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GEOMETRIC NONLINEAR ANALYSIS FOR STEEL FRAME STRUCTURE

HO WAI WAN

A project report submitted in fulfillment of the requirement for the award of the Degree
of Bachelor of Civil Engineering

Faculty of Civil Engineering
UNIVERSITI TEKNOLOGI MALAYSIA

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To my beloved parents and siblings.

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ABSTRACT

This paper presents two analysis methods and two calculation methods for simple steel frame with rigid joint connections and semi-rigid column bases in order to capture the changes in nodal displacement and bending moment. Basically the two calculation methods applied in this paper are to perform analysis by manually using Secant Stiffness Method and with computer-aided software which based on Newton-Raphson Method. The structural model formulated is utilised for several calculation methods which take into account first order elastic analysis and second order elastic analysis which also known as linear analysis and nonlinear analysis. Each analysis on steel frame was conducted taking into consideration about the different dimension cases and load cases. These factors control the behavior of frames and influence the sizing of the members and lastly contribute to the overall stability of the frame. Present study mostly deals with the effect of geometrical nonlinearity to the optimal design problem. The first order linear elastic analysis procedures and second order nonlinear analysis procedures with two different cases are presented to demonstrate the efficiency of both calculation methods by giving the critical results. The analysis results revealed that a rigorous analysis should be carried out rather than to use conventional analysis to analyse the entire structure and members strength

ABSTRAK

Projek ini mempersembahkan dua cara analisis dan dua cara pengiraan bagi kerangka keluli bersambungan tegar antara rasuk dengan tiang dan bersambungan separuh tegar antara tiang dengan penahan. Secara umumnya, dua cara pengiraan yang digunakan untuk analisis linear elastik peringkat pertama dan analisis bukan linear elastic peringkat kedua adalah Kaedah Kekukuhan Secant secara manual dan perisian analisis komputer yang berasaskan Keadah Newton-Raphson. Model kerangka keluli digunakan untuk menerangkan sifat bukan-linear yang dibawa oleh analisis bukan linear elastik peringkat kedua. Dua kes yang diambil kira adalah kes dimensi dan kes beban akan dianalisis dalam projek ini. Faktor-faktor ini mempengaruhi cirri-ciri kerangka keluli dan emenetukan saiz rasuk serta tiang bagi kerangka serta menyumbang kepada kestabilan kerangka. Projek semasa adalah mengkaji kesan bukan-linear yang dibawa kepada reka bentuk struktur. Analisis linear elastik peringkat pertama dan analisis bukan-linear elastik peringkat kedua bagi kedua-dua kes dipersembahkan untuk memodelkan keupayaan kedua-dua cara analisis dalam menganalisiskan kelakuan struktur dengan keputusan yang dapat. Keputusan analisis memberi kesimpulan bahawa analisis bukan-linear elastik peringkat kedua perlu diadakan semasa menjalankan prosidur analisis struktur supaya kekukuhan struktur dan ahli struktur dapat dikaji dengan terperinci sebelum sebarang reka bentuk struktur mula.

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LIST OF SYMBOLS

P	-	Axial load (Vertical and Horizontal)
Δ	-	External displacement
∂	-	Internal displacement
$[T]$	-	Transformation matrix
$[K]$	-	Global linear element stiffness matrix
$[k]$	-	Local linear element stiffness matrix
A	-	Area of specific member
E	-	Young Modulus of steel
L	-	Length of specific member
λ_x	-	Cosine angle between X-axis
λ_y	-	Cosine angle between Y-axis
I	-	Moment of Inertia
$\{F\}$	-	External forces
$[K_e]$	-	Global first order elastic stiffness matrix

$[K_g]$	-	Global geometric nonlinear stiffness matrix
M	-	Moment
H	-	Height of structure
W	-	Width of structure
f	-	Internal member forces
F_k	-	External known force
F_u	-	External unknown force
Δ_k	-	External known displacement
Δ_u	-	External unknown displacement
[S]	-	Structure stiffness matrix
$\{\Delta D\}$	-	Increment displacement vector
Φ	-	Angle of specific member
K_u	-	Unknown global linear element stiffness matrix
K_k	-	Known global linear element stiffness matrix
U	-	Nodal displacement
F	-	Axial force on member
K_a	-	Summation of global linear element stiffness matrix and global geometric nonlinear element stiffness matrix

Δ_x	-	Lateral displacement
Δ_y	-	Vertical displacement
Θ_z	-	Slope value
P_{cr}	-	Critical axial load

CHAPTER 1

INTRODUCTION

1.1 Introduction

Steel frame system with beams and columns is the conventional building structure in today construction world. As everyone knows, structural design and structural analysis are both of the criteria needed to create a structure that safely accomplish its function in order to produce structures in a stability condition. In civil engineering field, steel is widely used in building construction. Its popularity may be due to the various sizes and the shape of steel sections to be used for varies types of structure such as small and simple buildings as well as complicated infrastructures construction. Generally, steel frame not is only design to sustain vertical loads but also able to resist lateral loads.

It is well known for many years that first order elastic analysis with small displacement assumption of geometric changes does not predict the actual behavior of structures. Recently, structures were analysed by using first order elastic analysis. This happened because of the lack of knowledge of structural theory in the early years. Even later when theoretical work was advanced, due to the lack of the computational capability, only the structural analysis based on first order elastic analysis became the most familiar analysis method utilised in current engineering world. Besides that, engineers nowadays are generally familiar with calculating the deflection of member under load. Unlucky is, engineers might ignored the effects that come from the load act on the deformed shape structure which known as geometrical nonlinearity, termed as second order elastic analysis or P-Delta analysis. Geometrical nonlinearities occur when members bend and sway or deflect horizontally under loading. From here, it is obvious that second order elastic analysis play an important role in controlling the stability of the entire structure. This showed that first order elastic analysis cannot totally gives accurate solutions for practical analysis in engineering world. Hence, the inclusion of second order elastic analysis will represent the appropriate behavior of the planar steel frame structure.

Thus, the structural engineering community has developed a new generation of analysis method that incorporate performance based structures and is moving away from first order elastic analysis towards a more nonlinear technique which known as second order elastic analysis.

The important concern focus here is to do a study on the deflection of planar steel frame structure that involve first order elastic analysis and second order elastic analysis. One must be emphasised is the P-Delta effect becomes more severe with higher height. Therefore, commercial software will be programmed the whole structure completely. By carried out both analysis may provide an accurate understanding of planar steel frame behavior.

A simple flow chart is shown in Figure 1.1 to give a rough idea on what are the loadings and forces experienced in the structure, and how the P-Delta effects lead to the total effects on the structure.

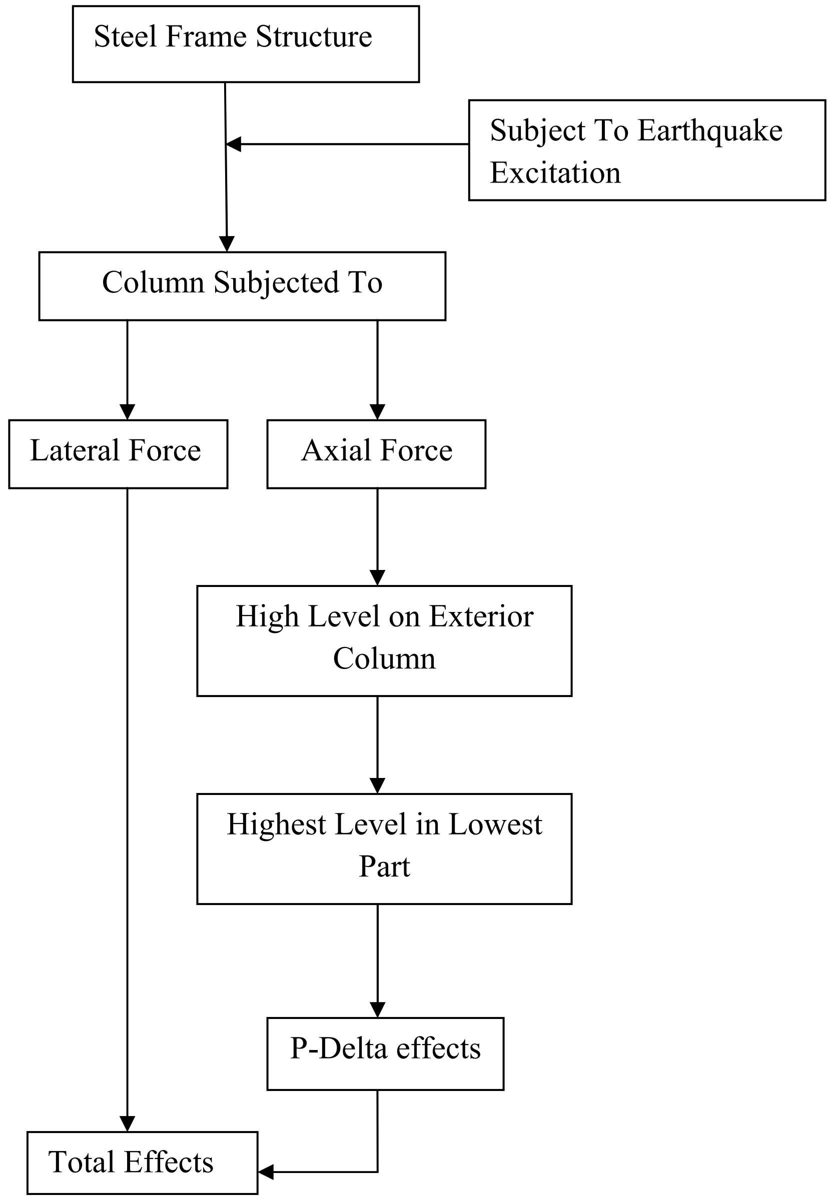

Figure 1.1 P-Delta effects lead to total effect

The important concern focus here is to do a study on the deflection of multi-storey planar steel frame structure that involve first order elastic analysis and second order elastic analysis. One must be emphasised is the P-Delta effect becomes more severe with higher story drift. Therefore, commercial software will be programmed the whole structure completely. By carried out both analysis may provide an accurate understanding of planar steel frame behavior.

Now, it is very clear that the aim of this paper is to highlight the P-Delta effects address to the steel frame structure.

1.2 Statement of Problem

In this paper, the main issue will be highlighted here is to take note on the effect which bring by second order elastic analysis by making a comparison between first order elastic analysis and second order elastic analysis. The effect given by the first order elastic analysis is the most common effect that obtained by using conventional analysis method that is Stiffness Method which considered the linearity effect in current practice. Contrary, the effects those contribute by second order elastic analysis may bring certain unexpected influences to the structure. In fact the term of "second order" reflects the higher degree of the behavior for the frame structure might be observed due to significant aspect which is excluded throughout first order elastic analysis.

So far, most of the engineers who considered just for the first order elastic analysis only surrounded with the linearity effect is captured and some of the researches showed that the first order elastic analysis not capable to give overall effects occurred on the structure. The effects mentioned are P-Delta effects which contribute by the changes in geometric stiffness of structure. As the height/width ratio of the structure increases, as well as increment of loadings, P-Delta effects will become more significant and give a chance for the structure to displace from its own pre-position and sway in unstable condition compare to the structure which directly to collapse. Since there has lot of researches shown the importance of second order elastic analysis should be concerned in structural analysis and design, therefore, a study about the different characteristics for both analyses which play important role in contributing the significant P-Delta effects will be carried out by both analysis methods and calculation methods. Furthermore, two cases which analysed the geometrical nonlinearity effects provide some useful information, and these give a hand in proceeding to the structural design section especially the critical bending moment comes from second order elastic analysis for the structural design purposes, in order that all the members are designed in an economical way.

1.3 Objective

The main objectives of this paper work are:

- To present manual analysis and software analysis with analytical results for linear problems and nonlinear problems,
- To study the response and behavior of simple one bay planar steel frame structure through a series of analysis under different load and dimension cases by adopting two analysis methods,
- To study the efficiency of Secant method (manual calculations) and Newton-Raphson method (software calculations) in analysing and providing an accurate result which able to reflect the real behavior of the frame structure.

1.4 Scope of Study

This paper focuses of the work is on a planar frame structure with the following characteristics:

- A simple one bay planar steel frame structure with rigid connection between beams and columns and semi-rigid column base supports.
- All connections between beams and columns for the planar frame building are fixed.
- The behavior of the planar frame building is elastic.
- The material nonlinearity due to the change in material property is not considered.
- The entire structure performs as static analysis.
- Static vertical loads and horizontal loads are applied as nodal forces.
- Distance between both columns of the planar frame structure is constant as structure's height increasing.
- $\alpha = 0.5P$ value for wind load according to Mcguire,2005.

1.5 Significant of Study

Stability is a fundamental to design and analysis, yet it becomes a challenging aspect for the current engineers since engineers fluent in first order elastic analysis rather than second order elastic analysis. Elastic method is preferred by the engineers for its simplicity, even for estimation of ultimate load and service load conditions. Nevertheless, first order elastic analysis insufficient captures the exact behavior of the building during the design stage. In order to perform the structure indeed, it is possible to take account of the second order effects as they will not be realised in first order elastic analysis.

In reality, multi-storey structure is essentially a vertical cantilever subjected to gravity loading and lateral loads. Horizontal loading for second order elastic analysis causes external shear, moment and torque which more complex than first order effects. It is dominant problem of analysis since it increases lateral sway stability of the structures. A significant lateral load might enhance moment resulting from eccentricity of axial loading at the design deflection under wind loading, commonly named P-Delta effects.

Engineers are borned to design and analyze a building structure for the benefit to provide a comfortable niche for the residents. In order to prevent the building free from collapse or unstable, it is necessary for these types of analysis to be applied optimally in practical design. Of course, second order elastic analysis involves more computational effort than conventional first order elastic analysis. It involved nonlinear effects and showed out the members which have potential tends to unstable the whole structure. However, it is clear that second order elastic analysis is necessary adopt in all cases since it does not make any simplification. Therefore it leads to a more accurate evaluation of the internal forces and moments than first order elastic analysis. Finally, second order elastic analysis is more efficient in capturing the real behavior of the building and by using the computer-aided analysis programs, no matter how complicated of such building is; there will be have a way to analyze it entirely to avoid it unstable.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

The deformation characteristics of a structure are concerned with stiffness rather than strength. The use of the geometric stiffness matrix is a general approach to include nonlinear effects in the static and dynamic analysis of all types of structural systems. However, in Civil Structural Engineering, it is commonly referred to as P-Delta analysis that is based on a more physical approach. This second order behavior has been termed the P-Delta effect since the additional overturning moments on the structure are equal to the sum of structure's weights, P times the lateral displacements, Delta. Thereby, excessive deformations may cause a number of undesirable effects occurred on the structure. In extreme case, a change in structural behavior is sufficient to cause structure to collapse.

According to research, first order elastic analysis is the most widely used method in the engineering analysis world due to its simplification in analysis and calculation since this analysis just considered the loads acted under the undeformed shape. Result obtained only valid for elastic structure with insignificant change in geometry; the resulting forces and moments do not contribute any effect to the deflection or deformation of the structure under load [1]. Due to its simplifications, it purposely make engineers analysed a structure in a comfortable way which cannot totally captured the real behavior of the structure. Insufficient structural analysis will caused structure to collapse and involved in fatal accidents.

The stiffness of the structure is an important characteristic, concerned with resistance to deformation rather than collapse. Thereby, to get a real approach on how the structure deformed, it is important to take into account the nonlinear behavior in analysis.

When the geometry change is significant, second order elastic analysis becomes an important issue that need to be considered in structural design in order to improve the accuracy of the results for geometric changes. Clearly there is a role for both approaches. Perhaps the most important aspect is the attitude of the engineers which should include a safety element during the analysis.

Figure 2.1 shows load-deflection behavior of plane frame. The bold line is obtained by first-order analysis ignoring the effects of the geometry change. The curve line is that of second-order elastic analysis that includes only the effect of change in geometry of the structure [1].

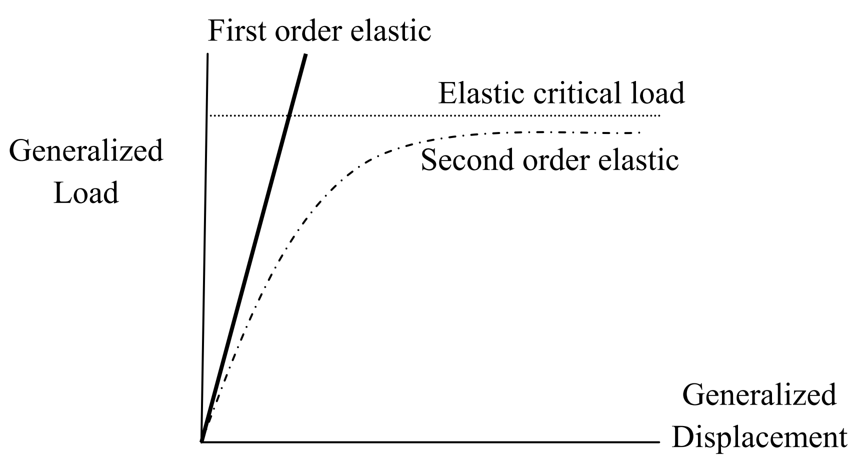

Figure 2.1 General analysis types for framed structure.

2.2 First-Order Elastic Analysis

Most of the civil engineering structures are analysed without considering the effect of the changes in geometry in order to simplify the numerous calculations. Normally deflection or deformation caused by applied loads that occurred is considered to be small; there by do not experience any significant nonlinearity. In “Limit state design”, structures are designed for limit state of strength and limit state of serviceability, leaving the structure with minimum reserve energy. If low horizontal loads act to a low-rise structure with small deflections and insignificant change in geometry of the structure, thus the reserve energy of the structure is sufficient to bring back the structure to equilibrium state after the load is removed, rendering stability to the structure as a whole. There by first-order elastic analysis of structures that satisfy the equilibrium conditions on the undeformed geometry is sufficient to verify the structural design [2]. From this approach, it can be assumed that the deformation or deflection is directly proportional to the applied load so that the relationship between the applied load and the deflection or deformation at any point is a straight line.

Engineers today typically use first-order elastic analysis to determine design forces and moments resulting from the loads that applied on the structures. It cannot be deny that first-order elastic analysis is the main profession in structural design and the solutions for these analyses are simple and straight forward although this type of analysis does not provide any information on the strength or stability of the frame.

Furthermore, first-order elastic analysis gains the advantages of being computationally simple, which is also efficient. A special benefit is the validity of the application of principle of superposition when subjected to multiple load cases. However, because of linearity, the stiffness of the structural member is taken as constant and independent of the presence of axial force. As a result, this linear approximation may not be accurate since second-order elastic effects due to geometrical changes are negligible.

2.3 Second-Order Elastic Analysis

In most practical designs, second-order elastic analysis not yet gains a wide acceptance among engineers. Even though most engineers are fluent in first order analysis but it is much important to conduct nonlinear analysis which will help analytically simulate more appropriate and perform a better behavior of the structure than the first order analysis. This will ultimately help in the optimal design of the structures by giving a sufficient resistance to the enhance moment.

Second-order elastic analysis considers the geometrical nonlinearity effects due to geometry changes in structures and members respectively, which considers the $P\text{-}\delta$ effect and $P\text{-}\Delta$ effect in the analysis. The analysis of the structure must be modified to capture the impact of these effects, as they will not be realized in a first-order elastic analysis. When allowing second-order effects due to a change in geometry by incorporating the variation of element stiffness in the presence of axial force, the calculated deflections, forces, and moments will be more accurate than the first-order elastic analysis.

