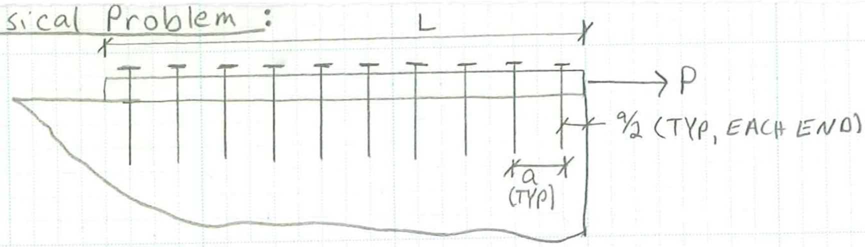
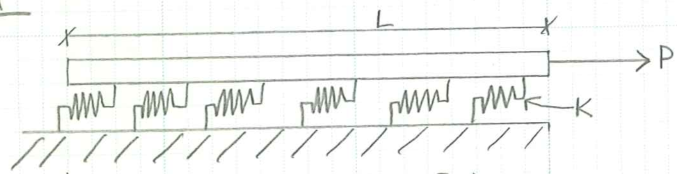
Physical Problem :



Ⓐ ANALYTICAL Model



$\longmapsto x$ $\quad r \rightarrow \quad r(x) = K \cdot u(x)$

$+u \rightarrow$

Ⓑ ANALYTICAL Formulation

Governing diff. eq. : $EA\,u''(x) - K\,u(x) = 0$

- Boundary conditions :

  1.) free end w/ P (tension)

  $N(L) = P$

  $u'(L) = \dfrac{P}{EA}$

  2.) @ far end (@ $x=0$)

  $N(0) = 0$

  $u'(0) = 0$

(2) continued

Using B.C.1 :

$$\frac{\cancel{EA}u''(x)}{\cancel{EA}} - \frac{k[u(x)]}{EA} = \frac{0}{EA}$$

$$\boxed{u''(x) - K^2\{u(x)\} = 0} \quad (1.)$$

assume a solution of the form

$$u(x) = Ae^{nx}$$
$$u'(x) = Ane^{nx}$$
$$u''(x) = An^2e^{nx}$$
$$An^2e^{nx} - K^2\{Ae^{nx}\} = 0$$
$$Ae^{nx}\{n^2 - K^2\} = 0$$

$$n^2 - K^2 = 0 \quad \therefore \quad n = \pm K$$
$$u(x) = A_1e^{-Kx} + A_2e^{Kx}$$

Using B.C.2:

$$u'(0) = A_1(-K)\cancel{e^{-Kx}}^{1} + A_2K\cancel{e^{Kx}}^{1} = 0$$
$$A_1(-K) + A_2K = 0$$
$$\therefore \quad A_1 = A_2$$

Using B.C.1 :

$$u'(L) = \frac{P}{EA}$$
$$u'(x) = A_1(-K)e^{-Kx} + A_2Ke^{Kx}$$
$$u'(L) = A_1(-K)e^{-KL} + A_2Ke^{KL}$$
$$A_1(-K)e^{-KL} + A_2Ke^{KL} = \frac{P}{EA}$$

Say $A_1 = A_2$

$$A_1K(-e^{-KL} + e^{KL}) = \frac{P}{EA}$$
$$A_1 = \frac{P}{KEA[-e^{-KL} + e^{KL}]}$$

$$u(x) = \frac{P}{KEA[-e^{-KL} + e^{KL}]}e^{-Kx} + \frac{P}{KEA[-e^{-KL} + e^{KL}]}e^{Kx}$$

$$u'(x) = A_1 K e^{Kx} - A_2 K e^{-Kx}$$

using B.C. 2

$$u'(0) = A_1 K \cancel{e^{K0}}^{1} - A_2 K \cancel{e^{-K0}}^{1} = 0$$

$$\underbrace{(A_1 - A_2)}\, K = 0$$

$$A_1 = A_2$$

using B.C. 1

$$u'(L) = A_1 K e^{KL} - A_1 K e^{-KL} = \frac{P}{EA}$$

$$A_1 (K e^{KL} - K e^{-KL}) = \frac{P}{EA}$$

$$A_1 = \frac{P}{KEA(e^{KL} - e^{-KL})}$$

$$u(x) = \frac{P}{KEA(e^{KL} - e^{-KL})}\left[e^{-Kx}\right] + \frac{P}{KEA(e^{KL} - e^{-KL})}\left[e^{Kx}\right]$$

$$u(x) = \left[\frac{P}{KEA(e^{KL} - e^{-KL})}\right]\left[e^{-Kx} + e^{Kx}\right]$$

$$N(x) = u'(x)\, EA$$

$$N(x) = \left[\frac{P \cancel{K} e^{Kx}}{\cancel{KEA}(e^{KL} - e^{-KL})} + \frac{P(-K)e^{-Kx}}{\cancel{KEA}(e^{KL} - e^{-KL})}\right] \cancel{EA}$$

$$N(x) = \frac{Pe^{Kx} - Pe^{-Kx}}{(e^{KL} - e^{-KL})}$$

$$N(0) = 0 \quad \text{, ok} \checkmark$$

$$N(L) = \frac{P\cancel{(e^{KL} - e^{-KL})}}{\cancel{(e^{KL} - e^{-KL})}} = P \quad \text{, ok} \checkmark$$

Resistance:

$$r(x) = K\,[u(x)]$$

$$= K\left\{\frac{P(e^{-Kx} + e^{Kx})}{KEA(e^{KL} - e^{-KL})}\right\}$$

$$= K^2 EA \left\{\frac{2P(e^{-Kx} + e^{Kx})}{\cancel{KEA}(e^{KL} - e^{-KL})}\right\}$$

$$r(x) = \frac{PK(e^{-Kx} + e^{Kx})}{(e^{KL} - e^{-KL})}$$

(C) I did not finish the math of this problem in time to get a plot out USING MATLAB (SEE ATTACHED EXCEL PRINTOUT)

(D) for $KL = 2$

Say $S_{10} = 0.1L\left[\frac{r(0.9L)+r(L)}{2}\right] = \left[\frac{\frac{PK(e^{-0.9(2)}+e^{0.9(2)})}{(e^{2}-e^{-2})} + \frac{PK(e^{-2}+e^{2})}{(e^{2}-e^{-2})}}{2}\right] = 0.1L\,(PK)\left(\frac{1.894109}{2}\right) \Rightarrow 0.1\left(\frac{\cancel{2}}{\cancel{2}}\right)P(1.894109)$

$$\boxed{S_{10} = 0.189411\ P}$$

$S_9 = 0.1L\,\frac{[r(0.8L)+r(0.9L)]}{2} = 0.156745P$ $\boxed{S_9 = 0.156745P}$

$S_8 = 0.1L\,\frac{[r(0.7L)+r(0.8L)]}{2} = 0.130731P$ $\boxed{S_8 = 0.13073\ P}$

$S_7 = 0.1L\,\frac{[r(0.6L)+r(0.7L)]}{2} = 0.109228P$ $\boxed{S_7 = 0.109228P}$

(E) As P is continually increased, the shearing forces in each bolt will continue to increase (with bolt 10 seeing the most load) until bolt 10 begins to yield at which time the remaining bolts will "catch up" to bolt 10, until bolt 9 yields. This process will continue until all bolts have yielded.