

TREATMENT OF TORSION OF REINFORCED CONCRETE BEAMS IN CURRENT STRUCTURAL STANDARDS

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ABSTRACT

Structural standards and codes of practice are reviewed continuously and improvements are implemented as research findings reveal more accurate methods of design. Design for torsion unlike design for shear, bending, and axial force, which have been perfected over the years, has not attracted as much attention and is in a rather weaker state. This paper reviews the provisions of the current standards in relation to torsion of reinforced concrete beams and highlights their weaknesses and strengths. A couple of important parameters are introduced and some limits on them are proposed outside of which methods of current standards are better not to be used.

Keywords: Torsion, reinforced concrete, standards, ultimate torque

1. INTRODUCTION

The interest in gaining better understanding of the torsional behaviour of reinforced concrete (RC) members has grown in the past decades. This may be due to the increasing use of structural members in which torsion is a central feature of behaviour such as curved bridge girders and helical slabs. The achievements, however, have not been as much as those made in the areas of shear and bending. Dealing with torsion in today's codes of practice is also very primitive and does not contain the more elaborate techniques. Predictions of current standards for the ultimate torsional capacity of RC beams are found to be either too conservative or slightly risky for certain geometry, dimensions and steel bar sizes and arrangements.

In order to have a closer view to the above mentioned, this paper firstly presents a brief review of the provisions of well-known international standards in relation to the design of reinforced concrete beams against torsion. The chosen standards are from Australia (AS 3600) [1], Britain (BS 8110) [2], the United States (ACI 318-02) [3], Europe (European Standard, Eurocode 2) [4], and Canada (CSA) [5]. Secondly, the accuracy of the standards in predicting the ultimate torque of RC beams is examined by comparing their predictions against

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experimental studies available in the literature. While the sources generating differences between these two are identified, a couple of parameters are introduced and limits to these parameters are presented, outside of which, the standards methods cannot be trusted.

2. TREATMENT OF TORSION IN THE STANDARDS

Provisions for torsional design of reinforced concrete members appear in majority of international standards of concrete design. While these provisions are conceptually similar, they contain variations that produce different results. Provisions of some of the more well-known standards are reviewed here in this section.

2.1 Australian Standard (AS3600)

According to the Australian standard for concrete structures, AS3600, the ultimate strength in pure torsion, T_{uc} , for a beam without closed ties can be calculated as

$$T_{uc} = J_t \left(0.3 \sqrt{f'_c} \right) \quad (1)$$

where f'_c is the compressive strength of concrete at 28 days and J_t is the torsional rigidity of the cross-section. This torsional rigidity for a rectangular cross-section with dimensions $x \times y$ (where $x < y$) can be determined as $0.4 x^2 y$. For beams with closed ties, the ultimate torsional strength, T_{us} , is

$$T_{us} = f_{ys} (A_{sw} / s) 2 A_t \cot \theta_t \quad (2)$$

where A_t is the area enclosed by the centre lines of longitudinal bars Figure 1, s is the centre-to-centre spacing of stirrups, f_{ys} is the yield strength of stirrups, A_{sw} is the cross-sectional area of stirrups, and θ_t is the crack angle which can be taken as 45° or can vary linearly between 30° when $T^* = \phi T_{uc}$ and 45° when $T^* = \phi T_{u.\max}$, where T^* is the factored design torque, T_{uc} is the ultimate torsional strength of a beam without torsional reinforcement, and ϕ is equal to 0.7. The term $T_{u.\max}$ is the ultimate torsional strength of a beam limited by web crushing failure and can be obtained from $T_{u.\max} = 0.2 f'_c J_t$. This is a simple equation to evaluate $T_{u.\max}$. Other more complicated equations have been presented in the literature but not adapted by the standard. For example, Warner et al. [6] present $T_{u.\max}$ as

