Several particular types of instability for bluff bodies have been defined. Three of these are summarized in Table 5.1.

Coupled aeroelastic instabilities in relation to long-span bridge decks, and flutter derivatives, are further discussed in Chapter 12, Bridges.

5.5.4 Lock-in

Motion-induced forces can occur during vibration produced by vortex shedding (Section 4.6.3). Through a feedback mechanism, the frequency of the shedding of vortices can 'lock-in' to the frequency of motion of the body. The strength of the vortices shed, and the fluctuating forces resulting are also enhanced. *Lock-in* has been observed many times during the vibration of lightly damped cylindrical structures such as steel chimneys, and occasionally during the vortex-induced vibration of long-span bridges.

5.6 Fatigue under wind loading

5.6.1 Metallic fatigue

The 'fatigue' of metallic materials under cyclic loading has been well researched, although the treatment of fatigue damage under the random dynamic loading characteristic of wind loading is less well developed.

In the usual failure model for the fatigue of metals it is assumed that each cycle of a sinusoidal stress response inflicts an increment of damage which depends on the amplitude of the stress. Each successive cycle then generates additional damage which accumulates in proportion to the number of cycles until failure occurs. The results of constant amplitude fatigue tests are usually expressed in the form of an *s-N* curve, where s is the stress amplitude, and N is the number of cycles until failure. For many materials, the *s-N* curve is well approximated by a straight line when log s is plotted against log N (Figure 5.15). This implies an equation of the form:

$$Ns^m = K \tag{5.44}$$

where K is a constant which depends on the material, and the exponent m varies between about 5 and 20.

A criterion for failure under repeated loading, with a range of different amplitudes is Miner's rule:

$$\sum\left(\frac{n_i}{N_i}\right) = 1 \tag{5.45}$$

Table 5.1 Types of aerodynamic instabilities

Name	*Conditions*	*Type of motion*	*Type of section*
Galloping	$H_1 > 0$	translational	Square section
'Stall' flutter	$A_2 > 0$	rotational	Rectangle, H-section
'Classical' flutter	$H_2 > 0$, $A_1 > 0$	coupled	Flat plate, airfoil

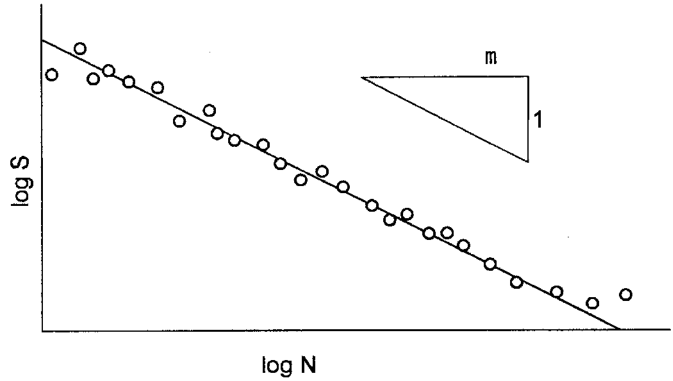

Figure 5.15 Form of a typical *s-N* curve.

where n_i is the number of stress cycles at an amplitude for which N_i cycles are required to cause failure. Thus failure is expected when the sum of the fractional damage for all stress levels is unity.

Note that there is no restriction on the *order* in which the various stress amplitudes are applied in Miner's rule. Thus we may apply it to a random loading process which can be considered as a series of cycles with randomly varying amplitudes.

5.6.2 Narrow band fatigue loading

Some wind loading situations produce resonant 'narrow-band' vibrations. For example, the along-wind response of structures with low natural frequencies (Section 5.3.1), and cross-wind vortex induced response of circular cylindrical structures with low damping. In these cases, the resulting stress variations can be regarded as quasi-sinusoidal with randomly varying amplitudes, as shown in Figure 5.16.

