fc240 compatibility of deformations STATUS of PC circular section subject to P_{uxy}

- 1 between 15 passive steels
- 1 between 19 active prestressing steels or materials
- concentrical array of strands

Setup for Units is the default to SI.

Initialization

$ORIGIN \equiv 1$ Count with fingers $TOL := 1$ $CTOL := 1$

$ton := 1000 \cdot kgf$ $ksi := 70.307 \cdot \frac{kgf}{cm^2}$ $psi := \frac{ksi}{1000}$ $kip := 453.592 \cdot kgf$ $MPa := \frac{N}{mm^2}$

$$AND2(a,b) := \left| \begin{array}{l} \text{if } a = 1 \\ \quad \left| \begin{array}{l} 1 \text{ if } b = 1 \\ 0 \text{ otherwise} \end{array} \right. \\ 0 \text{ otherwise} \end{array} \right.$$

$$OR2(a,b) := \left| \begin{array}{l} 1 \text{ if } a = 1 \\ \text{otherwise} \\ \quad \left| \begin{array}{l} 1 \text{ if } b = 1 \\ 0 \text{ otherwise} \end{array} \right. \end{array} \right.$$

$DIV(a,b) := floor\left(\frac{a}{b}\right)$ assumed both positive

Section

Circular section od D diameter $D := 50 \cdot cm$

Mesh

$n_{cL} := 5$ annular layers $n_{cM} := 12$ meridian pieces $n_c := n_{cL} \cdot n_{cM}$ total number of concrete elements in the mesh $n_c = 60$

$r := \frac{D}{2}$

$$A_c := \left| \begin{array}{l} k \leftarrow 1 \\ \text{while } k \le n_{cM} \\ \quad \left| \begin{array}{l} j \leftarrow 1 \\ \text{while } j \le n_{cL} \\ \quad \left| \begin{array}{l} A_{c_{(k-1) \cdot n_{cL}+j}} \leftarrow \frac{\pi}{n_{cM}} \cdot \left[\left(\frac{j}{n_{cL}} \cdot r \right)^2 - \left(\left| \begin{array}{l} \frac{j-1}{n_{cL}} \cdot r \text{ if } j > 1 \\ 0 \cdot m \text{ otherwise} \end{array} \right. \right)^2 \right] \\ j \leftarrow j + 1 \end{array} \right. \\ k \leftarrow k + 1 \end{array} \right. \\ \text{return } A_c \end{array} \right.$$

$$
R_c := \left|\begin{array}{l}
k \leftarrow 1 \\
\text{while } k \le n_{cM} \\
\quad \left|\begin{array}{l}
j \leftarrow 1 \\
\text{while } j \le n_{cL} \\
\quad \left|\begin{array}{l}
R_{c_{(k-1)\cdot n_{cL}+j}} \leftarrow \dfrac{2}{3}\cdot\dfrac{\sin\left(\dfrac{2\cdot\pi}{n_{cM}}\right)}{\dfrac{2\cdot\pi}{n_{cM}}}\cdot\left[\dfrac{\left[\left(\dfrac{j}{n_{cL}}\cdot r\right)^3-\left(\left|\begin{array}{ll}\dfrac{j-1}{n_{cL}}\cdot r & \text{if } j>1\\ 0\cdot m & \text{otherwise}\end{array}\right.\right)^3\right]}{\left[\left(\dfrac{j}{n_{cL}}\cdot r\right)^2-\left(\left|\begin{array}{ll}\dfrac{j-1}{n_{cL}}\cdot r & \text{if } j>1\\ 0\cdot m & \text{otherwise}\end{array}\right.\right)^2\right]}\right] \\
j \leftarrow j+1
\end{array}\right. \\
k \leftarrow k+1
\end{array}\right. \\
\text{return } R_c
\end{array}\right.
$$

$$
X_c := \left|\begin{array}{l}
k \leftarrow 1 \\
\text{while } k \le n_{cM} \\
\quad \left|\begin{array}{l}
j \leftarrow 1 \\
\text{while } j \le n_{cL} \\
\quad \left|\begin{array}{l}
X_{c_{(k-1)\cdot n_{cL}+j}} \leftarrow r+\sin\left(\dfrac{k-1}{n_{cM}}\cdot 2\cdot\pi\right)\cdot R_{c_{(k-1)\cdot n_{cL}+j}} \\
j \leftarrow j+1
\end{array}\right. \\
k \leftarrow k+1
\end{array}\right. \\
\text{return } X_c
\end{array}\right.
$$

$$
Y_c := \left|\begin{array}{l}
k \leftarrow 1 \\
\text{while } k \le n_{cM} \\
\quad \left|\begin{array}{l}
j \leftarrow 1 \\
\text{while } j \le n_{cL} \\
\quad \left|\begin{array}{l}
Y_{c_{(k-1)\cdot n_{cL}+j}} \leftarrow r-\cos\left(\dfrac{k-1}{n_{cM}}\cdot 2\cdot\pi\right)\cdot R_{c_{(k-1)\cdot n_{cL}+j}} \\
j \leftarrow j+1
\end{array}\right. \\
k \leftarrow k+1
\end{array}\right. \\
\text{return } Y_c
\end{array}\right.
$$

Solicited by

$P := 100 \cdot ton$

Enter (positive) compression

$M_x := 20 \cdot m \cdot ton$

$M_y := 6 \cdot m \cdot ton$

- enter both moments positive
- will be assumed to compress atop and towards right
- P assumed to act at center of gravity of gross section
- M_x is respect axis paralell to abscissas and M_y respect axis paralell to ordinates
- moments will be combined prior to solving for equilibrium; it is usually stated that otherwise solution is overconservative

Passive Steel

$Steel := 12$

Type following list

$\gamma_y := 1.15$

Steel Material Safety Factor

- Choose one passive Steel type from the list below.
- If you choose one Safety Factor for Steel γ_y (must be bigger than -or equal to- 1) the strength assumed in calculation will be the real one divided by the steel strength reduction factor. Reduction will be by affinity. This normally will be safe
- For earthquake loads safety factor must be 1 to properly capture behaviour
- You can assess the chosen steel performance by the stress-strain diagram as plotted below.
- Any number not corresponding to the list will default to case 1 (perfectly elastic-perfectly plastic steel)

Spanish MPa denoted as per code

1. Any perfectly elastic-perfectly plastic
2. AEH-400 N
3. AEH-500 N
4. AEH-600 N
5. AEH-400 S
6. AEH-500 S
7. B 400 S
8. B 500 S
9. AEH-400 F
10. AEH-500 F
11. AEH-600 F

US ksi denoted

12. Grade 60
13. Grade 65
14. Grade 70
15. Grade 75

Input for and if Steel=1 (Perfectly Elastic-Perfectly Plastic steel)

$f_y := 4100 \frac{kgf}{cm^2}$ will affect exclusively Steel type 1.

$$E_s := \begin{cases} 2100000 \cdot \frac{kgf}{cm^2} & \text{if } Steel \le 11 \\ 29000 \cdot ksi & \text{otherwise} \end{cases}$$

$E_s = 2038903 \frac{kgf}{cm^2}$

Will assume antimmetrical stress-strain laws

Any perfectly elastic, perfectly plastic steel (1)

$\varepsilon_y := \frac{f_y}{E_s}$ $\varepsilon_y = 0.00201$ yield strain when Steel =1

$\frac{f_y}{\gamma_y} = 3565.22 \frac{kgf}{cm^2}$ assumed yield stress when Steel=1

Cold strain-hardened deformed bar steels (9 to 11)

$\sigma x := 5000$ seed, implied kgf/cm2

Given The Ramberg-Osgood branch thing

$$\frac{\sigma x}{\frac{E_s}{\frac{kgf}{cm^2}}} + 0.823\cdot\gamma_y{}^5\cdot\left(\frac{\sigma x}{\frac{f_y}{\frac{kgf}{cm^2}}} - \frac{0.7}{\gamma_y}\right)^5 = \varepsilon$$

$$\sigma_{over\_prop}(\varepsilon) := \frac{kgf}{cm^2}\cdot \mathrm{Find}(\sigma x)$$

Ramberg-Osgood no closed form, and we want such, so we make a fit to it

$$\mathrm{Parts} := 200 \qquad j := 1 .. \mathrm{Parts} + 1 \qquad \varepsilon s_j := \frac{\frac{f_y}{\gamma_y}}{E_s} + \frac{0.035 - \frac{\frac{f_y}{\gamma_y}}{E_s}}{\mathrm{Parts}}\cdot(j-1) \qquad \sigma s_j := \sigma_{over\_prop}(\varepsilon s_j)$$

$$vs := \mathrm{cspline}(\varepsilon s, \sigma s) \qquad f_{ss}(\varepsilon) := \mathrm{interp}(vs, \varepsilon s, \sigma s, \varepsilon)$$

$$f_{scold}(\varepsilon) := \left| \begin{array}{ll} E_s\cdot\varepsilon & \text{if } \varepsilon \le 0.7\cdot\frac{\frac{f_y}{\gamma_y}}{E_s} \\ f_{ss}(\varepsilon) & \text{otherwise} \end{array} \right.$$

Spanish Steels whose stress-strain diagrams are formed by only 2 straight lines per quadrant (2 to 8)

New B 400 S and B 500 S are made equal to AEH-400 S and AEH-500 S which are very similar

$f_y := 4100 \cdot \frac{kgf}{cm^2} \qquad f_u := 4305 \cdot \frac{kgf}{cm^2} \qquad \varepsilon_u := 0.08$