For instance, when structures subjected to horizontal loads such as wind load and vertical loads such as applied loads included self-weight will bring significant deflection or deformation of the structure, same as geometric stiffness of the structure. The significant deflection or deformation and low reserve energy can prove to be cataclysmic if this small energy in the structure fails to sustain loads. Thus failing to satisfy the equilibrium conditions that formulated on the deformed configuration of the structure could become highly unstable. Therefore, a second order elastic analysis has to be carried out to determine the exact behavior of the structure. The main advantage of this analysis is their generality. It can serve as a powerful design tool for the irregular and complex structures that are sometimes encountered [1]. Lastly, second order elastic analysis accounts for $P\text{-}\Delta$ effect and $P\text{-}\delta$ effect in any situations where they may be significant.

2.4 What is P – Delta Effects?

P-delta is a nonlinear effect that occurs in every structure where flexure is introduced into an axially loaded member from the axial force acting through the side way of the frame and curvature of a member. The magnitude of the P-delta effects are related to the magnitude of axial load P, stiffness or slenderness of the structure as a whole and slenderness of individual elements [3].

As structures become slender and less resistant to deformation, P-delta effects increase. To reflect this, engineers explore to the use of second-order elastic analysis in order that P-delta effects take into account when appropriate in design. There are two P-delta effects which are *P-Δ* effect and *P-δ* effect which shown in Figure 2.2.

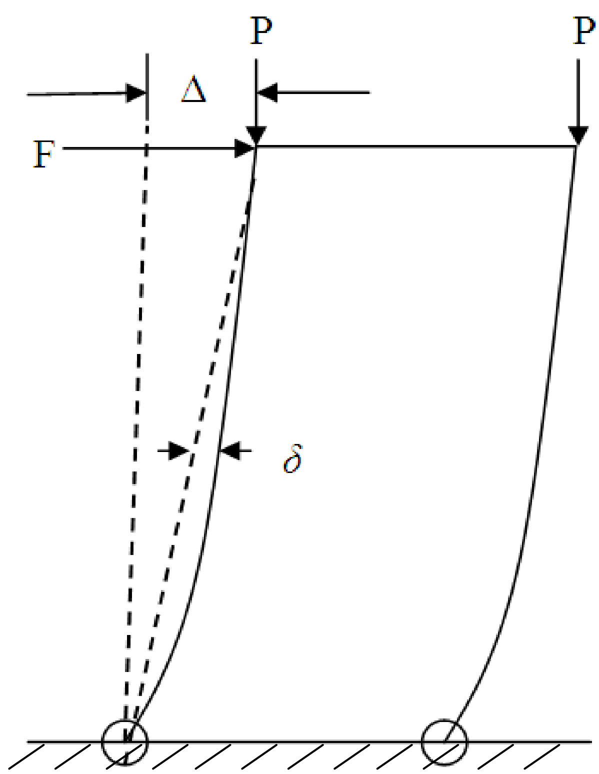

Figure 2.2 *P-Δ* effect and *P-δ* effect

2.4.1 *P-Δ* Effect

P-Δ effect is one part that should be included in the second-order elastic analysis. According to Henri Gavin Fall, defined that *P-Δ* effect is the potential energy function includes additional terms, which accounts for the interaction between the axial load effects on the frame element and the lateral deformation of the frame element in finite deformation analysis。

Meanwhile, Shankar Nair stated that *P-Δ* effect is the effect of loads acting on the displaced location of points of intersection of members in the structure [4]. According to Mcguire, the *P-Δ* effect destabilizing moment equal to gravity load times the horizontal displacement it undergoes as a result of lateral displacement [5].

To make *P-Δ* effect more understandable, *P-Δ* effect can be explain as the element flexural stiffness reduces against side way and the deformation shape occurred is shown in Figure 2.3.

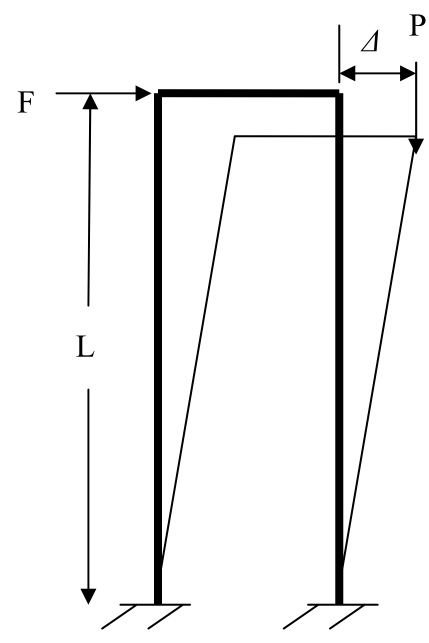

Figure 2.3 Deformation of *P-Δ* effect

2.4.2 *P-δ* Effect

P-δ effect which is the effect of loads acting on the deformed shape of individual members [4]. It is referred to as the geometrical nonlinearity effect due to the deflection along a member and the axial force. This force tends to reduce the flexural rigidity of the member, in the other words; the presence of axial force in a member is detrimental to the strength of the member. Meanwhile, Mcguire, stated that the *P-δ* effect is the influence of the axial on the flexural stiffness of the individual member of the structure [5]. Figure 2.4 demonstrate the behavior of the frame structure under P- *δ* effect.

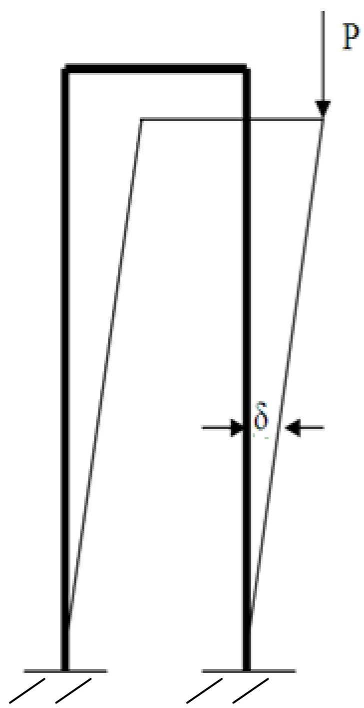

Figure 2.4 Deformation of *P-δ* effect

2.5 Stiffness Method for Second-Order Elastic Analysis

2.5.1 Transformation Matrix

Local coordinate system can be related to global coordinate system by a transformation matrix. The transformation matrix for two dimensional elements can be expressed as which shown in (2.1) [5].

$$[T] = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 & 0 & 0 \\ -\lambda_y & \lambda_x & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_x & \lambda_x & 0 \\ 0 & 0 & 0 & -\lambda_y & \lambda_y & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2.1)$$

The element stiffness matrix in the local coordinate system can be transformed into global coordinate system by using (2.2).

$$[K]=[T]^T[k][T] \qquad (2.2)$$

2.5.2 Global Element Stiffness Matrix [K] for First Order Elastic Analysis

From (2.2), the global linear stiffness matrix is obtained and shown in (2.3).

$$
[K] = \begin{bmatrix}
\frac{AE}{L}\lambda^2_x + \frac{12EI}{L^3}\lambda^2_y & (\frac{AE}{L} - \frac{12EI}{L^3})\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y & -\frac{AE}{L}\lambda^2_x & -(\frac{AE}{L} - \frac{12EI}{L^3})\lambda_x\lambda_y - \frac{12EI}{L^3}\lambda^2_y & -\frac{6EI}{L^2}\lambda_y \\
(\frac{AE}{L} - \frac{12EI}{L^3})\lambda_x\lambda_y & \frac{AE}{L}\lambda^2_y + \frac{12EI}{L^3}\lambda^2_x & \frac{6EI}{L^2}\lambda_x - \frac{12EI}{L^3}\lambda^2_x & -(\frac{AE}{L} - \frac{12EI}{L^3})\lambda_x\lambda_y & -\frac{AE}{L}\lambda^2\lambda_x & \frac{6EI}{L^2} \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{4EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} \\
-\frac{AE}{L}\lambda^2_x - \frac{12EI}{L^3}\lambda^2_y & -(\frac{AE}{L} - \frac{12EI}{L^3})\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_y + \frac{12EI}{L^3}\lambda^2_y & (\frac{AE}{L} - \frac{12EI}{L^3})\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y \\
-(\frac{AE}{L} - \frac{12EI}{L^3})\lambda_x\lambda_y & -\frac{AE}{L}\lambda^2_y - \frac{12EI}{L^3}\lambda^2_x & -\frac{6EI}{L^2}\lambda_x & (\frac{AE}{L} - \frac{12EI}{L^3})\lambda_x\lambda_y & \frac{AE}{L}\lambda^2_y + \frac{12EI}{L^3}\lambda^2_x & -\frac{6EI}{L^2}\lambda_x \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{4EI}{L}
\end{bmatrix} \quad (2.3)
$$

Where $\lambda_x = \frac{(\text{X−coordinate})\text{ending node} - (\text{X−coordinate})\text{starting node}}{\text{member length}}$

$$\lambda_y = \frac{(Y-\text{coordinate})\text{ending node} - (Y-\text{coordinate})\text{starting node}}{\text{member length}}$$

The [K] in (2.3) will be applied in the global equation as shown in (2.4) below.

$$\{F\} = [K]\ \{\Delta\} \qquad (2.4)$$

Where {F} = external forces

{Δ} = external displacements

[K] = structural stiffness matrix

The (2.4) used to determine the unknown support reactions and external displacements.

2.5.3 Global Element Stiffness Matrix [K] for Second Order Elastic Analysis

To derive the local element stiffness matrix of the two dimensional frame elements, the stiffness matrix of a planar element in member in local coordinates incorporating P-Delta effects can be expressed as in the following (2.5) [6].

$$[Ke + Kg]\{d\Delta\} = \{dP\} \qquad (2.5)$$

Where $[K_e]$ is first order elastic stiffness matrix

$[K_g]$ is geometric stiffness matrix

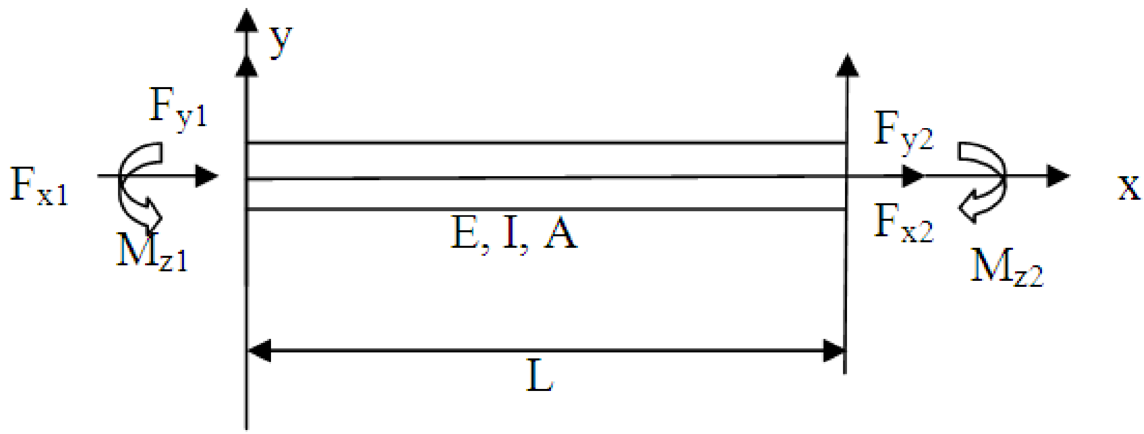

Figure 2.5 Two dimension of planar element

The global element stiffness matrix for first order linear elastic analysis $[K_e]$ can be obtained from the superposition of various planar elements by considering the end of the member supports is pinned support and vice versa. The element stiffness matrix for the two dimensional is recalled as shown in (2.6).

$$[K_e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{2.6}$$

For nonlinear, geometric stiffness matrix $[K_g]$ is needed for the analysis. This included the combination of bending moment and the axial force that act on the deformed shape of structure [5].

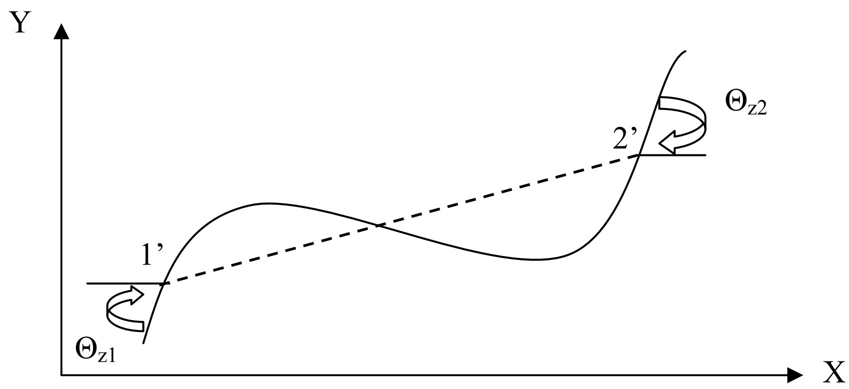

Figure 2.6 Beam members with rotational

Thus, combining the bending and the axial forces, the geometric stiffness matrix for planar frame will be as shown in (2.7).

$$[K_g] = \frac{P}{L}\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{6}{5} & \frac{L}{10} & 0 & -\frac{6}{5} & \frac{L}{10} \\ 0 & \frac{L}{10} & \frac{2L^2}{15} & 0 & -\frac{L}{10} & -\frac{L^2}{30} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{6}{5} & -\frac{L}{10} & 0 & \frac{6}{5} & -\frac{L}{10} \\ 0 & \frac{L}{10} & -\frac{L^2}{30} & 0 & \frac{L}{-10} & \frac{2L^2}{15} \end{bmatrix} \tag{2.7}$$

Where L is member length and P is the axial force in the member.

CHAPTER 3

FIRST ORDER ELASTIC ANALYSIS AND SECOND ORDER ELASTIC ANALYSIS

3.1 Introduction

In this chapter, the focus is on how an excellent structural analysis can be achieved through a rational analysis of considering various aspects of the structure's performance. It is worth to emphasize that the process of structural analysis should be concerned with defining the overall structural form included the whole structure and individual members. In analysis, detailed element analysis is more concerned about. The analysis detailing is to help ensure that structures are analysed to be safe, stable and fit for purpose.

No one exactly knows when and how is the concept of matrix methods being developed to analyse such numerous structures manually. Nowadays, behaviors of all types of structures become more significant and the probability of structures tend to collapse hike up due to the effect of deflections. Those deflections mentioned are P-delta effects. In practice, it is common to represent the frames as plane structures in which the results can be exactly reflect the nature of all individual members.

According to Amit Urs, by using appropriate solutions, the relationship between force and deflection the end of each member in combination with equilibrium and compatibility equations at the joints and supports can be written which yields a system of algebraic equations which shown in (2.4) that describe the behavior of the structure [2]. Therefore, simultaneous equations can be overwritten in the matrix form.

Methods of analysing structural behavior have advanced significantly in recent years. The huge matrix can be computerized by utilized software such as Mathcad. To obtain good precision, advanced commercial software that was regarded as research tools will brief and to the point discuss in this section. The steps for studying the planar frame structure can be summarized into few major steps such as methods used in solving first-order elastic analysis, second-order elastic analysis, evaluating on analysis software, and loading.

3.2 Analysis of Rigid and Simple Steel Frame Structure

A one bay simple steel frame structure will be analysed by using manual calculations and computer-aided software which are calculated by Secant Stiffness Method and Newton-Raphson Method respectively. The same structure with standard section size utilised for first order elastic analysis and second order elastic analysis. All the loads acting on the structure were considered as static loading and therefore the analysis carry out here perform as static analysis. Although Staadpro is fully recognized commercial software, a verification method needs to be carried to get the degree of reliability of StaadPro by comparing the results among manual calculations and software calculations. The diagram of the steel frame structure is as shown in Figure 3.1.

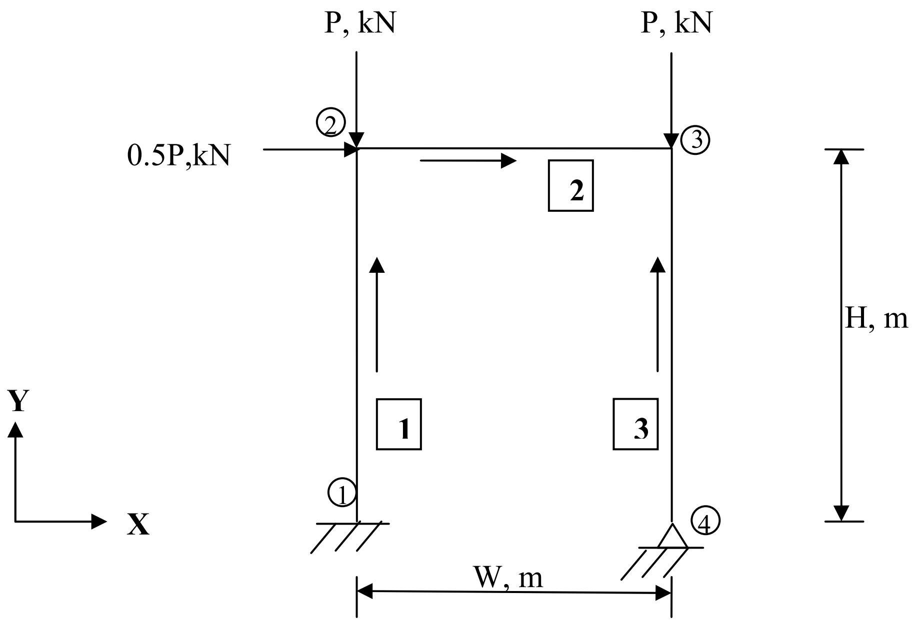

Figure 3.1 Simple planar frame structures with rigid connection and semi-rigid column base supports

The other design parameters which used in this paper for the steel frame structure are shown in Table 3.1.

Table 3.1 Design Parameters for Simple Frame Structure

Design Parameters	Value (unit)
Area	28 cm^2
Moment of Inertia	2841 cm^4
Young Modulus	205 kN/mm^2
Poisson's Ratio	300 * 10^{-3}
Density	7833.409 kg/m^3

3.3 Stiffness Method

In matrix method, generally there are two approaches; flexibility and stiffness approach. Here, stiffness method will be used in analysing the simple steel plane frame structure. The stiffness method is widely used for first order linear static analysis. It is generally considered the fundamental of finite element analysis and appropriate for the structures that have small deflections. Whereby the small changes in displacement are assumed to be neglected since such small displacements would not affect much to indicate the real behavior and response of structure. This is because the method just considered the geometrical linearity effects. Stiffness method involved here is used to analyse first order linear elastic analysis manually with Stiffness Method by Mathcad. The results obtained are displacement, support reactions and internal member forces. But then, horizontal displacements are only one factor to show the different between first order linear effects and second order nonlinear effects efficiently.

3.3.1 Procedures of Geometric Linear Analysis using Stiffness Method (With the Aid of Mathcad)

First order elastic analysis of rigid jointed steel frame can be carried out with the Stiffness Method. The steps toward to solution are shown as following [5]:

i. Node and member numbering system

   For a structure, nodes should be introduced to the joints, connections, supports or corners. They should be numbered in sequence. Members should be numbered too. Figure 3.2 shows node and numbering system for nodes and members.