For a narrow-band random stress $s(t)$, the proportion of cycles with amplitudes in the range from s to $s + \delta s$, is $f_p(s)$. δs, where $f_p(s)$ is the probability density of the peaks. The total number of cycles in a time period, T, is $\nu_o^+ T$, where ν_o^+ is the rate of crossing of the

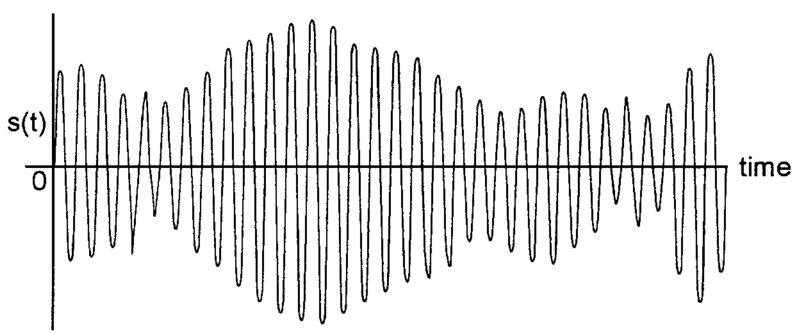

Figure 5.16 Stress-time history under narrow-band random vibrations.

mean stress. For narrow band resonant vibration, ν_o^+ may be taken to be equal to the natural frequency of vibration.

Then the total number of cycles with amplitudes in the range s to δs,

$$n(s) = \nu_o^+ T f_p(s).\delta s \tag{5.46}$$

If $N(s)$ is the number of cycles at amplitude s to cause failure, then the fractional damage at this stress level

$$= \frac{n(s)}{N(s)} = \frac{\nu_o^+ T f_p(s) s^m \delta s}{K}$$

where equation (5.46) has been used for $n(s)$, and equation (5.44) for $N(s)$.

The total expected fractional damage over all stress amplitudes is then, by Miner's rule:

$$D = \sum_0^\infty \frac{n(s)}{N(s)} = \frac{\nu_o^+ T \int_0^\infty f_p(s) s^m ds}{K} \tag{5.47}$$

Wind-induced narrow-band vibrations can be taken to have a normal or Gaussian probability distribution (Section C3.1, Appendix C). If this is the case then the peaks or amplitudes, s, have a Rayleigh distribution (e.g. Crandall and Mark, 1963):

$$f_p(s) = \frac{s}{\sigma^2} \exp\left(-\frac{s^2}{2\sigma^2} \right) \tag{5.48}$$

where σ is the standard deviation of the entire stress history. Derivation of equation (5.48) is based on the level crossing formula of Rice (1944–5).

Substituting into equation (5.47),

$$D = \frac{\nu_o^+ T}{K\sigma^2} \int_0^\infty s^{m+1} \exp\left(-\frac{s^2}{2\sigma^2} \right) ds = \frac{\nu_o^+ T}{K} (\sqrt{2}\sigma)^m \Gamma\left(\frac{m}{2} + 1 \right) \tag{5.49}$$

Here the following mathematical result has been used (Crandall and Mark, 1963):

$$\int_0^\infty x^n \exp\left(-\frac{x^2}{2\sigma^2} \right) dx = \frac{(\sqrt{2}\sigma)^{n+1}}{2} \Gamma\left(\frac{n+1}{2} \right) \tag{5.50}$$

$\Gamma(x)$ is the Gamma function.

Equation (5.49) is a very useful 'closed-form' result, but it is restricted by two important assumptions:

- 'high-cycle' fatigue behaviour in which steel is in the elastic range, and for which an s-N curve of the form of equation (5.44) is valid, has been assumed

- narrow band vibration in a single resonant mode of the form shown in Figure 5.16 has been assumed. In wind loading this is a good model of the behaviour for vortex-shedding induced vibrations in low turbulence conditions. For along-wind loading, the background (subresonant) components are almost always important and result in a random wide-band response of the structure.

5.6.3 Wide band fatigue loading

Wide band random vibration consists of contributions over a broad range of frequencies, with a large resonant peak – this type of response is typical for wind loading (Figure 5.7). A number of cycle counting methods for wide band stress variations have been proposed (Dowling, 1972). One of the most realistic of these is the 'rainflow' method proposed by Matsuishi and Endo (1968). In this method, which uses the analogy of rain flowing over the undulations of a roof, cycles associated with complete hysteresis cycles of the metal, are identified. Use of this method rather than a simple level-crossing approach which is the basis of the narrow-band approach described in Section 5.6.2, invariably results in fewer cycle counts.