$f_{inc}(f_u) := f_u - f_y$ increment of stress from turning point (fy is surmised data)

$$E_2(\varepsilon_y, \varepsilon_u, f_u) := \begin{cases} \dfrac{f_{inc}(f_u)}{\varepsilon_u - \varepsilon_y} & \text{if } \gamma_y = 1 \\[2ex] \dfrac{\dfrac{f_{inc}(f_u)}{\gamma_y}}{\left(\varepsilon_u - \dfrac{\dfrac{\gamma_y - 1}{\gamma_y} \cdot f_u}{E_s}\right) - \dfrac{\varepsilon_y}{\gamma_y}} & \text{otherwise} \end{cases}$$

slope at strain hardening if any

$$f_{s2lines}(\varepsilon, \varepsilon_y, \varepsilon_u, f_u) := \begin{cases} E_s \cdot \varepsilon & \text{if } \varepsilon \le \dfrac{\varepsilon_y}{\gamma_y} \\[2ex] \dfrac{f_y}{\gamma_y} + E_2(\varepsilon_y, \varepsilon_u, f_u) \cdot \left(\varepsilon - \dfrac{\varepsilon_y}{\gamma_y}\right) & \text{otherwise} \end{cases}$$

American Reinforcing Passive Deformed Bars (12 to 15)

These we rarely will use with any (steel) strength reduction factor, since this is not usual in american codes; still, we will formulate this also for consistency and completeness of formulation.

$$
USA := \begin{array}{c}
\begin{array}{ccccccccccc} \text{Gr} & f_{\psi} & f_{\sigma\upsilon} & D\phi_{\sigma} & e_{\psi} & e_{\sigma\eta} & e_{\sigma\upsilon} & \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\
\begin{pmatrix}
60 & 4218.42 & 7354.11 & 3135.69 & .002016 & .0091 & .0729 & 1.748272 & .173674 & -.251726 & 1.173672 \\
65 & 4569.96 & 7536.91 & 2966.96 & .002204 & .0086 & .0717 & 1.75624 & -.145637 & -.243758 & .854363 \\
70 & 4921.49 & 7719.71 & 2798.22 & .002396 & .0082 & .0706 & 1.766823 & -.416587 & -.233175 & .583412 \\
75 & 5273.03 & 7902.51 & 2629.48 & .002592 & .0077 & .0694 & 1.780521 & -.655189 & -.219478 & .344811
\end{pmatrix}
\end{array}
$$

The values are for static loads and don't take into account the higher values attainable under high strain loading rates

$$
XPAR(\varepsilon, Row) := \frac{\varepsilon - USA_{Row,6}}{USA_{Row,7} - USA_{Row,6}}
$$

$$
YPAR(\varepsilon, Row) := \frac{USA_{Row,8} \cdot XPAR(\varepsilon, Row) + USA_{Row,9} \cdot XPAR(\varepsilon, Row)^2}{1 + USA_{Row,9} \cdot USA_{Row,10} + USA_{Row,11} \cdot XPAR(\varepsilon, Row)^2}
$$

$$
f_{USA}(\varepsilon, Row) := \begin{cases} E_s \cdot \varepsilon & \text{if } \varepsilon \le \dfrac{USA_{Row,5}}{\gamma_y} \\ \dfrac{\frac{kgf}{cm^2}}{\gamma_y} \cdot \begin{cases} USA_{Row,2} & \text{if } \varepsilon \le USA_{Row,6} \\ \left[YPAR(\varepsilon, Row) \cdot \left(USA_{Row,3} - USA_{Row,2}\right)\right] + USA_{Row,2} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}
$$

$$
f_{s\_positive}(\varepsilon) := \left|
\begin{array}{l}
f_{s2lines}\left(\varepsilon, \dfrac{4100\cdot\dfrac{kgf}{cm^2}}{E_s}, 0.1, 4100\cdot\dfrac{kgf}{cm^2}\right) \ \text{ if } Steel = 2 \\
f_{s2lines}\left(\varepsilon, \dfrac{5100\cdot\dfrac{kgf}{cm^2}}{E_s}, 0.1, 5100\cdot\dfrac{kgf}{cm^2}\right) \ \text{ if } Steel = 3 \\
f_{s2lines}\left(\varepsilon, \dfrac{6100\cdot\dfrac{kgf}{cm^2}}{E_s}, 0.1, 6100\cdot\dfrac{kgf}{cm^2}\right) \ \text{ if } Steel = 4 \\
f_{s2lines}\left(\varepsilon, \dfrac{4100\cdot\dfrac{kgf}{cm^2}}{E_s}, 0.08, 4305\cdot\dfrac{kgf}{cm^2}\right) \ \text{ if } Steel = 5 \\
f_{s2lines}\left(\varepsilon, \dfrac{5100\cdot\dfrac{kgf}{cm^2}}{E_s}, 0.05, 5355\cdot\dfrac{kgf}{cm^2}\right) \ \text{ if } Steel = 6 \\
f_{s2lines}\left(\varepsilon, \dfrac{4100\cdot\dfrac{kgf}{cm^2}}{E_s}, 0.08, 4305\cdot\dfrac{kgf}{cm^2}\right) \ \text{ if } Steel = 7 \\
f_{s2lines}\left(\varepsilon, \dfrac{5100\cdot\dfrac{kgf}{cm^2}}{E_s}, 0.05, 5355\cdot\dfrac{kgf}{cm^2}\right) \ \text{ if } Steel = 8 \\
\text{if } Steel = 9 \\
\quad \left| \begin{array}{l} f_y \leftarrow 4100\cdot\dfrac{kgf}{cm^2} \\ \text{return } f_{scold}(\varepsilon) \end{array} \right. \\
\text{if } Steel = 10 \\
\quad \left| \begin{array}{l} f_y \leftarrow 5100\cdot\dfrac{kgf}{cm^2} \\ \text{return } f_{scold}(\varepsilon) \end{array} \right. \\
\text{if } Steel = 11 \\
\quad \left| \begin{array}{l} f_y \leftarrow 6100\cdot\dfrac{kgf}{cm^2} \\ \text{return } f_{scold}(\varepsilon) \end{array} \right. \\
\text{if } Steel = 12 \\
\quad \left| \begin{array}{l} Row \leftarrow 1 \\ \text{return } f_{USA}(\varepsilon, Row) \end{array} \right.
\end{array}
\right.
$$

$$
Passive\_Steel_{Type} := \left|
\begin{array}{l}
\text{"AEH-400 N" if } Steel = 2 \\
\text{"AEH-500 N" if } Steel = 3 \\
\text{"AEH-600 N" if } Steel = 4 \\
\text{"AEH-400 S" if } Steel = 5 \\
\text{"AEH-500 S" if } Steel = 6 \\
\text{"B 400 S" if } Steel = 7 \\
\text{"B 500 S" if } Steel = 8 \\
\text{"AEH-400 F" if } Steel = 9 \\
\text{"AEH-500 F" if } Steel = 10 \\
\text{"AEH-600 F" if } Steel = 11 \\
\text{"Grade 60" if } Steel = 12 \\
\text{"Grade 65" if } Steel = 13 \\
\text{"Grade 70" if } Steel = 14 \\
\text{"Grade 75" if } Steel = 15 \\
\text{return "Generic Perfectly Elastic - Perfectly Plastic" otherwise}
\end{array}
\right.
$$

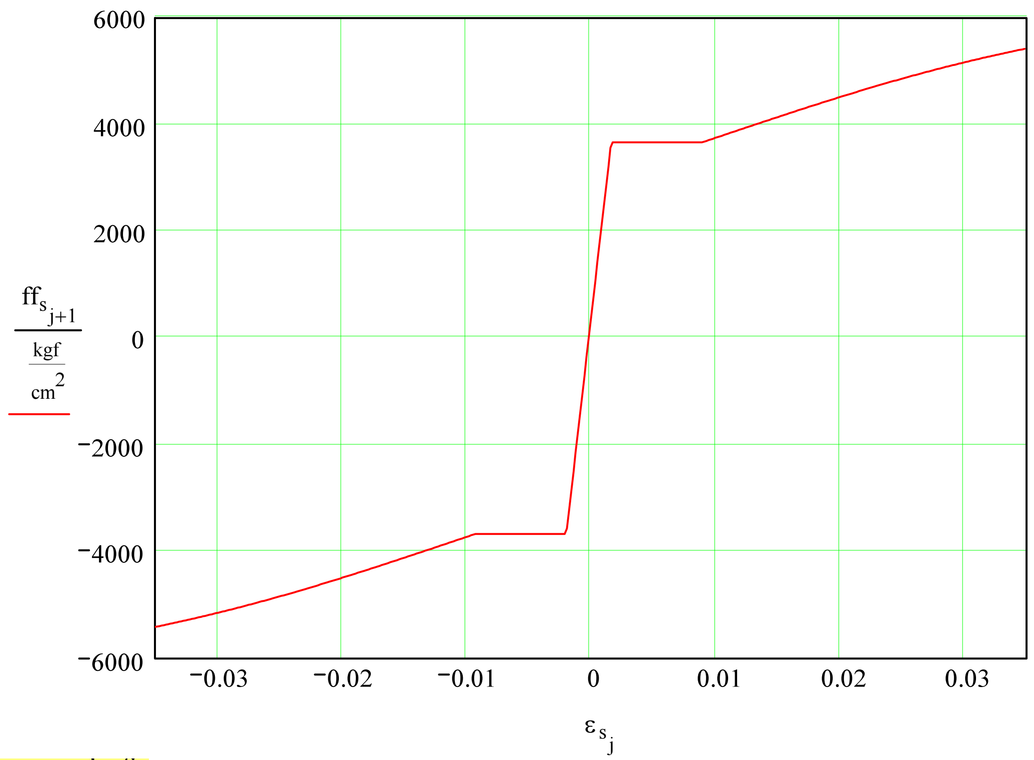
$$
\begin{array}{l}
\quad \text{return } f_{USA}(\varepsilon, Row) \\
\text{if Steel} = 13 \\
\quad \left| \begin{array}{l} Row \leftarrow 2 \\ \text{return } f_{USA}(\varepsilon, Row) \end{array} \right. \\
\text{if Steel} = 14 \\
\quad \left| \begin{array}{l} Row \leftarrow 3 \\ \text{return } f_{USA}(\varepsilon, Row) \end{array} \right. \\
\text{if Steel} = 15 \\
\quad \left| \begin{array}{l} Row \leftarrow 4 \\ \text{return } f_{USA}(\varepsilon, Row) \end{array} \right. \\
\text{otherwise} \\
\quad \left| \begin{array}{l} E_s \cdot \varepsilon \quad \text{if } \varepsilon \le \dfrac{\varepsilon_y}{\gamma_y} \\ \dfrac{E_s \cdot \varepsilon_y}{\gamma_y} \quad \text{otherwise} \end{array} \right.
\end{array}
$$

$$
f_y := \left| \begin{array}{l}
4100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 2 \\
5100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 3 \\
6100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 4 \\
4100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 5 \\
5100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 6 \\
4100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 7 \\
5100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 8 \\
4100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 9 \\
5100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 10 \\
6100 \cdot \dfrac{kgf}{cm^2} \quad \text{if Steel} = 11
\end{array} \right.
$$

$$\left|\begin{array}{l} \mathrm{cm}^{-} \\ 60\cdot\mathrm{ksi}\ \text{ if }\ Steel = 12 \\ 65\cdot\mathrm{ksi}\ \text{ if }\ Steel = 13 \\ 70\cdot\mathrm{ksi}\ \text{ if }\ Steel = 14 \\ 75\cdot\mathrm{ksi}\ \text{ if }\ Steel = 15 \\ f_y\ \text{ otherwise} \end{array}\right.$$

$$f_s(\varepsilon) := \left|\begin{array}{l} f_{s\_positive}(\varepsilon)\ \text{ if }\ \varepsilon \ge 0 \\ -f_{s\_positive}(-\varepsilon)\ \text{ otherwise} \end{array}\right.$$

