fc204

compatibility of deformations P - M actual status of RC circular section

Set Units to None at the Math...Options...Unit System Dialog. You will be using any of the units below.

Initialization

$ORIGIN \equiv 1$ Count with fingers $TOL := 1$ $CTOL := 1$

$ton := 1000 \cdot kgf$ $kip := 453.592 \cdot kgf$ $kN := 1000 \cdot N$

$MPa := \frac{N}{mm^2}$ $ksi := 70.307 \cdot \frac{kgf}{cm^2}$ $psi := \frac{ksi}{1000}$

$f_y := 4100 \frac{kgf}{cm^2}$ $E_s := 2100000 \cdot \frac{kgf}{cm^2}$

Steel

For this problem we'll have a perfectly elastic-perfectly plastic steel of f_y yield stress

So $\varepsilon_y := \frac{f_y}{E_s}$ $\varepsilon_y = 0$

$$f_s(\varepsilon) := \begin{cases} (-f_y) & \text{if } \varepsilon \le -\varepsilon_y \\ \text{otherwise} & \begin{cases} E_s \cdot \varepsilon & \text{if } \varepsilon \le \varepsilon_y \\ f_y & \text{otherwise} \end{cases} \end{cases}$$

This equation will rule steel stress-strain relationship

Say $\varepsilon := .0005$ $f_s(\varepsilon) = 1050 \frac{kgf}{cm^2}$

We won't state a limit strain for steel for the example

Concrete

$f_c := 250 \cdot \frac{kgf}{cm^2}$

You may feel adequate to enter a fcd reduced one, or a mean (probabilistic) real value

Take into account?

$Confinement := 0$ **1** for YES **0** for NO

confinement 1 only if per ACI 318

Take into account?

$Tensile\_stress := 0$ **1** for YES **0** for NO

Note
Formulation believed to be adequate even for the most exacting HPC, VHS concretes.

The feature that distinguishes the procedure of solving the merely flexural problem form the present flexocompression case is that in the flexocompression case upon growth of the force in the compressed block within the section we must impose concurrent decay of the limit strain that can be reached prior to failure at the most compressively strained fiber. This means than while solving for extreme strains we must simultaneously solve for the limit available compressive strain following the present compression to maximum compression ratio. This will be a fuction of current limit strains at surfaces, since concrete force is. To avoid constant redefinition of the decaying stress branch by the continuously varying 2nd point and being of little engineering significance (given tue o.91 to 0.98 of fc cutting point), for this first installment of the fxcomp programs we will merely curtail the constant formulation of s soon following at the appropriate level suggested by the compressed block.

e_{fc} evaluation

$$\varepsilon_{fc}(f_c) := .0015 + .002 \cdot \frac{f_c \cdot \frac{cm^2}{kgf}}{1300}$$

The strain at which concrete reachs its higher strenght f_c is $\varepsilon_{fc}(f_c) = 0.001885$

e_{fct} evaluation

$k_{fct} := 7.5$

You can alternatively make k_{fct}= 6.7 if for strenght or simply to be more conservative

$$f_{ct}(f_c) := k_{fct}\sqrt{f_c \cdot psi}$$

$f_{ct}(f_c) = 31.44 \frac{kgf}{cm^2}$ $f_{ct}(f_c) = 447.23\,psi$ $f_{ct}(f_c) = 3.08\,MPa$

We could get approximately the strain at which the ultimate tensile strain is reached, but will do exactly solving the equation in first quadrant:

$f_c = 250 \frac{kgf}{cm^2}$ Reminder

Our unwarranted guess $\varepsilon := .0005 \cdot \frac{cm}{cm}$

We'll solve the limit tensile strain without units since Mathcad 8 doesn't seem able to manage here properly these

$f_{ct}(f_c) := k_{fct}\sqrt{f_c \cdot psi}$ $f_{ct}(f_c) = 31.44 \frac{kgf}{cm^2}$

$$\varepsilon_{fc}(f_c) := .0015 + .002 \cdot \frac{f_c \cdot \frac{cm^2}{kgf}}{1300}$$

