $\omega = \omega_n$, therefore, we have

$$m\ddot{x} + k(1 - i\gamma)x = f(t) \quad (17)$$

where $\gamma = 2\zeta_{eq} = h/k$ is called the structural damping factor or modal damping ratio.

For the viscous damping, similarly, the viscous damping factor is $\gamma = 2\zeta$.

Complex Stiffness

The effect of polymer material on the damping of the whole structure is influenced by the material stiffness as well as by its damping. These two properties are conveniently quantified by the complex Young's modulus $E(1 - i\eta_e)$ or the complex shear modulus $G(1 - i\eta_g)$. η_g and η_e are usually assumed to be equal for a given material.

When the material is subjected to cyclic stress and strain with amplitude σ_0 and ϵ_0 , the maximum energy stored and dissipated per cycle in a unit volume are as

$$\text{Maximum energy stored per cycle} = E\epsilon_0^2/2 \quad (18)$$

$$\text{Energy dissipated per cycle} = \pi E\eta\epsilon_0^2 \quad (19)$$

Compared to equation 17, the complex stiffness $k(1 - ih/k)$ is similar to the complex modulus $E(1 - i\eta)$ or $G(1 - i\eta)$. Defining the loss factor $\eta = h/k$, the complex stiffness can be expressed as $k(1 - i\eta)$. η may vary from 2×10^{-5} pure aluminum to 1.0 for hard rubber. The structural damping factor γ is equivalent to loss factor η . Loss factor is a term used to quantify damping performance.

A physical interpretation of the loss factor can be obtained as follows. The energy dissipated per cycle for a structural damped system is

$$\Delta W = \pi h X^2 = \pi \eta k X^2 = 2\pi \eta \times \frac{1}{2} k X^2 = 2\pi \eta U_m \quad (20)$$

where U_m is the maximum strain energy stored. Therefore, we have

$$\eta = \frac{1}{2\pi} \frac{\Delta W}{U_m} = \frac{1}{2\pi} \frac{\text{energy dissipated per cycle}}{\text{maximum strain energy}} \quad (21)$$

From equation 21, it is found that the loss factor is a way to compare the damping of one material to another. It is a ratio of the amount of energy dissipated by the system at a certain frequency to the amount of the energy that remains in this system at the same frequency. The more damping a material has, the higher the loss factor will be. The method of representing the structural damping should only be used for frequency domain analysis where the excitation is harmonic [4].

Linear Mathematical Model [5]

Many nonlinear analyses of damped response of structures have been carried out using analytical representations of such a hysteresis loop as

$$\bar{\sigma} = E \left\{ \epsilon - \frac{V}{n} \left[(\epsilon_0 + \epsilon)^n - 2^{n-1} \epsilon_0^n \right] \right\} \quad (22)$$

$$\bar{\sigma} = E \left\{ \varepsilon + \frac{V}{n} \left[(\varepsilon_0 - \varepsilon)^n - 2^{n-1} \varepsilon_0^n \right] \right\} \quad (23)$$

where $\bar{\sigma}$ is the stress during the loading part of the cycle and $\bar{\sigma}$ is that during the unloading part. v is Poisson's ratio. ε_0 is the initial strain. n is the nondimensional parameter in description of hysteresis loop.

An alternate form, somewhat simpler, is

$$\bar{\sigma} = E(\varepsilon) \left\{ \varepsilon \pm \eta(\varepsilon) \varepsilon_0 \left| 1 - \frac{\varepsilon^2}{\varepsilon_0^2} \right|^n \right\} \text{ with } E(\varepsilon) = E / (1 + \alpha |\varepsilon|^\beta) \quad (24)$$

One of the best known representations of the state equation is known as the standard linear model, and it gives the following relationship between stress and strain:

$$\sigma + a \frac{d\sigma}{dt} = E\varepsilon + bE \frac{d\varepsilon}{dt} \quad (25)$$

This particular equation represents a more complex relationship between stress and strain than either Hooke's law $\sigma = E\varepsilon$ or the simple dashpot spring combinations, for which $\sigma = E\varepsilon + bE \frac{d\varepsilon}{dt}$. Two parameters a and b in equation 25 are the constants of stress and strain relaxation respectively. If the applied stress and strain vary harmonically, of the type $\sigma = \sigma_0 e^{-i\omega t}$ and $\varepsilon = \varepsilon_0 e^{-i\omega t}$, then the equation gives

$$\sigma_0 = E\varepsilon_0 \left(\frac{1 - i\omega a}{1 - i\omega b} \right); \quad \sigma_0 = E \left(\frac{1 + \omega^2 ab}{1 + \omega^2 a^2} - i\omega \frac{b - a}{1 + \omega^2 a^2} \right) \varepsilon_0 \quad (26, 27)$$

From equations 26 and 27, we can see that loss factor is usually dependent on the frequency. The modulus is also frequency dependence if the constant of stress a is not zero.