$j := 0..400$ $\qquad \varepsilon_{s_{j+1}} := \dfrac{0.035}{200}\cdot(j-200)$ $\qquad ff_{s_{j+1}} := f_s\left(\varepsilon_{s_{j+1}}\right)$



$f_y = 4218.42\,\dfrac{kgf}{cm^2}$ $\qquad \gamma_y = 1.15$ $\qquad Steel = 12$ $\qquad Passive_Steel_{Type} = $ "Grade 60"

Passive Steel Geometry

$n := 6$ number of bars $\qquad \Phi_{bar} := 20\cdot mm$ $\qquad Cover\_to\_axis := 5\cdot cm$

$\alpha_{0s} := 30\cdot deg$ centered angle, from (negative or) -Y axis towards lowest passive bar center, enter positive if such bar to be found counterclockwise

$\beta := \operatorname{atan}\left(\frac{M_y}{M_x}\right)$ $\beta = 16.7\,deg$ $\alpha_s := \alpha_{0s} - \beta$ $M_u := \sqrt{M_x^2 + M_y^2}$ $M_u = 20.88\,m\cdot ton$ the combination moment we will be checking

the above defined passive bar is found (respect rotated -Y' axis --plane of flexion--, from moment combination) at $\alpha_s = 13.3\,deg$ counterclockwise, if positive

Equilibrium will be defined respect such new X' and Y' axes. We will omit primes in our notation from now on.

$r := \frac{D}{2}$ $r_{to\_bar} := r - Cover\_to\_axis$ $r_{to\_bar} = 20\,cm$

$j := 1..n$ $X_{s_j} := r + r_{to\_bar}\cdot\sin\left[\alpha_s + (j-1)\cdot\frac{360\cdot deg}{n}\right]$ $Y_{s_j} := r - r_{to\_bar}\cdot\cos\left[\alpha_s + (j-1)\cdot\frac{360\cdot deg}{n}\right]$ perpendicular and paralell to combination moment Y plane

since origin at lower left corner of encasing adjusted square

Change of axis we have made only to use single Mu; equilibrium we will establish nevertheless biaxially to account for any dissymmetry

$A_{1\_bar} := \pi\cdot\frac{\Phi_{bar}^{\ 2}}{4}$ $A_{1\_bar} = 3.14\,cm^2$ $A_{s_j} := A_{1\_bar}$

Active (PRESTRESSING or POSTENSIONING) Steel (or FRP cable)

$Prest_{mat} := 11$ — Type following list

$\gamma_{py} := 1$ — Prest. Material Safety Factor

$fpyf := 0.915$ this is fpy/fpu

$k_{ps} := 0.55$ fraction of fpu at which the prestressing material is assumed to pass the section (without taking into account the moment effects brought by prestress) this percent permits to evaluate the prestress forces, initial without moment and then in equilibrium with the moment

- must be lower than $fpyf$ (the prestressing material is not allowed to undergo anelastic deformation at prestress nor service level limit states)
- it is assumed the same degree of prestress will be imparted to all prestressing material.

Input for material 1 (only affects it)

$E_{p1} := 20000\cdot ksi$ $f_{pu1} := 250\cdot ksi$ $\varepsilon_{pu1} := \frac{f_{pu1}}{E_{p1}}$

$f_{py1} := f_{pu1}$ per definition

$f_{pu1} = 17576.75\,\frac{kgf}{cm^2}$ warranted strength

$\varepsilon_{pu1} = 0.0125$

Spanish MPa denoted as per code

1. Any perfectly elastic material till fracture
2. Y 1570 C wire
3. Y 1670 C wire
4. Y 1770 C wire
5. Y 1860 C wire
6. Y 1770 S2-wires
7. Y 1860 S3-wires
8. Y 1960 S3-wires
9. Y 2060 S3-wires
10. Y 1770 S7 strand
11. Y 1860 S7 strand

US ksi denoted

12. ASTM A 421 Grade 270 wire
13. ASTM A 416 Grade 270 strand
14. Lo-rex 300 strand
15. ASTM A 722 Grade 150 bar

16. ASTM A 722 Grade 160 bar
17. ASTM A 722 Grade 157 bar
18. Leadline Grade 285 ksi FRP cable
19. CFCC Grade 250 ksi FRP cable

Made in Japan

- Choose one prestressing material from the list above.
- If you choose one Safety Factor for the prestressing material γ_{py} (must be bigger than -or equal to- 1) the strength assumed in calculation will be the real one divided by the steel strength reduction factor. Reduction will be by affinity for whenever the Ramberg Osgood formulation is used and "proportional" (see formulation) for others.
- You can asess the choosen prestressing material performance by the stress-strain diagram as plotted below.
- Any number not corresponding to the list will default to case 1 (perfectly elastic material)
- Ensure the existence of the prestressing part as assumed from catalog
- We conservatively take Ep=28500 ksi for american strands from PCI 4th ed Hbk. instead of 28600 of ASTM.
- Lo-rex 300 ksi strand is modeled following spanish's Ramberg-Osgood model. This may be too much conservative and you may be wanting to alter formulation.
- 7 wire Strands are usually 0.5" and 0.6", with respective areas 0.153 in2 and 0.217 in2
- The multi-strand cables made of paralell strands can take the same formulation than strands themselves. At least there are...
  multistrand 1/2" : 1 to 31, 37, 42, 55 and 61 strands
  multistrand 0.6" : 1 to 22, 27, 31, 37 and 42 strands
- Usual perstressing bars:
  5/8" (157 ksi)
  1", 1 1/4", 1 3/8" (150 and 160 ksi break strength)
- FRPs Fiber Reinforced Plastics cables I wouldn't let to go overly compressed in any state, so check if you find some in your checked section
- Typical FRPs
  Leadline Ø8 mm
  CFCC Ø15.2 mm
- We don't put a limit for the attainable strains for the given laws, but provide strength cutoffs fpr the prestressing materials; this will preclude the consideration of unattainable strains, given the biunivocal relationship between stress and strain..The assumption **for the perfectly elastic material 1** is that it breaks when it reaches it maximum strength, so **ensure by check the maximum strain prior to rupture is not attained** under the investigated limit load. This caution is extensive to almost any of the used prestressing materials, since may of them attain higher strength than its limit one under such strain-stress laws (that have or should have limit strain for their application). Such assumption is unrealistic and non conservative and you must **ensure by personal check the modeled and solved status is right in strength and strain.**
- Related with the previous paragraph, we limit our viewport for the assumed antimmetrical stress-strain prestressing material diagrams to the commonly assumed usable ranges Whenever the red line representing the stress-strain diagram cuts falls out of the viewport you might be over the real or purported warranted strength or outside the warranted attainable strain and you should not have your prestressing material at such (failure attained) strain.

(1) Any perfectly elastic prestressing material

$\varepsilon_{pu1} = 0.0125$ assumed real limit strain when Prestressed material set to 1

$\frac{f_{pu1}}{\gamma_{py}} = 17576.75 \frac{kgf}{cm^2}$ assumed limit stress when when Prestressed material set to 1

(18) Leadline

$E_{p18} := 21300 \cdot ksi$ $f_{pu18} := 285 \cdot ksi$ $\varepsilon_{pu18} := \frac{f_{pu18}}{E_{p18}}$

$f_{py18} := f_{pu18}$

per definition

$f_{pu18} = 20037.49 \frac{kgf}{cm^2}$

value shown not as if affected by safety factor

$\varepsilon_{pu18} = 0.01338$

assumed real limit strain when when Prestressed material set to 18

$\varepsilon_{pu18} = 0.01338$

$\frac{f_{pu18}}{\gamma_{py}} = 20037.49 \frac{kgf}{cm^2}$

assumed limit stress when when Prestressed material set to 18

(19) CFCC carbon fiber cable

$E_{p19} := 20000 \cdot ksi$ $f_{pu19} := 250 \cdot ksi$ $\varepsilon_{pu19} := \frac{f_{pu19}}{E_{p19}}$

$f_{py19} := f_{pu18}$

per definition

$f_{pu19} = 17576.75 \frac{kgf}{cm^2}$

value shown not as if affected by safety factor

$\varepsilon_{pu19} = 0.0125$

assumed real limit strain when when Prestressed material set to 19

$\varepsilon_{pu19} = 0.0125$

$\frac{f_{pu19}}{\gamma_{py}} = 17576.75 \frac{kgf}{cm^2}$

assumed limit stress when when Prestressed material set to 19

Prestressing steels here admitted ruled by the Ramberg-Osgood diagram in the spanish codes (art. 32 EHE)

