(3) Rack and Spur Gear

Table 4.5 presents the method for calculating the mesh of a rack and spur gear.

Figure 4.3(1) shows the the meshing of standard gear and a rack. In this meshing, the reference sircle of the gear touches the pitch lin of the rack.

Figure 4.3(2) shows a profile shifted spur gear, with positive correction xm, meshed with a rack. The spur gear has a larger pitch radius than standard, by the amount xm. Also, the pitch line of the rack has shifted outward by the amount xm.

Table 4.5 presents the calculation of a meshed profile shifted spur gear and rack. If the profile shift coefficient x_1 is 0, then it is the case of a standard gear meshed with the rack.

Table 4.5 The calculation of dimensions of a profile shifted spur gear and a rack

No.	Item	Symbol	Formula	Example: Spur gear	Example: Rack
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		12	—
4	Profile shift coefficient	x		0.6	
5	Height of pitch line	H		—	32.000
6	Working pressure angle	α'		20°	
7	Mounting distance	a	$\frac{zm}{2} + H + xm$	51.800	
8	Reference diameter	d	zm	36.000	—
9	Base diameter	d_b	$d \cos\alpha$	33.829	
10	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'}$	36.000	
11	Addendum	h_a	$m(1+x)$	4.800	3.000
12	Tooth depth	h	$2.25m$	6.750	
13	Tip diameter	d_a	$d + 2h_a$	45.600	—
14	Root diameter	d_f	$d_a - 2h$	32.100	

One rotation of the spur gear will displace the rack l one circumferential length of the gear's reference circle, per the formula:

$$l = \pi m z \qquad (4.2)$$

The rack displacement, l, is not changed in any way by the profile shifting. Equation (4.2) remains applicable for any amount of profile shift.

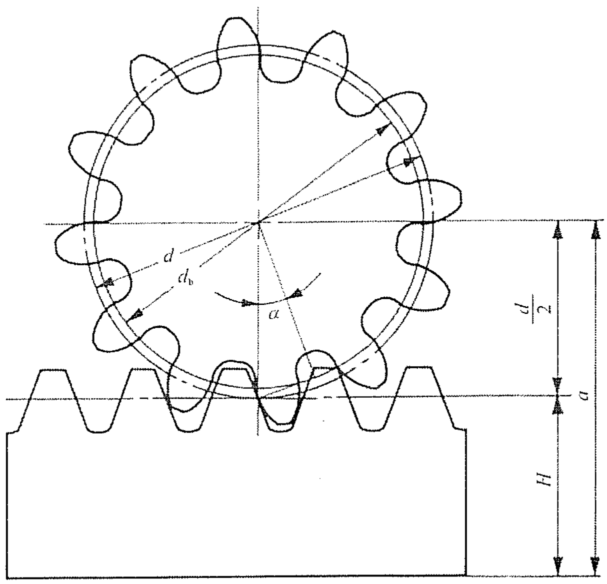

Fig.4.3(1) The meshing of standard spur gear and rack
($\alpha = 20°, z_1 = 12, x_1 = 0$)

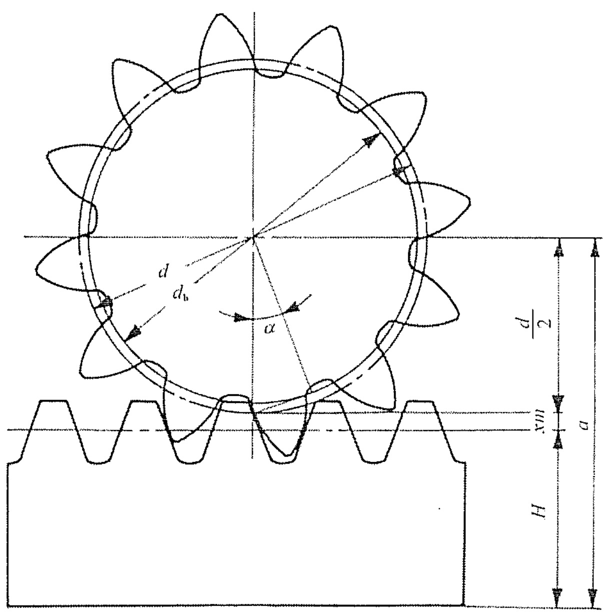

Fig.4.3(2) The meshing of profile shifted spur gear and rack
($\alpha = 20°, z_1 = 12, x_1 = +0.6$)

4.2 Internal Gears

(1) Internal Gear Calculations

Figure 4.4 presents the mesh of an internal gear and external gear. Of vital importance is the working pitch diameters (d') and working pressure angle (α'). They can be derived from center distance (a') and Equations (4.3).

$$\left.\begin{aligned} d'_1 &= 2a\frac{z_1}{z_2-z_1} \\ d'_2 &= 2a\frac{z_2}{z_2-z_1} \\ \alpha' &= \cos^{-1}\left(\frac{d_{b2}-d_{b1}}{2a}\right) \end{aligned}\right\} \quad (4.3)$$

Table 4.6 shows the calculation steps. It will become a standard gear calculation if $x_1 = x_2 = 0$.

