

by making a single horizontal cut (such as pp_1) through the beam. Also, since the force F_3 is the total horizontal shear force acting between the subelement and the rest of the beam, it may be distributed anywhere over the sides of the subelement, not just on its lower surface. These same comments apply to the shear flow f , since it is merely the force F_3 per unit distance.

Let us now return to the shear-flow formula $f = VQ/I$ (Eq. 5-52). The terms V and I have their usual meanings and are not affected by the choice of subelement. However, the first moment Q is a property of the cross-sectional face of the subelement. To illustrate how Q is determined, we will consider three specific examples of built-up beams (Fig. 5-43).

Areas Used When Calculating the First Moment Q

The first example of a built-up beam is a welded steel plate girder (Fig. 5-43a). The welds must transmit the horizontal shear forces that act between the flanges and the web. At the upper flange, the horizontal shear force (per unit distance along the axis of the beam) is the shear flow along the contact surface aa . This shear flow may be calculated by taking Q as the first moment of the cross-sectional area above the contact surface aa . In other words, Q is the first moment of the flange area (shown shaded in Fig. 5-43a), calculated with respect to the neutral axis. After calculating the shear flow, we can readily determine the amount of welding needed to resist the shear force, because the strength of a weld is usually specified in terms of force per unit distance along the weld.

The second example is a wide-flange beam that is strengthened by riveting a channel section to each channel and the main beam must transmit the shear force acting between each channel and the main beam by the rivets. This force is calculated from the shear-flow formula using Q as the first moment of the area of the entire channel (shown shaded in the figure). The resulting shear flow is the longitudinal force per unit distance acting along the contact surface bb , and the rivets must be of adequate size and longitudinal spacing to resist the force.

The last example is a wood box beam with two flanges and two webs that are connected by nails or screws (Fig. 5-43c). The total horizontal shear force between the upper flange and the webs is the shear flow acting along both contact surfaces cc and dd , and therefore the first moment Q is calculated for the upper flange (the shaded area). In other words, the shear flow calculated from the formula $f = VQ/I$ is the shear flow along all contact surfaces that surround the area for which Q is computed. In this case, the shear flow f is resisted by the action of the nails on both sides of the beam, that is, at both cc and dd , as illustrated in the following example.

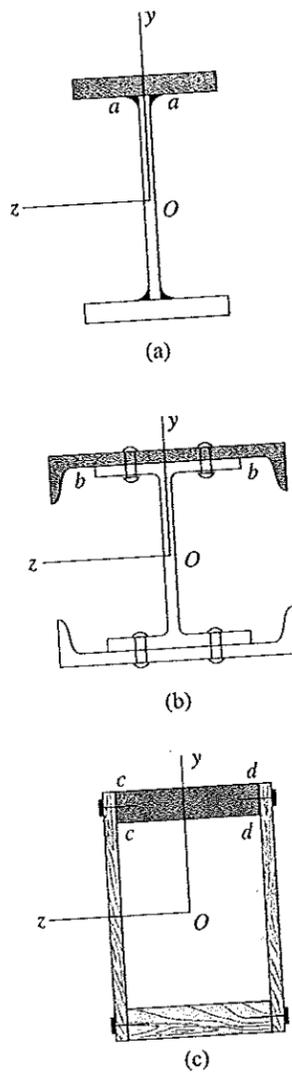
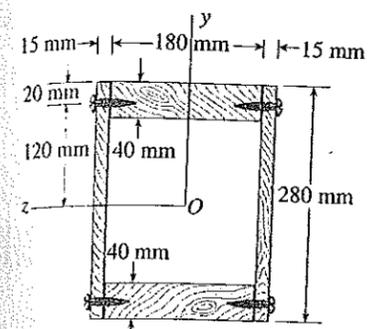
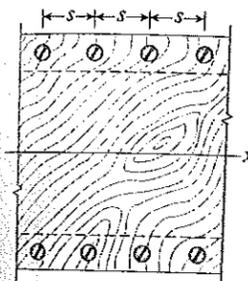


FIG. 5-43 Areas used when calculating the first moment Q

Example 5-16



(a) Cross section



(b) Side view

Fig. 5-44 Example 5-16. Wood box beam

A wood box beam (Fig. 5-44) is constructed of two boards, each 40×180 mm in cross section, that serve as flanges and two plywood webs, each 15 mm thick. The total height of the beam is 280 mm. The plywood is fastened to the flanges by wood screws having an allowable load in shear of $F = 800$ N each.

If the shear force V acting on the cross section is 10.5 kN, determine the maximum permissible longitudinal spacing s of the screws (Fig. 5-44b).

Solution

Shear flow. The horizontal shear force transmitted between the upper flange and the two webs can be found from the shear-flow formula $f = VQ/I$, in which Q is the first moment of the cross-sectional area of the flange. To find this first moment, we multiply the area A_f of the flange by the distance d_f from its centroid to the neutral axis:

$$A_f = 40 \text{ mm} \times 180 \text{ mm} = 7200 \text{ mm}^2 \quad d_f = 120 \text{ mm}$$

$$Q = A_f d_f = (7200 \text{ mm}^2)(120 \text{ mm}) = 864 \times 10^3 \text{ mm}^3$$

The moment of inertia of the entire cross-sectional area about the neutral axis is equal to the moment of inertia of the outer rectangle minus the moment of inertia of the "hole" (the inner rectangle):

$$I = \frac{1}{12}(210 \text{ mm})(280 \text{ mm})^3 - \frac{1}{12}(180 \text{ mm})(200 \text{ mm})^3 = 264.2 \times 10^6 \text{ mm}^4$$

Substituting V , Q , and I into the shear-flow formula (Eq. 5-52), we obtain

$$f = \frac{VQ}{I} = \frac{(10,500 \text{ N})(864 \times 10^3 \text{ mm}^3)}{264.2 \times 10^6 \text{ mm}^4} = 34.3 \text{ N/mm}$$

which is the horizontal shear force per millimeter of length that must be transmitted between the flange and the two webs.

Spacing of screws. Since the longitudinal spacing of the screws is s , and since there are two lines of screws (one on each side of the flange), it follows that the load capacity of the screws is $2F$ per distance s along the beam. Therefore, the capacity of the screws per unit distance along the beam is $2F/s$. Equating $2F/s$ to the shear flow f and solving for the spacing s , we get

$$s = \frac{2F}{f} = \frac{2(800 \text{ N})}{34.3 \text{ N/mm}} = 46.6 \text{ mm}$$

This value of s is the maximum permissible spacing of the screws, based upon the allowable load per screw. Any spacing greater than 46.6 mm would overload the screws. For convenience in fabrication, and to be on the safe side, a spacing such as $s = 45$ mm would be selected.