(2) Spanish prestressing steel designation (wire) Y 1570 C
(3) Spanish prestressing steel designation (wire) Y 1670 C
(4) Spanish prestressing steel designation (wire) Y 1770 C
(5) Spanish prestressing steel designation (wire) Y 1860 C
(6) Spanish prestressing steel designation (bi-wire) Y 1770 S2
(7) Spanish prestressing steel designation (tri-wire) Y 1860 S3
(8) Spanish prestressing steel designation (tri-wire) Y 1960 S3
(9) Spanish prestressing steel designation (tri-wire) Y 2060 S3
(10) Spanish prestressing steel designation (strand) Y 1770 S7
(11) Spanish prestressing steel designation (strand) Y 1860 S7
(12) ASTM A 421 Grade 270 wire
(13) ASTM A 416 Grade 270 strand
(14) Lo-rex 300 strand
(15) ASTM A 722 Grade 150 bar

- Lacking a stress-strain diagram we use EHE's Ramberg-Osgood as well for lo-rex 300 and all other american prestressing steels (be they bar, wire or strand).
- This may prove to be too much conservative, and you might want to substitute your own more correct formulation.
- 250 ksi strand would be similar to (10) material

(16) ASTM A 722 Grade 160 bar
(17) ASTM A 722 Grade 157 bar

$$E_p := \begin{cases} 200 \cdot \dfrac{1000 \cdot N}{mm^2} & \text{if } AND2\left(Prest_{mat} \ge 2, Prest_{mat} \le 5\right) \\ \text{otherwise} & \begin{cases} 190 \cdot \dfrac{1000 \cdot N}{mm^2} & \text{if } AND2\left(Prest_{mat} \ge 6, Prest_{mat} \le 11\right) \\ \text{otherwise} & \begin{cases} 29000 \cdot ksi & \text{if } AND2\left(Prest_{mat} \ge 15, Prest_{mat} \le 17\right) \\ \text{otherwise} & \begin{cases} 29000 \cdot ksi & \text{if } Prest_{mat} = 12 \\ \text{otherwise} & \begin{cases} 28500 \cdot ksi & \text{if } AND2\left(Prest_{mat} \ge 13, Prest_{mat} \le 14\right) \\ \text{otherwise} & \begin{cases} E_{p18} & \text{if } Prest_{mat} = 18 \\ \text{otherwise} & \begin{cases} E_{p19} & \text{if } Prest_{mat} = 19 \\ E_{p1} & \text{otherwise} \end{cases} \end{cases} \end{cases} \end{cases} \end{cases} \end{cases} \end{cases}$$

$$f_{pu} := \begin{cases} 1570\cdot MPa & \text{if } Prest_{mat} = 2 \\ 1670\cdot MPa & \text{if } Prest_{mat} = 3 \\ 1770\cdot MPa & \text{if } Prest_{mat} = 4 \\ 1860\cdot MPa & \text{if } Prest_{mat} = 5 \\ 1770\cdot MPa & \text{if } Prest_{mat} = 6 \\ 1860\cdot MPa & \text{if } Prest_{mat} = 7 \\ 1960\cdot MPa & \text{if } Prest_{mat} = 8 \\ 2060\cdot MPa & \text{if } Prest_{mat} = 9 \\ 1770\cdot MPa & \text{if } Prest_{mat} = 10 \\ 1860\cdot MPa & \text{if } Prest_{mat} = 11 \\ 270\cdot ksi & \text{if } Prest_{mat} = 12 \\ 270\cdot ksi & \text{if } Prest_{mat} = 13 \\ 300\cdot ksi & \text{if } Prest_{mat} = 14 \\ 150\cdot ksi & \text{if } Prest_{mat} = 15 \\ 160\cdot ksi & \text{if } Prest_{mat} = 16 \\ 157\cdot ksi & \text{if } Prest_{mat} = 17 \\ f_{pu19} & \text{if } Prest_{mat} = 19 \\ f_{pu18} & \text{if } Prest_{mat} = 18 \\ f_{pu1} & \text{otherwise} \end{cases}$$

$$f_{py} := \begin{cases} fpyf\cdot f_{pu} & \text{if } AND2\left(Prest_{mat} \geq 2, Prest_{mat} \leq 17\right) \\ f_{pu} & \text{otherwise} \end{cases}$$

$\sigma px := 15000$ seed, implied kgf/cm2

Given The Ramberg-Osgood branch thing

$$\frac{\sigma px}{\frac{E_p}{\frac{kgf}{cm^2}}} + 0.823\cdot\gamma_{py}{}^{5}\cdot\left(\frac{\sigma px}{\frac{f_{py}}{\frac{kgf}{cm^2}}} - \frac{0.7}{\gamma_{py}}\right)^5 = \varepsilon$$

$$\sigma_{over\_prop\_p}(\varepsilon) := \frac{kgf}{cm^2}\cdot Find(\sigma px)$$

Ramberg-Osgood no closed form and we want such so we build a fitted curve

$$\text{Parts} := 200 \qquad j := 1..\text{Parts}+1 \qquad \varepsilon s_j := \frac{\frac{f_{py}}{\gamma_{py}}}{E_p} + \frac{0.035 - \frac{\frac{f_{py}}{\gamma_{py}}}{E_p}}{\text{Parts}}\cdot(j-1) \qquad \sigma s_j := \sigma_{over\_prop\_p}(\varepsilon s_j)$$

$$vs := \text{cspline}(\varepsilon s, \sigma s) \qquad f_{pss}(\varepsilon) := \text{interp}(vs, \varepsilon s, \sigma s, \varepsilon)$$

$$f_{psRO}(\varepsilon) := \begin{cases} E_p\cdot\varepsilon & \text{if } \varepsilon \le 0.7\cdot\frac{\frac{f_{py}}{\gamma_{py}}}{E_p} \\ f_{pss}(\varepsilon) & \text{otherwise} \end{cases}$$

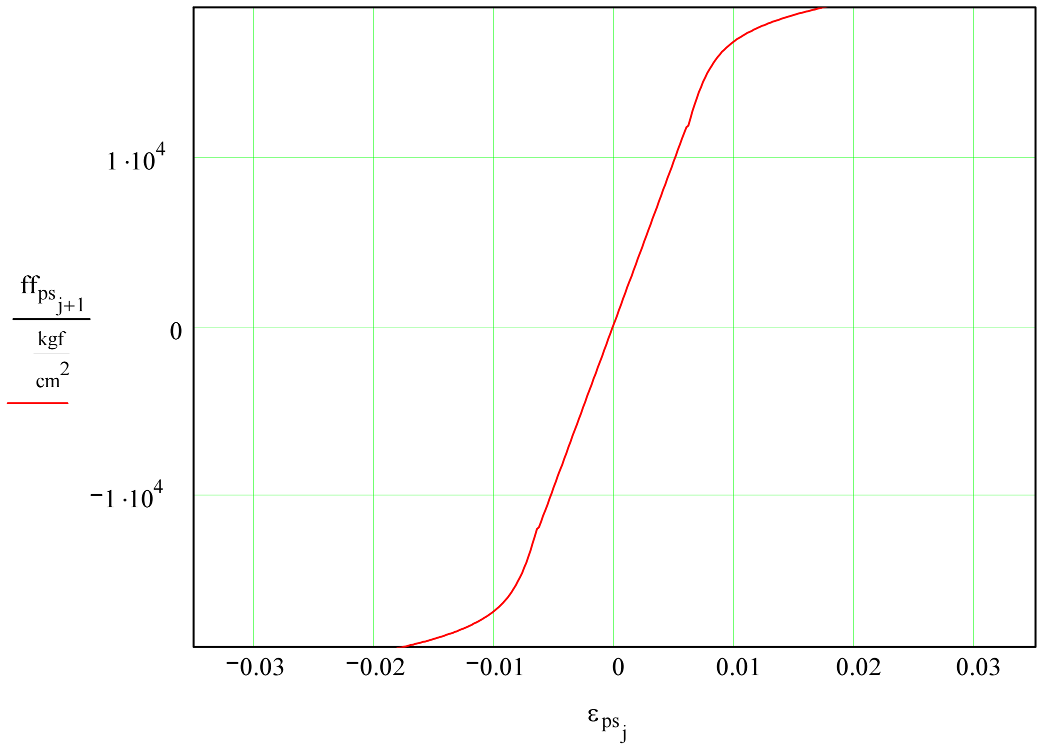
$$f_{ps\_positive}(\varepsilon) := \begin{cases} f_{psRO}(\varepsilon) & \text{if } \text{AND2}(\text{Prest}_{mat} \ge 2, \text{Prest}_{mat} \le 17) \\ E_p\cdot\varepsilon & \text{otherwise} \end{cases}$$