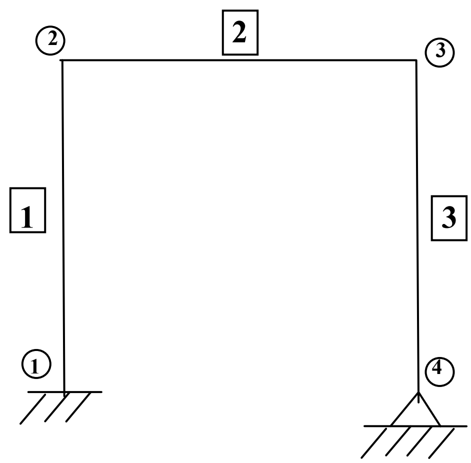

Figure 3.2 Nodes and members numbering system

ii. Coordinate systems

In structural analysis, a properly defined coordinate is essential in the process of calculating the loads and displacements. Loads and displacements are vector quantities and therefore the direction should be indentified based on standard reference axis. There are two coordinate systems such as global coordinate system and local coordinate system. For global coordinate system, it is used as a reference system for the formulation of stiffness and applied force matrices. Each structure should have one global axis only. Normally, the global X-axis is in horizontal direction whereas global Y-axis is in vertical direction. Hence, using the right hand screw rule to define the sign of moments and sets up the positive direction of X, Y and Z axis. The global coordinate system is shown in Figure 3.3.

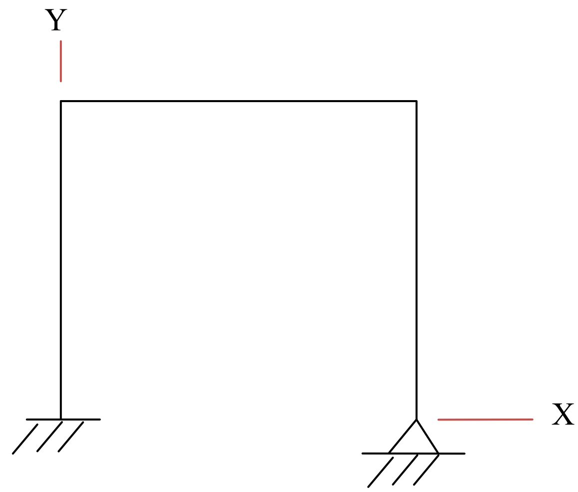

Figure 3.3 Global coordinate

For local coordinate system, a member is typically defined in its local coordinate system first. The local coordinate system is parallel and perpendicular to the member respectively. When a member does not have orientation, its local coordinate system coincides with the structure global coordinate system and member stiffness must be transformed to the global coordinate so that all the members are in the same coordinate system for further calculations. The local coordinate system is shown in Figure 3.4

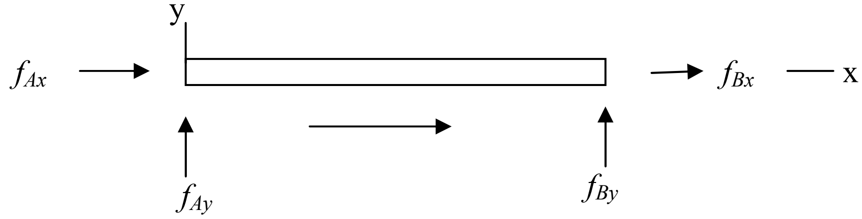

Figure 3.4 Local coordinate system

The local coordinate system transform to global coordinate system by a transformation matrix.

iii. Numbering system of degree of freedom

After nodes, members, and member directions are assigned to the structure, degree of freedom numbering system should be identified. For frame structure, there has three degree of freedoms in each node that is displacement in X and Y axis, and rotation about Z axis for planar structure. Unknown degree of freedoms are numbered first whereby start with the node with maximum number degree of freedom and then followed by known degree of freedoms whereby start with the minimum number degree of freedom.

iv. Loadings

If the uniform distribution load is acting on the member, it should convert to equivalent nodal loads because in stiffness method the solutions are carried out from one node to other nodes.

v. Structural stiffness matrix

The member stiffness matrix determined by using formula. After that, the entire stiffness matrix that calculated assembles and forms a structural stiffness matrix. Known displacements and known external forces can be obtained from the structure.

vi. Partitioning of matrix

This method is to separate the known and unknown parameters of matrix F and Δ as shown in (3.1).

$$\left[\frac{F_k}{F_u}\right] = \left[K\right]\left[\frac{\Delta_u}{\Delta_k}\right] \tag{3.1}$$

Where F_k = External known force on the nodes expressed in the global coordinate system

F_u = External unknown force on the nodes expressed in the global coordinate system

K = Stiffness matrix of the structure obtained by assembling the stiffness matrices of the different bars in the global coordinate system

Δ_k = External known displacement on the nodes expressed in the global coordinate system

Δ_u = External unknown displacement on the nodes expressed in the global coordinate system

Solving the above (3.1), unknown displacements and unknown forces can be obtained. Internal force for the member can be calculated from the following (3.2).

$$f = kT\Delta \tag{3.2}$$

Where f = internal force in the member

The solution of the system of equations in the unknown delta gives the global displacements. They enable, through the transfer matrix, the local displacements at each end of the elements to be derived. Hence the internal forces and moments are obtained through the element stiffness matrix.

For first order elastic analysis, there are no coefficients to take account of relative rotations at the joints and of the change of the flexural stiffness terms due to axial loads. Since it is a first order elastic analysis, the solution is one step process without any need for iteration of the external loads.

3.3.2 Procedures of Geometric Nonlinear Analysis Using Secant Stiffness Method (With the Aid of Mathcad)

The structure stiffness is constructed by superimposing the member stiffness matrices contain geometric nonlinearity. This matrix is substituted in the structural equilibrium equations, which are nonlinear and necessitate an iterative solution procedure. The applied loads are divided into a number of small-load increments and structural equilibrium equations are written in the incremental form shown in (3.3).

$$[S]\{\Delta D\} = \{\Delta F\} \qquad (3.3)$$

Where [S] is structure stiffness matrix, {F} is incremental load vector, and {ΔD} is incremental displacement vector. The incremental are iteratively solved by a sequence of linear steps.

For each load increment, structure stiffness matrix is formed at the start of each iterative cycle. This requires calculation of the nodal displacements at the beginning of each cycle, and changing of the latest geometry and member end forces based on information from previous cycle. The convergent Secant Stiffness related to all load increments are shown in Figure 3.5. Convergence is obtained when the difference between joint displacements of two consecutive cycles falls below a specified tolerance.

A convergent solution of a load increment forms initial values for the next iteration and the iterative procedure goes on until all load increments are taken into account. The solutions for all load increments are added up to acquire a total nonlinear response.

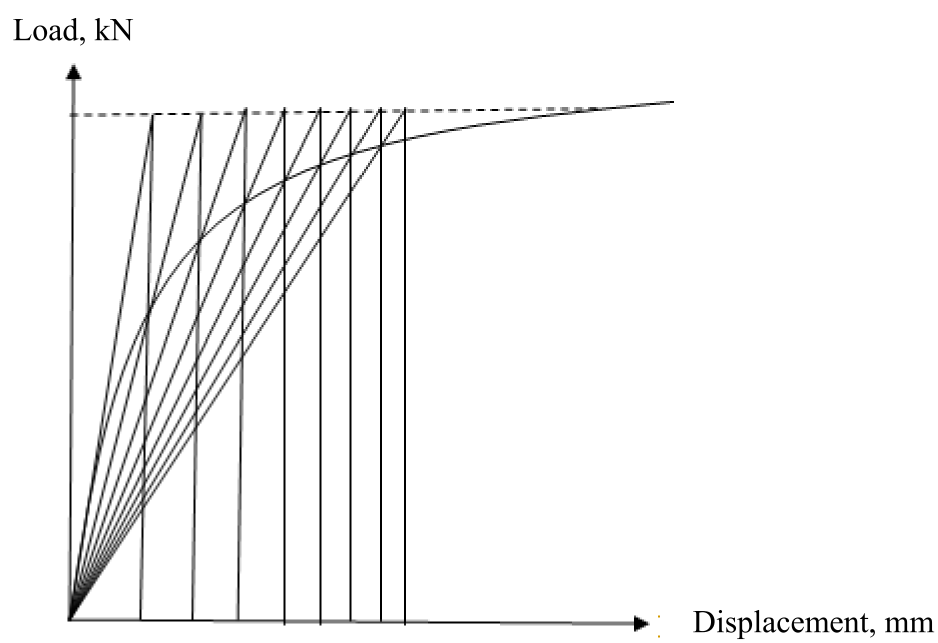

Figure 3.5 Conventional Secant Stiffness Method

The above mentioned analysis procedure can be summarized through the following steps:

1. Set up the member stiffness matrices for all members and assemble them in structure stiffness matrix $[K] = [K_e] + [K_g]$.

2. At the beginning of the calculations, applied loads are taken as zero.

3. Carry out the linear analysis under zero loads and obtain the response of the frame, which is an initial estimate for the nonlinear analysis.

4. Solve the equation $[K_e + K_g]\{d\Delta\} = \{dP\}$ for $\{d\Delta\}$ and then determine the incremental member end forces.

5. Solve the equation $[K_e + K_g]\{d\Delta\} = \{dP\}$iteratively by a sequence of linear steps.

6. The unknown nodal displacements and member end forces are obtained according to the zero applied loads.

7. Update the terms in member stiffness matrices, member forces and structure geometry.

8. Repeat steps 3 to 7 until convergence is attained.

9. Calculate accumulated displacements and member end forces at convergence.

3.3.3 Procedures of Geometric Nonlinear Analysis Using Newton-Raphson Method (With the Aid fo Staadpro)

In the Newton-Raphson Method, only one initial guess of the root is needed to get the iterative process started to find the root of an equation. In this method, the tangent stiffness is reformed at every iteration as shown in Figure 3.6. Alternatively, to simplify this method, the tangent stiffness is reformed at only the first iteration which is referred to the modified Newton-Raphson Method, Figure 3.7.

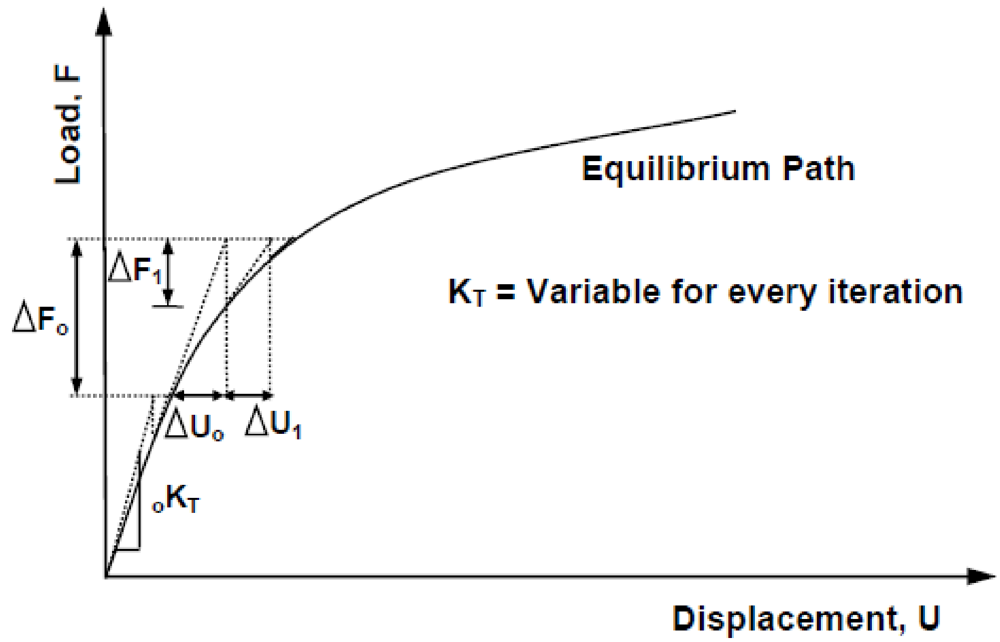

Figure 3.6 Conventional Newton-Raphson Method

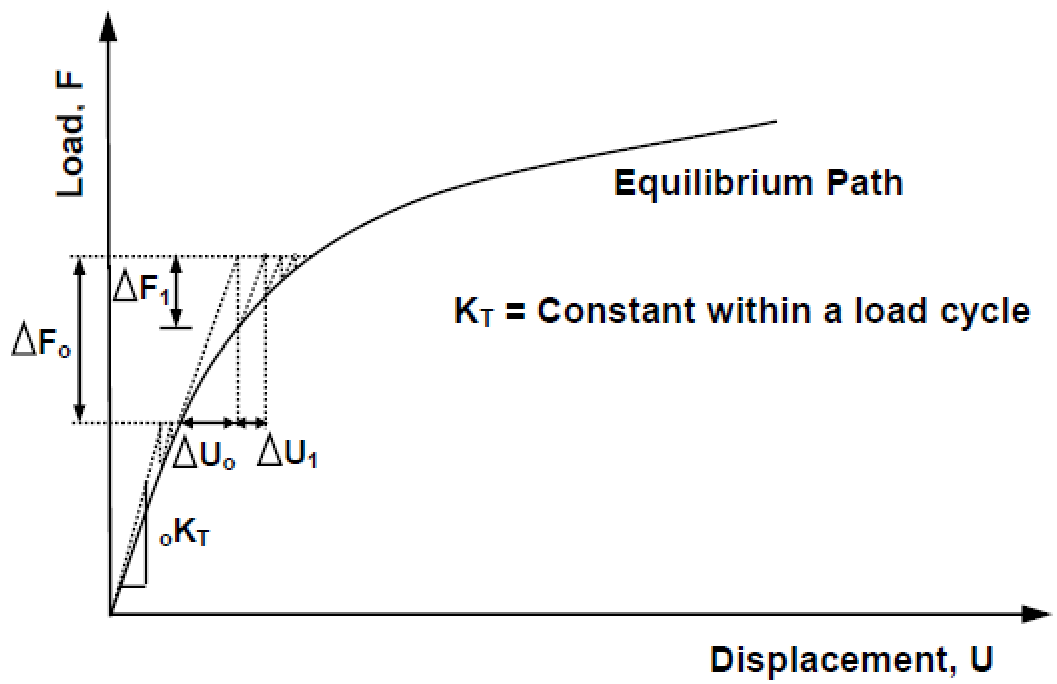

Figure 3.7 Modified Newton-Raphson Method

3.4 Description of Computer Analysis Programs (With the Aid of Staadpro)

Structural analysis with computer aided is needed for the study. Therefore, it is necessary to have a deep understanding on it. The software used for this paper is StaadPro. The ability and the features of software should be mastered before the real process is carried out.

StaadPro is the structural analysis and design software. It allows structural engineers to analyse and design virtually a wide variety of structures through its flexible modeling environment, advanced features and fluent data collaboration.

In today engineering world, most of the design offices of structural and Architectural Consultants, Private as well as Government Organisations is switching over to digital offices with Computer Aided Designing as the primary tool to make the design fast, efficient and competitive. So it has become essential for every Engineer or Architect to know the software that is used in their respective fields.

StaadPro is the professional's choice for steel, concrete, timber, aluminum and cold-formed steel design of low and high-rise buildings, culverts, petrochemical plants, tunnels, bridges, piles and much more.

3.4.1 Modeling Using STAAD Pro

Staadpro is the software used to analyse the response of the structure. The flow chart of modeling using Staadpro is shown in Figure 3.7.

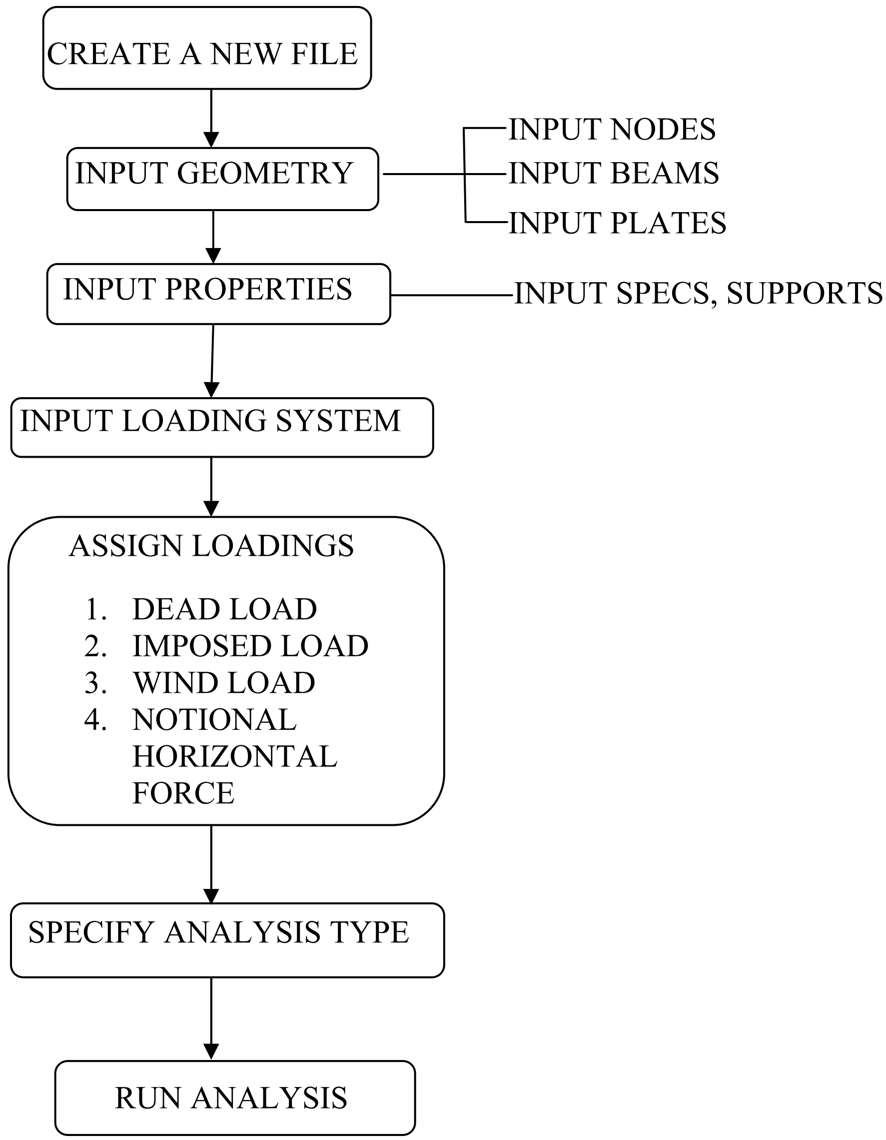

Figure 3.8 Summary procedures for StaadPro

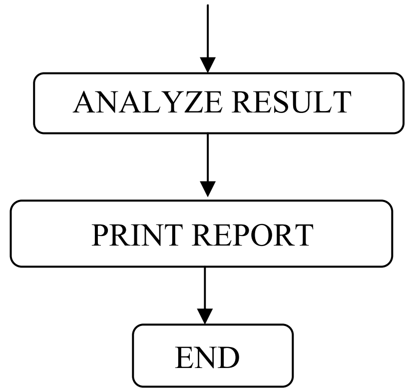

Figure 3.8 Summary procedures for StaadPro (cont)

3.5 MATHCAD

3.5.1 Procedures of Geometric Linear Analysis Using Secant Stiffness Method (With the Aid of Mathcad)

The following showed the calculations for frame structure with vertical loading, 60kN and horizontal loading, 30kN. The dimensions of the structure in height and width are 16000mm and 4000mm respectively.