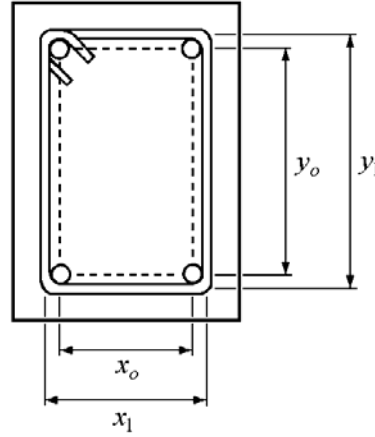


Figure 1. The cross-section of a rectangular reinforced concrete beam

$$T_{u.\max} = f'_c \left(A_{oh}^2 / p_h \right) \left(\frac{\sin \theta_t \cos \theta_t}{1.14 + 0.68 \cot^2 \theta_t} \right) \quad (3)$$

where A_{oh} is the area enclosed by the centreline of the exterior closed ties and p_h is the perimeter. AS3600 suggests that the total longitudinal steel area, A_s , shall be obtained by

$$A_s = (f_{ys} / f_y) (A_{sw} / s) u_t \cot^2 \theta_t \quad (4)$$

where u_t is the perimeter of A_t (in Eq. (2)) and f_y is the yield strength of longitudinal reinforcement. Furthermore, according to this standard, the spacing of stirrups shall not be greater than the lesser $0.12u_t$ and 300mm.

2.2 British Standard (BS8110)

The British standard for reinforced concrete structures, BS8110, indicates that the additional stirrups required to resist torsion in addition to what is required for shear shall be calculated from

$$\frac{A_{sv}}{s} > \frac{T_{us}}{0.8 x_1 y_1 (0.87 f_{ys})} \quad (5)$$

where A_{sv} is the area of the two legs of stirrups at a section, and x_1 and y_1 are the center-to-center of the shorter and longer legs of stirrups, Figure 1. Moreover, BS8110 suggests that additional longitudinal reinforcement A_s due to torsion should be provided as calculated by

$$A_s > \frac{A_{sw} f_{ys} (x_1 + y_1)}{s f_y} \quad (6)$$

This standard emphasises that the spacing of stirrups should not exceed the smallest of x_1 , $y_1/2$ or 200mm. BS8110 only allows the use of its provisions for torsional design when the yield stress of reinforcement is not more than 460MPa.

2.3 ACI Standard (ACI318-02)

ACI318-02 calculates the ultimate torsional strength of reinforced concrete beams as

$$T_{us} = f_{ys} (A_{sw} / s) 2 A_o \cot \theta_t \quad (7)$$

where A_o is the gross area enclosed by the shear flow path which can be taken equal to $0.85A_{oh}$, where A_{oh} is the area enclosed by the centre of stirrups.

ACI allows the crack angle θ_t of non-prestressed or low-prestressed members to be taken as 45° . Eq. (7) is based on the assumptions that all of the external torque is resisted by reinforcement and concrete resistance is negligible; that the concrete carries no tension; that the reinforcement yields, and that the concrete outside the stirrups is relatively ineffective.

The standard also indicates that the additional longitudinal reinforcement (A_s) required for torsion shall not be less than the value obtained from the following equation

$$A_s = (f_{ys} / f_y) (A_{sw} / s) u_t \cot^2 \theta_t \quad (8)$$

ACI318-02 recommends that the transverse torsional reinforcement (stirrup) shall be anchored by a 135° standard hook around a longitudinal bar and the spacing of transverse torsion reinforcement shall not exceed the smaller of $p_h/8$ or 12" (≈ 304 mm).

2.4 European Standard

According to the European Standard (Eurocode 2), three different ultimate values should be calculated and the minimum chosen. The first value is related to the stirrups contribution to the torsional resistance which can be calculated as

$$T_{u(1)} = f_{ys} (A_{sw} / s) 2 A_k \cot \theta_t \quad (9)$$

where A_k is the area enclosed by the centre-lines of the effective wall thickness. The effective wall thickness, t_{ef} , can be calculated as A/u where A is the total area and u is the perimeter of the cross-section. The second value of the torsional strength corresponds to the longitudinal bars as

$$T_{u(2)} = f_y (A_s / u_k) 2 A_k \tan \theta_t \quad (10)$$

where u_k is the perimeter of the area A_k .

Torsional capacity of the concrete struts is the third value. It can be derived from

$$T_{u(3)} = 2 \nu f_{ck} A_k t_{ef} \sin \theta_t \cos \theta_t \quad (11)$$

where f_{ck} is the compressive strength of concrete, and ν can be taken as $0.6(1 - f_{ck}/250)$. The least of these three values is the torsional strength of the member. The European Standard also indicates that the variation of crack angle is in the order of $1 \leq \cot \theta_t \leq 2.5$ but can be taken as $\theta_t = 45^\circ$.

