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STUDENT > #  Attempt to verify conclusion for light-bulb test of
          phase rotation using superposition
STUDENT > # see
          http://www.eng-tips.com/viewthread.cfm?qid=240139&page=1
STUDENT > restart;
STUDENT > # ===SYMBOLS =====================
STUDENT > # Va, Vb, Vc are voltages referenced to the POWER SUPPLY
          neutral (assumed balanced)
STUDENT > # Vn is the "FLOATING neutral" created at the wye point of
          the 2 R's and C(different than power supply neutral)
STUDENT > # (power supply neutral and this floating neutral are two
          different things)
STUDENT > # Phasors are assumed rotating CCW in the complex plane
STUDENT > # a terminal which leads another terminal has a more
          positive phase angle
STUDENT > # a terminal which lags another terminal has a more
          negative phase angle
STUDENT > # A:=exp(I*2*Pi/3) is the 120 deg phase shift between
          original vectors Va,Vb, Vc
STUDENT > # All voltage magnitudes Va Vb Vc assumed to have
          magnitude 1
STUDENT > # Arbitrarily start with phase B as our 0 phase angle
STUDENT > # As we go from A to B to C we are going in the lagging
          direction -> divide by A
STUDENT > # Solve Vn by superposition
STUDENT > # Vn = floating neutral voltage (different than power
          supply neutral)
STUDENT > # Vna = contribution to Vn from Va with other two sources
          shorted
STUDENT > # Vnb = contribution to Vn from Vb with other two sources
          shorted
STUDENT > # Vnc = contribution to Vn from Vc with other two sources
          shorted
STUDENT > # Zc = impedance of C
STUDENT > # Zp = impedance of Zc in parallel with R
STUDENT >
STUDENT >
STUDENT > # Find Vna by simple voltage divider
STUDENT > Vna:=Va*Zp/(R+Zp);
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$$Vna := \frac{Va\,Zp}{R + Zp}$$

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STUDENT > # Find Vnc by simple voltage divider
STUDENT > Vnc:=Vc*Zp/(R+Zp);
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$$Vnc := \frac{Vc\ Zp}{R + Zp}$$

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STUDENT > # Find Vnb by simple voltage divider
STUDENT > Vnb:=Vb*(R/2)/(R/2 + Zc);
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$$Vnb := \frac{1}{2} \frac{Vb\ R}{\frac{1}{2} R + Zc}$$

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STUDENT > # Vn is sum of Vna, Vnb, Vnc:
STUDENT > Vn:=Vna+Vnb+Vnc;
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$$Vn := \frac{Va\ Zp}{R + Zp} + \frac{1}{2} \frac{Vb\ R}{\frac{1}{2} R + Zc} + \frac{Vc\ Zp}{R + Zp}$$

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STUDENT > # Compute values of Zc and Zp which will be used in  Vn
STUDENT > Zc:=1/(I*w*C);
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$$Zc := -\frac{I}{w\ C}$$

```
STUDENT > Zp:=Zc*R/(Zc+R);
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$$Zp := -\frac{I\ R}{w\ C \left(-\frac{I}{w\ C} + R \right)}$$

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STUDENT > # Display Vn with new values Zc and Zp substituted
STUDENT > Vn;
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$$-\frac{I\ Va\ R}{w\ C \left(-\frac{I}{w\ C} + R \right) \left(R - \frac{I\ R}{w\ C \left(-\frac{I}{w\ C} + R \right)} \right)} + \frac{1}{2} \frac{Vb\ R}{\frac{1}{2} R - \frac{I}{w\ C}}$$
$$- \frac{I\ Vc\ R}{w\ C \left(-\frac{I}{w\ C} + R \right) \left(R - \frac{I\ R}{w\ C \left(-\frac{I}{w\ C} + R \right)} \right)}$$

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STUDENT >
STUDENT >
STUDENT >
STUDENT > # Simplify
STUDENT > Vn:=simplify(Vn);
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$$Vn := \frac{-I\ Va + Vb\ R\ w\ C - I\ Vc}{-2\ I + R\ w\ C}$$

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STUDENT > # THE ABOVE EXPRESSION IS ENOUGH TO ANSWER THE QUESTION
STUDENT > # ASSUME R*w*C <<1. =>  Denominator ~ -2 *I
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STUDENT > # Numerator is -I*(Va + Vc) + Vb * R * w * C
STUDENT > # When we divide numerator by -2*I we get
STUDENT > # Vn ~ 0.5*[(Va + Vc) + I*Vb * R * w * C]
STUDENT > # Now we will simply assume Vb = 1 ANGLE 0
STUDENT > # This covers both scenarios
STUDENT > #   ABC rotation: {Vb=1,Vc=1/A,Va=1/A^2};
STUDENT > #   ACB rotation: {Vb=1,Va=1/A,Vc=1/A^2};
STUDENT >
STUDENT > # Since Va and Vc appear symmetrically, we will get the
          same solution Vn
STUDENT > #  regardless of which orientation one we choose
STUDENT >
STUDENT > # To find out whether Vn is closer to Va or Vc we need
          only look at the imaginary part of
STUDENT > # Vn ~ 0.5*[(Va + Vc) + I*Vb * R * w * C]
STUDENT > # given that we assumed angle(Vb)=0, we know
STUDENT > #   Va+Vc is real
STUDENT > #   Vb is real and positive
STUDENT >
STUDENT > #  So we see imaginary part is
STUDENT > #  Im(Vb) =  I*Vb * R * w * C
STUDENT >
STUDENT > #  Since Vb is positive, the imaginary part of Vn is
          postive
STUDENT > #  If we choose ABC rotation, Vn is closer to Va and light
          attached to A will be dimmer
STUDENT > #  If we choose ACB rotation, Vn is closer to Vc and light
          attached to C will be dimmer
STUDENT >
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