$A := 0.0028$ $\quad E := 205000000$ $\quad I := 0.00002841$

$L_1 := 16$ $\quad L_2 := 4$ $\quad L_3 := 16$

Member 1

$$k1 := \begin{pmatrix} \frac{A\cdot E}{L1} & 0 & 0 & \frac{-A\cdot E}{L1} & 0 & 0 \\ 0 & 12\frac{E\cdot I}{L1^3} & 6\frac{E\cdot I}{L1^2} & 0 & -12\frac{E\cdot I}{L1^3} & 6\frac{E\cdot I}{L1^2} \\ 0 & 6\frac{E\cdot I}{L1^2} & 4\frac{E\cdot I}{L1} & 0 & -6\frac{E\cdot I}{L1^2} & 2\frac{E\cdot I}{L1} \\ \frac{-A\cdot E}{L1} & 0 & 0 & \frac{A\cdot E}{L1} & 0 & 0 \\ 0 & -12\frac{E\cdot I}{L1^3} & -6\frac{E\cdot I}{L1^2} & 0 & 12\frac{E\cdot I}{L1^3} & -6\frac{E\cdot I}{L1^2} \\ 0 & 6\frac{E\cdot I}{L1^2} & 2\frac{E\cdot I}{L1} & 0 & -6\frac{E\cdot I}{L1^2} & 4\frac{E\cdot I}{L1} \end{pmatrix}$$

$$\phi_1 := 90\cdot\frac{\pi}{180}$$

$$\Gamma_1 := \begin{pmatrix} \cos(\phi_1) & \sin(\phi_1) & 0 & 0 & 0 & 0 \\ -\sin(\phi_1) & \cos(\phi_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\phi_1) & \sin(\phi_1) & 0 \\ 0 & 0 & 0 & -\sin(\phi_1) & \cos(\phi_1) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K1 := \Gamma_1^{T}\cdot k1\cdot\Gamma_1$$

$$K1\ \text{float},5\ \rightarrow \begin{pmatrix} 17.063 & 0 & -136.5 & -17.063 & 0 & -136.5 \\ 0 & 35875.0 & 0 & 0 & -35875.0 & 0 \\ -136.5 & 0 & 1456.0 & 136.5 & 0 & 728.01 \\ -17.063 & 0 & 136.5 & 17.063 & 0 & 136.5 \\ 0 & -35875.0 & 0 & 0 & 35875.0 & 0 \\ -136.5 & 0 & 728.01 & 136.5 & 0 & 1456.0 \end{pmatrix}$$

Member 2

$$k2 := \begin{pmatrix} \frac{A \cdot E}{L2} & 0 & 0 & \frac{-A \cdot E}{L2} & 0 & 0 \\ 0 & 12\frac{E \cdot I}{L2^3} & 6\frac{E \cdot I}{L2^2} & 0 & -12\frac{E \cdot I}{L2^3} & 6\frac{E \cdot I}{L2^2} \\ 0 & 6\frac{E \cdot I}{L2^2} & 4\frac{E \cdot I}{L2} & 0 & -6\frac{E \cdot I}{L2^2} & 2\frac{E \cdot I}{L2} \\ \frac{-A \cdot E}{L2} & 0 & 0 & \frac{A \cdot E}{L2} & 0 & 0 \\ 0 & -12\frac{E \cdot I}{L2^3} & -6\frac{E \cdot I}{L2^2} & 0 & 12\frac{E \cdot I}{L2^3} & -6\frac{E \cdot I}{L2^2} \\ 0 & 6\frac{E \cdot I}{L2^2} & 2\frac{E \cdot I}{L2} & 0 & -6\frac{E \cdot I}{L2^2} & 4\frac{E \cdot I}{L2} \end{pmatrix}$$

$$\phi_2 := 0$$

$$\Gamma_2 := \begin{pmatrix} \cos(\phi_2) & \sin(\phi_2) & 0 & 0 & 0 & 0 \\ -\sin(\phi_2) & \cos(\phi_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\phi_2) & \sin(\phi_2) & 0 \\ 0 & 0 & 0 & -\sin(\phi_2) & \cos(\phi_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K2 := {\Gamma_2}^T \cdot k2 \cdot \Gamma_2$$

$$K2\ \text{float},5\ \rightarrow \begin{pmatrix} 143500.0 & 0 & 0 & -143500.0 & 0 & 0 \\ 0 & 1092.0 & 2184.0 & 0 & -1092.0 & 2184.0 \\ 0 & 2184.0 & 5824.0 & 0 & -2184.0 & 2912.0 \\ -143500.0 & 0 & 0 & 143500.0 & 0 & 0 \\ 0 & -1092.0 & -2184.0 & 0 & 1092.0 & -2184.0 \\ 0 & 2184.0 & 2912.0 & 0 & -2184.0 & 5824.0 \end{pmatrix}$$

Member 3

$$k3 := \begin{pmatrix} \frac{A\cdot E}{L3} & 0 & 0 & \frac{-A\cdot E}{L3} & 0 & 0 \\ 0 & 12\frac{E\cdot I}{L3^3} & 6\frac{E\cdot I}{L3^2} & 0 & -12\frac{E\cdot I}{L3^3} & 6\frac{E\cdot I}{L3^2} \\ 0 & 6\frac{E\cdot I}{L3^2} & 4\frac{E\cdot I}{L3} & 0 & -6\frac{E\cdot I}{L3^2} & 2\frac{E\cdot I}{L3} \\ \frac{-A\cdot E}{L3} & 0 & 0 & \frac{A\cdot E}{L3} & 0 & 0 \\ 0 & -12\frac{E\cdot I}{L3^3} & -6\frac{E\cdot I}{L3^2} & 0 & 12\frac{E\cdot I}{L3^3} & -6\frac{E\cdot I}{L3^2} \\ 0 & 6\frac{E\cdot I}{L3^2} & 2\frac{E\cdot I}{L3} & 0 & -6\frac{E\cdot I}{L3^2} & 4\frac{E\cdot I}{L3} \end{pmatrix}$$

$$\phi_3 := 90\cdot\frac{\pi}{180}$$

$$\Gamma_3 := \begin{pmatrix} \cos(\phi_3) & \sin(\phi_3) & 0 & 0 & 0 & 0 \\ -\sin(\phi_3) & \cos(\phi_3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\phi_3) & \sin(\phi_3) & 0 \\ 0 & 0 & 0 & -\sin(\phi_3) & \cos(\phi_3) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K3 := \Gamma_3^{T} \cdot k3 \cdot \Gamma_3$$

$$K3 \text{ float}, 5 \rightarrow \begin{pmatrix} 17.063 & 0 & -136.5 & -17.063 & 0 & -136.5 \\ 0 & 35875.0 & 0 & 0 & -35875.0 & 0 \\ -136.5 & 0 & 1456.0 & 136.5 & 0 & 728.01 \\ -17.063 & 0 & 136.5 & 17.063 & 0 & 136.5 \\ 0 & -35875.0 & 0 & 0 & 35875.0 & 0 \\ -136.5 & 0 & 728.01 & 136.5 & 0 & 1456.0 \end{pmatrix}$$

$$Ku := K1 + K2 + K3$$

$$Ku := \begin{pmatrix} K2_{3,3} + K3_{3,3} & K2_{3,4} + K3_{3,4} & K2_{3,5} + K3_{3,5} & K2_{3,0} & K2_{3,1} & K2_{3,2} & K3_{3,2} \\ K2_{4,3} + K3_{4,3} & K2_{4,4} + K3_{4,4} & K2_{4,5} + K3_{4,5} & K2_{4,0} & K2_{4,1} & K2_{4,2} & K3_{4,2} \\ K2_{5,3} + K3_{5,3} & K2_{5,4} + K3_{5,4} & K2_{5,5} + K3_{5,5} & K2_{5,0} & K2_{5,1} & K2_{5,2} & K3_{5,2} \\ K2_{0,3} & K2_{0,4} & K2_{0,5} & K1_{3,3} + K2_{0,0} & K1_{3,4} + K2_{0,1} & K1_{3,5} + K2_{0,2} & 0 \\ K2_{1,3} & K2_{1,4} & K2_{1,5} & K1_{4,3} + K2_{1,0} & K1_{4,4} + K2_{1,1} & K1_{4,5} + K2_{1,2} & 0 \\ K2_{2,3} & K2_{2,4} & K2_{2,5} & K1_{5,3} + K2_{2,0} & K1_{5,4} + K2_{2,1} & K1_{5,5} + K2_{2,2} & 0 \\ K3_{2,3} & K3_{2,4} & K3_{2,5} & 0 & 0 & 0 & K3_{2,2} \end{pmatrix}$$

$$Ku \text{ float}, 5 \rightarrow \begin{pmatrix} 143517.0 & 0 & 136.5 & -143500.0 & 0 & 0 & 136.5 \\ 0 & 36967.0 & -2184.0 & 0 & -1092.0 & -2184.0 & 0 \\ 136.5 & -2184.0 & 7280.1 & 0 & 2184.0 & 2912.0 & 728.01 \\ -143500.0 & 0 & 0 & 143517.0 & 0 & 136.5 & 0 \\ 0 & -1092.0 & 2184.0 & 0 & 36967.0 & 2184.0 & 0 \\ 0 & -2184.0 & 2912.0 & 136.5 & 2184.0 & 7280.1 & 0 \\ 136.5 & 0 & 728.01 & 0 & 0 & 0 & 1456.0 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} := Ku^{-1} \cdot \begin{pmatrix} 0 \\ -60 \\ 0 \\ 30 \\ -60 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} \text{float}, 5 \;\rightarrow \begin{pmatrix} 1.6103 \\ -0.0036352 \\ -0.004716 \\ 1.6104 \\ 0.00029021 \\ -0.029486 \\ -0.14861 \end{pmatrix}$$

$$Kk := K1 + K2 + K3$$

$$Kk := \begin{pmatrix} K3_{3,0} & K3_{4,0} & K3_{5,0} & 0 & 0 & 0 & K3_{2,0} \\ K3_{3,1} & K3_{4,1} & K3_{5,1} & 0 & 0 & 0 & K3_{2,1} \\ 0 & 0 & 0 & K1_{3,0} & K1_{4,0} & K1_{5,0} & 0 \\ 0 & 0 & 0 & K1_{3,1} & K1_{4,1} & K1_{5,1} & 0 \\ 0 & 0 & 0 & K1_{3,2} & K1_{4,2} & K1_{5,2} & 0 \end{pmatrix}$$

$$Kk\ \text{float}, 5 \;\rightarrow \begin{pmatrix} -17.063 & 0 & -136.5 & 0 & 0 & 0 & -136.5 \\ 0 & -35875.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -17.063 & 0 & -136.5 & 0 \\ 0 & 0 & 0 & 0 & -35875.0 & 0 & 0 \\ 0 & 0 & 0 & 136.5 & 0 & 728.01 & 0 \end{pmatrix}$$

$$\begin{pmatrix} F8 \\ F9 \\ F10 \\ F11 \\ F12 \end{pmatrix} := Kk \cdot \begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix}$$

$$\begin{pmatrix} F8 \\ F9 \\ F10 \\ F11 \\ F12 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} -6.5473 \\ 130.41 \\ -23.453 \\ -10.411 \\ 198.35 \end{pmatrix}$$

All units for forces, F are in kN and displacements in meter.

3.5.2 Procedures of Geometric Nonlinear Analysis Using Secant Stiffness Method (With the Aid of Mathcad)

The following showed the calculations for frame structure with vertical loading, 60kN and horizontal loading, 30kN. The dimensions of the structure in height and width are 16000mm and 4000mm.

$A := 0.0028$ $\quad E := 205000000$ $\quad I := 0.00002841$

$L_1 := 16$ $\quad L_2 := 4$ $\quad L_3 := 16$

Member 1

$$ke1 := \begin{pmatrix} \frac{A \cdot E}{L1} & 0 & 0 & \frac{-A \cdot E}{L1} & 0 & 0 \\ 0 & 12\frac{E \cdot I}{L1^3} & 6\frac{E \cdot I}{L1^2} & 0 & -12\frac{E \cdot I}{L1^3} & 6\frac{E \cdot I}{L1^2} \\ 0 & 6\frac{E \cdot I}{L1^2} & 4\frac{E \cdot I}{L1} & 0 & -6\frac{E \cdot I}{L1^2} & 2\frac{E \cdot I}{L1} \\ \frac{-A \cdot E}{L1} & 0 & 0 & \frac{A \cdot E}{L1} & 0 & 0 \\ 0 & -12\frac{E \cdot I}{L1^3} & -6\frac{E \cdot I}{L1^2} & 0 & 12\frac{E \cdot I}{L1^3} & -6\frac{E \cdot I}{L1^2} \\ 0 & 6\frac{E \cdot I}{L1^2} & 2\frac{E \cdot I}{L1} & 0 & -6\frac{E \cdot I}{L1^2} & 4\frac{E \cdot I}{L1} \end{pmatrix}$$

$$\phi_1 := 90 \cdot \frac{\pi}{180}$$

$$kg1 := \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{6 \cdot P1}{5 \cdot L1} & \frac{P1}{10} & 0 & \frac{-6 \cdot P1}{5 \cdot L1} & \frac{P1}{10} \\ 0 & \frac{P1}{10} & 2 \cdot P1 \cdot \frac{L1}{15} & 0 & \frac{-P1}{10} & -P1 \cdot \frac{L1}{30} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-6 \cdot P1}{5 \cdot L1} & \frac{-P1}{10} & 0 & \frac{6 \cdot P1}{5 \cdot L1} & \frac{-P1}{10} \\ 0 & \frac{P1}{10} & -P1 \cdot \frac{L1}{30} & 0 & \frac{-P1}{10} & 2 \cdot P1 \cdot \frac{L1}{15} \end{pmatrix}$$

$$\Gamma_1 := \begin{pmatrix} \cos(\phi_1) & \sin(\phi_1) & 0 & 0 & 0 & 0 \\ -\sin(\phi_1) & \cos(\phi_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\phi_1) & \sin(\phi_1) & 0 \\ 0 & 0 & 0 & -\sin(\phi_1) & \cos(\phi_1) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Ke1 := \Gamma_1^{T} \cdot ke1 \cdot \Gamma_1$$

$$Ke1 \text{ float}, 5 \rightarrow \begin{pmatrix} 17.063 & 0 & -136.5 & -17.063 & 0 & -136.5 \\ 0 & 35875.0 & 0 & 0 & -35875.0 & 0 \\ -136.5 & 0 & 1456.0 & 136.5 & 0 & 728.01 \\ -17.063 & 0 & 136.5 & 17.063 & 0 & 136.5 \\ 0 & -35875.0 & 0 & 0 & 35875.0 & 0 \\ -136.5 & 0 & 728.01 & 136.5 & 0 & 1456.0 \end{pmatrix}$$

$$Kg1 := \Gamma_1^{T} \cdot kg1 \cdot \Gamma_1$$

$$\text{Kg1 float},5 \rightarrow \begin{pmatrix} 0.075\cdot P1 & 0 & -0.1\cdot P1 & -0.075\cdot P1 & 0 & -0.1\cdot P1 \\ 0 & 1.0 & 0 & 0 & -1.0 & 0 \\ -0.1\cdot P1 & 0 & 2.1333\cdot P1 & 0.1\cdot P1 & 0 & -0.53333\cdot P1 \\ -0.075\cdot P1 & 0 & 0.1\cdot P1 & 0.075\cdot P1 & 0 & 0.1\cdot P1 \\ 0 & -1.0 & 0 & 0 & 1.0 & 0 \\ -0.1\cdot P1 & 0 & -0.53333\cdot P1 & 0.1\cdot P1 & 0 & 2.1333\cdot P1 \end{pmatrix}$$

Member 2

$$ke2 := \begin{pmatrix} \frac{A\cdot E}{L2} & 0 & 0 & \frac{-A\cdot E}{L2} & 0 & 0 \\ 0 & 12\frac{E\cdot I}{L2^3} & 6\frac{E\cdot I}{L2^2} & 0 & -12\frac{E\cdot I}{L2^3} & 6\frac{E\cdot I}{L2^2} \\ 0 & 6\frac{E\cdot I}{L2^2} & 4\frac{E\cdot I}{L2} & 0 & -6\frac{E\cdot I}{L2^2} & 2\frac{E\cdot I}{L2} \\ \frac{-A\cdot E}{L2} & 0 & 0 & \frac{A\cdot E}{L2} & 0 & 0 \\ 0 & -12\frac{E\cdot I}{L2^3} & -6\frac{E\cdot I}{L2^2} & 0 & 12\frac{E\cdot I}{L2^3} & -6\frac{E\cdot I}{L2^2} \\ 0 & 6\frac{E\cdot I}{L2^2} & 2\frac{E\cdot I}{L2} & 0 & -6\frac{E\cdot I}{L2^2} & 4\frac{E\cdot I}{L2} \end{pmatrix}$$

$$\phi_2 := 0$$

$$kg2 := \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{6\cdot P2}{5\cdot L2} & \frac{P2}{10} & 0 & \frac{-6\cdot P2}{5\cdot L2} & \frac{P2}{10} \\ 0 & \frac{P2}{10} & 2\cdot P2\cdot\frac{L2}{15} & 0 & \frac{-P2}{10} & -P2\cdot\frac{L2}{30} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-6\cdot P2}{5\cdot L2} & \frac{-P2}{10} & 0 & \frac{6\cdot P2}{5\cdot L2} & \frac{-P2}{10} \\ 0 & \frac{P2}{10} & -P2\cdot\frac{L2}{30} & 0 & \frac{-P2}{10} & 2\cdot P2\cdot\frac{L2}{15} \end{pmatrix}$$

$$\Gamma_2 := \begin{pmatrix} \cos(\phi_2) & \sin(\phi_2) & 0 & 0 & 0 & 0 \\ -\sin(\phi_2) & \cos(\phi_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\phi_2) & \sin(\phi_2) & 0 \\ 0 & 0 & 0 & -\sin(\phi_2) & \cos(\phi_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Ke2 := {\Gamma_2}^T \cdot ke2 \cdot \Gamma_2$$

$$Ke2 \text{ float}, 5 \rightarrow \begin{pmatrix} 143500.0 & 0 & 0 & -143500.0 & 0 & 0 \\ 0 & 1092.0 & 2184.0 & 0 & -1092.0 & 2184.0 \\ 0 & 2184.0 & 5824.0 & 0 & -2184.0 & 2912.0 \\ -143500.0 & 0 & 0 & 143500.0 & 0 & 0 \\ 0 & -1092.0 & -2184.0 & 0 & 1092.0 & -2184.0 \\ 0 & 2184.0 & 2912.0 & 0 & -2184.0 & 5824.0 \end{pmatrix}$$

$$Kg2 := {\Gamma_2}^T \cdot kg2 \cdot \Gamma_2$$

$$Kg2 \text{ float}, 5 \rightarrow \begin{pmatrix} 1.0 & 0 & 0 & -1.0 & 0 & 0 \\ 0 & 0.3 \cdot P2 & 0.1 \cdot P2 & 0 & -0.3 \cdot P2 & 0.1 \cdot P2 \\ 0 & 0.1 \cdot P2 & 0.53333P2 & 0 & -0.1 \cdot P2 & -0.13333P2 \\ -1.0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & -0.3 \cdot P2 & -0.1 \cdot P2 & 0 & 0.3 \cdot P2 & -0.1 \cdot P2 \\ 0 & 0.1 \cdot P2 & -0.13333P2 & 0 & -0.1 \cdot P2 & 0.53333P2 \end{pmatrix}$$