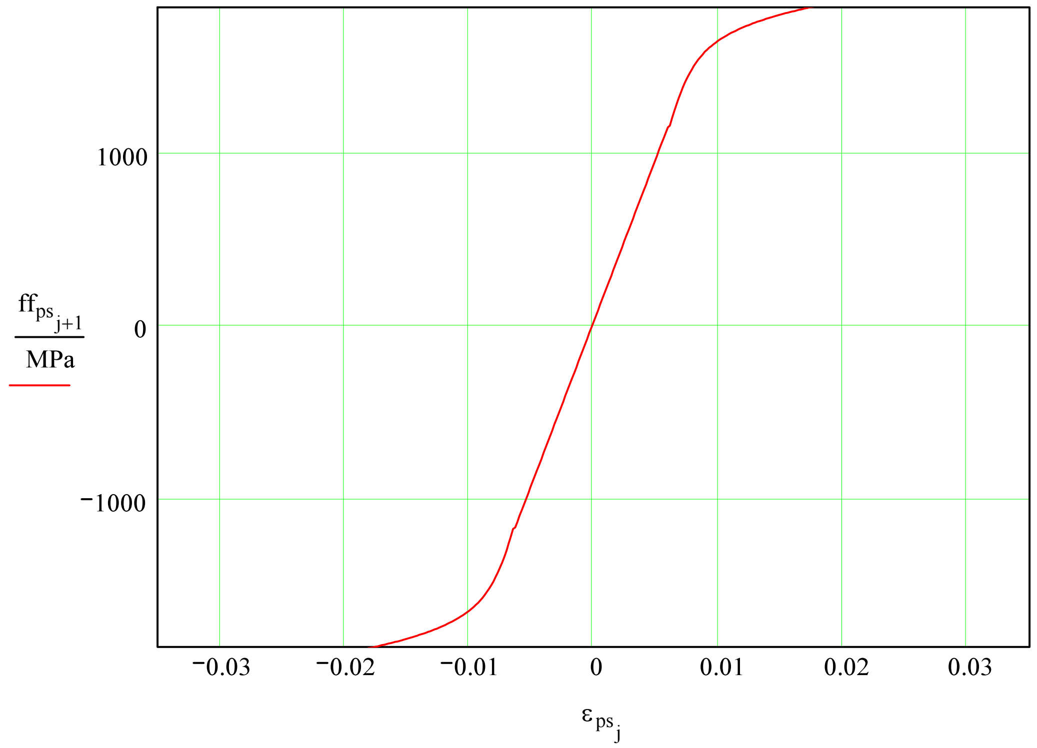
$$\varepsilon_{pu} := \begin{cases} 0.035 & \text{if } \text{AND2}(\text{Prest}_{mat} \ge 2, \text{Prest}_{mat} \le 17) \\ \text{otherwise} & \begin{cases} \frac{f_{pu19}}{E_{p19}} & \text{if } \text{Prest}_{mat} = 19 \\ \text{otherwise} & \begin{cases} \frac{f_{pu18}}{E_{p18}} & \text{if } \text{Prest}_{mat} = 18 \\ \frac{f_{pu1}}{E_{p1}} & \text{otherwise} \end{cases} \end{cases} \end{cases}$$

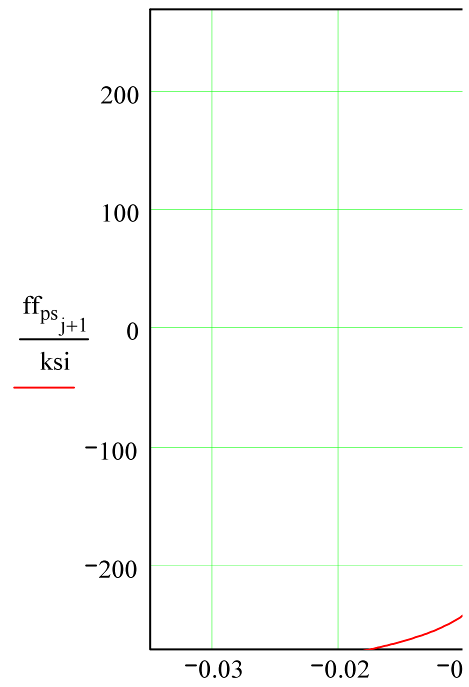
$$f_{ps}(\varepsilon) := \begin{cases} f_{ps\_positive}(\varepsilon) & \text{if } \varepsilon \ge 0 \\ -f_{ps\_positive}(-\varepsilon) & \text{otherwise} \end{cases}$$

$$j := 0..400$$

$$\varepsilon_{ps_{j+1}} := \frac{\varepsilon_{pu}}{200}\cdot(j-200) \qquad ff_{ps_{j+1}} := f_{ps}\left(\varepsilon_{ps_{j+1}}\right)$$







$Prest_{mat} = 11$

$\varepsilon_{pu} = 0.035$

$f_{pu} = 18966.72\,\frac{kgf}{cm^2}$ $f_{py} = 17354.55\,\frac{kgf}{cm^2}$ $E_p = 1.94\times10^6\,\frac{kgf}{cm^2}$

$f_{pu} = 1860\,MPa$ $f_{py} = 1701.9\,MPa$ $E_p = 190000\,MPa$

$f_{pu} = 269.77\,ksi$

Prestress

$\varepsilon_0 := -\frac{f_{pu}\cdot k_{ps}}{E_p}$ $\varepsilon_0 = -0.00538$ negative since initial tensile status of prestressing material

$f_{ps\_as\_prestressed} := f_{ps}(\varepsilon_0)$ $f_{ps\_as\_prestressed} = -1023\,MPa$ negative since tensile

Prestressing Materials Geometry

$n_p := 4$ number of strands or whatever $A_{p1} := 1\cdot cm^2$ $c_p := 10\cdot cm$

$\alpha_{0p} := 45\cdot deg$ centered angle, from (negative or) -Y axis towards lowest passive bar center, enter positive if such bar to be found counterclockwise

$\alpha_p := \alpha_{0p} - \beta$

the above defined passive bar is found (respect rotated -Y' axis –plane of flexion–, from moment combination) at $\alpha_p = 28.3\,deg$ counterclockwise, if positive

Equilibrium will be defined respect such new X' and Y' axes. We will omit primes in our notation from now on.

$r := \frac{D}{2}$ $r_{to\_p\_bar} := r - c_p$ $r_{to\_p\_bar} = 15\,cm$

$j := 1 .. n_p$ $X_{p_j} := r + r_{to\_p\_bar} \cdot \sin\left[\alpha_p + (j-1) \cdot \frac{360 \cdot deg}{n_p}\right]$ $Y_{p_j} := r - r_{to\_p\_bar} \cdot \cos\left[\alpha_p + (j-1) \cdot \frac{360 \cdot deg}{n_p}\right]$

perpendicular and paralell to combination moment Y plane

since origin at lower left corner of encasing adjusted square

Change of axis we have made only to use single Mu; equilibrium we will establish nevertheless biaxially to account for any dissymmetry

$$A_{p_j} := A_{p1}$$

Concrete

$f_{c28} := 35 \cdot MPa$

You may feel adequate to enter a fcd reduced one, or a mean (probabilistic) real value

Take into account?

$Confinement := 0$ **1** for YES **0** for NO

confinement 1 only if per ACI 318

Take into account?

$Tensile\_stress := 0$ **1** for YES **0** for NO

Note
Formulation believed to be adequate even for the most exacting HPC, VHS concretes.

$\nu_{SL} := 0.8$ Sustained Loading strength reduction factor (from 0.75 to 0.85) see fig 39.3 in EHE code (preferably not bigger than 0.8)

$f_c := f_{c28} \cdot \nu_{SL}$

the Sustained Loading strength reduction factor will have scarce effect in the strength of beams designed to fail by steel fracture, but will be essential to the safety of columns

ε_{fc} evaluation

$$\varepsilon_{fc}(f_c) := .0015 + .002 \cdot \frac{f_c \cdot \frac{cm^2}{kgf}}{1300}$$

The strain at which concrete reachs its higher strenght f_c is $\varepsilon_{fc}(f_c) = 0.00194$

ε_{fct} evaluation

$k_{fct} := 7.5$

You can alternatively make k_{fct}= 6.7 if for strenght or simply to be more conservative

$f_{ct}(f_c) := k_{fct} \sqrt{f_c \cdot psi}$

$f_{ct}(f_c) = 33.6 \frac{kgf}{cm^2}$ $f_{ct}(f_c) = 33.6 \frac{kgf}{cm^2}$ $f_{ct}(f_c) = 33.6 \frac{kgf}{cm^2}$

We could get approximately the strain at which the ultimate tensile strain is reached, but will do exactly solving the equation in first quadrant:

$f_c = 285.52 \frac{kgf}{cm^2}$ Reminder

Our unwarranted guess $\varepsilon := .0005 \cdot \frac{cm}{cm}$

We'll solve the limit tensile strain without units since Mathcad 8 doesn't seem able to manage here properly these

$f_{ct}(f_c) := k_{fct} \sqrt{f_c \cdot psi}$ $f_{ct}(f_c) = 33.6 \frac{kgf}{cm^2}$

$$\varepsilon_{fc}(f_c) := .0015 + .002 \cdot \frac{f_c \cdot \frac{cm^2}{kgf}}{1300}$$

$$\varepsilon_{fc}(f_c) = 0.00194 \frac{cm}{cm}$$

Given

$$\frac{f_{ct}(f_c)}{f_c} = \left(2 \cdot \frac{\varepsilon}{\varepsilon_{fc}(f_c)}\right) - \left(\frac{\varepsilon}{\varepsilon_{fc}(f_c)}\right)^2$$

$\varepsilon_{fct0} := -\mathrm{Find}(\varepsilon)$ $\qquad \varepsilon_{fct0} = -0.00012$

$\varepsilon_{fct}(f_c) := \mathrm{if}(\mathrm{Tensile\_stress}, \varepsilon_{fct0}, 0)$ $\qquad \varepsilon_{fct}(f_c) = 0$

ε_{cu} evaluation

kg/cm2 $\qquad \varepsilon_{cu}$

$$\mathrm{Stress} := \frac{kgf}{cm^2} \cdot \begin{pmatrix} 100 \\ 350 \\ 500 \\ 800 \\ 1200 \end{pmatrix} \qquad \mathrm{Strain} := \begin{pmatrix} .0039 \\ .0035 \\ .0028 \\ .0028 \\ .0034 \end{pmatrix}$$

$vs := \mathrm{lspline}(\mathrm{Stress}, \mathrm{Strain})$ $\qquad \varepsilon_{cu}(f_c) := \mathrm{interp}(vs, \mathrm{Stress}, \mathrm{Strain}, f_c)$

The ultimate strain for the given fc for a flexural condition like this is then $\varepsilon_{cu}(f_c) = 0.00371$

$\sigma(\varepsilon)$ evaluation

Stress in concrete corresponding to strain ε

$k_{\varepsilon cu} := \mathrm{if}(\mathrm{Confinement}, 0.98, 0.91)$ $k_{\varepsilon cu} = 0.91$ $\qquad$ as per disgression

