SHAFT SPILLWAYS

by M. G. Schaefer[1]

A shaft spillway is an uncontrolled spillway in which the water enters over a weir and drops through a vertical or sloping shaft into a conduit which discharges into the downstream channel (Figure 1).

The drop-inlet and the morning-glory (glory-hole) are the two most common shaft spillways. The drop-inlet spillway is a circular sharp crested weir which is normally constructed of either corrugated steel pipe or precast concrete pipe. The morning-glory spillway is a special case of the circular sharp crested weir in which the shape of the weir crest and the upper portion of the shaft are designed to follow the trajectory of the lower nappe (Figure 2). The crest and shaft are constructed from reinforced concrete, either precast or formed on site.

This paper discusses the hydraulic characteristics and the design for a drop-inlet spillway. The reader is referred to Design of Small Dams[3] for a discussion of the morning-glory spillway.

DISCHARGE CHARACTERTISTICS

The discharge characteristics of a shaft spillway are determined by that portion of the structure which is controlling the discharge. It can be seen in Figure 5 that there is the potential for as many as three separate controls and thus three distinct sections in the elevation discharge curve.

The first section is where the weir crest is the control, and the discharge can be described by the weir equation:

$$Q = C_W B H^{3/2} \tag{1}$$

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where Q = discharge, ft^3/sec

C_W = weir coefficient (Figure 3), $(ft/sec^2)^{1/2}$

B = perimeter length of weir, ft.

R = radius of shaft, ft.

H = head, depth of water over the weir, ft.

The second section is where the shaft opening responds as an orifice. The equation for orifice flow assuming no energy loss is:

$$Q = C_c A(2gH)^{1/2} \tag{2}$$

where C_c = contraction coefficient

A = cross-sectional area of pipe, ft^2

The transition point between weir and orifice flow can be approximated by assuming that the contraction of the flow is fully developed at the inlet. Assuming C_c = 0.52 (Borda mouthpiece) and realizing that at incipient transition the discharge for weir flow and orifice flow are the same, the respective areas of flow must then be equal (from continuity). Since the brink depth over the weir should be approximately critical depth (2/3 H), the area of flow for the weir and orifice can be equated, yielding:

$$2\pi R(\tfrac{2}{3}H) = 0.52\pi R^2$$

simplifying; the transition from weir to orifice flow should occur when

$$H/R = 0.40 \tag{3}$$

Referring to Figure 3, the weir coefficient is seen to sharply decrease at H/R values greater than 0.40. When $H/R > .5$ the inlet is submerged and operating under orifice control. To simplify the design of the spillway, equation 1 is used to predict the discharge. The weir coefficient, C_W, is adjusted to compensate for the differences between the two equations.

Section three occurs when the inlet is submerged and full pipe flow occurs (Figures 4 & 5). Writing the Bernoulli equation between the reservoir surface and the conduit exit downstream (constant diameter);

$$d_1 + H = d_2 + \frac{V_2^2}{2g} + h_{L_{1-2}} \tag{4}$$

from Figure 4:

$$H_T = d_1 + H - d_2 \tag{5}$$

$$H_T = \frac{V_2^2}{2g} + h_{L_{1-2}} \tag{6}$$

where H_T = difference in elevation between headwater and tailwater, ft.

$h_{L_{1-2}}$ = energy losses due to the trashrack, entrance, elbow, and friction, ft

A = area, ft^2

$K_t \frac{V_2^2}{2g}$ = trashrack loss, ft.

$K_e \frac{V_2^2}{2g}$ = entrance loss, ft.

$K_b \frac{V_2^2}{2g}$ = elbow loss, ft

$K_f \frac{V_2^2}{2g} = \frac{29.1 n^2 L}{r^{4/3}} \frac{V_2^2}{2g}$ = friction loss, ft.

K = loss coefficient

L = pipe length, ft

n = manning's n, pipe roughness

r = hydraulic radius (area/wetted perimeter), ft.

substitution yields:

$$H_T = \frac{Q^2}{2gA^2} \{1 + K_t + K_e + K_b + K_f\}$$

Utilizing equations 1, 5 and 7, the elevation discharge relationship can be determined.