Member 3

$$ke3 := \begin{pmatrix} \frac{A\cdot E}{L3} & 0 & 0 & \frac{-A\cdot E}{L3} & 0 & 0 \\ 0 & 12\frac{E\cdot I}{L3^3} & 6\frac{E\cdot I}{L3^2} & 0 & -12\frac{E\cdot I}{L3^3} & 6\frac{E\cdot I}{L3^2} \\ 0 & 6\frac{E\cdot I}{L3^2} & 4\frac{E\cdot I}{L3} & 0 & -6\frac{E\cdot I}{L3^2} & 2\frac{E\cdot I}{L3} \\ \frac{-A\cdot E}{L3} & 0 & 0 & \frac{A\cdot E}{L3} & 0 & 0 \\ 0 & -12\frac{E\cdot I}{L3^3} & -6\frac{E\cdot I}{L3^2} & 0 & 12\frac{E\cdot I}{L3^3} & -6\frac{E\cdot I}{L3^2} \\ 0 & 6\frac{E\cdot I}{L3^2} & 2\frac{E\cdot I}{L3} & 0 & -6\frac{E\cdot I}{L3^2} & 4\frac{E\cdot I}{L3} \end{pmatrix}$$

$$\phi_3 := 90\cdot\frac{\pi}{180}$$

$$kg3 := \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{6\cdot P3}{5\cdot L3} & \frac{P3}{10} & 0 & \frac{-6\cdot P3}{5\cdot L3} & \frac{P3}{10} \\ 0 & \frac{P3}{10} & 2\cdot P3\cdot\frac{L3}{15} & 0 & \frac{-P3}{10} & -P3\cdot\frac{L3}{30} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-6\cdot P3}{5\cdot L3} & \frac{-P3}{10} & 0 & \frac{6\cdot P3}{5\cdot L3} & \frac{-P3}{10} \\ 0 & \frac{P3}{10} & -P3\cdot\frac{L3}{30} & 0 & \frac{-P3}{10} & 2\cdot P3\cdot\frac{L3}{15} \end{pmatrix}$$

$$\Gamma_3 := \begin{pmatrix} \cos(\phi_3) & \sin(\phi_3) & 0 & 0 & 0 & 0 \\ -\sin(\phi_3) & \cos(\phi_3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\phi_3) & \sin(\phi_3) & 0 \\ 0 & 0 & 0 & -\sin(\phi_3) & \cos(\phi_3) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$Ke3 := \Gamma_3{}^T \cdot ke3 \cdot \Gamma_3$

$$Ke3 \text{ float}, 5 \rightarrow \begin{pmatrix} 17.063 & 0 & -136.5 & -17.063 & 0 & -136.5 \\ 0 & 35875.0 & 0 & 0 & -35875.0 & 0 \\ -136.5 & 0 & 1456.0 & 136.5 & 0 & 728.01 \\ -17.063 & 0 & 136.5 & 17.063 & 0 & 136.5 \\ 0 & -35875.0 & 0 & 0 & 35875.0 & 0 \\ -136.5 & 0 & 728.01 & 136.5 & 0 & 1456.0 \end{pmatrix}$$

$Kg3 := \Gamma_3{}^T \cdot kg3 \cdot \Gamma_3$

$$Kg3 \text{ float}, 5 \rightarrow \begin{pmatrix} 0.075 \cdot P3 & 0 & -0.1 \cdot P3 & -0.075 \cdot P3 & 0 & -0.1 \cdot P3 \\ 0 & 1.0 & 0 & 0 & -1.0 & 0 \\ -0.1 \cdot P3 & 0 & 2.1333 P3 & 0.1 \cdot P3 & 0 & -0.53333 P3 \\ -0.075 \cdot P3 & 0 & 0.1 \cdot P3 & 0.075 \cdot P3 & 0 & 0.1 \cdot P3 \\ 0 & -1.0 & 0 & 0 & 1.0 & 0 \\ -0.1 \cdot P3 & 0 & -0.53333 P3 & 0.1 \cdot P3 & 0 & 2.1333 P3 \end{pmatrix}$$

$Ke := Ke1 + Ke2 + Ke3$

$$Ke := \begin{pmatrix} Ke2_{3,3} + Ke3_{3,3} & Ke2_{3,4} + Ke3_{3,4} & Ke2_{3,5} + Ke3_{3,5} & Ke2_{3,0} & Ke2_{3,1} & Ke2_{3,2} & Ke3_{3,2} \\ Ke2_{4,3} + Ke3_{4,3} & Ke2_{4,4} + Ke3_{4,4} & Ke2_{4,5} + Ke3_{4,5} & Ke2_{4,0} & Ke2_{4,1} & Ke2_{4,2} & Ke3_{4,2} \\ Ke2_{5,3} + Ke3_{5,3} & Ke2_{5,4} + Ke3_{5,4} & Ke2_{5,5} + Ke3_{5,5} & Ke2_{5,0} & Ke2_{5,1} & Ke2_{5,2} & Ke3_{5,2} \\ Ke2_{0,3} & Ke2_{0,4} & Ke2_{0,5} & Ke1_{3,3} + Ke2_{0,0} & Ke1_{3,4} + Ke2_{0,1} & Ke1_{3,5} + Ke2_{0,2} & 0 \\ Ke2_{1,3} & Ke2_{1,4} & Ke2_{1,5} & Ke1_{4,3} + Ke2_{1,0} & Ke1_{4,4} + Ke2_{1,1} & Ke1_{4,5} + Ke2_{1,2} & 0 \\ Ke2_{2,3} & Ke2_{2,4} & Ke2_{2,5} & Ke1_{5,3} + Ke2_{2,0} & Ke1_{5,4} + Ke2_{2,1} & Ke1_{5,5} + Ke2_{2,2} & 0 \\ Ke3_{2,3} & Ke3_{2,4} & Ke3_{2,5} & 0 & 0 & 0 & Ke3_{2,2} \end{pmatrix}$$

$$Ke \text{ float}, 5 \rightarrow \begin{pmatrix} 143517.0 & 0 & 136.5 & -143500.0 & 0 & 0 & 136.5 \\ 0 & 36967.0 & -2184.0 & 0 & -1092.0 & -2184.0 & 0 \\ 136.5 & -2184.0 & 7280.1 & 0 & 2184.0 & 2912.0 & 728.01 \\ -143500.0 & 0 & 0 & 143517.0 & 0 & 136.5 & 0 \\ 0 & -1092.0 & 2184.0 & 0 & 36967.0 & 2184.0 & 0 \\ 0 & -2184.0 & 2912.0 & 136.5 & 2184.0 & 7280.1 & 0 \\ 136.5 & 0 & 728.01 & 0 & 0 & 0 & 1456.0 \end{pmatrix}$$

$$Kg := Kg1 + Kg2 + Kg3$$

$$
Kg := \begin{pmatrix}
Kg2_{3,3} + Kg3_{3,3} & Kg2_{3,4} + Kg3_{3,4} & Kg2_{3,5} + Kg3_{3,5} & Kg2_{3,0} & Kg2_{3,1} & Kg2_{3,2} & Kg3_{3,2} \\
Kg2_{4,3} + Kg3_{4,3} & Kg2_{4,4} + Kg3_{4,4} & Kg2_{4,5} + Kg3_{4,5} & Kg2_{4,0} & Kg2_{4,1} & Kg2_{4,2} & Kg3_{4,2} \\
Kg2_{5,3} + Kg3_{5,3} & Kg2_{5,4} + Kg3_{5,4} & Kg2_{5,5} + Kg3_{5,5} & Kg2_{5,0} & Kg2_{5,1} & Kg2_{5,2} & Kg3_{5,2} \\
Kg2_{0,3} & Kg2_{0,4} & Kg2_{0,5} & Kg1_{3,3} + Kg2_{0,0} & Kg1_{3,4} + Kg2_{0,1} & Kg1_{3,5} + Kg2_{0,2} & 0 \\
Kg2_{1,3} & Kg2_{1,4} & Kg2_{1,5} & Kg1_{4,3} + Kg2_{1,0} & Kg1_{4,4} + Kg2_{1,1} & Kg1_{4,5} + Kg2_{1,2} & 0 \\
Kg2_{2,3} & Kg2_{2,4} & Kg2_{2,5} & Kg1_{5,3} + Kg2_{2,0} & Kg1_{5,4} + Kg2_{2,1} & Kg1_{5,5} + Kg2_{2,2} & 0 \\
Kg3_{2,3} & Kg3_{2,4} & Kg3_{2,5} & 0 & 0 & 0 & Kg3_{2,2}
\end{pmatrix}
$$

$$
Kg \text{ float}, 5 \rightarrow \begin{pmatrix}
0.075 \cdot P3 + 1.0 & 0 & 0.1 \cdot P3 & -1.0 & 0 & 0 & 0.1 \cdot P3 \\
0 & 0.3 \cdot P2 + 1.0 & -0.1 \cdot P2 & 0 & -0.3 \cdot P2 & -0.1 \cdot P2 & 0 \\
0.1 \cdot P3 & -0.1 \cdot P2 & 0.53333 P2 + 2.1333 P3 & 0 & 0.1 \cdot P2 & -0.13333 P2 & -0.53333 P3 \\
-1.0 & 0 & 0 & 0.075 P1 + 1.0 & 0 & 0.1 \cdot P1 & 0 \\
0 & -0.3 \cdot P2 & 0.1 \cdot P2 & 0 & 0.3 \cdot P2 + 1.0 & 0.1 \cdot P2 & 0 \\
0 & -0.1 \cdot P2 & -0.13333 P2 & 0.1 \cdot P1 & 0.1 \cdot P2 & 2.1333 P1 + 0.53333 P2 & 0 \\
0.1 \cdot P3 & 0 & -0.53333 P3 & 0 & 0 & 0 & 2.1333 P3
\end{pmatrix}
$$

$Ka := Ke + Kg$

$$\mathrm{Ka}\ \mathrm{float},5\ \rightarrow \begin{pmatrix} 0.075\cdot P3 + 143518.0 & 0 & 0.1\cdot P3 + 136.5 & -143501.0 & 0 & 0 & 0.1\cdot P3 + 136.5 \\ 0 & 0.3\cdot P2 + 36968.0 & -0.1\cdot P2 - 2184.0 & 0 & -0.3\cdot P2 - 1092.0 & -0.1\cdot P2 - 2184.0 & 0 \\ 0.1\cdot P3 + 136.5 & -0.1\cdot P2 - 2184.0 & 0.53333 P2 + 2.1333 P3 + 7280.1 & 0 & 0.1\cdot P2 + 2184.0 & -0.13333 P2 + 2912.0 & -0.53333 P3 + 728.01 \\ -143501.0 & 0 & 0 & 0.075\cdot P1 + 143518.0 & 0 & 0.1\cdot P1 + 136.5 & 0 \\ 0 & -0.3\cdot P2 - 1092.0 & 0.1\cdot P2 + 2184.0 & 0 & 0.3\cdot P2 + 36968.0 & 0.1\cdot P2 + 2184.0 & 0 \\ 0 & -0.1\cdot P2 - 2184.0 & -0.13333 P2 + 2912.0 & 0.1\cdot P1 + 136.5 & 0.1\cdot P2 + 2184.0 & 2.1333 P1 + 0.53333 P2 + 7280.1 & 0 \\ 0.1\cdot P3 + 136.5 & 0 & -0.53333 P3 + 728.01 & 0 & 0 & 0 & 2.1333 P3 + 1456.0 \end{pmatrix}$$

$$\mathrm{Ka}(P1,P2,P3) := \begin{pmatrix} 0.075\cdot P3 + 143518.06 & 0 & 0.1\cdot P3 + 136.50117 & -143501.0 & 0 & 0 & 0.1\cdot P3 + 136.50117 \\ 0 & 0.3\cdot P2 + 36968.009 & -0.1\cdot P2 - 2184.0187 & 0 & -0.3\cdot P2 - 1092.0094 & -0.1\cdot P2 - 2184.0187 & 0 \\ 0.1\cdot P3 + 136.50117 & -0.1\cdot P2 - 2184.0187 & 0.53333333 P2 + 2.1333333 P3 + 7280.0625 & 0 & 0.1\cdot P2 + 2184.0187 & -0.13333333 P2 + 2912.025 & -0.53333333 P3 + 728.00625 \\ -143501.0 & 0 & 0 & 0.075\cdot P1 + 143518.06 & 0 & 0.1\cdot P1 + 136.50117 & 0 \\ 0 & -0.3\cdot P2 - 1092.0094 & 0.1\cdot P2 + 2184.0187 & 0 & 0.3\cdot P2 + 36968.009 & 0.1\cdot P2 + 2184.0187 & 0 \\ 0 & -0.1\cdot P2 - 2184.0187 & -0.13333333 P2 + 2912.025 & 0.1\cdot P1 + 136.50117 & 0.1\cdot P2 + 2184.0187 & 2.1333333 P1 + 0.53333333 P2 + 7280.0625 & 0 \\ 0.1\cdot P3 + 136.50117 & 0 & -0.53333333 P3 + 728.00625 & 0 & 0 & 0 & 2.1333333 P3 + 1456.0125 \end{pmatrix}$$

$$k1(P1) := \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{6\cdot P1}{5\cdot L1} & \frac{P1}{10} & 0 & \frac{-6\cdot P1}{5\cdot L1} & \frac{P1}{10} \\ 0 & \frac{P1}{10} & 2\cdot P1\cdot\frac{L1}{15} & 0 & \frac{-P1}{10} & -P1\cdot\frac{L1}{30} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-6\cdot P1}{5\cdot L1} & \frac{-P1}{10} & 0 & \frac{6\cdot P1}{5\cdot L1} & \frac{-P1}{10} \\ 0 & \frac{P1}{10} & -P1\cdot\frac{L1}{30} & 0 & \frac{-P1}{10} & 2\cdot P1\cdot\frac{L1}{15} \end{pmatrix} + \begin{pmatrix} \frac{A\cdot E}{L1} & 0 & 0 & \frac{-A\cdot E}{L1} & 0 & 0 \\ 0 & 12\frac{E\cdot I}{L1^3} & 6\frac{E\cdot I}{L1^2} & 0 & -12\frac{E\cdot I}{L1^3} & 6\frac{E\cdot I}{L1^2} \\ 0 & 6\frac{E\cdot I}{L1^2} & 4\frac{E\cdot I}{L1} & 0 & -6\frac{E\cdot I}{L1^2} & 2\frac{E\cdot I}{L1} \\ \frac{-A\cdot E}{L1} & 0 & 0 & \frac{A\cdot E}{L1} & 0 & 0 \\ 0 & -12\frac{E\cdot I}{L1^3} & -6\frac{E\cdot I}{L1^2} & 0 & 12\frac{E\cdot I}{L1^3} & -6\frac{E\cdot I}{L1^2} \\ 0 & 6\frac{E\cdot I}{L1^2} & 2\frac{E\cdot I}{L1} & 0 & -6\frac{E\cdot I}{L1^2} & 4\frac{E\cdot I}{L1} \end{pmatrix}$$

$$k2(P2) := \begin{pmatrix} \frac{A\cdot E}{L2} & 0 & 0 & \frac{-A\cdot E}{L2} & 0 & 0 \\ 0 & 12\frac{E\cdot I}{L2^3} & 6\frac{E\cdot I}{L2^2} & 0 & -12\frac{E\cdot I}{L2^3} & 6\frac{E\cdot I}{L2^2} \\ 0 & 6\frac{E\cdot I}{L2^2} & 4\frac{E\cdot I}{L2} & 0 & -6\frac{E\cdot I}{L2^2} & 2\frac{E\cdot I}{L2} \\ \frac{-A\cdot E}{L2} & 0 & 0 & \frac{A\cdot E}{L2} & 0 & 0 \\ 0 & -12\frac{E\cdot I}{L2^3} & -6\frac{E\cdot I}{L2^2} & 0 & 12\frac{E\cdot I}{L2^3} & -6\frac{E\cdot I}{L2^2} \\ 0 & 6\frac{E\cdot I}{L2^2} & 2\frac{E\cdot I}{L2} & 0 & -6\frac{E\cdot I}{L2^2} & 4\frac{E\cdot I}{L2} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{6\cdot P2}{5\cdot L2} & \frac{P2}{10} & 0 & \frac{-6\cdot P2}{5\cdot L2} & \frac{P2}{10} \\ 0 & \frac{P2}{10} & 2\cdot P2\cdot\frac{L2}{15} & 0 & \frac{-P2}{10} & -P2\cdot\frac{L2}{30} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-6\cdot P2}{5\cdot L2} & \frac{-P2}{10} & 0 & \frac{6\cdot P2}{5\cdot L2} & \frac{-P2}{10} \\ 0 & \frac{P2}{10} & -P2\cdot\frac{L2}{30} & 0 & \frac{-P2}{10} & 2\cdot P2\cdot\frac{L2}{15} \end{pmatrix}$$

$$k3(P3) := \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{6\cdot P3}{5\cdot L3} & \frac{P3}{10} & 0 & \frac{-6\cdot P3}{5\cdot L3} & \frac{P3}{10} \\ 0 & \frac{P3}{10} & 2\cdot P3\cdot\frac{L3}{15} & 0 & \frac{-P3}{10} & -P3\cdot\frac{L3}{30} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-6\cdot P3}{5\cdot L3} & \frac{-P3}{10} & 0 & \frac{6\cdot P3}{5\cdot L3} & \frac{-P3}{10} \\ 0 & \frac{P3}{10} & -P3\cdot\frac{L3}{30} & 0 & \frac{-P3}{10} & 2\cdot P3\cdot\frac{L3}{15} \end{pmatrix} + \begin{pmatrix} \frac{A\cdot E}{L3} & 0 & 0 & \frac{-A\cdot E}{L3} & 0 & 0 \\ 0 & 12\frac{E\cdot I}{L3^3} & 6\frac{E\cdot I}{L3^2} & 0 & -12\frac{E\cdot I}{L3^3} & 6\frac{E\cdot I}{L3^2} \\ 0 & 6\frac{E\cdot I}{L3^2} & 4\frac{E\cdot I}{L3} & 0 & -6\frac{E\cdot I}{L3^2} & 2\frac{E\cdot I}{L3} \\ \frac{-A\cdot E}{L3} & 0 & 0 & \frac{A\cdot E}{L3} & 0 & 0 \\ 0 & -12\frac{E\cdot I}{L3^3} & -6\frac{E\cdot I}{L3^2} & 0 & 12\frac{E\cdot I}{L3^3} & -6\frac{E\cdot I}{L3^2} \\ 0 & 6\frac{E\cdot I}{L3^2} & 2\frac{E\cdot I}{L3} & 0 & -6\frac{E\cdot I}{L3^2} & 4\frac{E\cdot I}{L3} \end{pmatrix}$$

First Iteration

$$Ka(P1,P2,P3)\ \text{float},5\ \rightarrow \begin{pmatrix} 143518.0 & 0 & 136.5 & -143501.0 & 0 & 0 & 136.5 \\ 0 & 36968.0 & -2184.0 & 0 & -1092.0 & -2184.0 & 0 \\ 136.5 & -2184.0 & 7280.1 & 0 & 2184.0 & 2912.0 & 728.01 \\ -143501.0 & 0 & 0 & 143518.0 & 0 & 136.5 & 0 \\ 0 & -1092.0 & 2184.0 & 0 & 36968.0 & 2184.0 & 0 \\ 0 & -2184.0 & 2912.0 & 136.5 & 2184.0 & 7280.1 & 0 \\ 136.5 & 0 & 728.01 & 0 & 0 & 0 & 1456.0 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} := Ka(P1,P2,P3)^{-1}\cdot\begin{pmatrix} 0 \\ -60 \\ 0 \\ 30 \\ -60 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 1.6108 \\ -0.0036356 \\ -0.0047173 \\ 1.6108 \\ 0.00029076 \\ -0.029494 \\ -0.14865 \end{pmatrix}$$

Member 1

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} := k1(P1) \cdot \Gamma_1 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ U4 \\ U5 \\ U6 \end{pmatrix}$$

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} -10.431 \\ 23.459 \\ 198.41 \\ 10.431 \\ -23.459 \\ 176.94 \end{pmatrix}$$

$$P1 := F5$$

Member 2

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k2(P2) \cdot \Gamma_2 \cdot \begin{pmatrix} U4 \\ U5 \\ U6 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 6.5449 \\ -70.431 \\ -176.94 \\ -6.5449 \\ 70.431 \\ -104.79 \end{pmatrix}$$

$P2 := F4$

Member 3

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k3(P3) \cdot \Gamma_3 \cdot \begin{pmatrix} 0 \\ 0 \\ U7 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 130.43 \\ 6.5492 \\ 0.0000030203 \\ -130.43 \\ -6.5492 \\ 104.79 \end{pmatrix}$$