Vibration Damping in software ANSYS [6]

The damping matrix C in ANSYS may be used in harmonic, damped modal and transient analysis as well as substructure generation. In its most general form, it is:

$$[C] = \alpha[M] + \beta[K] + \sum_{j=1}^{N_{ms}} \beta_j [K_j] + \beta_c [K] + [C_\zeta] + \sum_{k=1}^{N_{ds}} [C_k] \quad (28)$$

where

- α constant mass matrix multiplier (input on ALPHAD command)
- β constant stiffness matrix multiplier (input on BETAD command)
- β_j constant stiffness matrix multiplier (input on MP, DAMP command)-- material-dependent damping. It is noted that different damping parameters are defined for different types of analysis when using the material-dependent damping. For example, MP, DAMP in a spectrum analysis specifies a material-dependent damping ratio ζ , not β .
- β_c variable stiffness matrix multiplier: (available for the harmonic response analysis, is used to give a constant damping ratio, regardless of frequency)

$$\beta_c = \frac{\zeta}{\pi f} = \frac{2\zeta}{\omega} = \frac{\eta}{\omega} \quad (29)$$

- ζ constant damping ratio (input on DMPRAT command). From equation 29, the damping ratio ζ should be $\eta/2$ where η is the loss factor.

- f frequency in the range between f_b (beginning frequency) and f_e (end frequency);
- $[C_\zeta]$ frequency-dependent damping matrix
- $[C_\zeta]$ may be calculated from the specified ζ_r (damping ratio for mode shape r) and is never explicitly computed.
- $$\{u_r\}^T [C_\zeta] \{u_r\} = 4\pi f_r \zeta_r \quad (30)$$
- $\{u_r\}$ the r^{th} mode shape
- f_r frequency associated with mode shape r
- $$\zeta_r = \zeta + \zeta_{mr} \quad (31)$$
- ζ constant damping ratio (input on DMPRAT command)
- ζ_{mr} modal damping ratio for mode shape r (input on MDAMP command)
- $[C_k]$ element damping matrix

Rayleigh Damping α and β

The most common form of damping is the so-called Rayleigh type damping $[C] = \alpha[M] + \beta[K]$. The advantage of this representation is that the matrix becomes in modal coordinates

$$\bar{C} = \alpha \mathbf{I} + \beta \mathbf{\Lambda} \quad (32)$$

\bar{C} is diagonal. So for the r^{th} mode, the equation of motion (equation 15) can be uncoupled. Each one is of the form

$$\ddot{q}_r + (\alpha + \beta \omega_r^2) \dot{q}_r + \omega_r^2 q_r = Q_r \quad (33)$$

$$\text{Let } 2\zeta_{mr}\omega_r = (\alpha + \beta \omega_r^2) \quad (34)$$

The equation 33 reduces to

$$\ddot{q}_r + 2\zeta_{mr}\omega_r \dot{q}_r + \omega_r^2 q_r = Q_r \quad (35)$$

where ζ_{mr} is the r^{th} modal damping ratio.

The values of α and β are not generally known directly, but are calculated from modal damping ratios, ζ_{mr} . It is the ratio of actual damping to critical damping for a particular mode of vibration, r . From equation 34, we have

$$\zeta_{mr} = \frac{\alpha}{2\omega_r} + \frac{\beta}{2} \omega_r \quad (36)$$

In many practical structural problems, the α damping (or mass damping) which represents friction damping may be ignored ($\alpha = 0$). In such case, the β damping can be evaluated from known values of ζ_{mr} and ω_r , which represents material structural damping. It is noted that only one value of β can be input in a load step, so we should choose the most dominant frequency active in that load step to calculate β .

In case where the damping properties vary considerably in different parts of the structure, the above techniques cannot be used directly. An example is the analysis of soil-structure interaction problems, where there is significantly more damping in the soil than in the structure.