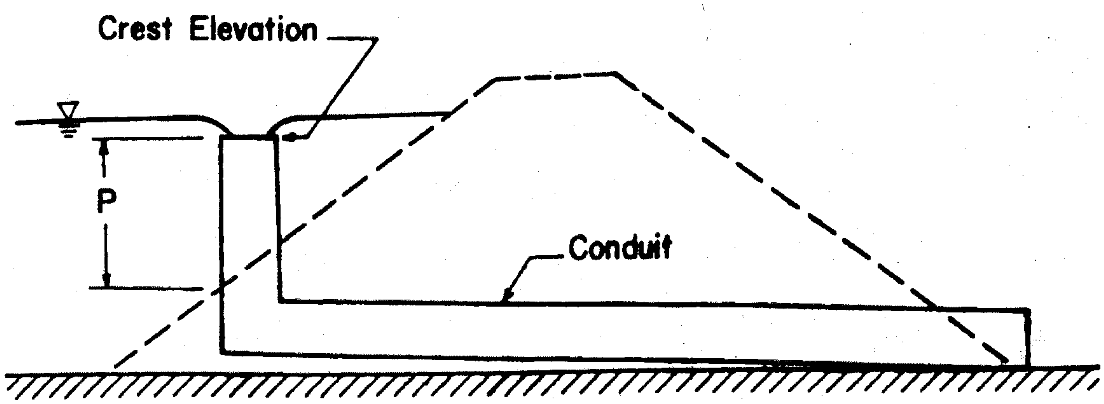


Figure 1. Profile-Shaft Spillway

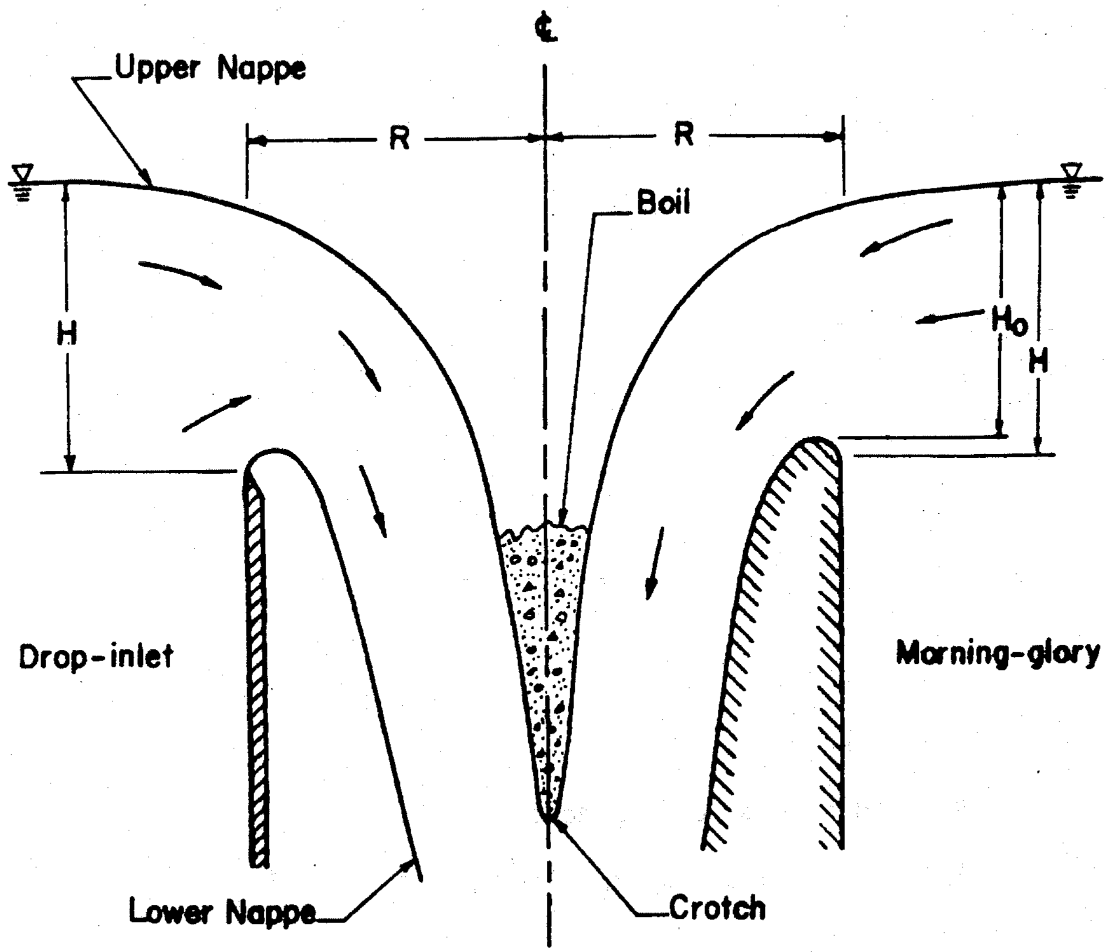


Figure 2. Cross-section Crest Section of a Shaft Spillway

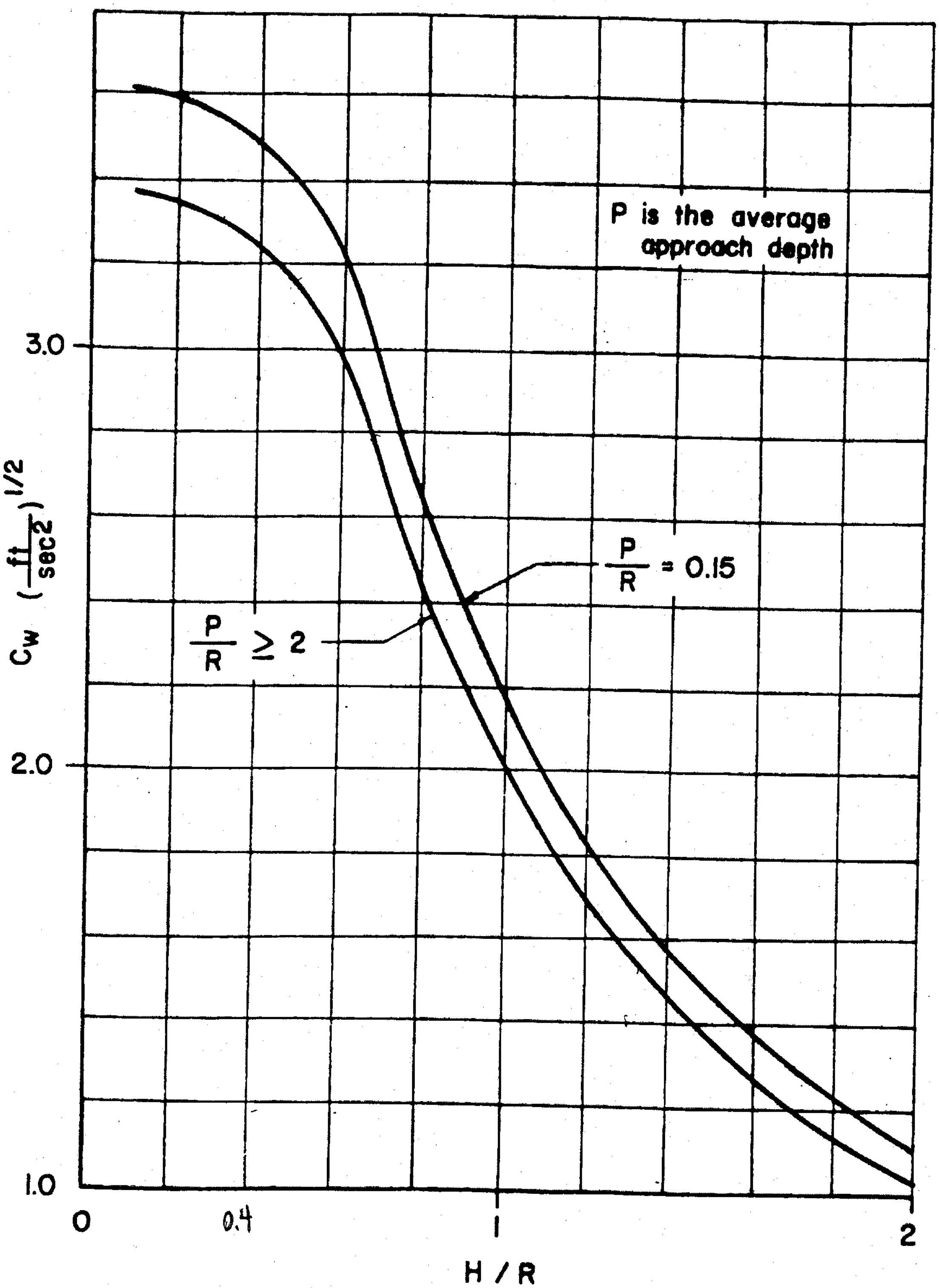


Figure 3. Relationship of Circular Sharp Crested Weir Coefficient C_w and H/R (aerated nappe)