$P3 := F2$

Second Iteration

$$Ka(P1,P2,P3) := \begin{pmatrix} 0.075P3 + 143518.06 & 0 & 0.1\cdot P3 + 136.50117 & -143501.0 & 0 & 0 & 0.1\cdot P3 + 136.50117 \\ 0 & 0.3\cdot P2 + 36968.009 & -0.1\cdot P2 - 2184.0187 & 0 & -0.3\cdot P2 - 1092.0094 & -0.1\cdot P2 - 2184.0187 & 0 \\ 0.1\cdot P3 + 136.50117 & -0.1\cdot P2 - 2184.0187 & 0.53333333P2 + 2.1333333P3 + 7280.0625 & 0 & 0.1\cdot P2 + 2184.0187 & -0.13333333P2 + 2912.025 & -0.53333333P3 + 728.00625 \\ -143501.0 & 0 & 0 & 0.075P1 + 143518.06 & 0 & 0.1\cdot P1 + 136.50117 & 0 \\ 0 & -0.3\cdot P2 - 1092.0094 & 0.1\cdot P2 + 2184.0187 & 0 & 0.3\cdot P2 + 36968.009 & 0.1\cdot P2 + 2184.0187 & 0 \\ 0 & -0.1\cdot P2 - 2184.0187 & -0.13333333P2 + 2912.025 & 0.1\cdot P1 + 136.50117 & 0.1\cdot P2 + 2184.0187 & 2.1333333P1 + 0.53333333P2 + 7280.0625 & 0 \\ 0.1\cdot P3 + 136.50117 & 0 & -0.53333333P3 + 728.00625 & 0 & 0 & 0 & 2.1333333P3 + 1456.0125 \end{pmatrix}$$

$$Ka(P1,P2,P3)\ \text{float},5 \rightarrow \begin{pmatrix} 143517.0 & 0 & 135.85 & -143501.0 & 0 & 0 & 135.85 \\ 0 & 36970.0 & -2184.7 & 0 & -1094.0 & -2184.7 & 0 \\ 135.85 & -2184.7 & 7269.6 & 0 & 2184.7 & 2911.2 & 731.5 \\ -143501.0 & 0 & 0 & 143516.0 & 0 & 134.16 & 0 \\ 0 & -1094.0 & 2184.7 & 0 & 36970.0 & 2184.7 & 0 \\ 0 & -2184.7 & 2911.2 & 134.16 & 2184.7 & 7233.5 & 0 \\ 135.85 & 0 & 731.5 & 0 & 0 & 0 & 1442.0 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} := Ka(P1, P2, P3)^{-1} \cdot \begin{pmatrix} 0 \\ -60 \\ 0 \\ 30 \\ -60 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 1.8237 \\ -0.0038654 \\ -0.0051214 \\ 1.8237 \\ 0.0005205 \\ -0.033086 \\ -0.1692 \end{pmatrix}$$

Member 1

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} := k1(P1) \cdot \Gamma_1 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ U4 \\ U5 \\ U6 \end{pmatrix}$$

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} -18.674 \\ 23.47 \\ 220.16 \\ 18.674 \\ -23.47 \\ 198.14 \end{pmatrix}$$

$P1 := F5$

Member 2

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k2(P2)\cdot\Gamma_2\cdot\begin{pmatrix} U4 \\ U5 \\ U6 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 6.5352 \\ -78.674 \\ -198.14 \\ -6.5352 \\ 78.674 \\ -116.58 \end{pmatrix}$$

$P2 := F4$

Member 3

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k3(P3)\cdot\Gamma_3\cdot\begin{pmatrix} 0 \\ 0 \\ U7 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 138.67 \\ 6.54 \\ 0.0000034562 \\ -138.67 \\ -6.54 \\ 116.58 \end{pmatrix}$$

$P3 := F2$

$$
\mathrm{Ka}(P1,P2,P3) := \begin{pmatrix}
0.075\cdot P3 + 143518.06 & 0 & 0.1\cdot P3 + 136.50117 & -143501.0 & 0 & 0 & 0.1\cdot P3 + 136.50117 \\
0 & 0.3\cdot P2 + 36968.009 & -0.1\cdot P2 - 2184.0187 & 0 & -0.3\cdot P2 - 1092.0094 & -0.1\cdot P2 - 2184.0187 & 0 \\
0.1\cdot P3 + 136.50117 & -0.1\cdot P2 - 2184.0187 & 0.53333333P2 + 2.1333333P3 + 7280.0625 & 0 & 0.1\cdot P2 + 2184.0187 & -0.13333333P2 + 2912.025 & -0.53333333P3 + 728.00625 \\
-143501.0 & 0 & 0 & 0.075\cdot P1 + 143518.06 & 0 & 0.1\cdot P1 + 136.50117 & 0 \\
0 & -0.3\cdot P2 - 1092.0094 & 0.1\cdot P2 + 2184.0187 & 0 & 0.3\cdot P2 + 36968.009 & 0.1\cdot P2 + 2184.0187 & 0 \\
0 & -0.1\cdot P2 - 2184.0187 & -0.13333333P2 + 2912.025 & 0.1\cdot P1 + 136.50117 & 0.1\cdot P2 + 2184.0187 & 2.1333333P1 + 0.53333333P2 + 7280.0625 & 0 \\
0.1\cdot P3 + 136.50117 & 0 & -0.53333333P3 + 728.00625 & 0 & 0 & 0 & 2.1333333P3 + 1456.0125
\end{pmatrix}
$$

$$
\mathrm{Ka}(P1,P2,P3)\ \text{float},5 \rightarrow \begin{pmatrix}
143517.0 & 0 & 135.85 & -143501.0 & 0 & 0 & 135.85 \\
0 & 36970.0 & -2184.7 & 0 & -1094.0 & -2184.7 & 0 \\
135.85 & -2184.7 & 7269.6 & 0 & 2184.7 & 2911.2 & 731.49 \\
-143501.0 & 0 & 0 & 143516.0 & 0 & 134.15 & 0 \\
0 & -1094.0 & 2184.7 & 0 & 36970.0 & 2184.7 & 0 \\
0 & -2184.7 & 2911.2 & 134.15 & 2184.7 & 7233.5 & 0 \\
135.85 & 0 & 731.49 & 0 & 0 & 0 & 1442.1
\end{pmatrix}
$$

$$
\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} := \mathrm{Ka}(P1,P2,P3)^{-1}\cdot\begin{pmatrix} 0 \\ -60 \\ 0 \\ 30 \\ -60 \\ 0 \\ 0 \end{pmatrix}
$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} \text{float}, 5 \;\rightarrow \begin{pmatrix} 1.8237 \\ -0.0038654 \\ -0.005122 \\ 1.8237 \\ 0.00052053 \\ -0.033086 \\ -0.1692 \end{pmatrix}$$

Member 1

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} := k1(P1)\cdot\Gamma_1\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \\ U4 \\ U5 \\ U6 \end{pmatrix}$$

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} \text{float}, 5 \;\rightarrow \begin{pmatrix} -18.674 \\ 23.468 \\ 220.16 \\ 18.674 \\ -23.468 \\ 198.14 \end{pmatrix}$$

$$P1 := F5$$

Member 2

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k2(P2)\cdot\Gamma_2\cdot\begin{pmatrix} U4 \\ U5 \\ U6 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 6.5364 \\ -78.674 \\ -198.14 \\ -6.5364 \\ 78.674 \\ -116.59 \end{pmatrix}$$

$P2 := F4$

Member 3

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k3(P3) \cdot \Gamma_3 \cdot \begin{pmatrix} 0 \\ 0 \\ U7 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 138.67 \\ 6.5412 \\ 0.0000034561 \\ -138.67 \\ -6.5412 \\ 116.59 \end{pmatrix}$$

$P3 := F2$

Forth Iteration

$$Ka(P1,P2,P3) := \begin{pmatrix} 0.075P3 + 143518.06 & 0 & 0.1\cdot P3 + 136.50117 & -143501.0 & 0 & 0 & 0.1\cdot P3 + 136.50117 \\ 0 & 0.3\cdot P2 + 36968.009 & -0.1\cdot P2 - 2184.0187 & 0 & -0.3\cdot P2 - 1092.0094 & -0.1\cdot P2 - 2184.0187 & 0 \\ 0.1\cdot P3 + 136.50117 & -0.1\cdot P2 - 2184.0187 & 0.53333333P2 + 2.1333333P3 + 7280.0625 & 0 & 0.1\cdot P2 + 2184.0187 & -0.13333333P2 + 2912.025 & -0.53333333P3 + 728.00625 \\ -143501.0 & 0 & 0 & 0.075P1 + 143518.06 & 0 & 0.1\cdot P1 + 136.50117 & 0 \\ 0 & -0.3\cdot P2 - 1092.0094 & 0.1\cdot P2 + 2184.0187 & 0 & 0.3\cdot P2 + 36968.009 & 0.1\cdot P2 + 2184.0187 & 0 \\ 0 & -0.1\cdot P2 - 2184.0187 & -0.13333333P2 + 2912.025 & 0.1\cdot P1 + 136.50117 & 0.1\cdot P2 + 2184.0187 & 2.1333333P1 + 0.53333333P2 + 7280.0625 & 0 \\ 0.1\cdot P3 + 136.50117 & 0 & -0.53333333P3 + 728.00625 & 0 & 0 & 0 & 2.1333333P3 + 1456.0125 \end{pmatrix}$$

$$Ka(P1,P2,P3)\ \text{float},5\ \rightarrow \begin{pmatrix} 143517.0 & 0 & 135.85 & -143501.0 & 0 & 0 & 135.85 \\ 0 & 36970.0 & -2184.7 & 0 & -1094.0 & -2184.7 & 0 \\ 135.85 & -2184.7 & 7269.6 & 0 & 2184.7 & 2911.2 & 731.49 \\ -143501.0 & 0 & 0 & 143516.0 & 0 & 134.15 & 0 \\ 0 & -1094.0 & 2184.7 & 0 & 36970.0 & 2184.7 & 0 \\ 0 & -2184.7 & 2911.2 & 134.15 & 2184.7 & 7233.5 & 0 \\ 135.85 & 0 & 731.49 & 0 & 0 & 0 & 1442.1 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} := Ka(P1, P2, P3)^{-1} \cdot \begin{pmatrix} 0 \\ -60 \\ 0 \\ 30 \\ -60 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 1.8237 \\ -0.0038654 \\ -0.0051219 \\ 1.8237 \\ 0.00052053 \\ -0.033086 \\ -0.1692 \end{pmatrix}$$

Member 1

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} := k1(P1) \cdot \Gamma_1 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ U4 \\ U5 \\ U6 \end{pmatrix}$$

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} -18.674 \\ 23.469 \\ 220.16 \\ 18.674 \\ -23.469 \\ 198.14 \end{pmatrix}$$

$$P1 := F5$$

Member 2

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k2(P2)\cdot\Gamma_2\cdot\begin{pmatrix} U4 \\ U5 \\ U6 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 6.5362 \\ -78.674 \\ -198.14 \\ -6.5362 \\ 78.674 \\ -116.59 \end{pmatrix}$$

$P2 := F4$

Member 3

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k3(P3)\cdot\Gamma_3\cdot\begin{pmatrix} 0 \\ 0 \\ U7 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 138.67 \\ 6.5411 \\ 0.0000034561 \\ -138.67 \\ -6.5411 \\ 116.59 \end{pmatrix}$$

$P3 := F2$

Fifth Iteration

$$Ka(P1, P2, P3) := \begin{pmatrix} 0.075 \cdot P3 + 143518.06 & 0 & 0.1 \cdot P3 + 136.50117 & -143501.0 & 0 & 0 & 0.1 \cdot P3 + 136.50117 \\ 0 & 0.3 \cdot P2 + 36968.009 & -0.1 \cdot P2 - 2184.0187 & 0 & -0.3 \cdot P2 - 1092.0094 & -0.1 \cdot P2 - 2184.0187 & 0 \\ 0.1 \cdot P3 + 136.50117 & -0.1 \cdot P2 - 2184.0187 & 0.53333333 P2 + 2.1333333 P3 + 7280.0625 & 0 & 0.1 \cdot P2 + 2184.0187 & -0.13333333 P2 + 2912.025 & -0.53333333 P3 + 728.00625 \\ -143501.0 & 0 & 0 & 0.075 \cdot P1 + 143518.06 & 0 & 0.1 \cdot P1 + 136.50117 & 0 \\ 0 & -0.3 \cdot P2 - 1092.0094 & 0.1 \cdot P2 + 2184.0187 & 0 & 0.3 \cdot P2 + 36968.009 & 0.1 \cdot P2 + 2184.0187 & 0 \\ 0 & -0.1 \cdot P2 - 2184.0187 & -0.13333333 P2 + 2912.025 & 0.1 \cdot P1 + 136.50117 & 0.1 \cdot P2 + 2184.0187 & 2.1333333 P1 + 0.53333333 P2 + 7280.0625 & 0 \\ 0.1 \cdot P3 + 136.50117 & 0 & -0.53333333 P3 + 728.00625 & 0 & 0 & 0 & 2.1333333 P3 + 1456.0125 \end{pmatrix}$$

$$Ka(P1, P2, P3) \text{ float}, 5 \rightarrow \begin{pmatrix} 143517.0 & 0 & 135.85 & -143501.0 & 0 & 0 & 135.85 \\ 0 & 36970.0 & -2184.7 & 0 & -1094.0 & -2184.7 & 0 \\ 135.85 & -2184.7 & 7269.6 & 0 & 2184.7 & 2911.2 & 731.49 \\ -143501.0 & 0 & 0 & 143516.0 & 0 & 134.15 & 0 \\ 0 & -1094.0 & 2184.7 & 0 & 36970.0 & 2184.7 & 0 \\ 0 & -2184.7 & 2911.2 & 134.15 & 2184.7 & 7233.5 & 0 \\ 135.85 & 0 & 731.49 & 0 & 0 & 0 & 1442.1 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} := Ka(P1, P2, P3)^{-1} \cdot \begin{pmatrix} 0 \\ -60 \\ 0 \\ 30 \\ -60 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 1.8237 \\ -0.0038654 \\ -0.005122 \\ 1.8237 \\ 0.00052053 \\ -0.033086 \\ -0.1692 \end{pmatrix}$$

Member 1

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} := k1(P1) \cdot \Gamma_1 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ U4 \\ U5 \\ U6 \end{pmatrix}$$

$$\begin{pmatrix} F10 \\ F11 \\ F12 \\ F4 \\ F5 \\ F6 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} -18.674 \\ 23.469 \\ 220.16 \\ 18.674 \\ -23.469 \\ 198.14 \end{pmatrix}$$

$P1 := F5$

Member 2

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k2(P2)\cdot\Gamma_2\cdot\begin{pmatrix} U4 \\ U5 \\ U6 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F4 \\ F5 \\ F6 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 6.5362 \\ -78.674 \\ -198.14 \\ -6.5362 \\ 78.674 \\ -116.59 \end{pmatrix}$$

$P2 := F4$

Member 3

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} := k3(P3)\cdot\Gamma_3\cdot\begin{pmatrix} 0 \\ 0 \\ U7 \\ U1 \\ U2 \\ U3 \end{pmatrix}$$

$$\begin{pmatrix} F8 \\ F9 \\ F7 \\ F1 \\ F2 \\ F3 \end{pmatrix} \text{float}, 5 \rightarrow \begin{pmatrix} 138.67 \\ 6.5411 \\ 0.0000034561 \\ -138.67 \\ -6.5411 \\ 116.59 \end{pmatrix}$$

$P3 := F2$

P1 := F5

P1 float, 5 → −78.674

P2 := F4

P2 float, 5 → 6.5362

P3 := F2

P3 float, 5 → −6.5411

All units for forces, F are in kN and displacements in meter.

CHAPTER 4

RESULTS OF FIRST ORDER ELASTIC ANALYSIS AND SECOND ORDER ELASTIC ANALYSIS

4.1 Introduction

In the analysis, the responses of the simple two dimensional steel frames draw out from the software and manual analysis. In order to perform the objectives of this paper, the model of simple steel frames have been developed using manual and software analysis. Those analyses started with the study of behavior of steel frames due to different load and dimension cases. Fundamentally, the responses of the steel frames controlled by lateral and gravity loadings are analysed and the comparisons between linear and nonlinear are carried out.

In this paper, the analysis is divided into two sections, which are manual calculation and software calculation which categorized in different load and dimension cases. Both analyses carry out are to assess the trend of linear and nonlinear effects.

The behavior of simple steel frame is evaluated in two analyses and each analysis consists with two cases respectively. All the members have the same cross section which is UB 254 x 102 x 22, whereby its area is 28 cm^2; Young modulus is 205 kN/mm^2; second moment of area is 2841 cm^4.

The two analyses are:

- First order elastic analysis:
  The change in stiffness is small enough, and assumes that does not have any nonlinearity change during deformation process.

- Second order elastic analysis:
  The changes in stiffness under loadings, which come only from changes in shape which also known as geometrical nonlinearity.

The two cases are:

- Height /width ratio = 1, 2, 3, and 4

- Various load cases

4.2 Results of Manual Calculation Using Secant Stiffness Method

All the following calculation' results shown below are according to node 1 and only the horizontal displacements will be analysed.

4.2.1 First Order Elastic Analysis (Height / Width Ratio)

Four height / width ratio cases have been analysed and the respective results are shown. The loadings considered for this analysis are vertical loading and horizontal loading which are 60kN and 30kN.

1. Results for Height/Width = 1 (Height = 4000mm, Width = 4000mm)

   Horizontal displacement, Δ_x = 32.554mm
   Vertical displacement, Δ_y = -0.532mm
   Moment, θ_z = -0.002radian

2. Results for Height/Width = 2 (Height = 8000mm, Width = 4000mm)

   Horizontal displacement, Δ_x = 222.753mm
   Vertical displacement, Δ_y = -1.315mm
   Moment, θ_z = -0.003radian

3. Results for Height/Width = 3 (Height = 12000mm, Width = 4000mm)

   Horizontal displacement, Δ_x = 704.342mm
   Vertical displacement, Δ_y = -2.350mm
   Moment, θ_z = -0.004radian

4. Results for Height/Width = 4 (Height = 16000mm, Width = 4000mm)

   Horizontal displacement, Δ_x = 1610.347mm
   Vertical displacement, Δ_y = -3.635m
   Moment, θ_z = -0.005radian

The above results for first order elastic analysis can be simplified into a table as shown in Table 4.1.

Table 4.1 Summary results for First Order Elastic Analysis by Secant Stiffness Method (Height / Width Ratio)

Dimension		Ratio	Linear		
Height (mm)	Width (mm)		Δx (mm)	Δy (mm)	θz (radian)
4000	4000	1	32.555	-0.532	-0.002
8000	4000	2	222.753	-1.315	-0.003
12000	4000	3	704.342	-2.350	-0.004
16000	4000	4	1610.347	-3.635	-0.005

All the results for the height / width cases are shown in Table 4.1. From the results obtained, the flow obviously is increasing as the height / width ratio increases from height to height. The table showed that the lateral displacements are displaced the most compare to vertical displacements. This meant that the structure tends to sway in horizontal direction. Meanwhile, the deformation shape of structure is keep bending which results in stiffness of the structure degrade inconsistently. This contributes instability of the entire structure due to a higher proportion in height.

4.2.2 First Order Elastic Analysis (Load case)

A structure with height / width ratio of four is analysed with various load cases and the respective results are shown in Table 4.2. The dimensions of the structure are 16000mm of height and 4000mm of width.

