

Hertzian Contacts

Surfaces of engineering components are routinely subjected to contact loading, where large stresses are applied over highly localized areas, e.g., bearings, rails, valves, sliders, rollers, armor, wear-parts, micro-electronic devices, dental implants, prostheses, etc. These kinds of loading configurations in the elastic limit are typified by Hertzian contacts. In its narrow definition, a Hertzian contact refers to two non-conforming bodies in contact (Hertz 1896, Johnson 1985, Fischer-Cripps 2000), where: (i) the surfaces under normal loading have continuous profiles, (ii) the area of contact is finite, and is significantly smaller than the dimensions of the bodies and the radii of surface curvatures, (iii) the contact is purely elastic, i.e., the bodies do not undergo plastic or inelastic deformation, and (iv) friction is absent at the contact interface. Although such an idealized situation occurs infrequently in engineering applications, the elegant theory developed by Hertz in the 1890s to describe such a contact laid the foundation for contact mechanics used to analyze general contacts. The Hertzian theory enables one to calculate: (i) the shape of the area of contact and its evolution with increasing load, (ii) the magnitude and distribution of normal and tangential surface tractions transmitted across the contact interface, and (iii) the components of elastic stresses and deformations in both bodies in the vicinity of the contact (Hertz 1896, Johnson 1985, Fischer-Cripps 2000).

1. Stresses

Although the general Hertzian theory applies to arbitrary-geometry surfaces in contact (nonconforming and continuous), the following simple geometries are considered here: (i) sphere-on-sphere, (ii) cylinder-on-cylinder, and (iii) sphere-on-flat. Figure 1 is a schematic diagram showing the first two geometries, loaded normally with a load P . The maximum contact radius (a), contact pressure (p_0), and principal shear stress (τ_{13} ; at cylindrical coordinates $r = 0, \theta = 0$; $v_1 = v_2 = 0.3$) are also given in Fig. 1. The analytical solutions describing the elastic stress fields are complex, even for such simple geometries and are cited in the Bibliography (Hertz 1896, Johnson 1985, Fischer-Cripps 2000, Lawn 1998). Selected components of elastic stress field for the simplest case—sphere-on-flat—are presented in Fig. 2 (for $v_2 = 0.22$). The principal normal stress σ_{11} in the flat is compressive under the spherical indenter, and is mildly tensile in the interior. The magnitude of the maximum tensile stress is substantial just outside the contact radius, which determines the initiation and propagation of cracks in brittle solids, but drops off quickly into the interior. The principal normal stress σ_{22} (not shown here) is a hoop stress, whereas σ_{33} is compressive

everywhere. Generally speaking, the magnitudes of $\sigma_{11} > \sigma_{22} > \sigma_{33}$ nearly everywhere. The principal shear stress τ_{13} [$= 0.5(\sigma_{11} - \sigma_{33})$] is maximum subsurface, the magnitude and location of which is dependent on the Poisson ratios. For flat solids subjected to Hertzian indentation, it is the maximum τ_{13} that governs the onset of shear inelasticity. Note that stress contours (as in Fig. 2) give only the magnitudes of the stress; one needs to refer to stress trajectories to determine the stress directions (Fischer-Cripps 2000).

2. Onset of Plasticity in Metals

As alluded to above, the Hertzian analysis is useful in predicting the limit of elasticity, or onset of plasticity, for ductile metals in normal contact (Johnson 1985, Tabor 1951). Since shear stress is responsible for dislocation plasticity in metals, its onset occurs where the subsurface maximum shear stress (τ_{13}) satisfies an appropriate yield criterion. For example, in the sphere-

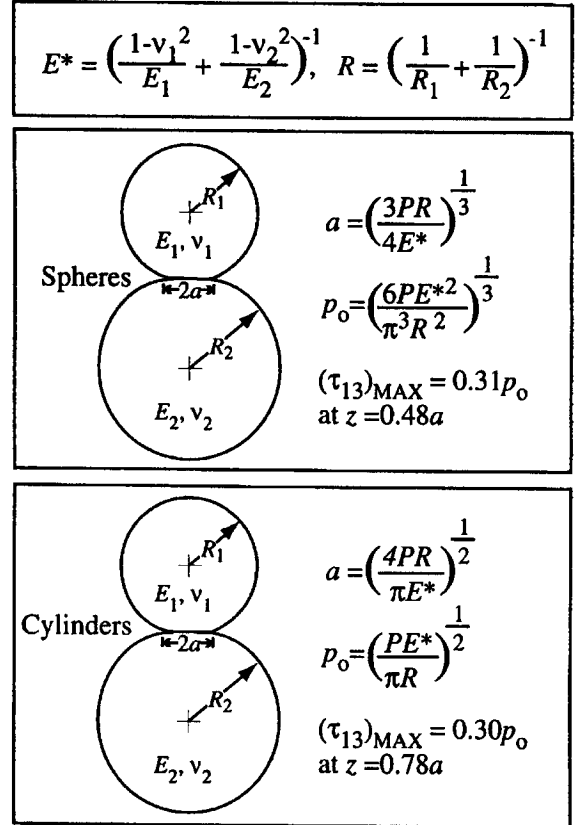
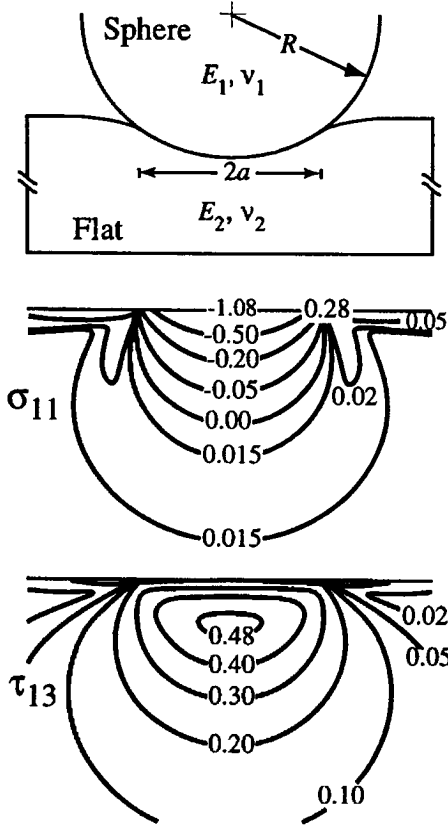