A useful empirical approach has been proposed by Wirsching and Light (1980). They proposed that the fractional fatigue damage under a wide-band random stress variation can be written as:

$$D = \lambda D_{nb} \tag{5.51}$$

where, D_{nb} is the damage calculated for narrow-band vibration with the same standard deviation, σ (equation 5.49). λ is a parameter determined empirically. The approach used to determine λ was to use simulations of wide-band processes with spectral densities of various shapes and bandwidths, and rainflow counting for fatigue cycles.

The formula proposed by Wirsching and Light to estimate λ was:

$$\lambda = a + (1 - a)(1 - \varepsilon)^b \tag{5.52}$$

where a and b are functions of the exponent m (equation 5.44), obtained by least-squares fitting, as follows:

$$a \cong 0.926 - 0.033\ \mathrm{m} \tag{5.53}$$

$$b \cong 1.587\ \mathrm{m} - 2.323 \tag{5.54}$$

ε is a spectral bandwidth parameter equal to:

$$\varepsilon = 1 - \frac{\mu_2^2}{\mu_0\mu_4} \tag{5.55}$$

where, μ_k is the kth moment of the spectral density defined by:

$$\mu_k = \int_0^\infty n^k S(n)dn \tag{5.56}$$

For narrow band vibration ε tends to zero, and, from equation (5.52), λ approaches 1. As ε tends to its maximum possible value of 1, λ approaches a, given by equation (5.53). These values enable upper and lower limits on the damage to be determined.

5.6.4 Effect of varying wind speed

Equation (5.49) applies to a particular standard deviation of stress, σ, which in turn is a function of mean wind speed, $\bar{U}$ This relationship can be written in the form:

$$\sigma = A\bar{U}^n \tag{5.57}$$

The mean wind speed, $\bar{U}$, itself, is a random variable. Its probability distribution can be represented by a Weibull distribution (see Section 2.5 and C.3.4):

$$f_U(\bar{U}) = \frac{k\bar{U}^{k-1}}{c^k}\exp\left[-\left(\frac{\bar{U}}{c}\right)^k\right] \tag{5.58}$$

The total damage from narrow-band vibration for all possible mean wind speeds is obtained from equations (5.49), (5.57) and (5.58) and integrating.

The fraction of the time T during which the mean wind speed falls between U and $U + \delta U$ is $f_U(U).\delta U$.

Hence the amount of damage generated while this range of wind speed occurs is from equations (5.49) and (5.57):

$$D_U = \frac{\nu_o^+ T f_U(U)\delta U}{K}(\sqrt{2}AU^n)^m\Gamma\left(\frac{m}{2}+1\right)$$

The total damage in time T during all mean wind speeds between 0 and ∞,

$$D = \frac{\nu_o^+ T(\sqrt{2}A)^m}{K}\Gamma\left(\frac{m}{2}+1\right)\int_0^\infty U^{mn} f_U(U)dU$$

$$= \frac{\nu_o^+ T(\sqrt{2}A)^m}{K}\Gamma\left(\frac{m}{2}+1\right)\int_0^\infty U^{mn+k-1}\frac{k}{c^k}\exp\left[-\left(\frac{U}{c}\right)^k\right]dU \tag{5.59}$$

This can be integrated numerically for general values of k. Usually k is around 2, in which case,

$$D = \frac{2\nu_o^+ T(\sqrt{2}A)^m}{Kc^2}\Gamma\left(\frac{m}{2}+1\right)\int_0^\infty U^{mn+1}\exp\left[-\left(\frac{U}{c}\right)^2\right]dU$$

This is now of the form of equation (5.50), so that:

$$D = \frac{2\nu_o^+ T(\sqrt{2}A)^m}{Kc^2}\Gamma\left(\frac{m}{2}+1\right)\frac{c^{mn+2}}{2}\Gamma\left(\frac{mn+2}{2}\right)$$

$$= \frac{\nu_o^+ T(\sqrt{2}A)^m c^{mn}}{K} \Gamma\left(\frac{m}{2} + 1\right) \Gamma\left(\frac{mn + 2}{2}\right) \tag{5.60}$$

This is a useful closed form expression for the fatigue damage over a lifetime of wind speeds, assuming narrow band vibration.