Material-dependent Damping β_j

It specifies beta damping β_j as a material property (input on MP, DAMP command). It is noticed that MP, DAMP in a spectrum analysis [ANTYPE,SPECTR] specifies a material-dependent damping ratio ζ_i , not β_j .

Constant Damping Ratio ζ

The command DMPRAT is used to represent the loss factor when ζ is set to be $\eta/2$. It is specified as a decimal number with the DMPRAT command and is the simplest way of specifying damping in the structure. It represents the ratio of actual damping to critical damping. DMPRAT is available only for spectrum, harmonic response, and mode superposition transient dynamic analyses. As stated in equation 29, the constant damping ratio is used to calculate β_c .

Modal damping ζ_{mr}

It is specified with the MDAMP command and gives us the ability to specify different damping ratios for different modes of vibration. MDAMP is available only for the spectrum and mode superposition method of solution (transient dynamic and harmonic response analyses). Together with equations 30 and 31, ζ_{mr} is used to compute the frequency dependent damping matrix $[C_\zeta]$.

Element damping $[C_k]$

Element damping involves using some special element types having viscous damping characteristics, such as COMBIN7, COMBIN14, COMBIN37, COMBIN40 and so on. Element damping is applied via element real constant.

Vibration Damping for Harmonic Analysis

Several discrete vibration systems have been used for checking how to consider the damping properties when using ANSYS for harmonic analysis.

Viscous Damping

A single mass spring system (referring to Figure 1)

- By the method of real constant of element (Figure 3)

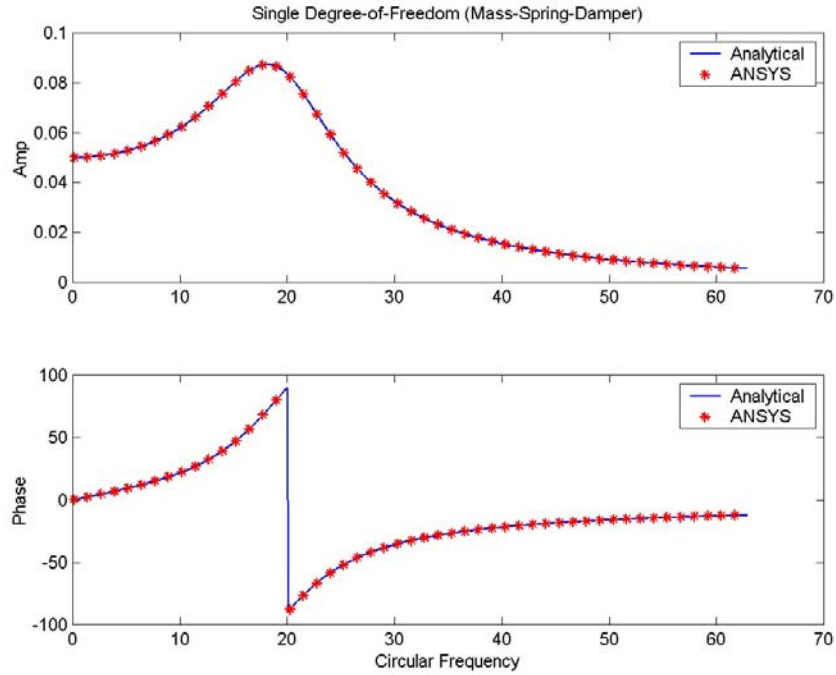
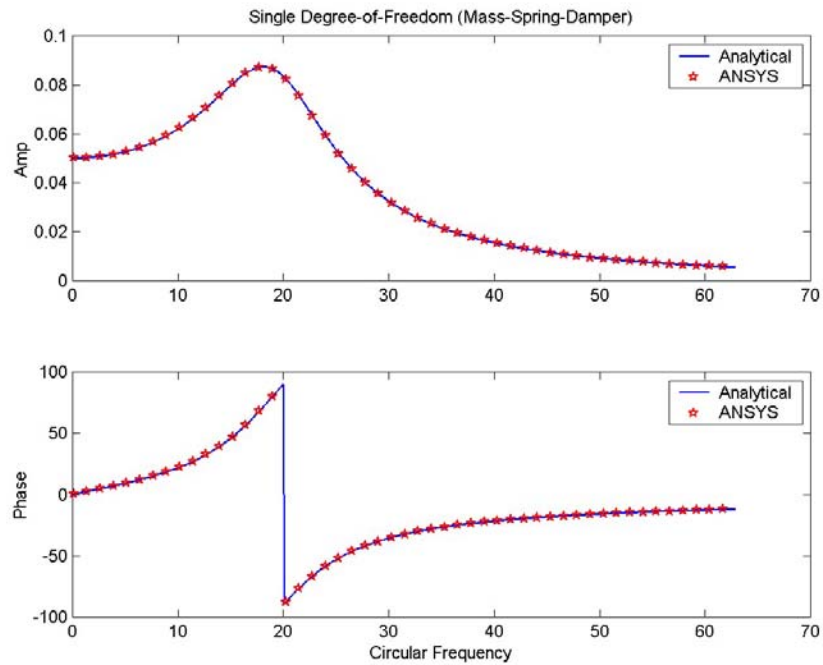