$\varepsilon_{fc}(f_c) = 0 \frac{cm}{cm}$

Given

$$\frac{f_{ct}(f_c)}{f_c} = \left(2 \cdot \frac{\varepsilon}{\varepsilon_{fc}(f_c)}\right) - \left(\frac{\varepsilon}{\varepsilon_{fc}(f_c)}\right)^2$$

$\varepsilon_{fct0} := -\mathrm{Find}(\varepsilon)$ $\varepsilon_{fct0} = -0.00012$

$\varepsilon_{fct}(f_c) := \mathrm{if}(\mathrm{Tensile\_stress}, \varepsilon_{fct0}, 0)$ $\varepsilon_{fct}(f_c) = 0$

e_{cu} evaluation

kgf/cm2 $\qquad$ e_{cu}

$$Stress := \frac{kgf}{cm^2}\cdot\begin{pmatrix}100\\350\\500\\800\\1200\end{pmatrix} \qquad Strain := \begin{pmatrix}.0039\\.0035\\.0028\\.0028\\.0034\end{pmatrix}$$

$$vs := lspline(Stress, Strain) \qquad \varepsilon_{cu}(f_c) := interp(vs, Stress, Strain, f_c)$$

The ultimate strain for the given fc for a flexural condition like this is then $\varepsilon_{cu}(f_c) = 0.003779$

s(e) evaluation

Stress in concrete corresponding to strain e

$k_{\varepsilon cu} := if(Confinement, 0.98, 0.91)\; k_{\varepsilon cu} = 0.91$ $\qquad$ as per disgression

$$\sigma(\varepsilon) := f_c\cdot \begin{cases} 0 & \text{if } \varepsilon < \varepsilon_{fct}(f_c) \\ \text{otherwise} & \begin{cases} -1\cdot\left[\left(2\cdot\frac{-\varepsilon}{\varepsilon_{fc}(f_c)}\right)-\left(\frac{-\varepsilon}{\varepsilon_{fc}(f_c)}\right)^2\right] & \text{if } \varepsilon_{fct}(f_c) \le \varepsilon \le 0 \\ \text{otherwise} & \begin{cases} \left(2\cdot\frac{\varepsilon}{\varepsilon_{fc}(f_c)}\right)-\left(\frac{\varepsilon}{\varepsilon_{fc}(f_c)}\right)^2 & \text{if } 0 < \varepsilon \le \varepsilon_{fc}(f_c) \\ \text{otherwise} & \begin{cases} 1-(1-k_{\varepsilon cu})\dfrac{\varepsilon^2-2\cdot\varepsilon\cdot\varepsilon_{fc}(f_c)+\varepsilon_{fc}(f_c)^2}{\varepsilon_{cu}(f_c)^2-2\cdot\varepsilon_{fc}(f_c)\cdot\varepsilon_{cu}(f_c)+\varepsilon_{fc}(f_c)^2} & \text{if } \varepsilon_{fc}(f_c) < \varepsilon \le \varepsilon_{cu}(f_c) \\ 0 \text{ if } \varepsilon > \varepsilon_{cu}(f_c) & \text{otherwise} \end{cases} \end{cases} \end{cases} \end{cases}$$

For this implementation, the formulation will rule stress determination in concrete for any input e

In spite of this and as discussed above, the ultimate strain won't be that of flexure once the resultant of the compression block attains a value bigger than the one the (we will accept) brute section has when it has 0 stress in one face and fc at the other (concrete block resultant at decompression of face value), which for a rectangular section is equivalent to the full section at 2/3 of fc, given the parabolic law of the first branch of our law for stress-strain

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Circular section od D diameter $\quad D := 40\cdot cm$

$M := 8 \cdot m \cdot ton$ present moment, factored or non factored, enter positive and compressing atop

$P := 94 \cdot ton$ present axial load, factored or non factored, compressive and positive till we further check the implementation for tensile load equilibrium

Reinforced as follows $n := 6$ number of bars $\Phi_{bar} := 20 \cdot mm$ $Cover\_to\_axis := 5 \cdot cm$