2.5 Canadian Standard

The method of calculating torsional strength of reinforced concrete beams in the Canadian Standard, CSA, is similar to ACI. In addition, CSA advises that the stirrups must be anchored by 135° hooks, the nominal diameter of the bar or tendon shall not be less than $s/16$, and the total area of longitudinal bars required around the section, A_l , (with a spacing not exceeding 300 mm) shall be calculated from $A_l p_h / s$, where A_l is the area of a stirrup, p_h is the perimeter of the centre line of the stirrups, and s is the spacing of stirrups.

In the above mentioned standards, the method of evaluating the ultimate torsional capacity of reinforced concrete beams is essentially similar to the analytical method of skew bending model [7]. This universal choice probably is because of the simplicity of this model. The accuracy of predictions based on standards' methods, however, may not be as favourable as will be seen in the following.

3. STANDARDS VERSUS EXPERIMENTAL RESULTS

In order to investigate the accuracy of standards' provisions for torsion, they are compared with some experimental results in this section. Table 1 contains the chosen test beams extracted from different sources [8-14]. These beams cover a wide range in terms of dimensions, concrete compressive strength (f'_c), stirrups and rebars diameters, yield stress, and spacing of stirrups along the length. Beams are identified using the notations used in the original papers. The results of torsional strength of the beams predicted by different standards and the corresponding strength obtained from the tests, are presented in Table 2. Ratios of $T_{Standard}/T_{Experimental}$ are calculated from these and recorded in Table 3. As is seen, means of the torque ratios for all standards are less than one except for ACI. This indicates that except for ACI, all other standards have predicted the torsional capacities conservatively. The average of torque ratios show that Eurocode 2 and CSA are more successful in predicting the ultimate torques comparing other standards. Standard deviations, however, demonstrate that Australian standard is least deviated from the average value with a standard deviation of 0.256 and thus its conservativeness can be trusted more confidently.

Table 1. Characteristics of tested beams

Test beam	Cross-Section <i>mm</i> × <i>mm</i>	Length <i>mm</i>	f'_c MPa	Diameter of stirrups <i>mm</i>	Diameter of rebars <i>mm</i>	f_{yt} MPa	f_{yl} MPa	S <i>mm</i>
1-1	151×308	2745	35.8	9.5	9.5&19.1	379	344	83
E-3	157.5×310	3100	32.4	9.5	12.7	395	402	114
B3	254×381	3100	27.6	12.7	19.1	320	327.5	127
L1-1	127×203	3100	28.3	6.35	12.7&15.9	345	345	102
R4	127×203	3100	35.6	6.35	12.7&15.9	345	345	76
RC	150×350	1600	39	6	16	251	502	80
SI	100×150	1800	42.4	6	12	307	478	50
IIA	100×150	1800	34	6	12	419	357	40
IIC	100×150	1800	22.8	6	12	424	371	40
B-1	130×130	2000	20	6	6	240	240	130
C-2	130×130	2000	20	6	8.5	240	240	65

Table 2. Torsional strength, T_{us} , of beams in Table 1, according to standards' predictions and test results, *kNm*

Test beam	AS3600, T_{AS}	BS 8110, T_{BS}	ACI 318-02, T_{ACI}	European Standard, T_{ES}	CSA, T_{CSA}	Test result, T_E
1-1	13.74	13.49	16.47	16.74	16.47	13.78
E-3	11.29	13.27	16.21	13.33	16.21	13.67
B3	32.01	29.35	42.16	34.57	35.83	37.51
L1-1	2.41	1.95	2.39	3.09	2.39	3.28
R4	3.47	2.61	3.20	4.15	3.2	3.39
RC	3.46	3.39	4.88	5.12	4.15	15.00
SI	1.81	1.76	2.53	2.92	2.15	2.7
IIA	2.55	2.56	3.68	2.96	3.13	2.3
IIC	2.03	2.66	3.83	2.09	3.26	2.08
B-1	1.00	0.79	1.13	0.99	0.96	2.00

C-2	1.88	1.58	2.26	1.32	1.92	2.25
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Table 3. Comparison of the ultimate torques between standards' predictions and experimental results *