Figure 3 - Response of Viscous-damped Single DOF System

Directly input the viscous damping coefficient c as real constant of the element (COMBIN14).

- By the method of Rayleigh Beta damping (input on BETAD command) $\beta = 2\zeta/\omega_n$ (Figure 4)

Calculate the damping ratio ζ which depends on the natural frequency of the system. Then compute the parameter β with formula $\beta = 2\zeta/\omega_n$.



**Figure 4 - Response of Viscous-damped Single DOF System
($\beta=0.03s$)**

- By the method of material properties (input on MP, DAMP command), $\beta_j = 2\zeta_j / \omega_n$ (Figure 5)

Similar to the input of the parameter β above, the damping properties of the damped structure can be defined via assigning the related element (COMBIN14, for example) on a specific damping value as the material properties [MP,DAMP].

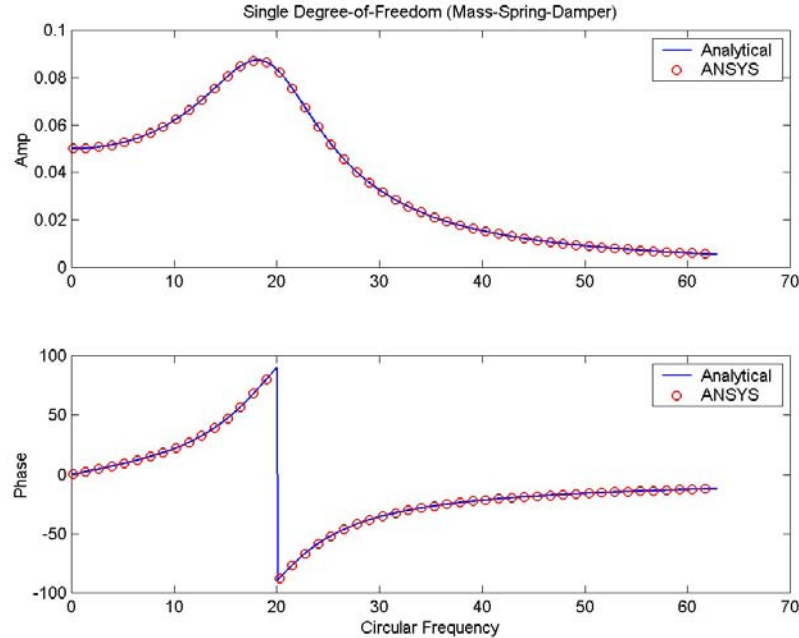


Figure 5 - Response of viscous-damped single DOF system ($\beta_j=0.03s$)

Multiple mass spring system (referring to Figure 6)

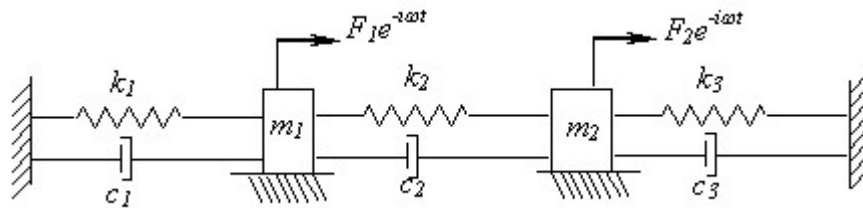


Figure 6 - A forced two DOFs of damped vibration (figure6.jpg)
 $[m_1=m_2=0.5 \text{ (kg)}, k_1=k_2=k_3=150 \text{ (N/m)} \text{ } c_1=5 \text{ (N}\cdot\text{s/m)}, c_2=0.1 \text{ (N}\cdot\text{s/m)}, c_3=0.05 \text{ (N}\cdot\text{s/m)}$
 $F_1=10 \text{ (N)}, F_2=500 \text{ (N)}$]

- By the method of real constant of element (Figure 7)

It is straightforward to specify the viscous damping coefficients c_i of each damper via real constants of the elements used, respectively.