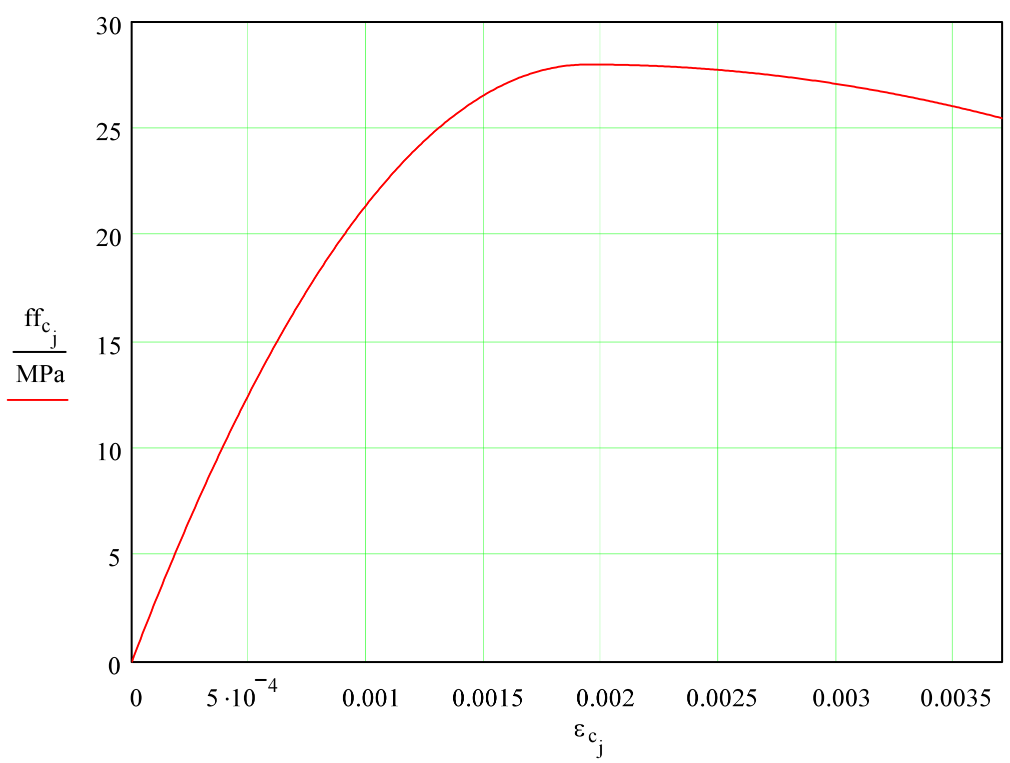
$$\sigma(\varepsilon) := f_c \cdot \left| \begin{array}{l} 0 \ \text{ if } \varepsilon < \varepsilon_{fct}(f_c) \\ \text{otherwise} \\ \left| \begin{array}{l} -1 \cdot \left[\left(2 \cdot \frac{-\varepsilon}{\varepsilon_{fc}(f_c)} \right) - \left(\frac{-\varepsilon}{\varepsilon_{fc}(f_c)} \right)^2 \right] \ \text{ if } \varepsilon_{fct}(f_c) \le \varepsilon \le 0 \\ \text{otherwise} \\ \left| \begin{array}{l} \left(2 \cdot \frac{\varepsilon}{\varepsilon_{fc}(f_c)} \right) - \left(\frac{\varepsilon}{\varepsilon_{fc}(f_c)} \right)^2 \ \text{ if } 0 < \varepsilon \le \varepsilon_{fc}(f_c) \\ \text{otherwise} \\ \left| \begin{array}{l} 1 - (1 - k_{\varepsilon cu}) \frac{\varepsilon^2 - 2 \cdot \varepsilon \cdot \varepsilon_{fc}(f_c) + \varepsilon_{fc}(f_c)^2}{\varepsilon_{cu}(f_c)^2 - 2 \cdot \varepsilon_{fc}(f_c) \cdot \varepsilon_{cu}(f_c) + \varepsilon_{fc}(f_c)^2} \ \text{ if } \varepsilon_{fc}(f_c) < \varepsilon \le \varepsilon_{cu}(f_c) \\ 0 \ \text{ if } \varepsilon > \varepsilon_{cu}(f_c) \ \text{ otherwise} \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \right.$$

will rule stress determination in concrete for input ε

Say $\quad \varepsilon := 0.00114 \quad \sigma(\varepsilon) = 237.02 \frac{\text{kgf}}{\text{cm}^2}$

$\text{Parts} := 400 \quad j := 1 .. \text{Parts} + 1 \quad \varepsilon_{c_j} := \frac{\varepsilon_{cu}(f_c) - \varepsilon_{fct}(f_c)}{\text{Parts}} \cdot (j - 1) \quad ff_{c_j} := \sigma(\varepsilon_{c_j})$

$f_{c28} = 35\,\text{MPa} \quad f_c = 28\,\text{MPa}$



- the stress-strain diagram needs to be scaled down to give the effective (really available) strenght for sustained loads.
- any load that must be held about two hours is to strength effects a sustained load, and for what I know most structures are used for, if any overload can occur it is almost sure it can also stay for such time or more, i.e., the sustained load strenght reduction factor is required.
- this means you can only count on about 80% of the average strength you would get from probes tested at the normal rate.
- probability considerations, the fact of that part of the load is live load and if it grows it won't know when to stop (so the failure is likely to be by instantaneous overload), the safety factors, and the growth of strength of concrete with age make that a sustained load reduction factor bigger than 0.8 can be taken without much structural concern; in fact 0.9 is explicitly permitted by some codes (may be undermining a bit safety for short term sustained loads); we prefer our more substantiated value of 0.8 applied to specified strength
- this sustained loads strength reduction factor is completely different from any within codes; this is required in a compatibility of deformations setup to get the real strength of the structural member, while those of codes must further diminish the resulting strength to compare with factored loads; not all code writers seem fully aware of this.

$$P_{max} := \pi \cdot \left(\frac{D}{2}\right)^2 \cdot f_c$$

we accept to no make it interact with atop and at bottom strains for steel voids deductions

$$P_{max} = 560.62\,\text{ton}$$

full section at fc, a reference value

$$b(z,r) := 2 \cdot r \cdot \sin\left(\operatorname{acos}\left(\frac{r-z}{r}\right)\right) \qquad b_1(z,r) := b[r-(z-r),r]$$

$$P_{ref} := \int_{0 \cdot m}^{r} \frac{f_c}{4 \cdot r^2} \cdot z \cdot (4 \cdot r - z) \cdot b(z,r)\,dz + \int_{r}^{2 \cdot r} \frac{f_c}{4 \cdot r^2} \cdot z \cdot (4 \cdot r - z) \cdot b_1(z,r)\,dz$$

$$P_{ref} = 385.43\,\text{ton}$$

Integration of the first branch parabolic fc growth of strength along D for a circular section, the circular section decompression milestone against which to gauge how much we will be curtailing the strain towards that of at maximum compressive strength from that of a non compressively loaded case.

$$e_{to\_center} := \frac{M_u}{P} \qquad e_{to\_center} = 20.88\,\text{cm}$$

$$e\_to\_bottom := e_{to\_center} + \frac{D}{2} \qquad e\_to\_bottom = 45.88\,\text{cm}$$

We will be assuming P, Mu data referred to center of brute section as usual and will establish equilibrium integrating moments respect bottom of the section; that is, the moment of the P as per above implied positioned will be in place equilibrated by the moments of inner forces in steel and concrete; all moments will be referred to bottom edge of section.

We take compression stresses positive, and tension stresses negative.
We dump the areas of both steel reinforcement and concrete layers at their c.o.g.

We need 3 epsilons to define a status of the section in the plane remain plane hypothesis.

We set these unwarranted assumptions

$\varepsilon_1 := 0.000$ at common origin (bottom, left)

$\varepsilon_2 := 0.0005$ top left vertex (on ordinate axis) referred to bottom left corner of the encasing adjusted square

$\varepsilon_3 := 0.0005$ bottom right vertex (on abscissas axis)

Now we establish the strain in any point in the plane by interpolation

$$\varepsilon(x,y,\varepsilon_1,\varepsilon_2,\varepsilon_3) := \varepsilon_1 + \frac{y}{D} \cdot (\varepsilon_2 - \varepsilon_1) + \frac{x}{D} \cdot (\varepsilon_3 - \varepsilon_1)$$

We will take into account the displaced concrete diminishing the ability of steel to contribute to equilibrium in exactly the value of the displaced (absent) concrete force. So, the corresponding total forces for passive steel with the effect of displaced concrete dumped unto them are

$$\mathrm{Steel}_{\mathrm{Force}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n} A_{s_k}\cdot\left(f_s\left(\varepsilon\left(X_{s_k},Y_{s_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)-\sigma\left(\varepsilon\left(X_{s_k},Y_{s_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)\right)$$

$$\mathrm{Steel}_{\mathrm{Moment\_X}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n} Y_{s_k}\cdot\left[A_{s_k}\cdot\left(f_s\left(\varepsilon\left(X_{s_k},Y_{s_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)-\sigma\left(\varepsilon\left(X_{s_k},Y_{s_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)\right)\right]$$

respect bottom (abscissas) axis

$$\mathrm{Steel}_{\mathrm{Moment\_Y}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n} X_{s_k}\cdot\left[A_{s_k}\cdot\left(f_s\left(\varepsilon\left(X_{s_k},Y_{s_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)-\sigma\left(\varepsilon\left(X_{s_k},Y_{s_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)\right)\right]$$

respect left (ordinates) axis

$$\mathrm{Prestress}_{\mathrm{Force}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n_p} A_{p_k}\cdot\left(f_{ps}\left(\varepsilon_0+\varepsilon\left(X_{p_k},Y_{p_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)-\sigma\left(\varepsilon\left(X_{p_k},Y_{p_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)\right)$$

$$\mathrm{Prestress}_{\mathrm{Moment\_X}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n_p} Y_{p_k}\cdot A_{p_k}\cdot\left(f_{ps}\left(\varepsilon_0+\varepsilon\left(X_{p_k},Y_{p_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)-\sigma\left(\varepsilon\left(X_{p_k},Y_{p_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)\right)$$

$$\mathrm{Prestress}_{\mathrm{Moment\_Y}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n_p} X_{p_k}\cdot A_{p_k}\cdot\left(f_{ps}\left(\varepsilon_0+\varepsilon\left(X_{p_k},Y_{p_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)-\sigma\left(\varepsilon\left(X_{p_k},Y_{p_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)\right)$$