Test beam	T_{AS} / T_E	T_{BS} / T_E	T_{ACI} / T_E	T_{ES} / T_E	T_{CSA} / T_E
1-1	0.997	0.979	1.195	1.215	1.195
E-3	0.826	0.971	1.186	0.975	1.186
B3	0.853	0.782	1.124	0.922	0.955
L1-1	0.735	0.595	0.729	0.942	0.729
R4	1.024	0.770	0.944	1.224	0.944
RC	0.231	0.226	0.325	0.341	0.277
SI	0.670	0.652	0.937	1.081	0.796
IIA	1.109	1.113	1.600	1.287	1.361
IIC	0.976	1.279	1.841	1.005	1.567
B-1	0.500	0.395	0.565	0.495	0.480
C-2	0.836	0.702	1.004	0.587	0.853
Average	0.796	0.769	1.041	0.916	0.940
Sta. dev.	0.256	0.308	0.432	0.313	0.377

* Corresponding torques are obtained from Table 2.

While average values are not too much of a problem (apart from ACI's average of 1.041), the results for some beams are so out of range that they may make the designers too uncomfortable. For instance, the torque ratio for beams B-1 and SI in Table 3 obtained from the Australian standard are 0.500 and 0.670 respectively which are too conservative. The worst risky cases for ACI are related to beams IIC and IIA with the torque ratios of 1.841 and 1.600 respectively. Unfortunately not all standards present unsatisfactory results for similar beams and the accuracy of predictions is not uniform across all of the test beams. The discussion below may provide some useful information on the sources of these problems.

One of the parameters that plays a major role in the procedure of evaluating the torsional strength is the area used in the shear flow calculations. This area determined differently in

different standards is calculated for the beams of Table 1 as presented in Table 4. As seen in this table, in almost all cases the minimum values are related to AS3600 and the maximum values to the European Standard. The exception is beam B3 for which the maximum belongs to ACI (or CSA). In different standards, the position of sides of the area used in the shear flow calculations, is chosen differently. Taking the centres of longitudinal bars or centre-to-centre of stirrups for this calculation will make a (sometimes considerable) difference in size of the “area”. This difference in cases where the diameter of steel bars is larger, would be more pronounced.

Table 4. The value of area related to shear flow obtained from the standards, mm^2

Test beam	AS3600, A_t	BS 8110, $0.8(x_l \times y_l)$	ACI 318-02 & CSA, A_o	European Standard, A_k
1-1	21228	23950	25447	26941
E-3	23787	25535	27131	27146
B3	50172	52869	56174	54193
L1-1	8616	10490	11145	14432
R4	8616	10490	11145	14432
RC	19584	19085	27636	28966
SI	4416	5837	6202	8400
IIA	4416	5837	6202	8400
IIC	4416	5837	6202	8400
B-1	8464	8653	9194	9506
C-2	8464	8653	9194	9506

Another issue to be noted is that yielding of reinforcements at ultimate torque, is a “take it for granted” assumption in most of the standards. The reality is rather more complicated, as the yielding would be very much affected by the amount of reinforcement relative to the size of the cross-section and also depends on the relative sizes of longitudinal bars and the stirrups. In general, three different conditions can be assumed for the longitudinal bars and the stirrups at ultimate torque as: 1) both longitudinal bars and stirrups are yielded, 2) either longitudinal bars or stirrups are yielded, and 3) neither longitudinal bars nor stirrups are yielded.

The design procedure used by AS3600, ACI, and CSA assumes the first condition, which is yielding of both longitudinal bars and stirrups, for all types and sizes of reinforcements and for any compressive strength. This generalisation does not seem appropriate. Analytical investigations as well as numerical and experimental studies available in the literature show that yielding of stirrups or longitudinal reinforcements are very much affected by dimensions, the amount of reinforcement, and the concrete compressive strength. High values of yield stresses, larger sizes of reinforcement and weaker concrete gives way to the dominance of the third condition in which neither the longitudinal bars nor stirrups are yielded at ultimate torque. In the case of smaller sizes of reinforcement, lower yield stress and stronger concrete, the first and the second conditions may govern the ultimate mode of failure.