For wide band vibration, equation (5.60) can be modified, following equation (5.51), to:

$$D = \frac{\lambda \nu_o^+ T(\sqrt{2}A)^m c^{mn}}{K} \Gamma\left(\frac{m}{2} + 1\right) \Gamma\left(\frac{mn + 2}{2}\right) \tag{5.61}$$

By setting D equal to 1 in equations (5.60) and (5.61), we can obtain lower and upper limits to the fatigue life as follows:

$$T_{lower} = \frac{K}{\nu_o^+ (\sqrt{2}A)^m c^{mn} \Gamma\left(\frac{m}{2} + 1\right) \Gamma\left(\frac{mn + 2}{2}\right)} \tag{5.62}$$

$$T_{upper} = \frac{K}{\lambda \nu_o^+ (\sqrt{2}A)^m c^{mn} \Gamma\left(\frac{m}{2} + 1\right) \Gamma\left(\frac{mn + 2}{2}\right)} \tag{5.63}$$

Example

To enable the calculation of fatigue life of a welded connection at the base of a steel pole, using equations (5.62) and (5.63), the following values are assumed:

$m = 5$; $n = 2$; $\nu_0^+ = 1.0$ Hertz (the natural frequency of the pole) for the lower limit; 0.5 Hertz (one half the natural frequency) for the upper limit of fatigue life

$$K = 2 \times 10^{15}\ [\text{MPa}]^{1/5};\ c = 8\ \text{m/s};\ A = 0.1 \frac{\text{MPa}}{(\text{m/s})^2}$$

$$\Gamma\left(\frac{m}{2} + 1\right) = \Gamma(3.5) = e^{1.201} = 3.323$$

$$\Gamma\left(\frac{mn + 2}{2}\right) = \Gamma(6) = 5! = 120$$

Then from equation (5.62),

$$T_{lower} = \frac{2 \times 10^{15}}{1.0 \times (\sqrt{2} \times 0.1)^5 \times 8^{10} \times 3.323 \times 120.0}$$

$$= 0.826 \times 10^8 \text{ secs} = \frac{0.826 \times 10^8}{365 \times 24 \times 3600} \text{ years} = 2.62 \text{ years}$$

From equation (5.53), $a = 0.926 - 0.033\text{m} = 0.761$.
From equation (5.52), this is a lower limit for λ

$$T_{upper} = \frac{2T_{lower}}{\lambda} = \frac{5.24}{0.761} \text{ years} = 6.88 \text{ years}$$

This example illustrates the sensitivity of the estimates of fatigue life to the values of both A and c. For example, increasing A to $0.15\frac{\text{MPa}}{(\text{m/s})^2}$ would decrease the fatigue life by 7.6 times (1.5^5). Decreasing c from 8 to 7 m/s will increase the fatigue life by 3.8 times $(8/7)^{10}$.

5.7 Summary

This chapter has covered a wide range of topics relating to the dynamic response of structures to wind forces. For wind loading, the subresonant or background response should be distinguished from the contributions at the resonant frequencies and calculated separately.

The along-wind response of structures that can be represented as single- and multi-degree-of-freedom systems has been considered. The effective static load approach in which the distributions of the mean, background and resonant contributions to the loading are considered separately, and assembled as a combined effective static wind load, has been presented.

Aeroelastic effects such as aerodynamic damping, and the instabilities of galloping and flutter have been introduced. Finally wind-induced fatigue has been treated resulting in usable formulae for the calculation of fatigue life of a structure under along-wind loading.

Cross-wind dynamic response from vortex shedding has not been treated in this chapter, but is discussed in Chapters 9 and 11.

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