Shell 4. Concrete Shells' Buckling

reference is *Láminas de Hormigón*, Haas, p. 383...

0. Important Notes

- It is typical from shells of sound design if something to fail by local buckling failure under a concentrated load and maybe time dependent effects. The buckling studies here won't prevent local buckling coming from such concentrated local loading, causing excess of flexure where a membranal status is wanted per design.
- So inaccessibility of shells susceptible to such buckling damage must be established by design and warranted by owner's agents inspection during useful life. Maintenance will take care of not disturbing the membranal status to such degree as to cause local or general buckling failure.
- All holes made in the membrane by design will be properly rounded by rings which effects on the shell will have been taken into account (long term) and as to prevent buckling failure originating in them.
- Our recommendation is: make a FEM model and subject it to the load case more likely to produce buckling. Include in it concomitant (augmenting the natural deflections of the load case) likely or worst case imperfections. Check you don't have a inestable model by running the model with second order effects taken into account.

$\gamma_c := 2400 \cdot \frac{kgf}{m^3}$ concrete specific weight

$f_c := 35 \cdot MPa$ specified strength

$\nu := 0.20$ coeff. of Poisson of concrete

$$E_c := 33 \cdot \left(\frac{\gamma_c}{\frac{lbf}{ft^3}} \right)^{1.5} \cdot \left(\frac{f_c}{psi} \right)^{0.5} \cdot psi$$

$$E_{cLT} := \frac{E_c}{5}$$

Long Term modulus of deformation

- You will see we will be using one long term degraded modulus of elasticity to take into account time dependent rheological effects.
- This may be necessary for cases where the stresses are those common in beams and colums (shells like folded plates, flexural cylindrical beams, compressed circular piles, etc), but might not be needed for all cases where the stresses are so low (many domes and shells) that the additional time-dependent effects may not be significant.
- Note however that lantern inclusions and other irregularities may cause the referred high stresses and ignite the buckling failure, long term.
- Whenever you have disruptions to the basic geometry of the shell, I would recommend to use the long term degraded modulus of deformation.

1. Folded plates

Assuming long term loading and simply supported conditions at all sides (which is conservative) and considering some (short) length at which stresses can be assumed to remain constant...

$b := 3 \cdot m$ width of an element component of the folded plate section, assumed uniformly loaded

$t := 12 \cdot cm$ its thickness

$$\frac{b}{t} = 25$$

$$\sigma_{cr} := \frac{\pi^2 \cdot E_{cLT} \cdot t^2}{3 \cdot \left(1 - \nu^2\right) \cdot b^2}$$

$\sigma_{cr} = 32.6\,MPa$ Remain under this value at the factored loads level

- Most folded plates won't exceed these dimensions nor materials will be worse or real coercions more exacting than considered, so it is unlikely that folded plates have serious buckling problems. We are in the 1/25 slenderness range.
- Note however that box piles are some kind of folded plates and for them no more slenderness than b/t=15 is currently being recommended.
- We could roughly say that current recommendations for both steel and concrete are b/t=30 for static loads and b/t=15 or lower for dynamic.

2. Cylindrical shells / Haas

From the practical approach in p. 391...

$r := 15 \cdot m$ radius of the cylindrical shell

$t := 12 \cdot cm$ thickness of the shell

$b := 3 \cdot m$ arc length (measured on it) (within transversal section, uniformly loaded)

$$K := \frac{b^2}{r \cdot t} \qquad K = 5 \qquad \frac{4 \cdot \pi^2}{\sqrt{12 \cdot \left(1 - \nu^2\right)}} = 11.63$$

$$K \geq \frac{4 \cdot \pi^2}{\sqrt{12 \cdot \left(1 - \nu^2\right)}} = 0$$

must be 1 for OK, and then...

$$\sigma_{cr\_Haas} := \frac{E_{cLT} \cdot t^2}{b^2} \cdot \frac{K}{\sqrt{3\left(1 - \nu^2\right)}}$$

$$\sigma_{cr\_Haas} = 28.03\,\text{MPa}$$

having curvature enhances very much the buckling strength.