$\alpha := 30 \cdot deg$ centered angle, from 6 'o clock vector towards lowest bar center, clock or counterclockwise
(any effect of dissymmetry respect vertical axis is dismissed)

$$P_{max} := \pi \cdot \left(\frac{D}{2}\right)^2 \cdot f_c$$

we accept to no make it interact with atop and at bottom strains for steel voids deductions

$P_{max} = 314.16\,ton$ full section at fc, a reference value

$$r := \frac{D}{2} \qquad r_{to\_bar} := r - Cover\_to\_axis \qquad r_{to\_bar} = 15\,cm$$

$$b(z,r) := 2 \cdot r \cdot \sin\left(\operatorname{acos}\left(\frac{r-z}{r}\right)\right) \qquad b_1(z,r) := b[r-(z-r),r]$$

$$P_{ref} := \int_{0 \cdot m}^{r} \frac{f_c}{4 \cdot r^2} \cdot z \cdot (4 \cdot r - z) \cdot b(z,r)\,dz + \int_{r}^{2 \cdot r} \frac{f_c}{4 \cdot r^2} \cdot z \cdot (4 \cdot r - z) \cdot b_1(z,r)\,dz$$

$P_{ref} = 215.99\,ton$

Integration of the first branch parabolic fc growth of strength along D for a circular section, the circular section decompression milestone against which to gauge how much we will be curtailing the strain towards that of at maximum compressive strength from that of a non compressively loaded case.

$$e_{to\_center} := \frac{M}{P} \qquad e_{to\_center} = 8.51\,cm$$

$$e\_to\_bottom := e_{to\_center} + \frac{D}{2} \qquad e\_to\_bottom = 28.51\,cm$$

We will be assuming P, M data referred to center of brute section as usual and will establish equilibrium integrating moments respect bottom of the section; that is, the moment of the P as per above implied positioned will be in place equilibrated by the moments of inner forces in steel and concrete; all moments will be referred to bottom edge of section.

We take compression stresses positive, and tension stresses negative.
We dump the areas of both steel reinforcement and concrete layers at layer's c.o.g.

Say e_1 represents strain at bottom surface and e_2 is strain at top surface

$$A_{1\_bar} := \pi \cdot \frac{\Phi_{bar}^{\ 2}}{4} \qquad A_{1\_bar} = 3.14\,cm^2$$

$A_{1\_bar} := \pi \cdot \frac{\ }{4} \qquad A_{1\_bar} = 3.14cm$

Elements of Equilibrium

$$\varepsilon_2 := 0.000189 \cdot \frac{cm}{cm}$$

guess values

$$\varepsilon_1 := -0.00033 \cdot \frac{cm}{cm}$$

Determination of the steel force and moment as a function of surface strains

$$Steel_{force}(\varepsilon_1, \varepsilon_2) := A_{1\_bar} \cdot \sum_{k=1}^{n} \left[f_s\left[\varepsilon_1 + \frac{r - r_{to\_bar} \cdot \cos\left[\alpha + (k-1) \cdot \frac{360 \cdot deg}{n}\right]}{D} \cdot (\varepsilon_2 - \varepsilon_1) \right] - \sigma\left[\varepsilon_1 + \frac{r - r_{to\_bar} \cdot \cos\left[\alpha + (k-1) \cdot \frac{360 \cdot deg}{n}\right]}{D} \cdot (\varepsilon_2 - \varepsilon_1) \right] \right]$$

effective force and moment, so that concrete can be taken brute section in integration

$$Steel_{moment}(\varepsilon_1, \varepsilon_2) := A_{1\_bar} \cdot \sum_{k=1}^{n} \left[r - r_{to\_bar} \cdot \cos\left[\alpha + (k-1) \cdot \frac{360 \cdot deg}{n}\right] \right] \cdot \left[f_s\left[\varepsilon_1 + \frac{r - r_{to\_bar} \cdot \cos\left[\alpha + (k-1) \cdot \frac{360 \cdot deg}{n}\right]}{D} \cdot (\varepsilon_2 - \varepsilon_1) \right] - \sigma\left[\varepsilon_1 + \frac{r - r_{to\_bar} \cdot \cos\left[\alpha + (k-1) \cdot \frac{360 \cdot deg}{n}\right]}{D} \cdot (\varepsilon_2 - \varepsilon_1) \right] \right]$$