The angle of cracks θ_i may also affect the torsional strength of reinforced concrete beams. Among the standards that are considered in this study, ACI, CSA and BS do not have any particular procedure to evaluate the crack angle. In AS3600, however, the angle of cracks varies from 30° to 45° , and in the European Standard, from 22° to 45° . In the calculation presented here in Table 2, the values of torsional strength were calculated assuming a $\theta_i = 45^\circ$ except for AS3600 for which the angle was varied as suggested by the standard.

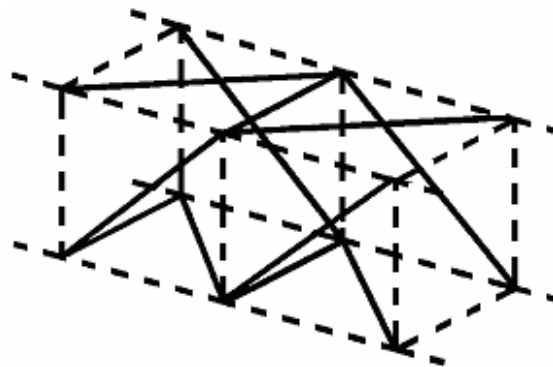


Figure 2. The torsion truss

A method which is sometimes used to determine the ultimate torsional capacity of reinforced concrete beams is the Compression Field Theory (CFT) of Collins & Mitchell [15]. The Compression Field Theory (CFT) [15] incorporates both equilibrium and compatibility conditions. In this method, the RC beam is assumed to carry the torque through a truss-shaped structure, elements of which are comprised of longitudinal chords, diagonal concrete struts, and transverse steel ties, Figure 2. CFT assumes that concrete carries no tension after cracking and the inclination angle of the diagonal compressive stress is the same as the inclination angle of the principal compressive strain. This assumption provides a means to relate the angle of cracks and the strains together as:

$$\tan^2 \theta = \frac{\varepsilon_l - \varepsilon_{ds}}{\varepsilon_t + \varepsilon_{ds}} \quad (12)$$

where θ is the angle of cracks measured from the horizontal beam axis, ε_l and ε_t are the strains in the longitudinal and transverse steel bars, respectively, and ε_{ds} is the maximum diagonal compressive strain in concrete. The analysis procedure by this method involves trial and error on the value of ε_{ds} until both equilibrium and compatibility conditions are satisfied. The Compression Field Theory is often very accurate. Table 5 shows the ultimate torques predicted by CFT for the above mentioned beams in comparison to the corresponding experimental torques. As seen in this table, the ultimate torsional strengths, T_{CFT} , predicted by CFT are in most cases reasonably close to the experimental results. The average of torque ratio for all beams is 0.912. While the average is comparable to the averages of Eurocode 2 and CSA (0.916 and 0.940 respectively), the standard deviation is only 0.115, which is smallest compared to all other standard deviations presented in Table 3. This clearly shows the consistency of CFT in predicting the torques. While CSA and Eurocode 2 are both performing rather weakly for beam B-1, the performance of CFT is adequate in predicting the theoretical ultimate torque at 86% of the experimental torque.

Table 5. The ultimate torques and strains of stirrups predicted by Compression Field Theory versus experimental results for test beams

Test beam	Ultimate torsional capacity, $kN.m$		Ratio of torque, T_{CFT} / T_E
	Experimental Results, T_E	CFT ^a T_{CFT}	
1-1	13.78	14.06	1.020
E-3	13.67	12.92	0.945
B3	37.51	36.47	0.972
L1-1	3.28	3.05	0.929
R4	3.39	3.82	1.127
RC	15.00	14.3	0.953
SI	2.70	2.12	0.785
IIA	2.30	2.02	0.878
IIC	2.08	1.46	0.702
B-1	2.00	1.72	0.860
C-2	2.25	1.94	0.862
		Average	0.912
		Sta. dev.	0.115