Figure 1

Schematic illustration of Hertzian contact between spheres and cylinders, and corresponding relations for contact radius (a), contact pressure (p_0), and maximum principal shear stress ($(\tau_{13})_{MAX}$) (after Johnson 1985).


Figure 2

Schematic, cross-sectional illustration of sphere-on-flat (cross-section) Hertzian indentation (top). Corresponding stress contours for principal normal stress (σ_{11}) and principal shear stress (τ_{13}) in the flat (for $\nu_2 = 0.22$). Stress values normalized to p_0 (after Lawn 1998).

on-flat case, onset of plasticity in the flat occurs at a depth $0.48a$ ($\nu_2 = 0.3$) for a normal load given by (Johnson 1985):

$$P_Y = \frac{\pi^3 R^2}{6E^{*2}} (p_0)_Y^3 \quad (1)$$

where R and E^* are defined in Fig. 1, and the subscript Y refers to the critical value, and

$$(p_0)_Y = 3.23\tau_Y = 1.62\sigma_Y, \quad (\text{Tresca criterion; } 2\tau_Y = \sigma_Y) \quad (2a)$$

$$(p_0)_Y = 2.80\tau_Y = 1.62\sigma_Y, \quad (\text{von Mises criterion; } \tau_Y^2 = 0.33\sigma_Y^2) \quad (2b)$$

with τ_Y and σ_Y being shear and tensile (or compressive) yield points of the metal, respectively.

Note that for maximum contact loads well below

the elastic limit, a small amount of irreversibility occurs during a contact cycle between metals. This is particularly true for contact between dissimilar metals, where interfacial slip can occur. This “elastic hysteresis” effect becomes significant when a large number of contact cycles (fatigue) are involved (Johnson 1985).

In the case of brittle, heterogeneous materials that are weak in shear, a similar analysis can be used to predict onset of shear failure using an appropriate failure criterion (Lawn 1998, Lawn *et al.* 1994).

3. Hertzian Cone Fracture in Brittle Solids

Beyond a critical load (P_c) during the Hertzian indentation of a brittle solid (hard sphere on a glass or ceramic substrate) a stable crack forms at the edge of the contact and it propagates downward and outward into that solid in the form of a truncated cone (frustum). As mentioned earlier, the tensile σ_{11} outside the contact circle is responsible for this fracture. This cone fracture originates from a shallow ring crack, which forms at the edge of the contact (Frank and Lawn 1967). This precursor ring crack initiates at flaws on the surface near the edge of contact where the tensile stress is concentrated. The ring crack then flares out into a frustum, following a path determined by the stress trajectories and other effects including stress relaxation. The critical load for cone fracture follows the Auerbach law, $P_c \propto R$, for a wide range of sphere sizes (Frank and Lawn 1967, Auerbach 1891). As the applied contact load is increased beyond P_c , the contact circle engulfs the surface ring crack, resulting in the generation of secondary, concentric ring, and zone cracks. The Auerbach law may be used to determine the toughness, K_{IC} , of a brittle solid using the following relation:

$$\frac{P_c}{R} = \frac{AK_{IC}^2}{E_2} = \text{constant} \quad (3a)$$

where A is a constant containing the elastic properties of the materials and a dimensionless weight function for the cone-crack system ($\phi \approx 0.0011$), and is given by (Fischer-Cripps 2000):

$$A = \frac{k\pi^3}{3(1-\nu_2^2)\phi} \quad (3b)$$

The constant k is given by (Fischer-Cripps 2000):

$$k = \frac{9}{16} \left[(1-\nu_2^2) + \frac{E_2}{E_1}(1-\nu_1^2) \right] \quad (3c)$$

where the subscripts 1 and 2 represent the spherical indenter and the flat specimen, respectively. However,

due to the simplifying assumptions made in deriving Eqn. (3a), the toughness values determined using that equation may not be very accurate, and more complex analyses may be required (Lawn 1998). Also, note that the critical load P_c for a particular solid and loading configuration is not unique, but depends on the distribution of the surface flaws in the vicinity of the edge of contact (Lawn 1993). In fact, Hertzian indentation can be used to determine the surface-flaw distributions (Lawn 1993).

4. Non-Hertzian Contacts

The above effects can be analyzed for other contact geometries with appropriate modifications to Hertzian theory (Johnson 1985). These geometries include blunt wedge-on-flat, cone-on-flat, punch (e.g., cylinder end)-on-flat, two conforming surfaces (e.g., pin in a hole with small clearance), etc. The Hertzian theory can also be modified to consider friction at the contact interface, sliding contacts, rolling contacts, torsional contacts, etc. Furthermore, analyses of contacts between solids with inhomogeneous elastic properties, such as multi-phase composites (Lawn *et al.* 1994), layered materials (Lawn 1998), thin films (Weppelman and Swain 1996), coatings (Ramsey *et al.* 1991), and graded materials (Suresh and Mortensen 1998) find their origins in the Hertzian theory. Note that it is not always possible to obtain analytical solutions for these

non-Hertzian contact situations: one needs to resort to numerical and/or finite element methods.

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