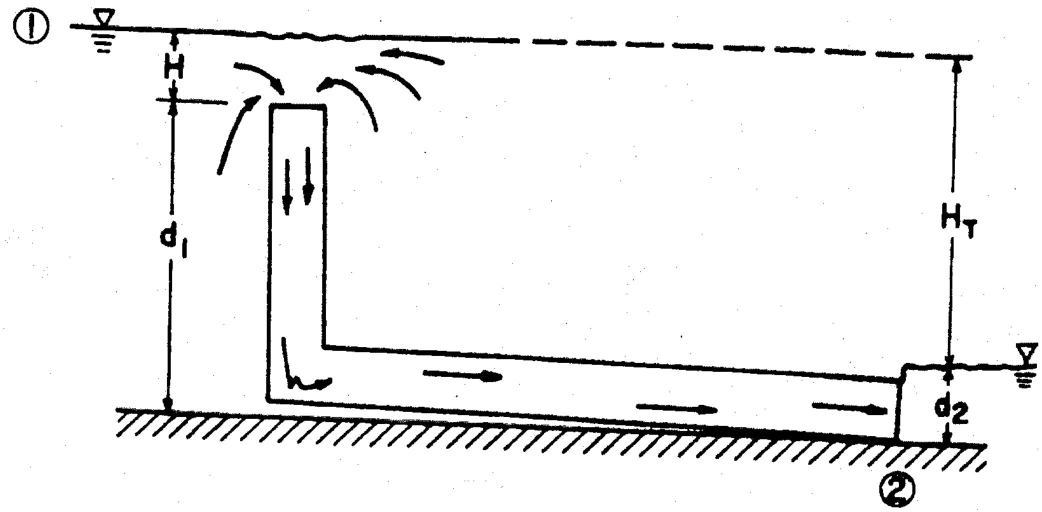


Figure 4. Complete Weir Submergence, Full Pipe Flow

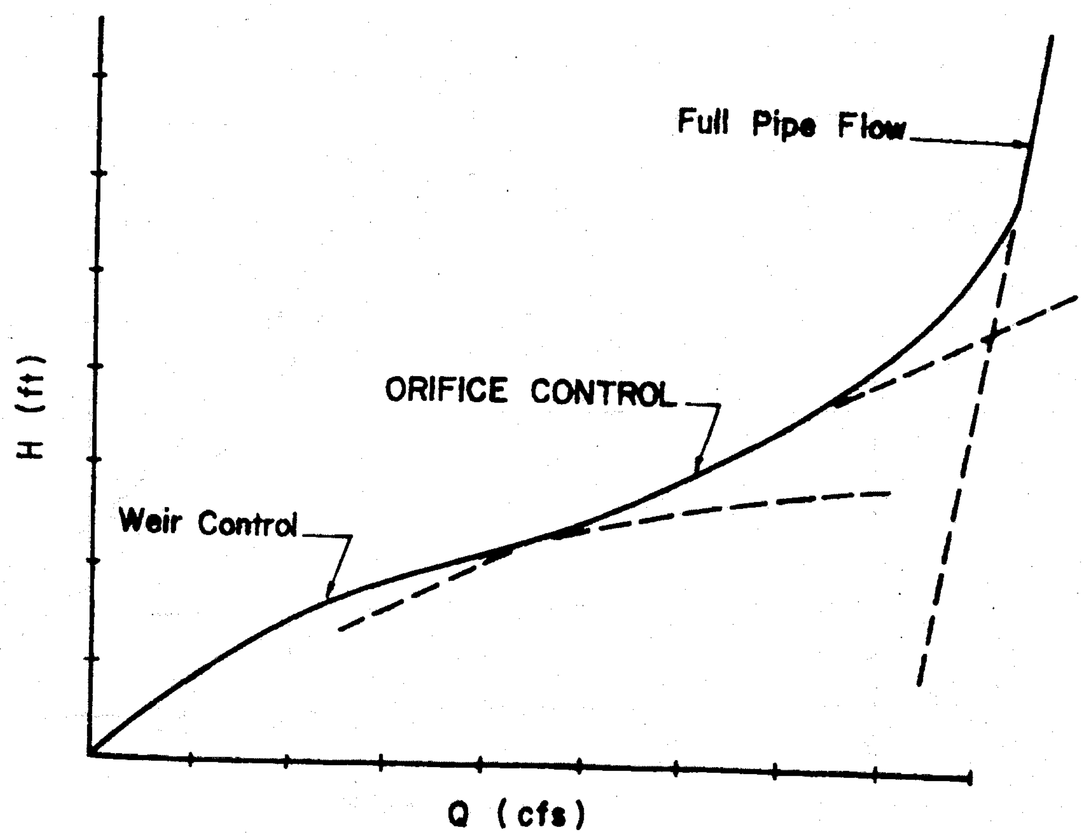


Figure 5. Discharge Characteristics for a Shaft Spillway

Example #1

Determine the elevation-discharge relationship for a drop-inlet spillway constructed from corrugated steel pipe having a constant diameter of 5 ft, given:

$d_1 = 25$ ft $d_2 = 5$ ft $R = 2.5$ ft $L = 150$ ft

$n = 0.022$ $A = 19.6\ \text{ft}^2$

$Q = C_W BH^{3/2}$, $B = 2\pi R = 15.71$ ft, using equation 1 and Figure 3

Table 1 Elevation-Discharge Data

H (ft)	H/R	$C_W\ (\frac{\text{ft}}{\text{sec}^2})^{1/2}$	Q (cfs)	Remarks
0.5	0.2	3.33	18.5	weir flow
1.0	0.4	3.24	50.9	weir flow
1.5	0.6	2.95	85.1	transition
2.0	0.8	2.45	109	orifice flow
2.5	1.0	2.01	125	orifice flow
3.0	1.2	1.70	139	orifice flow
3.5	1.4	1.47	151	orifice flow
4.0	1.6	1.28	161	orifice flow
4.5	1.8	1.14	170	orifice flow
5.0	2.0	1.02	179	orifice flow

For full pipe flow with pipe D = 5.0 ft, using equations 5 and 7 with : $K_t = 0.25$, $K_e = 1.0$, $K_b = 1.0$,

$$K_f = \frac{29.1 n^2 L}{r^{4/3}} = 1.57 \qquad H_T = 20 + H = \frac{Q^2}{2g(19.6)^2}[4.82]$$

Table 2 Elevation-Discharge Data for Full Pipe Flow

Q	H_T	H	Remarks
300	17.53	-2.57	not possible
325	20.58	0.58	weir flow still controlling
350	23.86	3.86	orifice control, possible full pipe flow
375	27.40	7.40	probable full pipe flow

It is obvious from Figure 6 that there is a large gap in the elevation-discharge curve. If air is purged from the discharge conduit the control can quickly switch from inlet causing large nearly instantaneous increases in the discharge.

Assume further that the tower and outlet conduit in example #1 are not vented to the atmosphere. The following scenario describes the events which would occur as the water level in the reservoir rose and covered the spillway:

When $\frac{H}{R}$ becomes greater than .5 (H = 1.25 ft) the inlet becomes sealed. The high turbulence at the base of the tower entrains air and removes portions of the trapped air from the tower. The pessure inside the tower becomes negative creating a pressure differential across the inlet which increases the discharge through the inlet. The increased discharge causes the driving head (the depth of water in the tower) to increase and increase the discharge down the outlet conduit. During this process the unsteady nature of the flow causes waves to travel down the conduit. Usually a wave will seal the conduit and the large negative pressure created as it moves down the conduit will cause the system to siphon and flow full. The discharge can be seen to increase abruptly as the flow jumps from inlet to outlet control. Dynamic pressures are created throughout the system. Often times vortexs and/or other