$$B(z,r) := \begin{cases} b(z,r) & \text{if } z \le r \\ b_1(z,r) & \text{otherwise} \end{cases}$$

$$Concrete_{force}(\varepsilon_1, \varepsilon_2) := \int_{0 \cdot m}^{D} B(z,r) \cdot \sigma\left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{D} \cdot z \right) dz$$

$$Concrete_{moment}(\varepsilon_1, \varepsilon_2) := \int_{0 \cdot m}^{D} z \cdot B(z,r) \cdot \sigma\left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{D} \cdot z \right) dz$$

$$Total_{force}(\varepsilon_1, \varepsilon_2) := Steel_{force}(\varepsilon_1, \varepsilon_2) + Concrete_{force}(\varepsilon_1, \varepsilon_2)$$

$$Total_{moment}(\varepsilon_1, \varepsilon_2) := Steel_{moment}(\varepsilon_1, \varepsilon_2) + Concrete_{moment}(\varepsilon_1, \varepsilon_2)$$

We need also to define here what will be being taken as the ultimate compressive strain for the case, that will vary with $\varepsilon_1, \varepsilon_2$, since the present compressive block does.

$$\varepsilon_{cu\_current}(\varepsilon_1, \varepsilon_2) := \begin{cases} \varepsilon_{cu}(f_c) & \text{if } Concrete_{force}(\varepsilon_1, \varepsilon_2) \le P_{ref} \\ \ldots \end{cases}$$

$$\left\| \left[\varepsilon_{cu}(f_c) + \left(\varepsilon_{fc}(f_c) - \varepsilon_{cu}(f_c)\right) \frac{\left(\frac{Concrete_{force}(\varepsilon_1, \varepsilon_2)}{P_{max}} - \frac{P_{ref}}{P_{max}}\right)^2}{\left(1 - \frac{P_{ref}}{P_{max}}\right)^2} \right] \right. \quad \text{otherwise}$$

Solving the problem

Given

$$Total_{force}(\varepsilon_1, \varepsilon_2) = P$$

$$Total_{moment}(\varepsilon_1, \varepsilon_2) = P \cdot e\_to\_bottom$$

$$\varepsilon_2 \le \varepsilon_{cu\_current}(\varepsilon_1, \varepsilon_2)$$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} := Find(\varepsilon_1, \varepsilon_2)$$

$$k := 1..n$$

$$y_k := r - r_{to\_bar} \cdot \cos\left[\alpha + (k-1) \cdot \frac{360 \cdot deg}{n}\right] \qquad Vector_{angle_k} := \alpha + (k-1) \cdot \frac{360 \cdot deg}{n}$$

$$\varepsilon_k := \varepsilon_1 + \frac{r - r_{to\_bar} \cdot \cos\left[\alpha + (k-1) \cdot \frac{360 \cdot deg}{n}\right]}{D} \cdot (\varepsilon_2 - \varepsilon \quad F_{s_k} := f_s\left[\varepsilon_1 + \frac{r - r_{to\_bar} \cdot \cos\left[\alpha + (k-1) \cdot \frac{360 \cdot deg}{n}\right]}{D} \cdot (\varepsilon_2 - \varepsilon_1)\right]$$

$Total_{force}(\varepsilon_1, \varepsilon_2) = 94\,ton$ $\quad P = 94\,ton$ $\quad \varepsilon_1 = -0.00032$ at bottom surface

$Total_{moment}(\varepsilon_1, \varepsilon_2) = 26.8\,m \cdot ton$ $\quad P \cdot e\_to\_bottom = 26.8\,m \cdot ton$ $\quad \varepsilon_2 = 0.00085$ at top surface $\quad \frac{\varepsilon_2}{\varepsilon_{cu\_current}(\varepsilon_1, \varepsilon_2)} = 22.55\%$