Table 4.2 Various loadings with respective displacements for First Order Elastic Analysis by Mathcad (Load case)

Loading (kN)	**Horizontal Displacement (mm)**
30	805.17
60	1610.35
90	2415.52
120	3220.69
150	4025.87
180	4831.04
210	5636.21
240	6441.39
270	7246.56
300	8051.73
330	8856.91
360	9662.08
390	10467.26
420	11272.43
450	12077.60
460	12345.99
470	12614.38
472	12668.06
474	12721.74
476	12775.42
477	12802.26
478	12829.10
479	12855.94
480	12882.78

The results shown in Table 4.2 can be plotted in graph which shown in Figure 4.1.

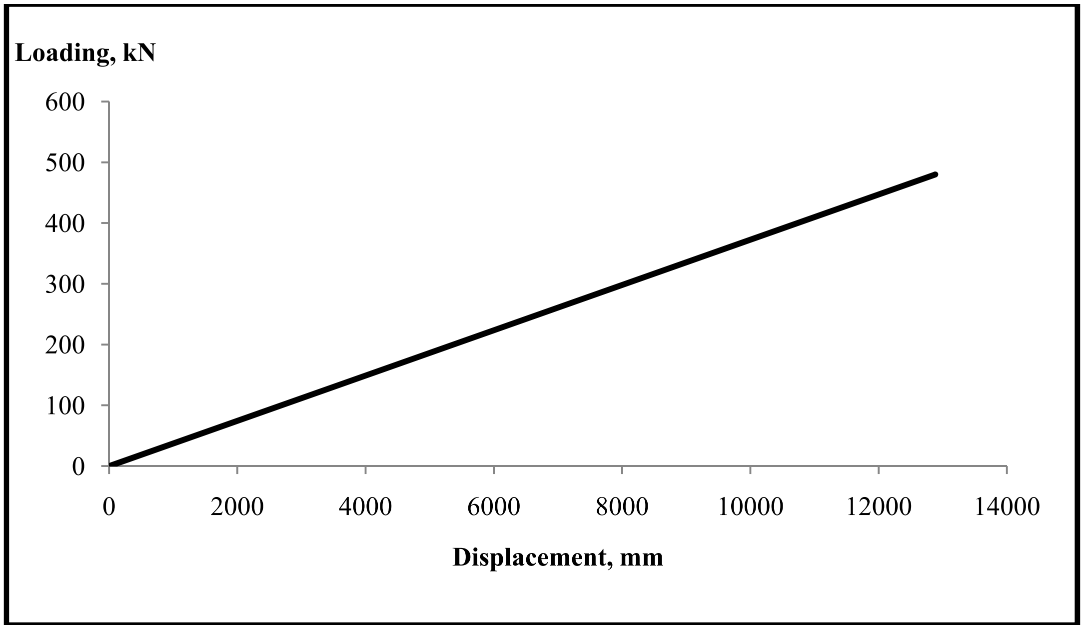

Figure 4.1 First Order Elastic Analysis by Secant Stiffness Method

All the results for various load cases are showed in Table 4.2 and also represented in graph which shown in Figure 4.1. From Figure 4.1, the loadings are directly proportional to the displacements. As the loadings increase, there is an increment in displacements. There did not have much change in the deformation shape of the member and structure.

4.2.3 Second Order Elastic Analysis (Height / Width Ratio)

Four height / width ratio cases have been analysed and the respective results are shown. The loadings considered for this analysis are vertical loading and horizontal loading which are 60kN and 30kN.

1. Results for Height/Width = 1 (Height = 4000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 32.852mm

Vertical displacement, Δ_y = -0.533mm

Moment, θ_z = -0.002radian

2 Results for Height/Width = 2 (Height = 8000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 230.055mm

Vertical displacement, Δ_y = -1.330mm

Moment, θ_z = -0.003radian

3 Results for Height/Width = 3 (Height = 12000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 755.559mm

Vertical displacement, Δ_y = -2.421mm

Moment, θ_z = -0.004radian

4 Results for Height/Width = 4 (Height = 16000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 1823.660mm

Vertical displacement, Δ_y = -3.865m

Moment, θ_z = -0.005radian

The above results can be simplified in table as shown in Table 4.3.

Table 4.3 Summary results for Second Order Elastic Analysis by Secant Stiffness Method (Height / Width ratio)

Dimension		Ratio	Nonlinear		
Height (mm)	Width (mm)		Δx (mm)	Δy (mm)	θz (radian)
4000	4000	1	32.852	-0.533	-0.002
8000	4000	2	230.055	-1.330	-0.003
12000	4000	3	755.559	-2.421	-0.004
16000	4000	4	1823.660	-3.865	-0.005

Analysis results for the four ratios are shown in Table 4.3. Through the results, it noted that a bit huge increment among each ratio, whereby these hike up values are coming from the geometrical nonlinearity of the structure contribute by P-Delta effects.

4.2.4 Second Order Elastic Analysis (Load Case)

A structure with height / width ratio of four is analysed with various load cases and the respective results are shown in Table 4.4. The dimensions of the structure are in 16000mm height and 4000mm width.

Table 4.4 Various loadings with respective displacements Second Order Elastic Analysis by Mathcad (Load case)

Loading (kN)	Horizontal Displacement (mm)
30	855.00
60	1824.00
90	2929.13
120	4203.00
150	5688.54
180	7439.11
210	9537.62
240	12097.83
270	15292.57
300	19405.89
330	24835.10
360	32228.02
390	43318.11
420	60991.58
450	108533.16
460	134986.96
470	206652.05
472	238267.64
474	288145.39
476	381277.57
477	468075.67
478	626822.74
479	1014149.30
480	3441879.50

A graph with loadings, kN versus displacements, mm is plotted in Figure 4.2 according to Table 4.4.

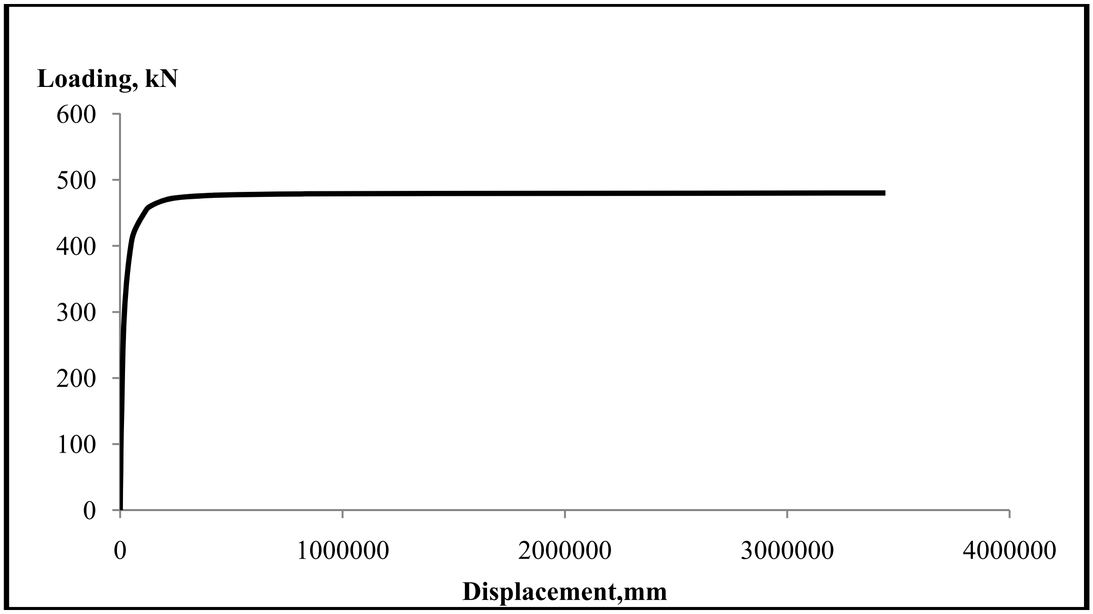

Figure 4.2 Second Order Elastic Analysis by Secant Stiffness Method

Figure 4.2 captured the changing in structure shape start at load 400kN. The change in geometry is due to the nonlinear effects driven by second order elastic analysis. At load about 480kN, a straight horizontal line is observed for the P_{cr} for elastic behavior.

4.2.5 Comparisons Analysis Results between Linear Analysis and Nonlinear for Manual Calculations (Secant Stiffness Method)

Case 1: Dimension

Loading for each ratio: Vertical loading = 60 kN

Horizontal Loading = 30 kN

Table 4.5 showed the results for both analyses of height / width cases in order to make a comparison between them by using Secant stiffness Method.

Table 4.5 Results for First Order Elastic Analysis and Second Order Elastic Analysis by Secant Stiffness Method (Height / Width Ratio)

Dimension		Ratio	Linear Analysis	Nonlinear Analysis	Difference
Height (mm)	Width (mm)		Δx (mm)	Δx (mm)	Δx (%)
4000	4000	1	32.555	32.852	0.912
8000	4000	2	222.753	230.055	3.278
12000	4000	3	704.342	755.559	7.272
16000	4000	4	1610.347	1823.66	13.246

According to the results in Table 4.5, obviously the nonlinear displacements are greater than linear displacements. This reflects that a geometry change in structure is occurred and stiffness of the structure starts to degrade. As the ratio increases, the percentage difference between linear analysis and nonlinear analysis are drawn further and further.

Case 2: Loading

Dimension for each loading: Height = 16 m
Width = 4 m

Table 4.6 showed the results for both analyses of load cases in order to make a comparison between them by using Secant stiffness Method.

Table 4.6 Results for First Order Elastic Analysis and Second Order Elastic Analysis by Secant Stiffness Method (Load case)

Loading (kN)	Displacement (Linear, mm)	Displacement (Nonlinear, mm)	Difference, %
0	0.00	0.00	0.00
30	805.17	855.00	6.19
60	1610.35	1824.00	13.27
90	2415.52	2929.13	21.26
120	3220.69	4203.00	30.50
150	4025.87	5688.54	41.30
180	4831.04	7439.11	53.99
210	5636.21	9537.62	69.22
240	6441.39	12097.83	87.81
270	7246.56	15292.57	111.03
300	8051.73	19405.89	-
330	8856.91	24835.10	-
360	9662.08	32228.02	-
390	10467.26	43318.11	-
420	11272.43	60991.58	-
450	12077.60	108533.16	-
460	12345.99	134986.96	-
470	12614.38	206652.05	-
472	12668.06	238267.64	-
474	12721.74	288145.39	-
476	12775.42	381277.57	-
477	12802.26	468075.67	-
478	12829.10	626822.74	-
479	12855.94	1014149.30	-
480	12882.78	3441879.50	-

*- indicate that the difference between linear and nonlinear more than 100%

In order to show clearly the relationship between first order elastic analysis and second order elastic analysis, a graph of both analyses is plotted in Figure 4.3.

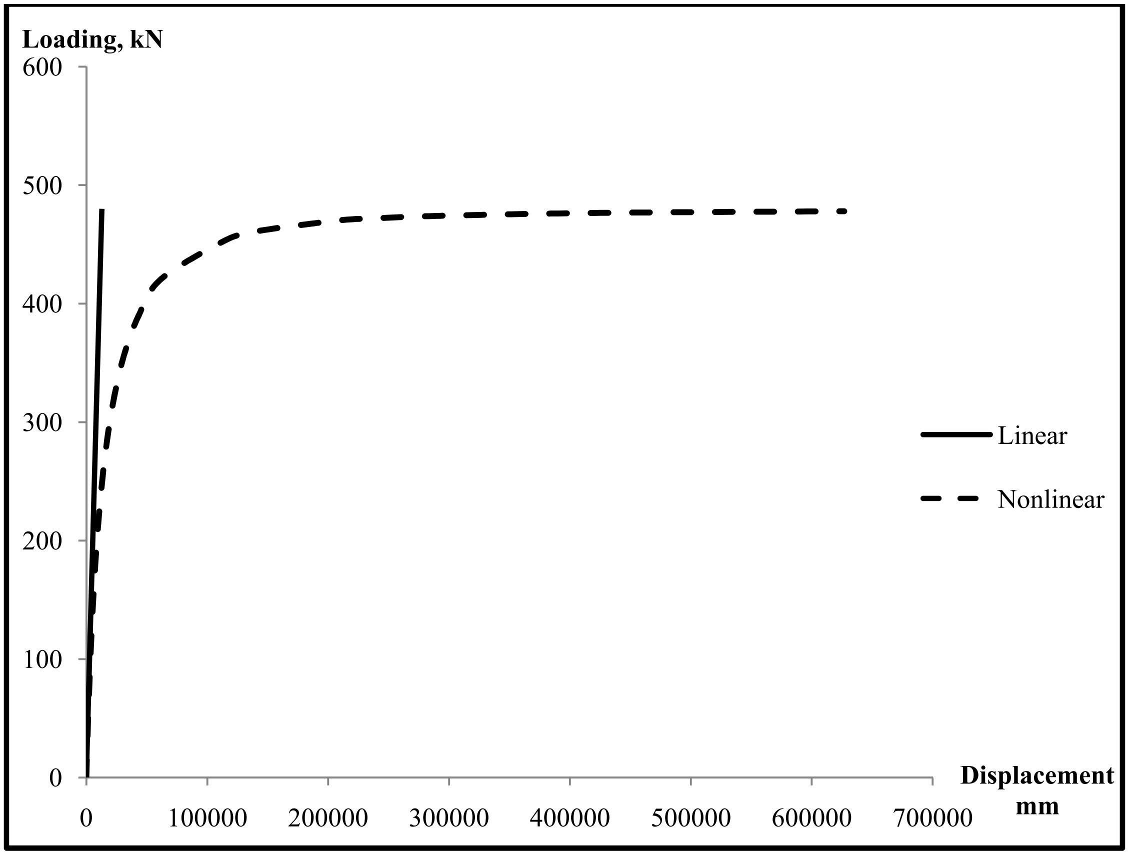

Figure 4.3 Relationships between Linear Analysis and Nonlinear Analysis by Secant Stiffness Method.

Figure 4.3 showed there is a difference exists between both analyses' curves. It implied that second order elastic analysis analysed there had a change in shape of structure. The displacements for nonlinear analysis indicated that nonlinear effects have a rigorous impact for the deformation of the structure. As the variable magnitude loadings keep acting on the structure, the structure easy to sway as the stiffness of structure decreased.

4.3 Results of Software Calculations Using Newton-Raphson Method

All the following calculations' results shown below are according to node 1 and only the horizontal displacements will be analyzed.

4.3.1 First Order Elastic Analysis (Height / Width Ratio)

Four height / width ratio cases have been analysed and the respective results are shown. The loadings considered for this analysis are vertical loading and horizontal loading which are 60kN and 30kN.

1 Results for Height/Width = 1 (Height = 4000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 33.548mm
Vertical displacement, Δ_y = -0.532mm
Moment, θ_z = -0.002radian

2 Results for Height/Width = 2 (Height = 8000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 225.359mm
Vertical displacement, Δ_y = -1.314mm
Moment, θ_z = -0.003radian

3 Results for Height/Width = 3 (Height = 12000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 709.611mm
Vertical displacement, Δ_y = -2.348mm

Moment, θ_z = -0.004radian

4 Results for Height/Width = 4 (Height = 16000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 1618.902mm
Vertical displacement, Δ_y = -3.633mm
Moment, θ_z = -0.005radian

Table 4.7 Summary results for First Order Elastic Analysis by Newton-Raphson Method (Height / Width Ratio)

Dimension		Ratio	Linear		
Height (mm)	Width (mm)		Δx (mm)	Δy (mm)	θz (radian)
4000	4000	1	33.548	-0.532	-0.002
8000	4000	2	225.459	-1.314	-0.003
12000	4000	3	709.611	-2.348	-0.004
16000	4000	4	1618.902	-3.633	-0.005

All the results for the height / width cases are shown in Table 4.7. A typical increment in lateral displacements is observed as he ratio increases from one to another. This analysed that as the height varies, structure displaced far away from its original position due to P-Delta effects. Table also gave an information on which is the most important impact affect the overall stability of the structure.

4.3.2 First Order Elastic Analysis (Load Case)

A structure with height / width ratio of four is analysed with various load cases and the respective results are shown in Table 4.8. The dimensions of the structure are 16000mm of height and 4000mm of width.

Table 4.8 Various loadings with respective displacements First Order Elastic Analysis by Newton-Raphson Method (Load case)

Loading (kN)	Horizontal Displacement (mm)
30	809.45
60	1618.90
90	2428.35
120	3237.81
150	4047.26
180	4856.71
210	5666.16
240	6475.61
270	7285.06
300	8094.51
330	8903.96
360	9713.415
390	10522.87
420	11332.32
450	12141.77
460	12411.59
470	12681.40
472	12735.37
474	12789.33
476	12843.29
477	12870.28
478	12897.26
479	12924.24
480	12951.22

A graph with loadings, kN versus displacements, mm is plotted in Figure 4.4 according to Table 4.8.

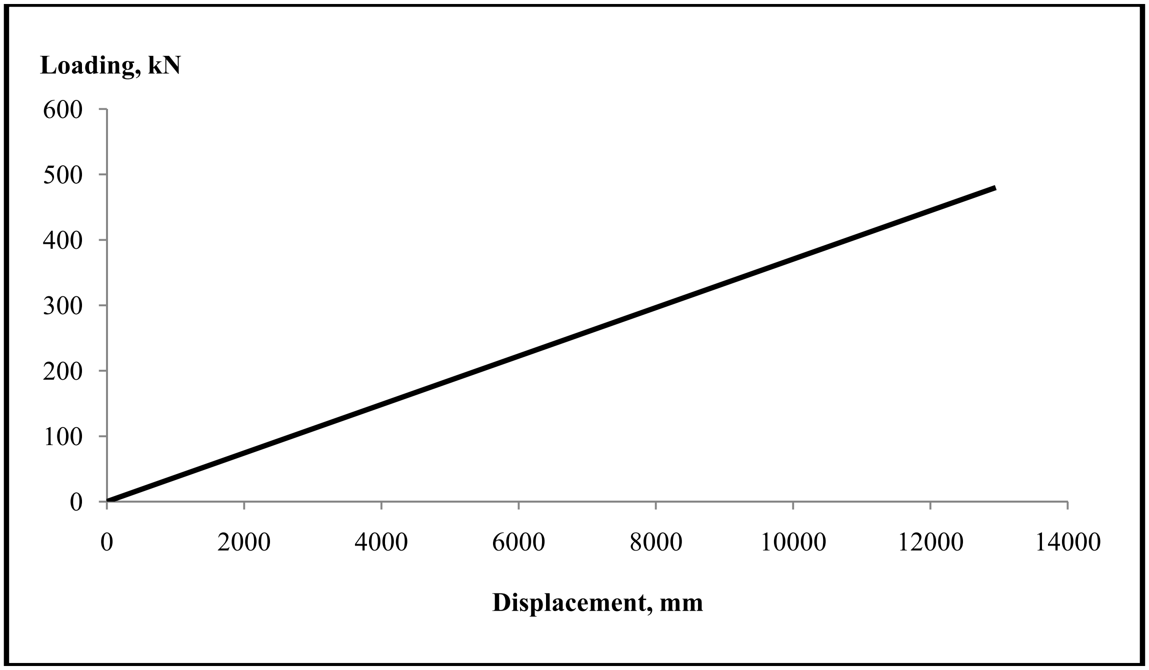

Figure 4.4 First Order Elastic Analysis by Newton-Raphson Method

From Figure 4.4, the loadings are directly proportional to the displacements. As the loadings increase, there is an increment in displacements. There did not have much change in the deformation shape of the member and structure since this analysis only concerned the linear effects.

4.3.3 Second Order Elastic Analysis (Height / Width Ratio)

Four height / width ratio cases have been analysed and the respective results are shown. The loadings considered for this analysis are vertical loading and horizontal loading which are 60kN and 30kN.