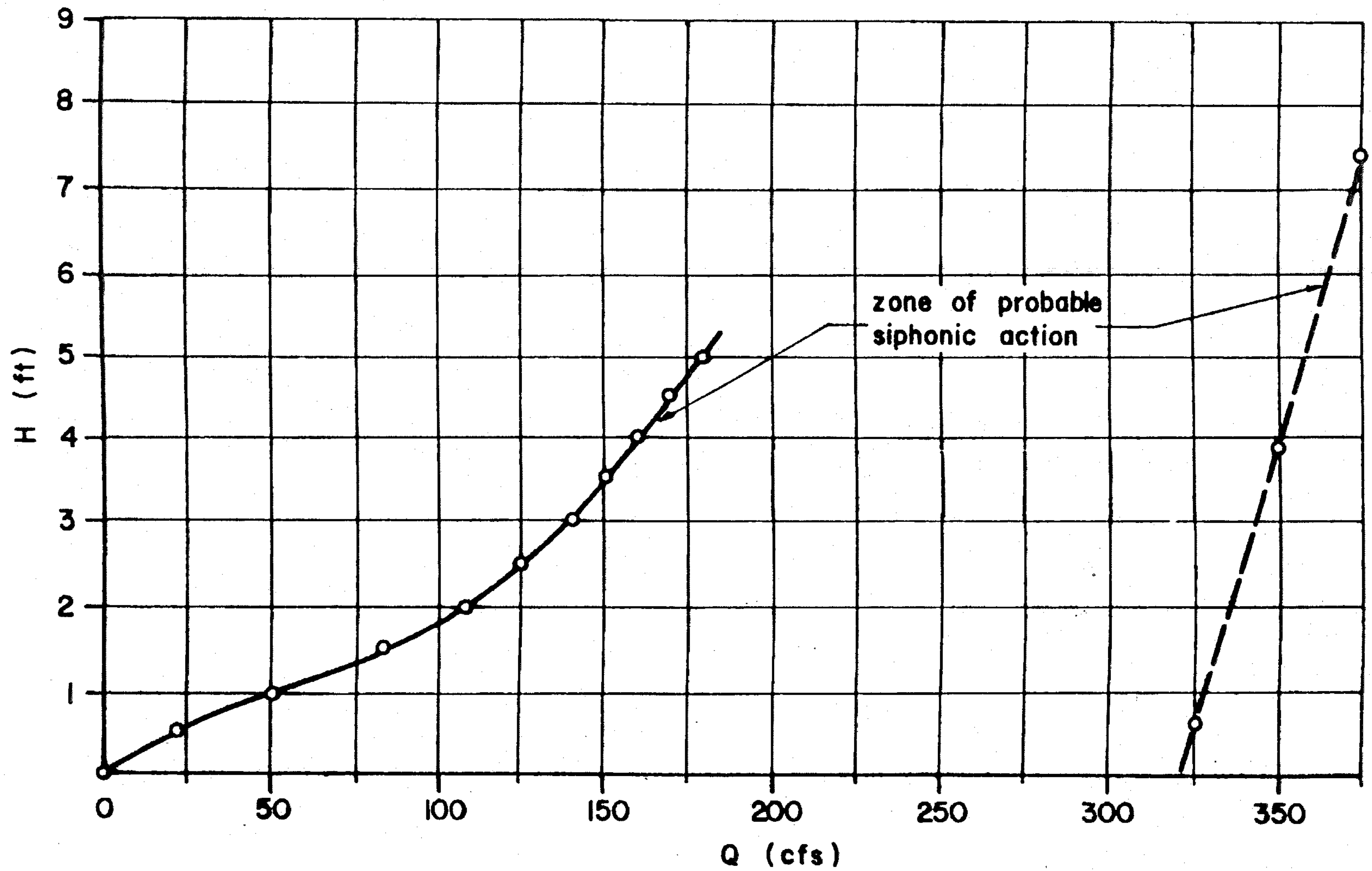


Figure 6. Elevation-discharge Relationship for Example #1

disturbances at the inlet will bring air into the system and break the siphon starting the cycle over.

Sometimes the siphon action only partially occurs. The driving head is drawn down in the tower as the system begins to siphon but the wave which had sealed the conduit allows air back up the conduit. The driving head quickly rises and forces the trapped air in the tower up and through the inlet "belching" out into the atmosphere. The water at the inlet quickly replaces the belched air as it passes out, and falls into the tower as a slug causing dynamic pressure surges.

Obviously this is not an ideal design situation and measures must be taken to avoid these problems and insure the proper operation of the system.

DESIGNING THE INLET

One way to avoid the problems previously outlined is to design the inlet to pass the inflow design flood with $H/R < .5$. However, this is usually not economically desireable.

The most positive way to eliminate the pressure flucuations problems which can occur in the tower is to vent the tower to the atmosphere. The best location for the vents is just below the nappe of the incoming flow. This insures the entering flow is responding as predicted by Figure 3. Likewise any tendencies for pressure differentials to develop inside the tower will be eliminated by the vents.

A list of the design considerations for the inlet would include:

1) the range of expected discharges usually encountered by the spillway
2) the peak discharge resulting from the inflow design flood
3) the tower should be vented to the atmosphere with some venting provided just beneath the incoming nappe

4) tower location in reservoir to allow full radial inflow and unrestricted approach

5) trash-rack with anti-vortex vanes--the trashrack should extend beyond the entrance to allow inflow should the trash-rack become clogged (Figure 7)