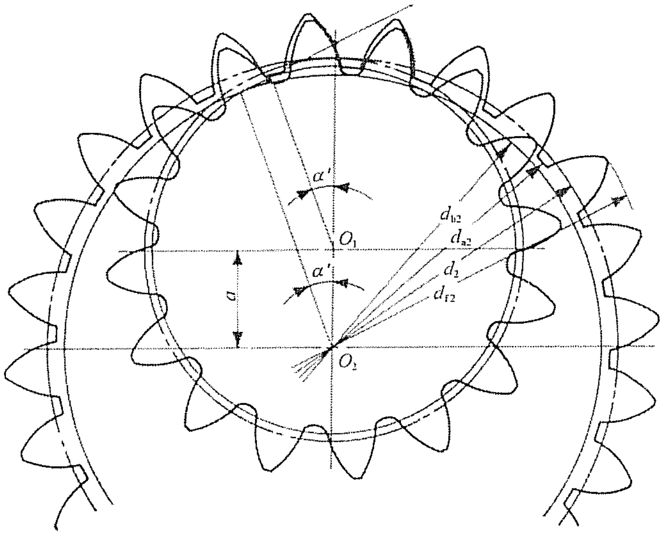

Fig.4.4 The meshing of internal gear and external gear
($\alpha = 20°$, $z_1 = 16$, $z_2 = 24$, $x_1 = x_2 = +0.5$)

Table 4.6 The calculation of a profile shifted internal gear and externl gear (1)

No.	Item	Symbol	Formula	Example	
				External gear	Internal gear
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		16	24
4	Profile shift coefficient	x		0	+ 0.5
5	Involute function α'	invα'	$2\tan\alpha\left(\frac{x_2-x_1}{z_2-z_1}\right)+\text{inv}\alpha$	0.060401	
6	Working pressure angle	α'	Find from involute Function Table	31.0937°	
7	Center distance modification coefficient	y	$\frac{z_2-z_1}{2}\left(\frac{\cos\alpha}{\cos\alpha'}-1\right)$	0.389426	
8	Center distance	a	$\left(\frac{z_2-z_1}{2}+y\right)m$	13.1683	
9	Reference diameter	d	zm	48.000	72.000
10	Base diameter	d_b	$d\cos\alpha$	45.105	67.658
11	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'}$	52.673	79.010
12	Addendum	h_{a1} h_{a2}	$(1+x_1)m$ $(1-x_2)m$	3.000	1.500
13	Tooth depth	h	$2.25m$	6.75	
14	Tip diameter	d_{a1} d_{a2}	d_1+2h_{a1} d_2-2h_{a2}	54.000	69.000
15	Root diameter	d_{f1} d_{f2}	$d_{a1}-2h$ $d_{a2}+2h$	40.500	82.500

(2) Profile Shifted Spur Gear

Figure 4.2 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating (working) pitch diameters (d') and the working (operating) pressure angle (α').
These values are obtainable from the modified center distance and the following formulas:

$$\left.\begin{aligned} d'_1 &= 2a\frac{z_1}{z_1+z_2} \\ d'_2 &= 2a\frac{z_2}{z_1+z_2} \\ \alpha' &= \cos^{-1}\left(\frac{d_{b1}+d_{b2}}{2a}\right) \end{aligned}\right\} \quad (4.1)$$

In the meshing of profile shifted gears, it is the operating pitch circle that are in contact and roll on each other that portrays gear action.
Table 4.3 presents the calculation where the profile shiht coefficient has been set at x_1 and x_2 at the beginning. This calculation is based on the idea that the amount of the tip and root clearance should be 0.25 m.

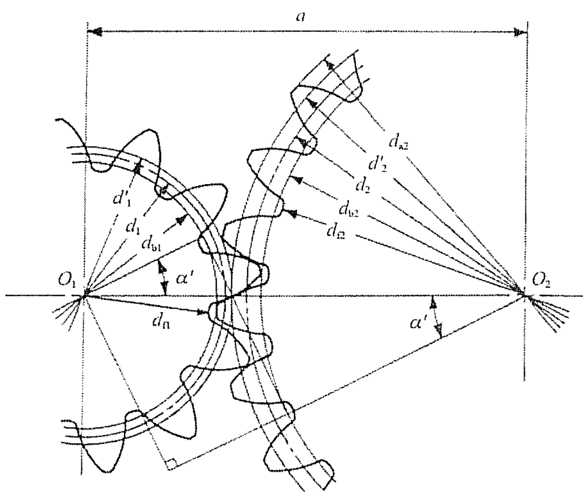

Fig. 4.2 The meshing of profile shifted gears
($\alpha = 20^\circ$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

Table 4.3 The calculation of profile shifted spur gear (1)