^a Compression Field Theory

One of the reasons for the better performance of Compression Field Theory for beam B-1

is that the yielding of reinforcement is not assumed; rather the state of strain is calculated according to the equilibrium and compatibility equations. In order to study the strain values in stirrups at the ultimate torque, ϵ_{yt} , the ratios of this strain to the yield strain of stirrups, ϵ_{ys} , obtained from CFT for all tested beams are presented in Table 6. This table also shows the average of torque ratios for different beams. Results presented in Table 6 show that one of the most influential parameters affecting the ultimate torque is the ultimate strain of stirrups. While this parameter is very much affected by the complex interaction between the concrete strut, the longitudinal steel and the stirrups which in turn depend on the material properties, the main effect for common dimensions and properties of steel and concrete is probably attributed to ratios of $Af'_c / A_l f_{yl}$ and $Af'_c s / p A_{sw} f_{ys}$ where A is the area of the cross-section of the beam, f'_c is the concrete compressive strength, A_l is the area of all longitudinal bars in the cross-section, f_{yl} is yield stress of longitudinal bars, s is the space of stirrups along the beam, p is the perimeter of the cross-section, A_{sw} is the area of the stirrup, and f_{ys} is the yield stress of the stirrup. As is seen in Table 6, for beams IIA and IIC, the stirrups are between 50% to 65% of the yield strain at ultimate. The unconservative results of almost all standards for these beams may be attributed partly to this effect. On the other hand the hugely over-conservative values of all standards for beam B-1 and RC may be related to the fact that this beam is considerably under-reinforced and therefore would experience significant yielding.

Table 6. Effective ratios $Af'_c / A_l f_{yl}$ and $Af'_c s / p A_{sw} f_{ys}$ in values of strain of stirrups at ultimate torque

Test beam	Ave. of $T_{Standards}/T_E^a$	$\epsilon_{ut}/\epsilon_{yt}^b$	$\frac{Af'_c}{A_l f_{yl}}$	$\frac{Af'_c s}{p A_{sw} f_{ys}}$
1-1	1.116	0.959	6.8	5.6
E-3	1.029	1.428	7.7	6.9
B3	0.927	1.807	7.1	6.6
L1-1	0.746	2.798	3.3	10.3
R4	0.981	2.401	4.1	12.9
RC	0.280	10.4	5.1	22.9
SI	0.827	1.171	2.9	9.1
IIA	1.294	0.650	3.1	4.3
IIC	1.335	0.509	2.0	2.9
B-1	0.487	6.778	9.4	12.4
C-2	0.796	2.139	6.3	6.3

^a Average of all five torque ratios for each row presented in Table 3.

^b ϵ_{ut} is the strain of stirrups at the ultimate torque obtained from Compression Field Theory and ϵ_{yt} is the yield strain of stirrups.

Correlating the ratios in columns 4 and 5 of Table 6 with column 2 that contains the ratio of average of the predictions of current standards to experimental torques and observing the corresponding value of $\epsilon_{ut} / \epsilon_{yt}$ (column 3) reveals that when both ratios $Af'_c / A_l f_{yl}$ and $Af'_c s / p A_{sw} f_{ys}$ are smaller than 4 to 5, the predictions of standards could be too risky. The failure mode is a concrete crushing mode and is not associated with the yielding of stirrups. As many of the aforementioned standards do not present a formula to calculate this and the ones that do present it, are too simple in this regard, appropriate methods of calculating the torque associated with concrete crushing failure should be sought. On the other hand when both of the aforementioned ratios $Af'_c / A_l f_{yl}$ and $Af'_c s / p A_{sw} f_{ys}$ are greater than 9 or $Af'_c s / p A_{sw} f_{ys}$ is greater than 20, the standards' predictions could be too conservative following which would unnecessarily add to the cost. Better methods of evaluating the capacity should be utilised in those situations. It is recommended that these limits (possibly after further fine tuning) are introduced in the standards in addition to more elaborate formulae for the ultimate torque based on concrete crushing.

4. CONCLUSION

This paper reviewed the provisions of some of the well known international standards in relation to torsion of reinforced concrete beams. In order to highlight various standards' weaknesses and strengths, the ultimate torques predicted by the standards for a set of tested beams were calculated and the accuracy of the results was discussed. It was shown that none of the standards were successful in predicting the ultimate torque accurately for all beams. For some beams, standards' predictions were too conservative and for some too risky.

The ultimate torques calculated by the analytical method of Compression Field Theory (CFT) demonstrated much better agreement with the experiments. While reasons for this better performance were highlighted, a couple of parameters influential in the ultimate torques were identified and limits to their values were proposed beyond which the standards' methods may become unreliable. These limits could be used in the standards after further refinement.

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