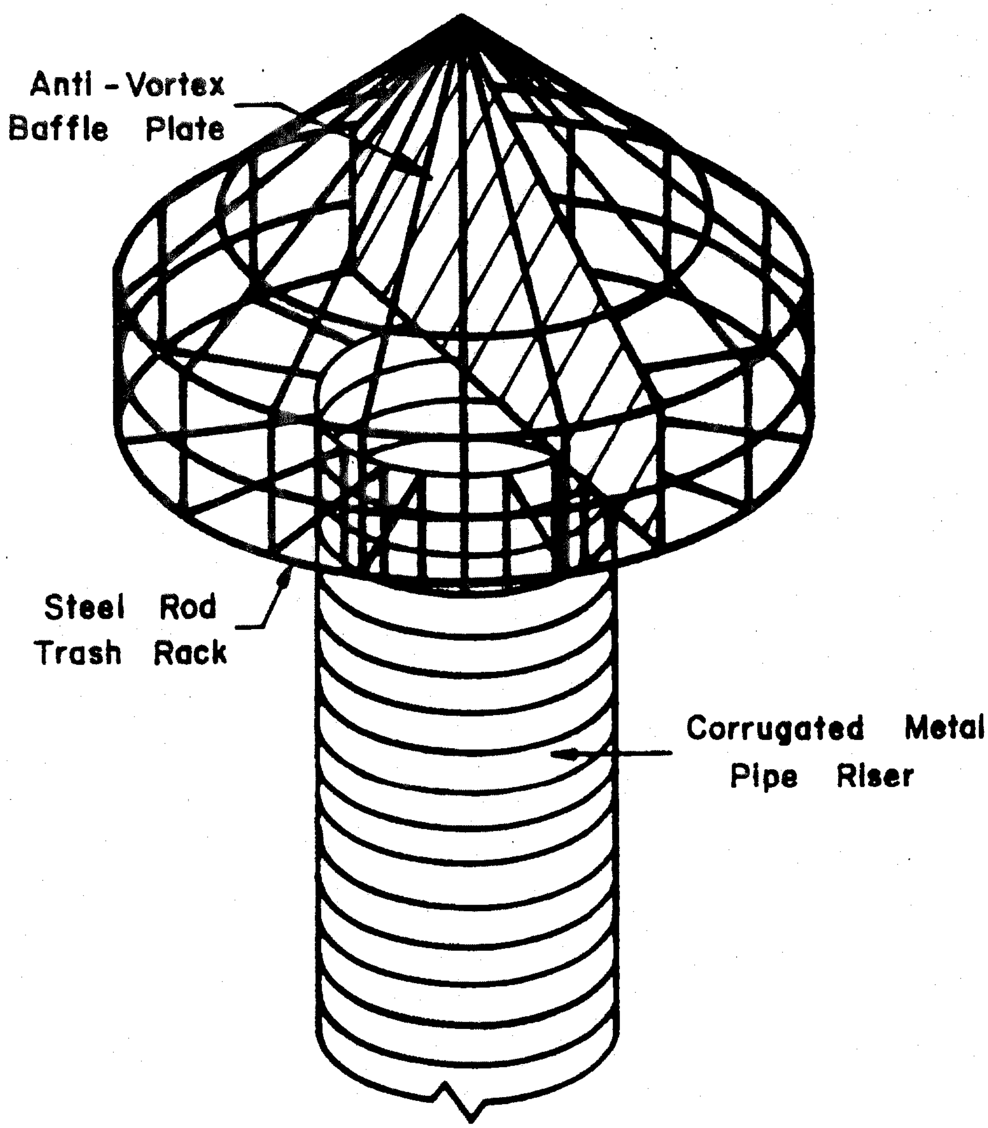


Figure 7 - Trash Rack and Anti-Vortex Baffle

DESIGNING THE OUTLET CONDUIT

The outlet conduit should be designed with a free air path along its entire length to insure that siphon flow does not occur. This can be accomplished by providing a positive control at the entrance to the conduit, by venting the conduit to the atmosphere and by sizing the conduit to flow no more than 75% full (by area) at the outlet.

As seen in Figure 8 the deflector provides the positive control at the entrance to the conduit where the flow is forced to enter at a depth which does not seal off the conduit. A vent immediately behind the deflector provides a means of supplying the air necessary for maintaining a free air path throughout the conduit.

The elevation-discharge relationship for this new situation can be determined as follows:

The area of flow, A_2, at point 2 is

$A_2 = C_c A_o$

where: C_c = contraction coefficient, $\approx$ .70

A_o = cross-sectional area of opening below deflector, ft^2

Writing the Bernoulli equation for the case where the water surface (driving head) is at some intermediate position in the tower:

$$H_w + \frac{Q^2}{2gA_1^2} = y_2 + \frac{Q^2}{2gA_2^2} + h_{L_{1-2}} \qquad (8)$$

where: H_w = height of water (ft), in the tower above the invert at point 2

A_1 = cross-sectional area of flow in tower, ft^2

y_2 = depth of flow at point 2, ft

$h_{L_{1-1}}$ = energy losses between points 1 and 2, ft, which would include the losses due to friction, bend and contraction of flow.

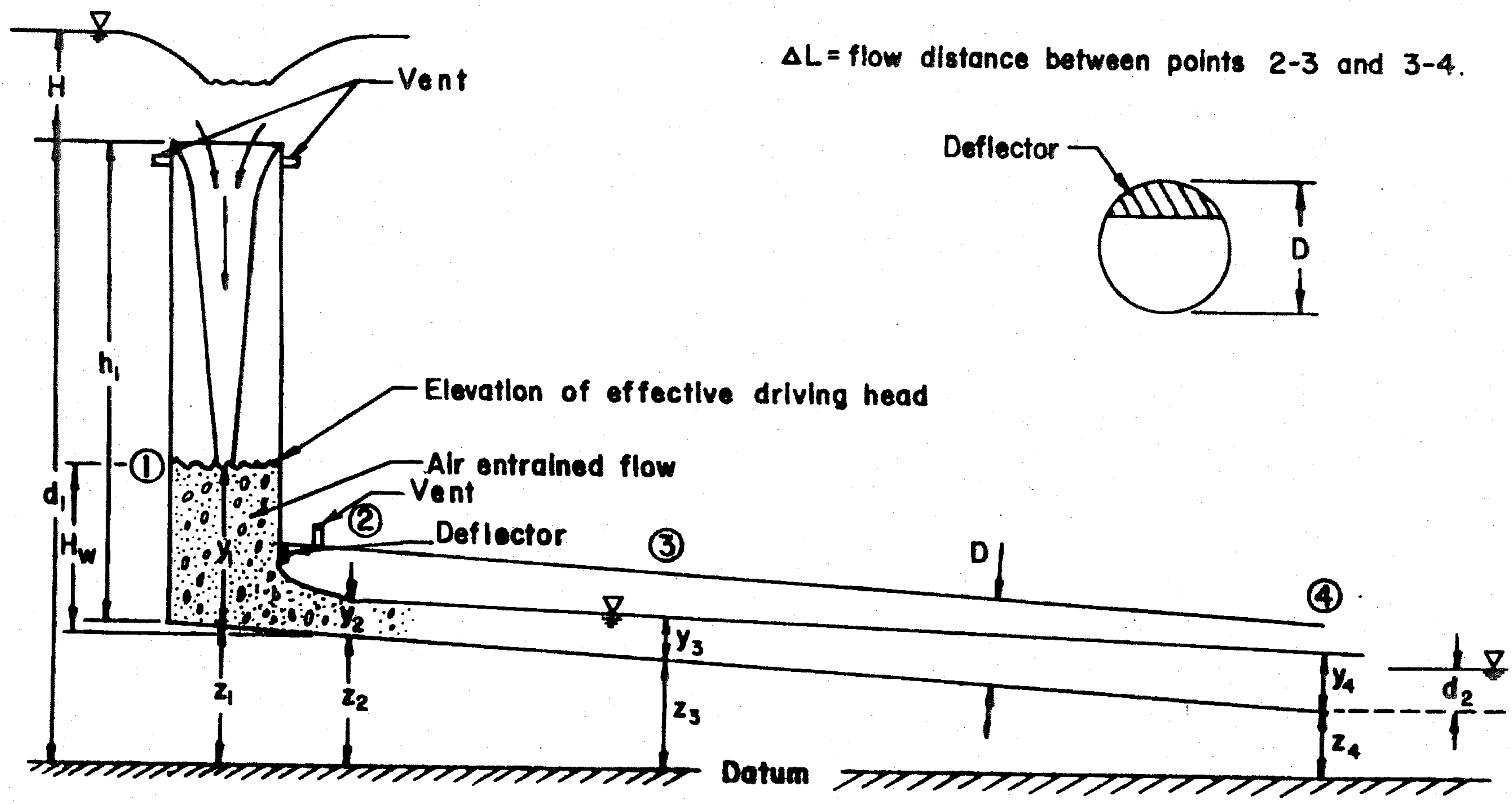


Figure 8 - Discharge Conduit - Water Surface Profile

For practical purposes the magnitude of the energy losses are about equal to the velocity head in the tower and thus equation 8 can be simplified to:

$$H_w = y_2 + \frac{Q^2}{2gA_2{}^2} \qquad (9)$$

Equation 9 is used to select the size of the outlet conduit and deflector and to establish the elevation discharge relationship for sluice gate control.

However, before the conduit size can be determined a decision must be made as to how the system is to operate. One way the spillway system can function is by sizing the deflector large enough so that the control is smoothly transfered from weir to sluice gate control at some point in the range of operation; the second way is to keep entrance control (weir and orifice) throughout the range of operation.