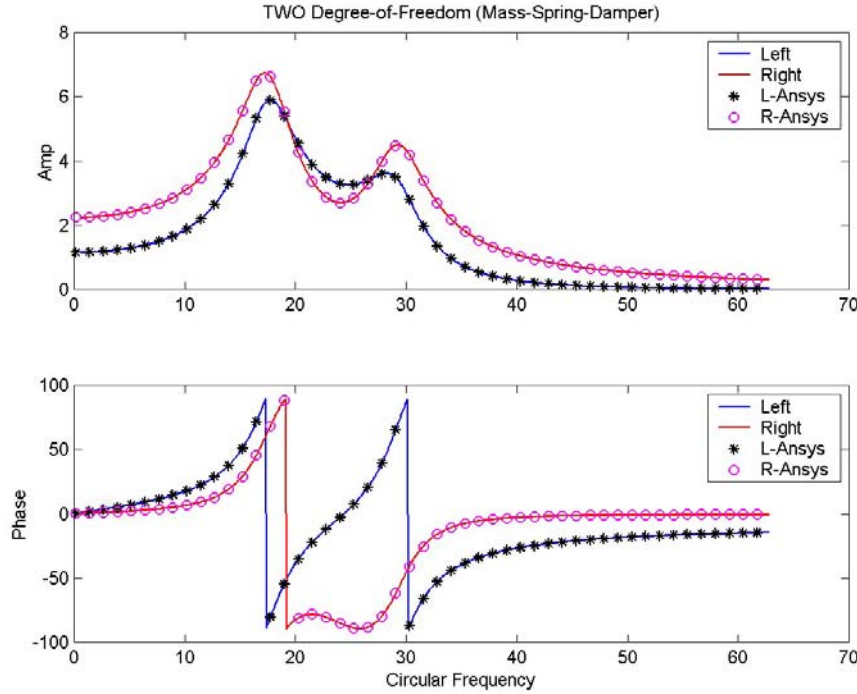


Figure 7 - Response of Viscous-damped Two DOF System

- By material properties (input on MP, DAMP command), $\beta_j = 2\zeta_j / \omega_n$.

It is needed to calculate the natural frequencies of the system and define the most significant mode for the problem at hand because there are more than one natural frequency for the multiple-DOF vibration system. Say the dominant or major natural frequency be ω_d , the material-dependent damping β_i can be calculated respectively with this natural frequency. It is expected that it is hard to specify the dominant natural frequency with increase of the number of degree-of-freedom of vibration system. Figure 8 shows the responses of two masses when the first natural frequency $\omega_1=17.32(\text{Hz})$ is used as dominant frequency. Figure 9 shows the response of two masses when the second natural frequency $\omega_2=30.00(\text{Hz})$ is used as dominant frequency. It is seen that obvious discrepancy is observed when an appropriate natural frequency is selected as the dominant frequency as shown in Figure 9.

It is noted that only one value of β can be input in a load step, therefore, the method by Rayleigh Beta damping can not be used directly to specify the damping properties of the vibration system with different damping materials. Figure 10 shows the response of two masses when $\beta=0.03333$.