Observe that respect bottom and so different of data

consider only if positive

$\text{Steel}_{force}(\varepsilon_1,\varepsilon_2) = 9.18\,\text{ton}$

$\sigma(\varepsilon_2) = 174.98\,\frac{kgf}{cm^2}$

$\varepsilon_{cu\_current}(\varepsilon_1,\varepsilon_2) = 0.00378$

actual

vs that of flexure

$\varepsilon_{cu}(f_c) = 0.00378$

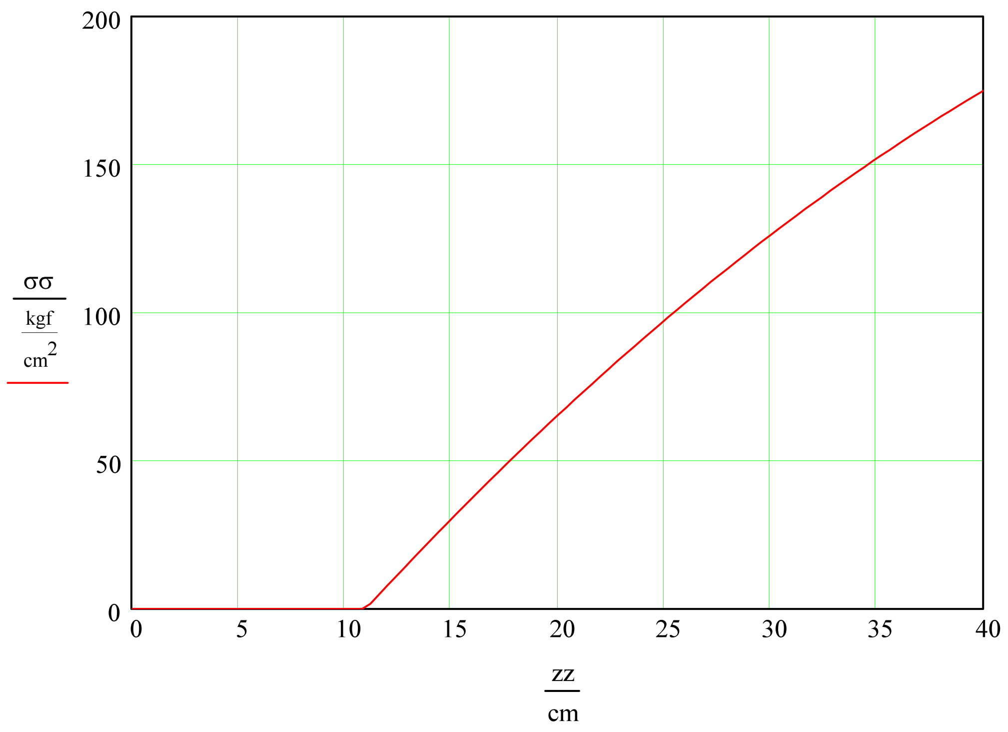
bar results

$$\text{Vector}_{angle} = \begin{pmatrix} 30 \\ 90 \\ 150 \\ 210 \\ 270 \\ 330 \end{pmatrix} deg \quad y = \begin{pmatrix} 7.01 \\ 20 \\ 32.99 \\ 32.99 \\ 20 \\ 7.01 \end{pmatrix} cm \quad \varepsilon = \begin{pmatrix} -0.000117 \\ 0.000265 \\ 0.000646 \\ 0.000646 \\ 0.000265 \\ -0.000117 \end{pmatrix} \quad F_s = \begin{pmatrix} -245.18 \\ 556.05 \\ 1357.28 \\ 1357.28 \\ 556.05 \\ -245.18 \end{pmatrix} \frac{kgf}{cm^2}$$

compression positive

$N_{parts} := 100$ $\quad j := 1..N_{parts} + 1$ $\quad zz_j := \frac{D}{N_{parts}} \cdot (j-1)$ $\quad \sigma\sigma_j := \sigma\left(\varepsilon_1 + \frac{\varepsilon_2 - \varepsilon_1}{D} \cdot zz_j\right)$

Chart concrete stresses



Note how circular columns show to be worse in a compatibility of deformations setup, due mainly to loss of efficiently positioned area.