The first method is demonstrated in example #2 and the second method is explained later and demonstrated in example 3.

Example #2

Rework the outlet conduit from Example #1 to provide a smooth transition from weir flow to sluice gate control at H/R = .5.

Given: h_1 = 19 ft, $Z_1 - Z_2$ = .5 ft, at H/R = .5 H = 1.25 ft Q = 74 cfs

therefore at transition: $H_w \approx h_1 + Z_1 - Z_2 + H$

then H_w = 20.75 ft

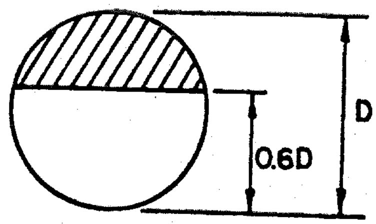
from equation 9

$$H_w = Y_2 + \frac{Q_2^2}{2gA_2{}^2}$$

and

$$20.75 = y_2 + \frac{85.0}{A_2{}^2}$$

y_2 and A_2 will be functions of the pipe and deflector sizes choose opening = .6D



therefore $A_o = .492\ D^2$

$A_2 = .7\ (.492\ D^2) = .344\ D^2$

$y_2 = .45\ D$

substituting $20.75 = .45\ D + \frac{85.0}{(.344D^2)^2}$

and D = 2.46 ft, use D = <u>2.50 ft</u>

thus $H_w = 1.13 + \frac{Q^2}{2g(2.15)^2}$

and $Q = 17.3\ [H_w - 1.13]^{\frac{1}{2}}$

The elevation discharge curve for sluice gate control will be:

H_w (ft)	H (ft)	Q(cfs)
0	--	0
5	--	34
10	--	52
15	--	64
18	--	71
19	--	73
19.5	0	74
20.0	.5	75
20.75	1.25	77
22.5	3.00	80
25.5	6.00	85

As seen in Figure 9 there is now a smooth transition from weir to sluice gate control. And with venting in the upper shaft and outlet conduit this system will operate as desired.

The design is completed by selecting a slope on which to place the conduit to insure that the flow remains supercritical and flows no more than 75% full at the outlet.

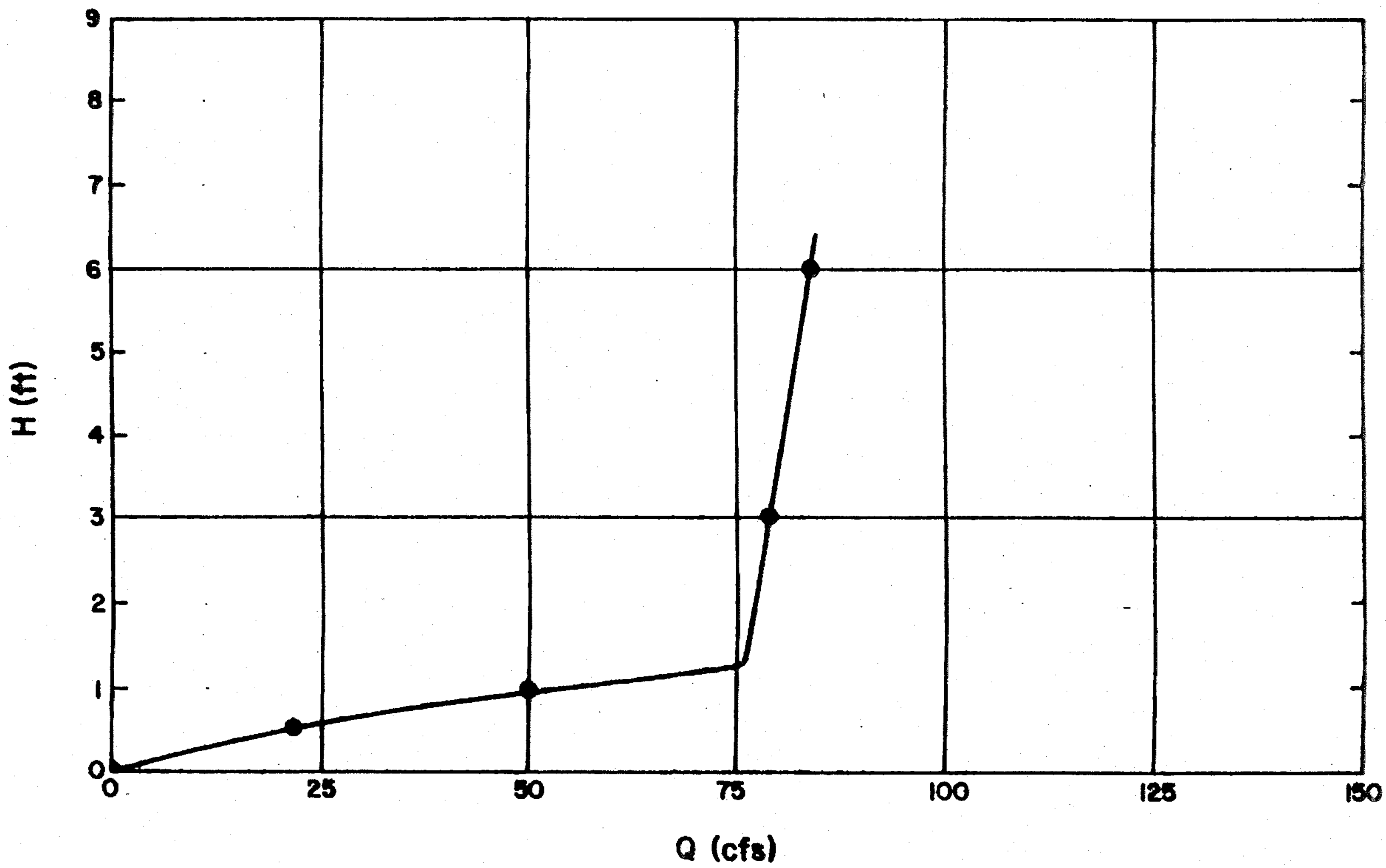


Figure 9. Elevation - discharge Relationship for Example #2

Example #2 continued

Assume max water surface during design flood gives

$H_w = 25$ ft

thus design $Q \approx 85$ cfs

then $A = .75\ A_{full} = 3.68\ ft^2$

$y = .7D = 1.75$ ft

$r = .2962\ D = .741$ ft

$n = .014$

using Mannings equation slope required is $S = .071$ ft/ft

and flow is supercritical, $F = 3.3$

therefore a slope of .071 ft/ft or greater will insure that the pipe flows no more than 75% full at any point.

Oftentimes entrance control is desired so that a larger discharge capacity can be obtained. This means that the opening below the deflector and the outlet conduit must be large enough to pass the flow that the inlet is discharging.

At first glance this doesn't appear to be a problem because this is similar to the situation in example #2 except that now the driving head H_w is not allowed to rise high enough to interfere with the flow at the entrance.

In some cases that is the solution. More frequently, because of economic constraints in selecting the conduit size and/or physical constraints in the available slope for placing the conduit a more detailed analysis must be made of the flow conditions in the outlet conduit. The problem at hand is one of juggling discharge efficiency (capacity) and outlet conduit cost while meeting the criterion of a maximum of 75% full flow in the outlet conduit.