1 Results for Height/Width = 1 (Height = 4000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 34.717mm

Vertical displacement, Δ_y = -0.536mm

Moment, θ_z = -0.002radian

2 Results for Height/Width = 2 (Height = 8000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 254.174mm

Vertical displacement, Δ_y = -1.375mm

Moment, θ_z = -0.004radian

3 Results for Height/Width = 3 (Height = 12000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 929.535mm

Vertical displacement, Δ_y = -2.687mm

Moment, θ_z = -0.006radian

4 Results for Height/Width = 4 (Height = 16000mm, Width = 4000mm)

Horizontal displacement, Δ_x = 2707.927mm

Vertical displacement, Δ_y = -4.951mm

Moment, θ_z = -0.009radian

The above results can be simplified in table as shown in Table 4.9.

Table 4.9 Summary results for Second Order Elastic Analysis by Newton-Raphson Method (Height / Width Ratio)

Dimension		Ratio	Nonlinear		
Height (mm)	Width (mm)		Δx (mm)	Δy (mm)	θz (radian)
4000	4000	1	34.717	-0.536	-0.002
8000	4000	2	254.174	-1.375	-0.004
12000	4000	3	929.535	-2.687	-0.006
16000	4000	4	2707.927	-4.951	-0.009

Analysis results for the four ratios are shown in Table 4.9. Through the results, it noted that a bit huge increment among each ratio, whereby these hike up values are coming from the geometrical nonlinearity of the structure contribute by P-Delta effects. It can be seen that the different between ratio three and ratio four is bigger than the others.

4.3.4 Second Order Elastic Analysis (Load Case)

A structure with height / width ratio of four is analysed with various load cases and the respective results are shown in Table 4.10. The dimensions of the structure are in 16000mm height and 4000mm width.

Table 4.10 Various loadings with respective displacements Second Order Elastic Analysis by Newton-Raphson Method (Load case)

Loading (kN)	Horizontal Displacement (mm)
30	1014.77
60	2707.93
90	5872.72
120	12221.00
150	25049.61
180	50124.54
210	96817.83
240	179578.40
270	319904.40
300	549123.78
330	912463.62
360	1475000.00
390	2329910.00
420	3608230.00
450	5485880.00
460	6281290.00
470	7175330.00
472	7366630.00
474	7562280.00
476	7762370.00
477	7864018.57
478	7966788.88
479	8070739.19
480	8175749.51

A graph with loadings, kN versus displacements, mm is plotted in Figure 4.5 according to Table 4.10.

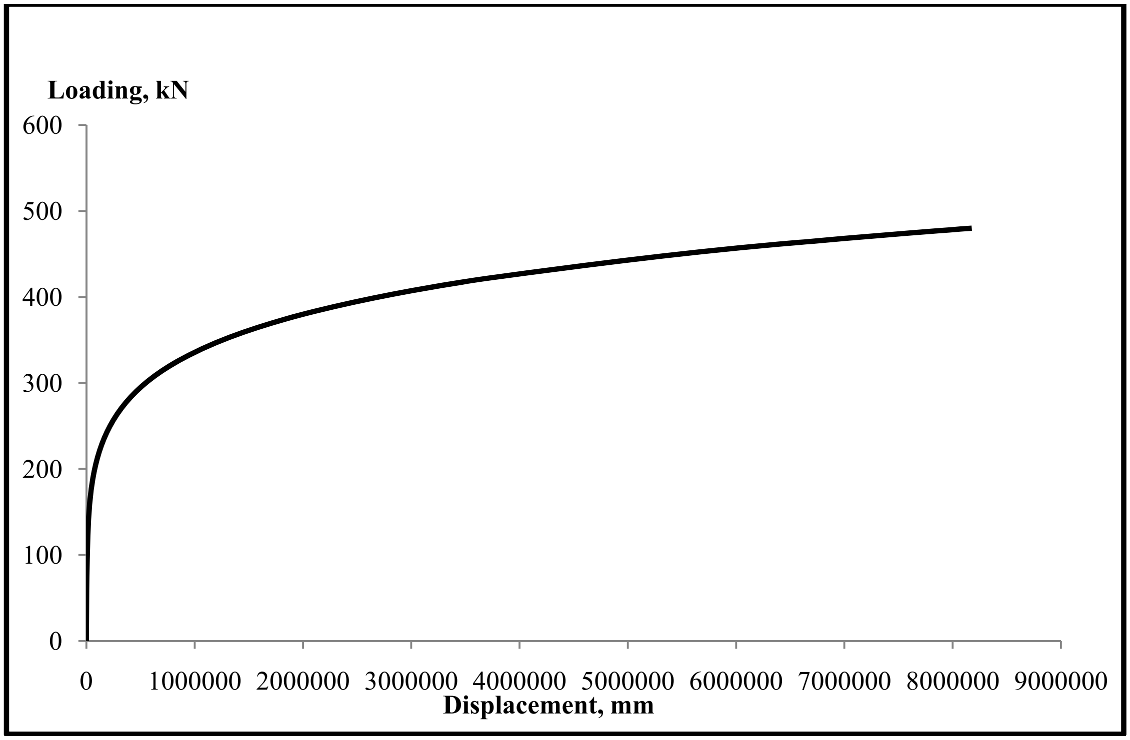

Figure 4.5 Second Order Elastic Analysis by Newton-Raphson Method

From Figure 4.5, there is a big change in the curve shape. The gradient of the curve increase with a constant rate as it almost approach to reach plastic behavior. When allowing second-order effects due to a change in geometry by incorporating the variation of element stiffness in the presence of axial force, the calculated deflections, forces, and moments will be more accurate than the linear analysis.

4.3.5 Comparisons Analysis Results between Linear Analysis and Nonlinear for Software Calculations (Newton-Raphson Method)

Case 1: Dimension

Loading for each ratio: Vertical loading = 60 kN

Horizontal Loading = 30

Table 4.11 showed the results for both analyses of height / width cases in order to make a comparison between them by using Newton-Raphson Method.

Table 4.11 Results for First Order Elastic Analysis and Second Order Elastic Analysis by Newton-Raphson Method (Height / Width Ratio)

Dimension		Ratio	Linear Analysis	Nonlinear Analysis	Difference
Height (mm)	Width (mm)		Δx (mm)	Δx (mm)	Δx (%)
4000	4000	1	33.548	34.717	3.485
8000	4000	2	225.459	254.174	12.736
12000	4000	3	709.611	929.535	30.992
16000	4000	4	1618.902	2707.927	67.269

According to the results in Table 4.11, obviously the nonlinear displacements are greater than linear displacements. This reflects that a geometry change in structure is occurred and stiffness of the structure starts to degrade. As the ratio increases, the percentage difference between linear analysis and nonlinear analysis are drawn further and further as it shown for the ratio of four.

Case 2: Loading

Dimension for each loading: Height = 16 m
Width = 4 m

All results are shown in Table 4.12.

Table 4.12 Results for First Order Elastic analysis and Second Order Elastic Analysis by Newton-Raphson Method (Load case)

Loading (kN)	Displacement (Linear, mm)	Displacement (Nonlinear, mm)	Difference, %
0	0.00	0.00	0.00
30	809.45	1014.77	25.37
60	1618.90	2707.93	67.27
90	2428.35	5872.72	141.84
120	3237.81	12221.00	-
150	4047.26	25049.61	-
180	4856.71	50124.54	-
210	5666.16	96817.83	-
240	6475.61	179578.40	-
270	7285.06	319904.40	-
300	8094.512	549123.78	-
330	8903.963	912463.62	-
360	9713.415	1475000.00	-
390	10522.87	2329910.00	-
420	11332.32	3608230.00	-
450	12141.77	5485880.00	-
460	12411.59	6281290.00	-
470	12681.40	7175330.00	-
472	12735.37	7366630.00	-
474	12789.33	7562280.00	-
476	12843.29	7762370.00	-
477	12870.28	7864018.57	-
478	12897.26	7966788.88	-
479	12924.24	8070739.19	-
480	12951.22	8175749.51	-

*- indicate that the difference between linear and nonlinear more than 100%

In order to show clearly the relationship between first order elastic analysis and second order elastic analysis, a graph of both analyses is plotted in Figure 4.6.

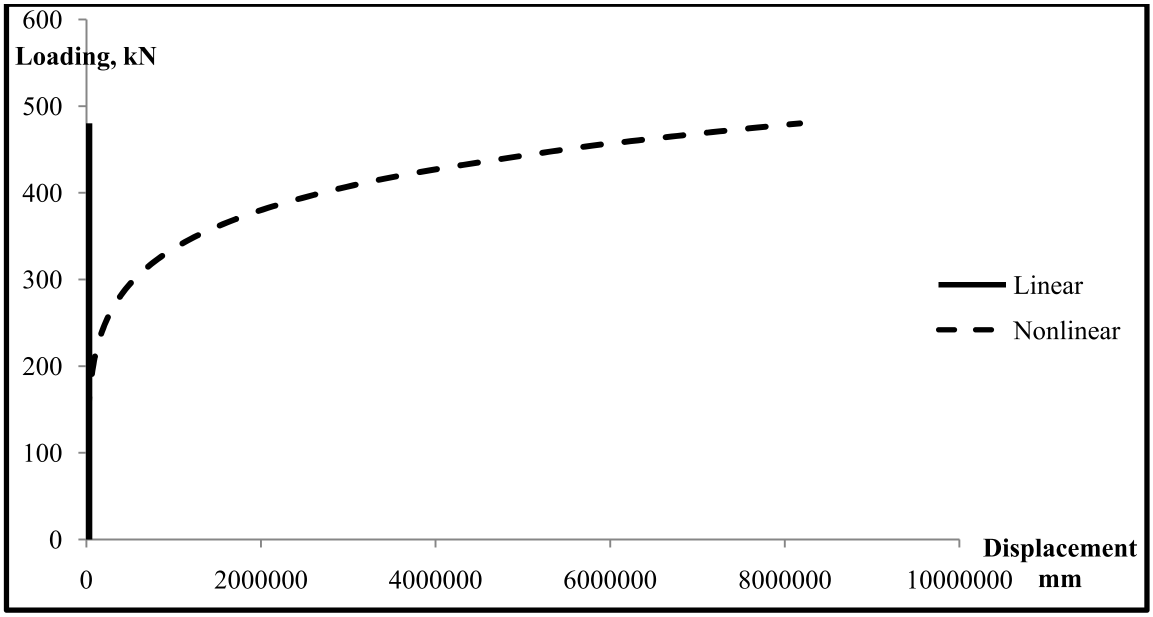

Figure 4.6 Relationships between Linear and Nonlinear Analysis by Newton-Raphson Method.

Figure 4.6 showed there is a difference exists between both analyses' curves. It implied that second order nonlinear elastic analysis analysed there had a change in shape of structure. The displacements for nonlinear analysis indicated that nonlinear effects have a rigorous impact for the deformation of the structure. As the variable magnitude loadings keep acting on the structure, the structure easy to sway as the stiffness of structure decreased. As shown in Fig. 4.6, the load-deflection curve for second-order elastic analysis follows the exact linear equilibrium path before yielding, and then it deviates and becomes nonlinear due to geometric nonlinearity.

4.4 Comparisons Efficiency between Secant Stiffness Method and Newton-Raphson Method

A comparison on the degree of efficiency of both calculation methods between Figure 4.3 and Figure 4.6 is discussed. By looking on the shape of the nonlinear curve, it can be seen that nonlinear curve in Figure 4.6 is much significant than the nonlinear curve in Figure 4.3. This showed that Newton-Raphson Method captured the unexpected effects which first order analysis did not undertake those nonlinear effects. As a result, Newton-Raphson Method provides a clear structure behavior.

4.5 Internal Forces and Bending Moment Responses on Frame Structure by Secant Stiffness Method

Vertical loading with 200kN and horizontal loading with 100kN are applied on the frame structure which with dimensions of 16000mm in height and 4000mm in width. The following tables which are Table 4.13, Table 4.14, Table 4.15 and Table 4.16 are the results gained from the Secant Stiffness Method.

Table 4.13 Node displacements of Linear Analysis and Nonlinear Analysis

Node	Node Displacement	
	Linear Analysis	Nonlinear Analysis
1	5368.000	8793.500
2	-12.100	-15.800
3	-0.016	-0.022
4	5368.000	8793.600
5	1.000	4.700
6	-0.098	-0.156
7	-0.495	-0.826

Table 4.14 Internal forces and Bending Moment at Member 1

Internal Forces	Linear Anlaysis (kN)	Nonlinear Analysis (kN)
f_{10}	-78.18	-167.17
f_{11}	-34.70	78.33
θ_{12}	661.18	1011.39
f_4	78.17	167.17
f_5	34.70	-78.33
θ_6	589.63	930.38

Table 4.15 Internal forces and Bending Moment at Member 2

Internal Forces	Linear Anlaysis (kN)	Nonlinear Analysis (kN)
f_4	21.82	21.70
f_5	-234.70	-367.17
θ_6	-589.63	-930.37
f_1	-21.82	-21.70
f_2	234.70	367.17
θ_3	-349.19	-538.76

Table 4.16 Internal forces and Bending Moment at Member 3

Internal Forces	Linear Anlaysis (kN)	Nonlinear Analysis (kN)
f_8	-21.82	567.17
f_9	434.70	21.72
θ_7	9.47	0.00
f_1	21.82	-567.17
f_2	434.70	-21.72
f_3	349.19	538.76

In Table 4.13, there is a slightly increase in nodal displacements between both analyses. Definitely, there was an increase in internal forces and bending moments for all members. The purpose internal forces and bending moments take into consideration is to aware engineers that geometry nonlinearity not only affects the deformation shape f structure, it also increases the internal force and bending moment values.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The nature of frequently encountered analysis problems should be the yard-stick by which to justify a decision to add nonlinear analysis for all structural analysis. If day-to-day work requires nonlinear analysis only occasionally, then the quality of analysis is be suspicious of.

Every day, engineers are introduced to design analysis problems involve large deformations, buckling, geometrical nonlinearity, material nonlinearity and etc, then nonlinear analysis capabilities should be added to analysis routine intended for design engineers to consider the most critical value for any structure design.

The stiffness and stability of steel frame structure are dominant consideration in structure response. It is observed that substantial differences exist between linear and nonlinear analysis due to two cases such as dimension case and loading case. Significant differences also exist between method calculations that are difference between linear and linear analysis; and nonlinear and nonlinear analysis. More accurate difference for the geometrical nonlinearity of frame members after the analysis is required to for both analyses in order to discover a better estimate of sway. The present work has been carried out to study deflections amplification on the response and behavior of simple frame structure. The significant results are summarized below.

1. The significant increase in deflections is observed as dimension and loading increase especially for nonlinear analysis of both calculation methods. A multiply increase in percentage is predicted throughout the analyses.

2. The effect of slenderness is enlarged due to the horizontal loading as the stiffness of the structure decreases.

3. Newton-Raphson Method proposed the minimum most critical load where characteristic of nonlinearity starts to become one of the factors in contributing a large displacement to the frame structure. This showed that Newton-Raphson Method provides a critical data more than Secant Method.

As the structure sways to a specific horizontal distance, which refer to Δ and δ, under the effect of lateral force, the product of P by horizontal distance produce an additional moment at each column base, which named as overturning moment. It showed that P-delta effects give rigorous impacts on the stability of a structure. A more realistic evaluation of structure stiffness against lateral loadings can be reached only by analysis that takes into account nonlinear effects. There is a need to consider the P-delta effects in the appropriate design for structures. In the past, this consideration in design has been simple and limited in engineering analysis.

Times have changed. Engineers today may have to master the application of the second-order elastic analysis so to obtain accurate design forces and moments, which accommodate all the P-delta effects, then the analysis method used should account for both P-Δ and P-δ; both the deltas (Δ and δ) are inextricably linked which means an increase in one brings about an increase in the other.

Although the second order elastic analysis of frame structure is time consuming and costly, it is about time that engineers are encouraged to use more accurate analysis methods and computer programs to predict more accurately structure. Such advanced analysis will exhibit a more uniform level of safety and provide a better long-term serviceability and maintainability.

5.2 Recommendation

In this paper, the dimension and loading are limited to simple steel frame structure in order to give an idea about P-Delta analysis which might brings and creates a threaten effect on the structure. P-Delta analysis studied here is to improve the analysis procedures besides for linear analysis. For further studies, it is recommended that a detailed parametric study be conducted to:

1. Identify the effect of different joint connections for beam-column members and based-supports on the overall frame behavior.
2. Identify the effects of dynamic loadings on two-dimensional and three-dimensional frames.
3. Highlight the other nonlinearity such as material nonlinear, buckling, nonlinear supports and etc.
4. Outline the type of analysis procedure which is necessary for P-Delta analysis to be applied optimally in practical.
5. Indicate the computer-aided analysis software capabilities needed to support P-Delta analysis.

REFERENCES

1. Donald W. White and Herome F. Hajjar (1991). *Application of Second-Order Elastic Analysis in LRFD: Research to Practice.* Engineering Journal/American Institute of Steel Construction pp 133-148.

2. Amit Urs (Oct. 2002). *Stability Analysis of Frame Tube Tall Buildings.* Department of Civil and Environmental Engineering, Worcester Polytechnic Institute.

3. Christopher M. Hewitt, S.E. (September 2008). *Stability Analysis: It's not as hard as you think.* Modern Steel Construction pp 44-46.

4. R. Shankar Nair, PH.D., P.E., S.E. (May 2007). *Stability Analysis and the 2005 AISC Specification*. Modern Steel Construction pp 48-50.

5. William McGuire (2005). *Structural Analysis*.

6. S.O. Degertekin and M.S. Hayalioglu (2004). *Design of Nonlinear Semi-Rigid Steel Frames with Semi-Rigid Column Bases.* Electronic Journal of Structural Engineering, 4(2004).

7. M.A.A. Mollick (1997). *Experimental Study on P-Delta Effect in RC High Rise Building.* Journal of Civil Engineering Vol. CE 25, No. 2.

8. Lim Pui Yan (2006/2007). *Nonlinear Behavior of One-Bay Steel Frames With Semi-Rigid Connections.*

9. Hirotaka Oda (1997) and Tsutomu Usami(1997). *Stability Design of Steel Plane Frames by Second-Order Elastic Analysis.* Engineering Structures, Vol. 19, No. 8, pp. 617-627, 1997.

10. Fumio Nishino and Walid Attia (January 1992). *A Proposal for In-Plane Stability Design of Steel Framed Structures.* Structural Engineering/Earthquake Engineering Vol. 8, No. 4, 169s-178s, January 1992.

11. Ong Lee Chin (2006/2007). *Stability of Rigid and Simple Steel Framed Systems.*

12. Angela M. Schimizze (April 23, 2001). *Comparison of P-Delta Analysis of Plane Frames Using commercial Structural Analysis Programs and Current AISC Design Specifications.*

13. **R**icardo A. M. Silveira, Alexandre S. Galvao and Paulo B. Goncalves (2000). *A Comparative of the Analysis of the Analysis of Plane Frames.* European Congress on Computational Methods in Applied Sciences and Engineering, Barcelona, 11-14 September 2000.

14. Atsumi OHTSUKI and Fernand ELLYIN (Oktober 29, 1999). *Analytical Approach to Large Deformation Problems of Frame Structures (In Case of A Square Frame with Rigid Joints).* Department of Mechanical Engineering JSME International Journal Series A Vol. 44 No 1, 2001 pp 89-93.

15. Leonardo Pinheiro and Richardo A. M. Silveira (2005). *Computational Procedures for Nonlinear Analysis of Frames with Semi-Rigid Connections.* Civil Engineering Program. Latin American Journal of Solids and Structures pp339-367.