

General form of Navier Stokes Equation

$$\rho \left( \frac{\delta u}{\delta t} + u \frac{\delta u}{\delta x} + v \frac{\delta v}{\delta y} + w \frac{\delta w}{\delta z} \right) = \rho g_x - \frac{\delta p}{\delta x} + \frac{\delta}{\delta x} \left[ \mu \left( 2 \frac{\delta u}{\delta x} \right) - \frac{2}{3} \mu (\nabla \cdot \vec{u}) \right] + \frac{\delta}{\delta y} \left[ \mu \left( \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} \right) \right] + \frac{\delta}{\delta z} \left[ \mu \left( \frac{\delta u}{\delta z} + \frac{\delta w}{\delta x} \right) \right]$$

$$\rho \left( \frac{\delta v}{\delta t} + u \frac{\delta u}{\delta x} + v \frac{\delta v}{\delta y} + w \frac{\delta w}{\delta z} \right) = \rho g_y - \frac{\delta p}{\delta y} + \frac{\delta}{\delta x} \left[ \mu \left( \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} \right) \right] + \frac{\delta}{\delta y} \left[ \mu \left( 2 \frac{\delta v}{\delta y} \right) - \frac{2}{3} \mu (\nabla \cdot \vec{u}) \right] + \frac{\delta}{\delta z} \left[ \mu \left( \frac{\delta v}{\delta z} + \frac{\delta w}{\delta y} \right) \right]$$

$$\rho \left( \frac{\delta w}{\delta t} + u \frac{\delta u}{\delta x} + v \frac{\delta v}{\delta y} + w \frac{\delta w}{\delta z} \right) = \rho g_z - \frac{\delta p}{\delta z} + \frac{\delta}{\delta x} \left[ \mu \left( \frac{\delta u}{\delta z} + \frac{\delta w}{\delta x} \right) \right] + \frac{\delta}{\delta y} \left[ \mu \left( \frac{\delta v}{\delta z} + \frac{\delta w}{\delta y} \right) \right] + \frac{\delta}{\delta z} \left[ \mu \left( 2 \frac{\delta w}{\delta z} \right) - \frac{2}{3} \mu (\nabla \cdot \vec{u}) \right]$$

Conditions:

- Newtonian Fluid
- Isotropic Fluid
  - Stress and stress tensor unchanged by rotation of coordinate system or by interchanging axis
  - Principle axis of stress same as principle axis of stress tensor
- Gravity only body force