An analysis of the surface water profile inside the conduit can be made by using the hydraulics principles of gradually varied flow.

Referring to Figure 8 and writing the energy equation from points 2 to 3 yields

$$Z_2 + Y_2 + \alpha_2 \frac{V_2^2}{2g} = Z_3 + Y_3 + \alpha_3 \frac{V_3^2}{2g} + h_{L_{2-3}} \tag{10}$$

using Manning's equation to determine the friction loss gives

$$Z_2 + Y_2 + \alpha_2 \frac{V_2^2}{2g} = Z_3 + Y_3 + \alpha_3 \frac{V_3^2}{2g} + \frac{n^2 \Delta L}{4.416} \left[\frac{V_2^2}{r_2^{4/3}} + \frac{V_3^2}{r_3^{4/3}} \right] \tag{11}$$

the depth at point 4 may be computed by equation 11 by substituting points 3 and 4 for points 2 to 3 respectively.

This procedure can best be demonstrated with an example problem.

Example #3.

A drop-inlet spillway has been selected as the primary spillway for a proposed reservoir. It has also been decided that the spillway shall operate with entrance control. After routing the 100 yr storm and the P.M.F. it was determined that an inlet with radius of 5 ft would be adequate.

What is the smallest outlet conduit which can be used?

The following information is available:

spillway inlet invert 1332 msl

100 yr flood pool elevation 1334.6 msl

outlet conduit exit invert 1280 msl

outlet conduit slope restricted to .020 ft/ft

desire to use sectioned elbow as part of tower configuration

general layout as shown in Figure 10

First develop the Elevation-Discharge Curve from equation 1 and figure 3 with $P/R > 2$

$Q = C_w BH^{3/2}$ $\qquad$ $B = 2\pi R = 31.42$ ft

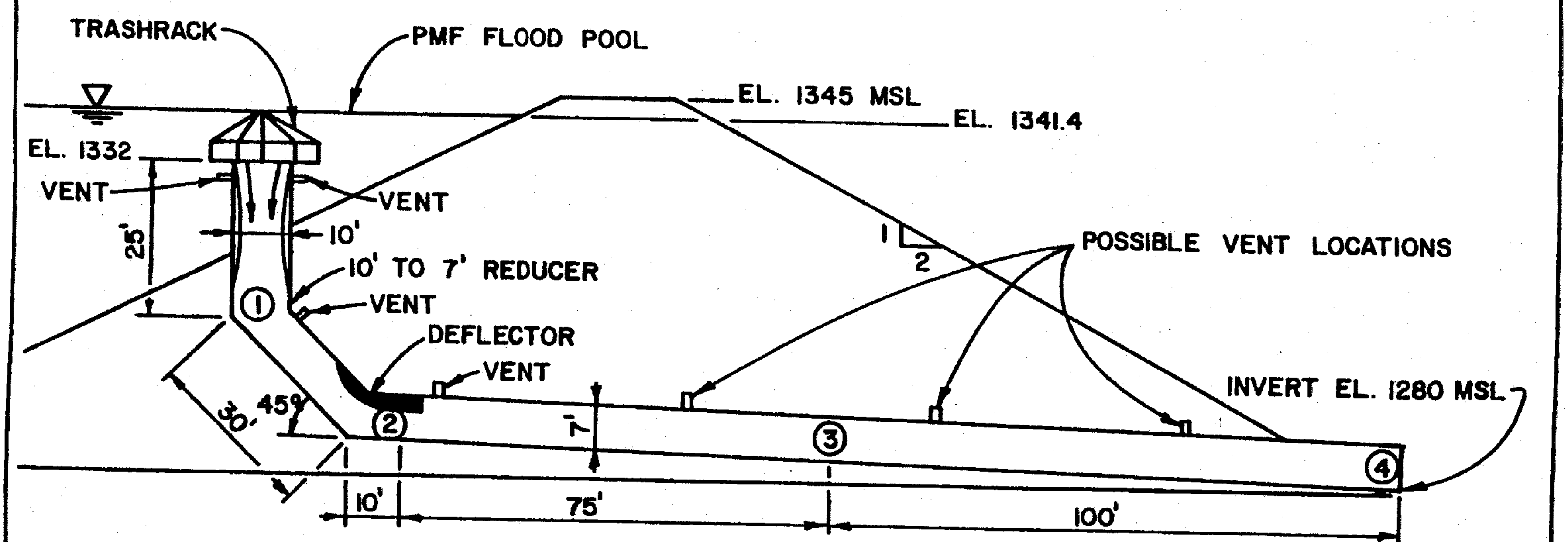


Figure 10 - Design Information for Example #3

Elevation-Discharge Computations

H (ft)	H/R	$C_w (\frac{ft}{sec^2})$	Q(cfs)
0			0
1	0.20	3.33	105
2	0.40	3.24	288
3	0.60	2.95	482
4	0.80	2.46	618
5	1.00	2.01	706
6	1.20	1.70	785
7	1.40	1.46	850
8	1.60	1.26	895
9	1.80	1.13	959
10	2.00	1.02	1014

From the elevation discharge computations the maximum discharge is 980 cfs which corresponds to 9.4 ft of head.

Now that the design discharge is known the problem is to select the smallest conduit which can pass the flow given the previous constraints.

Estimate size required using Manning's equation:

$n = .014$

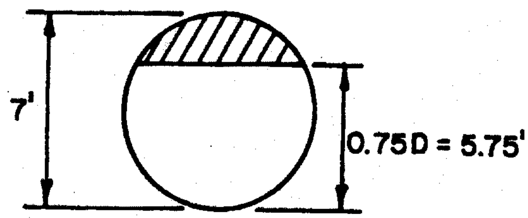
$$\left.\begin{aligned} y &= .7\ D \\ A &= .5872\ D^2 \\ r &= .2462\ D \end{aligned}\right\}\ 75\%\ \text{full}$$

$$980 = \frac{1.486}{.014}\ (.5872\ D^2)\ (.2962\ D)^{2/3}\ (.02)^{1/2}$$

$$D = 7.93\ \text{ft}$$

try D = 7 ft, since uniform flow probably won't establish itself within the 175 ft length of pipe.

choose deflector size of



so $A_2 = C_c A_o = .7(.6320\ D^2) = 21.7\ ft^2$

$Y_2 = .550\ D = 3.85\ ft$

from equation 9.

$$H_w = 3.85 + \frac{980^2}{2g(21.7)^2}$$

$H_w = 35.5\ ft$

$H_w \rightarrow$ 1319 ft msl which is 13 ft below spillway inlet when passing the design discharge, should provide adequate margin of safety to insure inlet maintains control.

Next check that the flow in the conduit doesn't exceed 75% full.