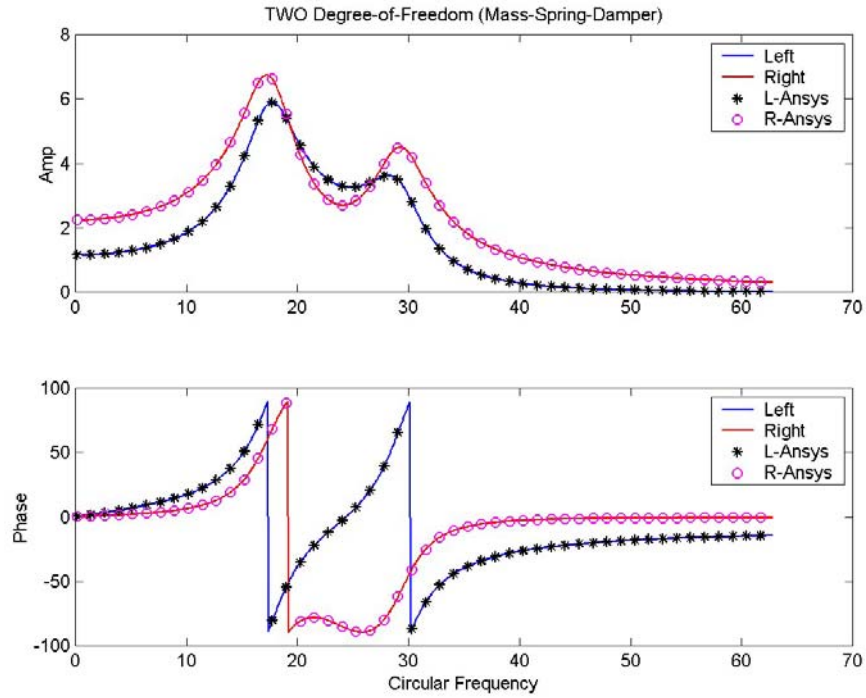


Figure 8 - Response of viscous-damped two DOF system
 $(\beta_1=0.033, \beta_2=6.666 \times 10^{-4}, \beta_3=3.333 \times 10^{-4})$

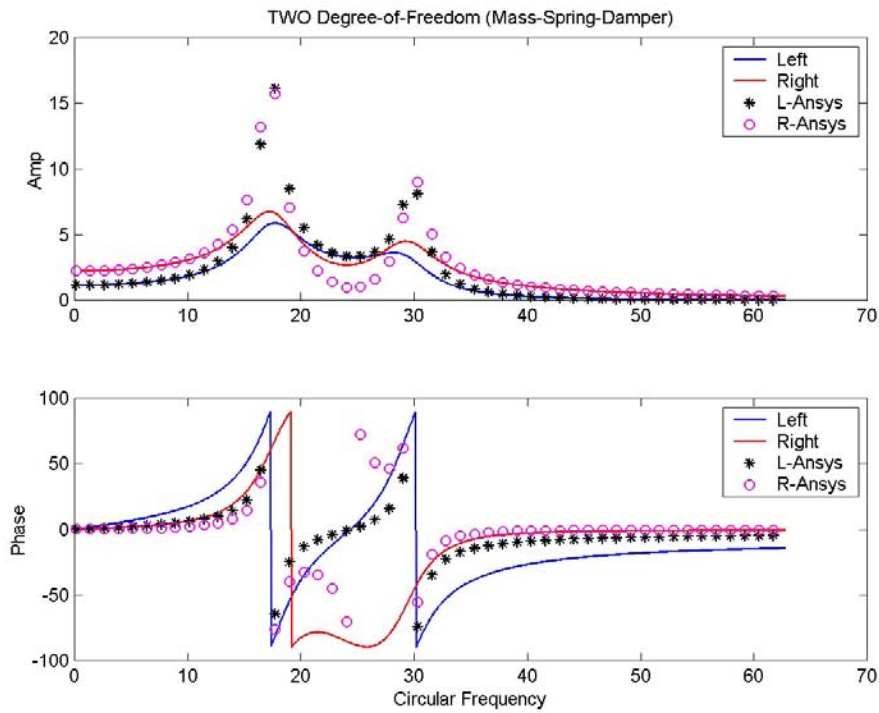
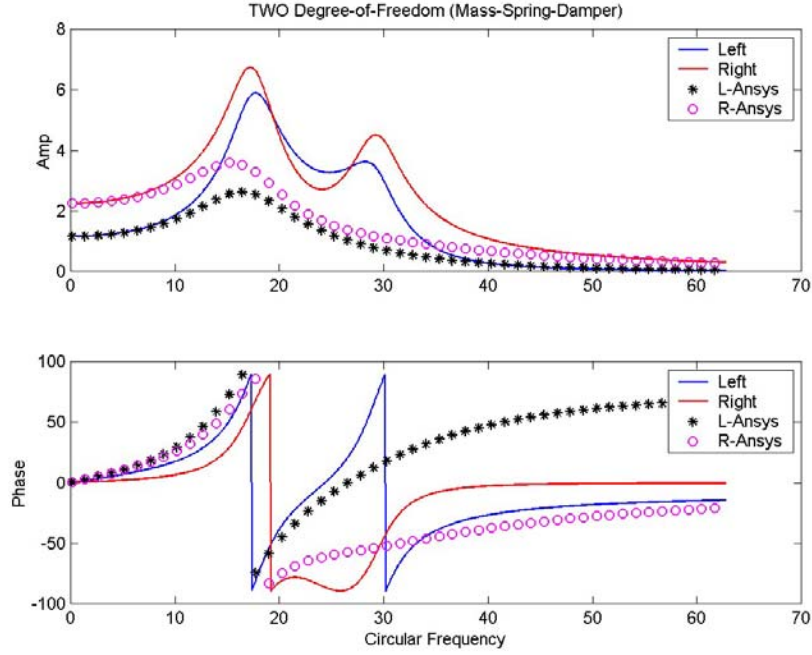