With discretization, integration of stresses for concrete becomes summation

$$\mathrm{Concrete}_{\mathrm{Force}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n_c} A_{c_k}\cdot\sigma\left(\varepsilon\left(X_{c_k},Y_{c_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)$$

$$\mathrm{Concrete}_{\mathrm{Moment\_X}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n_c} Y_{c_k}\cdot A_{c_k}\cdot\sigma\left(\varepsilon\left(X_{c_k},Y_{c_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)$$

respect bottom (abscissas) axis

$$\mathrm{Concrete}_{\mathrm{Moment\_Y}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \sum_{k=1}^{n_c} X_{c_k}\cdot A_{c_k}\cdot\sigma\left(\varepsilon\left(X_{c_k},Y_{c_k},\varepsilon_1,\varepsilon_2,\varepsilon_3\right)\right)$$

respect left (ordinates) axis

$$\mathrm{Total}_{\mathrm{force}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \mathrm{Steel}_{\mathrm{Force}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)+\mathrm{Concrete}_{\mathrm{Force}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)+\mathrm{Prestress}_{\mathrm{Force}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)$$

$$\mathrm{Total}_{\mathrm{moment\_X}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \mathrm{Steel}_{\mathrm{Moment\_X}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)+\mathrm{Concrete}_{\mathrm{Moment\_X}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)+\mathrm{Prestress}_{\mathrm{Moment\_X}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)$$

$$\mathrm{Total}_{\mathrm{moment\_Y}}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \mathrm{Steel}_{\mathrm{Moment\_Y}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)+\mathrm{Concrete}_{\mathrm{Moment\_Y}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)+\mathrm{Prestress}_{\mathrm{Moment\_Y}}(\varepsilon_1,\varepsilon_2,\varepsilon_3)$$

$$\mathrm{MaxC}_{strain}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \left| \begin{array}{l} maxe \leftarrow 0 \\ \text{for } j \in 1..n_c \\ \quad maxe \leftarrow \varepsilon\left(X_{c_j}, Y_{c_j}, \varepsilon_1, \varepsilon_2, \varepsilon_3\right) \text{ if } \varepsilon\left(X_{c_j}, Y_{c_j}, \varepsilon_1, \varepsilon_2, \varepsilon_3\right) \ge maxe \\ maxe \end{array} \right.$$

ne maximum (mean) compresive stress
n our meshed elements

$$\varepsilon_{cu\_current}(\varepsilon_1,\varepsilon_2,\varepsilon_3) := \left| \begin{array}{l} \varepsilon_{cu}(f_c) \text{ if } \mathrm{Concrete}_{Force}(\varepsilon_1,\varepsilon_2,\varepsilon_3) \le P_{ref} \\ \left[\varepsilon_{cu}(f_c) + \left(\varepsilon_{fc}(f_c) - \varepsilon_{cu}(f_c)\right) \dfrac{\left(\dfrac{\mathrm{Concrete}_{Force}(\varepsilon_1,\varepsilon_2,\varepsilon_3)}{P_{max}} - \dfrac{P_{ref}}{P_{max}}\right)^2}{\left(1 - \dfrac{P_{ref}}{P_{max}}\right)^2} \right] \text{ otherwise} \end{array} \right.$$

Solving the problem

Given

$$\mathrm{Total}_{force}(\varepsilon_1,\varepsilon_2,\varepsilon_3) = P$$

$$\mathrm{Total}_{moment\_X}(\varepsilon_1,\varepsilon_2,\varepsilon_3) = P\cdot\left(r + \frac{M_u}{P}\right)$$

$$\mathrm{Total}_{moment\_Y}(\varepsilon_1,\varepsilon_2,\varepsilon_3) = P\cdot r$$ since we have reduced the problem to plane P, M

$$\mathrm{MaxC}_{strain}(\varepsilon_1,\varepsilon_2,\varepsilon_3) \le \varepsilon_{cu\_current}(\varepsilon_1,\varepsilon_2,\varepsilon_3)$$

$$\mathrm{Result} := \mathrm{Find}(\varepsilon_1,\varepsilon_2,\varepsilon_3)$$

$\varepsilon_1 := \mathrm{Result}_1$ $\quad\varepsilon_2 := \mathrm{Result}_2$ $\quad\varepsilon_3 := \mathrm{Result}_3$

$$\varepsilon_4 := \varepsilon(D, D, \varepsilon_1, \varepsilon_2, \varepsilon_3)$$

$$f_{conc}(x,y) := \sigma\left(\varepsilon(x, y, \varepsilon_1, \varepsilon_2, \varepsilon_3)\right)$$

$\mathrm{Max\_Concrete\_stress} := \sigma\left(\mathrm{MaxC}_{strain}(\varepsilon_1,\varepsilon_2,\varepsilon_3)\right)$ $\quad f_{c\_max\_inferred} := \sigma\left(\varepsilon(0\cdot m, D, \varepsilon_1, \varepsilon_2, \varepsilon_3)\right)$

$f_{steel}(xa, ya) := f_s(\varepsilon(xa, ya, \varepsilon_1, \varepsilon_2, \varepsilon_3))$

$\varepsilon_{c\_max\_inferred} := \varepsilon(0 \cdot m, D, \varepsilon_1, \varepsilon_2, \varepsilon_3)$

$Attack_{Angle} := 0 \cdot deg$

$MaxC_{strain}(\varepsilon_1, \varepsilon_2, \varepsilon_3) = 0.0011$

$$Trace_{Angle} := atan\left[\frac{D}{D} \cdot \left(\frac{\varepsilon_1}{\varepsilon_1 - \varepsilon_2} - \frac{\varepsilon_3}{\varepsilon_3 - \varepsilon_4}\right)\right]$$

$f_{ps\_at\_equilibrium} := f_{ps}(\varepsilon_0 + \varepsilon(r, r, \varepsilon_1, \varepsilon_2, \varepsilon_3))$

Please note we have solved equilibrium in axes different from those data, so rotated $\beta = 16.7 deg$ where $\beta := atan\left(\frac{M_y}{M_x}\right)$

Equilibrium

$Total_{force}(\varepsilon_1, \varepsilon_2, \varepsilon_3) = 100 ton$

$P = 100 ton$

$Total_{moment\_X}(\varepsilon_1, \varepsilon_2, \varepsilon_3) = 45.88 m \cdot ton$

$M_u + P \cdot r = 45.88 m \cdot ton$

$Total_{moment\_Y}(\varepsilon_1, \varepsilon_2, \varepsilon_3) = 25 m \cdot ton$

$P \cdot r = 25 m \cdot ton$

Attack and Response Angles

$Attack_{Angle} = 0 deg$

$Trace_{Angle} = -0.02 deg$

Concrete Strains and Stresses

in calculated fibers

inferred worst solicitations, derived from solution

value atop, extrapolated from the plane that produces equilibrium

$MaxC_{strain}(\varepsilon_1, \varepsilon_2, \varepsilon_3) = 0.0011$

$\varepsilon_{c\_max\_inferred} = 0.00126$

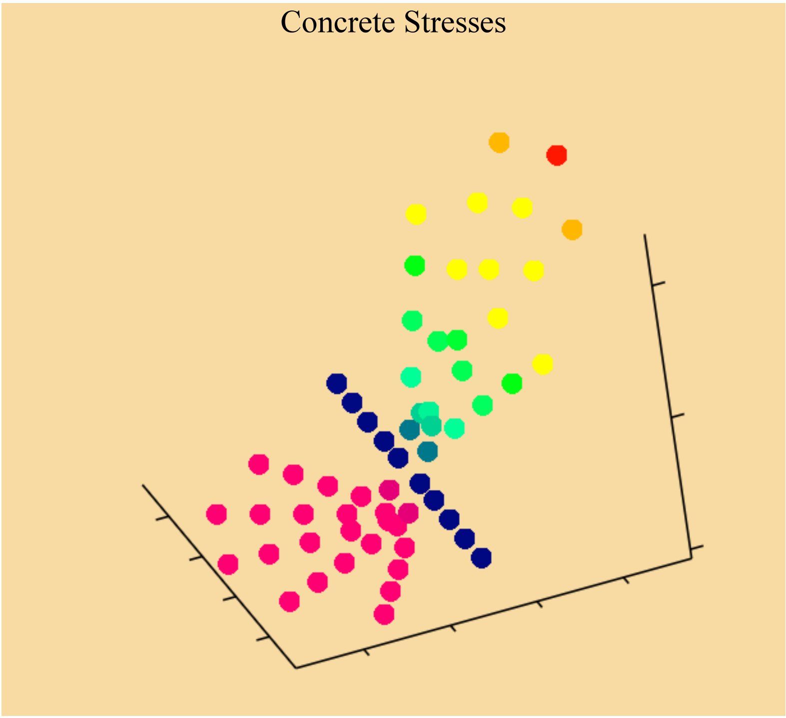
$Max\_Concrete\_stress = 232.09 \frac{kgf}{cm^2}$

$f_{c\_max\_inferred} = 250.23 \frac{kgf}{cm^2}$

$$\frac{MaxC_{strain}(\varepsilon_1, \varepsilon_2, \varepsilon_3)}{\varepsilon_{cu\_current}(\varepsilon_1, \varepsilon_2, \varepsilon_3)} = 29.68\%$$

$$\frac{\varepsilon_{c\_max\_inferred}}{\varepsilon_{cu\_current}(\varepsilon_1, \varepsilon_2, \varepsilon_3)} = 33.92\%$$

$i := 1 .. n_c$ $x_i := X_{c_i}$ $y_i := Y_{c_i}$ $z_i := f_{conc}(x_i, y_i)$



$\left(\frac{x}{cm}, \frac{y}{cm}, \frac{z}{\frac{kgf}{cm^2}}\right)$

- Plot curtailed to centers of considered concrete elements
- If you want more precision use elements of lesser size