Establish depths of flow throughout conduit using equation 11. Assume $\alpha \approx 1.0$, $n = .014$

Item	Z (ft)	Y (ft)	A (ft^2)	V (ft/sec)	$\alpha V^2/2g$ (ft)	r (ft)	$\frac{V^2 n^2 \Delta L}{4.416 r^{4/3}}$	Σh_L	Comment
1st reach from pt 2 → 3 ΔL = 75 ft									
Pt. 2	3.5	3.85	21.7	45.2	31.7	1.85	2.99	--	--
Pt. 3 trial 1	2.0	4.13	23.6	41.5	26.7	1.93	2.39	5.38	39.05 ≠ 38.21
trial 2	2.0	<u>4.08</u>	23.3	42.1	27.5	1.91	2.49	5.48	39.05 = 39.06 OK
2nd reach from pt 3 → 4 ΔL = 100 ft									
Pt. 3	2.0	4.08	23.3	42.1	27.5	1.91	3.32	--	--
Pt. 4 trial 1	0	4.15	23.8	41.2	26.4	1.93	3.14	6.46	33.58 ≠ 37.01
trial 2	0	4.34	25.1	39.0	23.6	1.97	2.73	6.05	33.58 ≠ 33.99
trial 3	0	<u>4.38</u>	25.3	38.7	23.3	1.98	2.67	5.99	33.58 = 33.67 OK

therefore

$y_2 = 3.85$ ft

$y_3 = 4.08$ ft

$y_4 = 4.38$ ft

and 75% full → 4.90 ft OK

at exit $F = \dfrac{V_4}{\sqrt{g\,D_4}}$

$V_4 = 38.7$ ft/sec

$D_4 = \dfrac{A_4}{T_4} = \dfrac{25.3}{6.78} = 3.73$ ft

$F = 3.53$ supercritical OK

design is OK

SUMMARY OF CONSIDERATIONS FOR OUTLET CONDUIT DESIGN

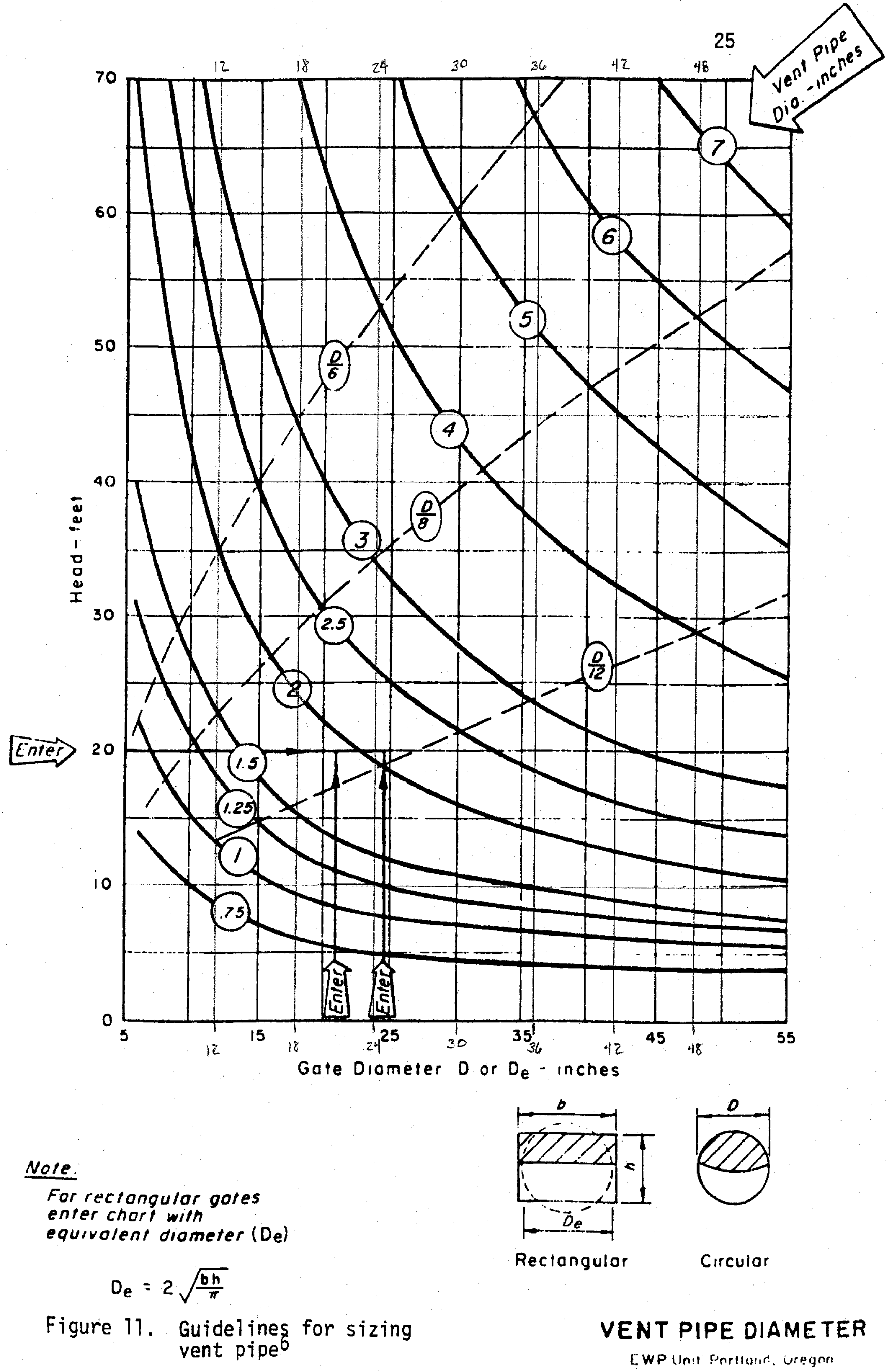
1) Tailwater conditions (stilling basin) must not submerge the exit. This avoid the possibility of siphon flow.

2) Provide a deflector or other control at the inlet to the outlet conduit to initiate the flow at a depth which doesn't seal off the conduit.

3) Place an atmospheric vent (Figure 11) immediately behind the deflector and if appropriate at other locations along the conduit. This provides the means for a free air path in the conduit and avoids the possibility of siphon flow.

4) Develop the elevation discharge curves for both the tower inlet and the sluice gate to check the positions of the driving heads (H and H_w) and determine if the system is operating as desired.

5) Check that the conduit does not flow more than 75% full, $y \leq .7\ D$. This allows room for wave action and for an increased volume of flow due to air entrainment.



Note.

For rectangular gates enter chart with equivalent diameter (D_e)

$$D_e = 2\sqrt{\frac{bh}{\pi}}$$

Figure 11. Guidelines for sizing vent pipe[6]

NOTATIONS

The following terms are used in this paper

A = area, ft^2

B = perimeter length of weir, ft

C_c = contraction coefficient

C_w = weir coefficient, $(ft/sec^2)^{1/2}$

d = water depth, ft

D = pipe diamter, ft

D = hydraulic depth, (Area/Top width), ft

F = froude number

g = acceleration of gravity, ft/sec^2

h = vertical length of the shaft, ft

h_L = head loss, ft

H = head, depth of water over the weir, ft

H_W = effective driving head, height of water in tower, ft

H_T = elevation difference between head water & tailwater, ft

K = minor loss coefficient

L = length, ft

ΔL = change in length, ft

n = Mannings's n, roughness coefficient

P = average approach depth, ft

Q = discharge, ft^3/sec

r = hydraulic radius (area/wetted perimeter),ft

R = radius, ft

S = conduit slope, ft/ft

T = width of the free water surface, ft

V = velocity, ft/sec

y = depth of flow, ft

Z = height above datum, ft

α = velocity distribution coefficient

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