Figure 9 - Response of viscous-damped two DOF system
 $(\beta_1=0.011, \beta_2=2.222 \times 10^{-4}, \beta_3=1.111 \times 10^{-4})$



**Figure 10 - Response of Viscous-damped Two DOF System
(via Rayleigh damping method: $\beta=0.03333$)**

Hysteretic Damping

- β_c via constant damping ratio ζ (input on DMPRAT command)

As mentioned above, the command DMPRAT can be used for the hysteretic damping input for single degree of freedom of vibration system. Again from equation 29, the damping ratio ζ should be $\eta/2$ if the loss factor is η . Figure 11 shows the response of single DOF of vibration system with a hysteretic damper.

- β_j via material dependent damping (input on MP, DAMP command)

Material-dependent damping β_j can be used for specifying the hysteretic damping properties both in single and multiple DOF of vibration systems. The β_j should be calculated with equation 29 if the loss factors η_j is known.

$$\beta_j = \frac{\eta_j(\omega)}{\omega} \quad (37)$$

where $\eta_j(\omega)$ is the frequency-dependent loss factor of the j^{th} material. ω is the circular frequency.

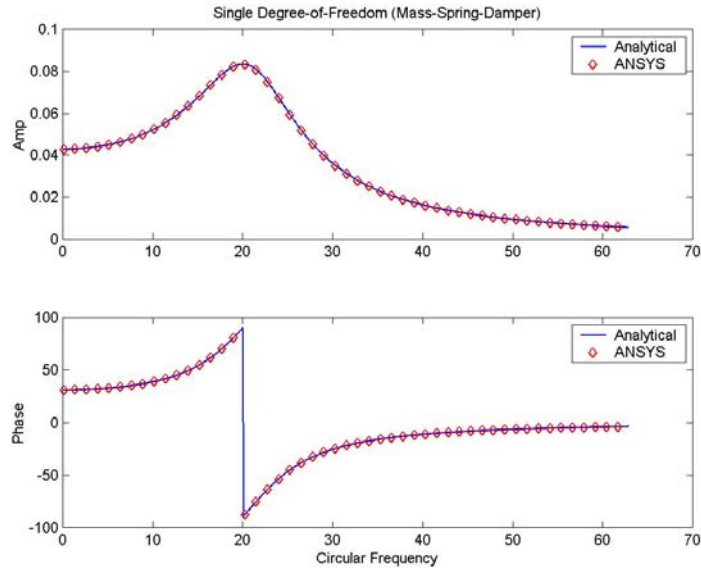


Figure 11 - Response of Hysteretic-damped Single DOF System
[$m=0.5$ (kg), $k=200$ (N/m), $\eta=0.6$]

For harmonic vibration analysis, equation 37 should be applied frequency by frequency. In other words, the damping β_f needs to be given for each analysis circular frequency, respectively.

Figure 12 shows the responses of two DOF of vibration systems (Figure 6) with three hysteretic damping dampers.