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		12	24
4	Profile shift coefficient	x		0.6	0.36
5	Involute function α'	inv α'	$2\tan\alpha\left(\frac{x_1+x_2}{z_1+z_2}\right)+\text{inv}\alpha$	0.034316	
6	Working pressure angle	α'	Find from Involute Function Table	26.0886°	
7	Center distance modification coefficient	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha'}-1\right)$	0.83329	
8	Center distance	a	$\left(\frac{z_1+z_2}{2}+y\right)m$	56.4999	
9	Reference diameter	d	zm	36.000	72.000
10	Base diameter	d_b	$d\cos\alpha$	33.8289	67.6579
11	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'}$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1+y-x_2)\,m$ $(1+y-x_1)\,m$	4.420	3.700
13	Tooth depth	h	$\{2.25+y-(x_1+x_2)\}m$	6.370	
14	Tip diameter	d_a	$d+2h_a$	44.840	79.400
15	Root diameter	d_f	d_a-2h	32.100	66.660

A standard spur gear is, according to Table 4.3, a profile shifted gear with 0 coefficient of shift;
that is , $x_1 = x_2 = 0$.

Table 4.4 is the inverse formula of items from 4 to 8 of Table 4.3.

Table 4.4 The calculation of profile shifted spur gear (2)

No.	Item	Symbol	Formula	Example	
1	Center distance	a		56.4999	
2	Center distance modification coefficient	y	$\frac{a}{m} - \frac{z_1 + z_2}{2}$	0.8333	
3	Working pressure angle	α'	$\cos^{-1}\left(\frac{\cos\alpha}{\frac{2y}{z_1 + z_2} + 1}\right)$	26.0886°	
4	Sum of profile shift coefficient	$x_1 + x_2$	$\frac{(z_1 + z_2)(\mathrm{inv}\alpha' - \mathrm{inv}\alpha)}{2\tan\alpha}$	0.9600	
5	Profile shift coefficient	x		0.6000	0.3600

There are several theories concerning how to distribute the sum of profile shift coefficient $(x_1 + x_2)$ into pinion (x_1) and gear (x_2) separately. BSS (British) and DIN (German) standards are the most often used. In the example above, the 12 tooth pinion was given sufficient correction to prevent undercut, and the residual profile shift was given to the mating gear.

(2) Profile Shifted Spur Gear

Figure 4.2 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating (working) pitch diameters (d') and the working (operating) pressure angle (α').
These values are obtainable from the modified center distance and the following formulas:

$$\left.\begin{aligned} d'_1 &= 2a\frac{z_1}{z_1+z_2} \\ d'_2 &= 2a\frac{z_2}{z_1+z_2} \\ \alpha' &= \cos^{-1}\left(\frac{d_{b1}+d_{b2}}{2a}\right) \end{aligned}\right\} \qquad (4.1)$$

In the meshing of profile shifted gears, it is the operating pitch circle that are in contact and roll on each other that portrays gear action.
Table 4.3 presents the calculation where the profile shiht coefficient has been set at x_1 and x_2 at the beginning. This calculation is based on the idea that the amount of the tip and root clearance should be 0.25 m.

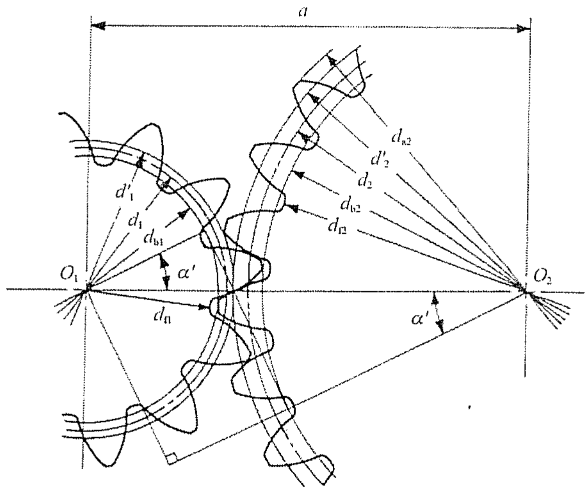

Fig. 4.2 The meshing of profile shifted gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

Table 4.3 The calculation of profile shifted spur gear (1)

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		12	24
4	Profile shift coefficient	x		0.6	0.36
5	Involute function α'	inv α'	$2\tan\alpha\left(\frac{x_1+x_2}{z_1+z_2}\right)+\text{inv}\alpha$	0.034316	
6	Working pressure angle	α'	Find from Involute Function Table	26.0886°	
7	Center distance modification coefficient	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha'}-1\right)$	0.83329	
8	Center distance	a	$\left(\frac{z_1+z_2}{2}+y\right)m$	56.4999	
9	Reference diameter	d	zm	36.000	72.000
10	Base diameter	d_b	$d\cos\alpha$	33.8289	67.6579
11	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'}$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1+y-x_2)\,m$ $(1+y-x_1)\,m$	4.420	3.700
13	Tooth depth	h	$\{2.25+y-(x_1+x_2)\}m$	6.370	
14	Tip diameter	d_a	$d+2h_a$	44.840	79.400
15	Root diameter	d_f	d_a-2h	32.100	66.660

A standard spur gear is, according to Table 4.3, a profile shifted gear with 0 coefficient of shift; that is , $x_1 = x_2 = 0$.