$j := 1..n \qquad xx_j := X_{s_j} \qquad yy_j := Y_{s_j} \qquad zz_j := f_{steel}(xx_j, yy_j)$

$Lower\_Steel\_stress := \min(zz)$

$Higher\_Steel\_stress := \max(zz)$

Steel stresses

- if negative tensile
- if positive compressive

$Lower\_Steel\_stress = -1586.61\,\frac{kgf}{cm^2}$

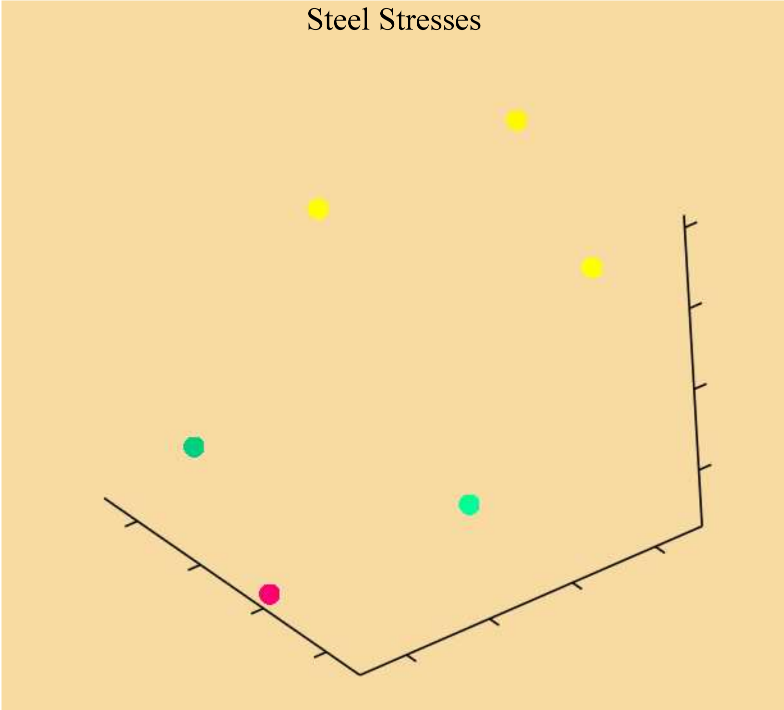
If negative tensile

$Higher\_Steel\_stress = 2046.6\,\frac{kgf}{cm^2}$

if positive compressive

$\frac{Lower\_Steel\_stress}{f_y} = -37.61\,\%$

$\frac{Higher\_Steel\_stress}{f_y} = 48.52\,\%$



$$\left(\frac{xx}{cm}, \frac{yy}{cm}, \frac{zz}{\frac{kgf}{cm^2}}\right)$$

Prestressing Material Stresses

Prior to apply moments and axial force $f_{ps\_as\_prestressed} = -1023\,MPa$

$j := 1..n_p$ $\quad xxx_j := X_{p_j}$ $\quad yyy_j := Y_{p_j}$ $\quad zzz_j := f_{ps}\left(\varepsilon_0 + \varepsilon\left(xxx_j, yyy_j, \varepsilon_1, \varepsilon_2, \varepsilon_3\right)\right)$

$Lower\_Prestress\_Mat\_stress := \min(zzz)$ $\quad \Delta p_1 := -\left(f_{ps\_as\_prestressed} - Lower\_Prestress\_Mat\_stress\right)$

$Higher\_Prestress\_Mat\_stress := \max(zzz)$ $\quad \Delta p_2 := -\left(f_{ps\_as\_prestressed} - Higher\_Prestress\_Mat\_stress\right)$

- if negative tensile
- if positive compressive

$Lower\_Prestress\_Mat\_stress = -1116.45\,MPa$

If negative tensile

$$\frac{Lower\_Prestress\_Mat\_stress}{f_{pu}} = -60.02\,\%$$

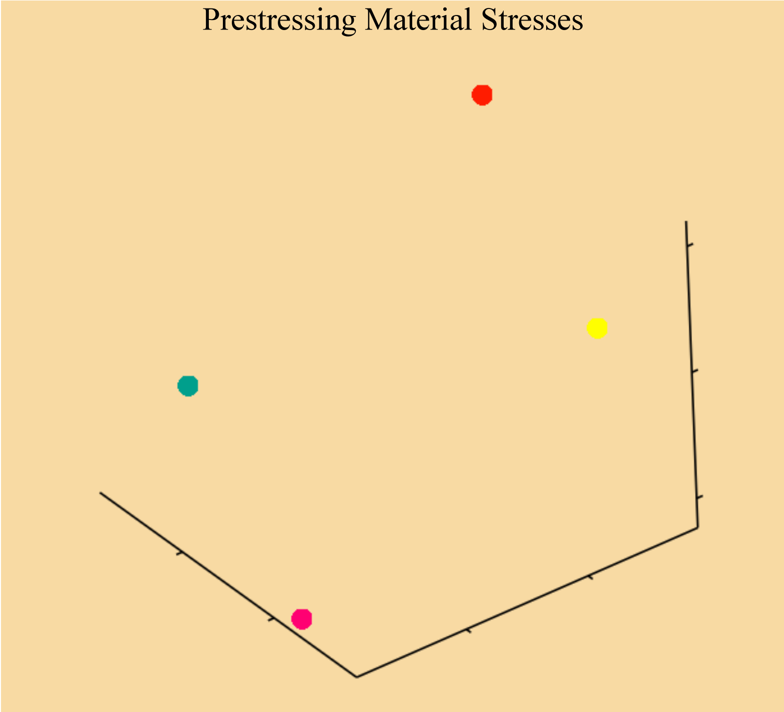
$\Delta p_1 = -93.45\,MPa$

$Higher\_Prestress\_Mat\_stress = -886.69\,MPa$

if positive compressive,
otherwise remains tensile

$$\frac{Higher\_Prestress\_Mat\_stress}{f_{pu}} = -47.67\,\%$$

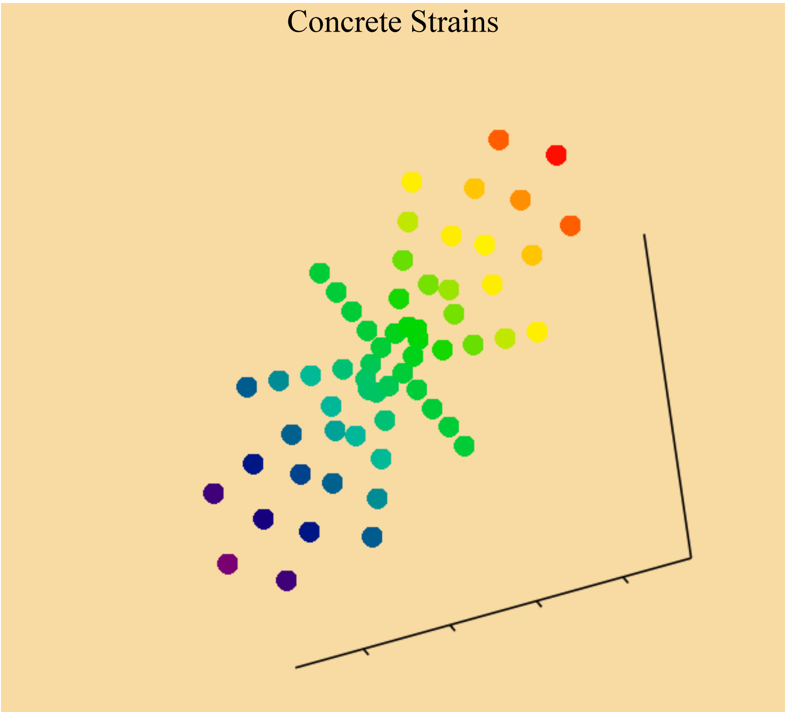
$\Delta p_2 = 136.31\,MPa$



$\left(\frac{xxx}{cm}, \frac{yyy}{cm}, \frac{zzz}{MPa}\right)$

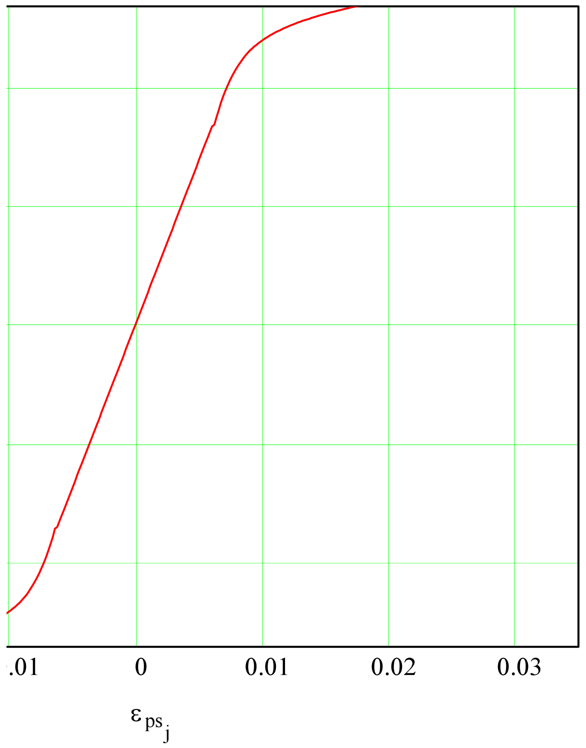
Strains

$i := 1..n_c$ $\quad x_i := X_{c_i}$ $\quad y_i := Y_{c_i}$ $\quad epsilon_i := \varepsilon\left(x_i, y_i, \varepsilon_1, \varepsilon_2, \varepsilon_3\right)$



$\left(\frac{x}{cm}, \frac{y}{cm}, epsilon\right)$

- Plot curtailed to centers of considered concrete elements
- If you want more precision use elements of lesser size



$f_{py} = 246.84\,ksi$ $E_p = 27557.15\,ksi$