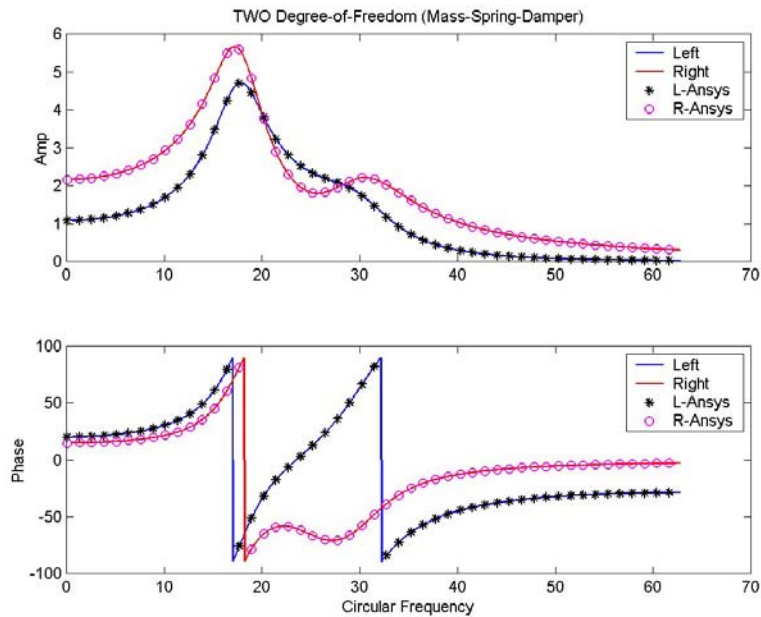


Figure 12 - Response of Hysteretic-damped Two DOF System
(Via material-dependent properties method: $\eta_1=0.5$, $\eta_2=0.3$, $\eta_3=0.2$)

Vibration Damping for Modal Analysis

Modal analysis is used to determine the vibration characteristics (natural frequencies and mode shapes) of a vibration system. The results from modal analysis may be further applied for another dynamic analysis via mode superposition method.

Alpha (mass) damping, Beta (stiffness) damping, Material-dependent damping ratio (input on MP, DAMP command) and element damping (applied via element real constant) can be specified in modal analysis.

It is noticed that in the mode superposition method, only Rayleigh or constant damping is allowed, and explicit element damping in such elements as COMBIN 14 is not allowed. Considering equations 30 and 31, the modal damping, ζ_r , is the combination of several ANSYS damping inputs as

$$\zeta_r = \left(\frac{\alpha}{2\omega_r} \right) + \left(\frac{\beta}{2} \omega_r \right) + \zeta + \zeta_{mr} \quad (38)$$

where α is uniform mass damping multiplier (input on ALPHAD command); β uniform stiffness damping multiplier (input on BETAD command); ζ is constant damping ratio (input on DMPRAT command) and ζ_{mr} is the modal damping ratio (input on MDAMP command).

For the vibration structure consisting of single material only, material-dependent damping, β_j , provides the same results as specified Beta (stiffness) damping, β , since $\beta_j = 2\zeta_j/\omega_{nj}$. It is seen that the frequency-dependent damping treatment such as hysteretic damping in equation 37 is not applicable for modal analysis. In practice, Rayleigh damping parameters of continuum damping materials can not be read from the vendor's damping data sheet (modulus and loss factors). Therefore, Rayleigh damping may be only suitable for the vibration system with single degree of freedom.

Summaries

There are several methods to include the material damping in ANSYS. The following points are drawn out for using damping in ANSYS:

- Element related damping input is only suitable for some special elements such as COMBIN7, COMBIN 14, COMBIN37, COMBIN40 and so on. The viscous damping coefficients can input by the real constants directly. It is suitable for single and multiple DOF vibration systems;
- Rayleigh damping (input on BETAD and ALFAD commands) is suitable for single DOF vibration system because it depends on the dominant natural frequency and damping ratio. For multiple DOF systems and continuum vibration system, it is difficult to identify the dominant natural frequency and modal damping ratio. $\beta = 2\zeta_r/\omega_r$ if α is assumed to be zero;
- Constant damping ratio (input on DMPRAT command) is used to specify the hysteretic damping directly for a single DOF of vibration system and/or the system consisting of only one kind of material. The input constant damping ratio is half of the loss factor. $\zeta = 0.5\eta$;
- Material dependant damping (input on MP, DAMP command) can be used to input the viscous damping $\beta_j = 2\zeta_j/\omega_{nj}$ and/or hysteretic damping $\beta_j = \eta_i(\omega)/\omega$. With the similar limitation of Rayleigh damping, it can only used to represent the viscous damping of single DOF vibration system and/or the continuum system consisting of only one material. However, it can be used to represent hysteretic damping of multiple DOF system and other multiple-material continuum vibration system under harmonic analysis based on each analysis frequency. Obviously, this method is not applicable to damped modal analysis.
- For damped modal analysis, all four methods above to represent the material damping can be used with corresponding applications. Rayleigh Beta damping is only applicable for the natural frequency analysis of single DOF vibration systems in practical use. Constant damping ratio (input on DMPRAT command) is applicable for modal analysis of the continuum vibration system with

only one kind of hysteretic damping material. Element related damping input methods via real constants can be used for modal analysis of both single and multiple DOF discrete vibration systems with viscous damping properties.

The special attention should be paid when we use ANSYS for damped vibration and acoustic analysis.

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