We can make better and automatize the search for the lesser buckling strength of a uniformly compressed pipe by the above formulations. Will accept t, r dimensions can remain constant

$$b := 3 \cdot m$$

$$\sigma_{cr}(b) := \frac{E_{cLT} \cdot t^2}{b^2} \cdot \frac{\frac{b^2}{r \cdot t}}{\sqrt{3\left(1 - \nu^2\right)}}$$

Given

$b \geq 0 \cdot m$ $\qquad$ $b \leq \pi \cdot r$

$b := \text{Minimize}\left(\sigma_{cr}, b\right)$ $\qquad$ $b = 300\,\text{cm}$

$\sigma_{cr}(b) = 28.03\,\text{MPa}$ $\qquad$ not surprisingly equal to the already obtained result since the formulation for critical stress is independent of b, as can be seen once K is expressed explicitly

So change the blue input to guess acceptable proportions of compressed pipes or cylindrical shells from the buckling standpoint

See below other evaluations

3. Spherical domes

$t := 12 \cdot cm$ thickness

$r := 15 \cdot m$ radius

$\alpha := 0.15$ buckling parameter, about 0.58/4 of p. 395 or select from p. 399. It seems 0.30 could also be accepted.

0.06 minimum, 0.16 typical, and never more than 0.32

$\sigma_{cr} := \frac{\alpha}{2} \cdot E_{cLT} \cdot \frac{t}{r}$ $\sigma_{cr} = 3.57\,MPa$ Remain under this value at the factored loads level

Note this formulation of the buckling risk seems not consistent with that of cylindrical shells, which portrait as unsecure when compared to spherical domes, even the differences of loading (radial pressure on the shell for the dome and longitudinal compression for the cylinder) are accounted for.

The above formulation in terms of supported loas is

$q_{cr} := \alpha \cdot E_{cLT} \cdot \frac{t^2}{r^2}$ $q_{cr} = 5820.63\,\frac{kgf}{m^2}$ Remain under this value at the factored loads level

4. Double curvature shells

I think referred to both principal stresses compressive, hence mostly for sinclastic shells.
Otherwise the tension if kept in place would be increasing buckling strength.

$t := 12 \cdot cm$ thickness

$r_1 := 15 \cdot m$ radius 1

$r_2 := 5 \cdot m$ radius 2

$\alpha := 0.15$ buckling parameter, about 0.58/4 of p. 395 or select from p. 399. It seems 0.30 could also be accepted.

0.06 minimum, 0.16 typical, and never more than 0.32

In this case the formulation (p. 397) is given in critical load terms...

$$q_{cr} := \alpha \cdot E_{cLT} \cdot \frac{t^2}{r_1 \cdot r_2}$$ $$q_{cr} = 17461.9 \frac{kgf}{m^2}$$ Remain under this value at the factored loads level

This formulation is subject to the same critic than the spherical dome above; furthermore, it puts the critical (but buckling) load higher when one of the radiuses is bigger than the other, which seems contrary to experience; one shell of one radius equal to the lower in that of two radiuses would be tearing before the one having the bigger radius in place, which seems unrealistic.

2.b Cylindrical shells' buckling strength / Spampinato

reference is *Teoría y Cálculo de las Bóvedas Cáscaras Cilíndricas*, Agripino R. Spampinato p. 258...

These refer to half cylinder roofs of eliptical section acting as beams, and the compression is along the length of the cylinder.
Spampinato values seem more reasonable than those of Haas for these shells in reason of the given critical stresses values for spherical domes

for a elliptical section of a cylindrical shell of half horizontal breadth of ellipse a an half vertical breadth b is

$a := 10 \cdot m$ $b := 6.5 \cdot m$ $r := \frac{a^2}{b}$ $r = 15.38\,m$

$t := 12 \cdot cm$

Long cylindrical shells

$$\sigma_{xk} := \frac{80 \cdot \frac{kgf}{cm^2}}{1 + \frac{r}{200 \cdot t}}$$ $$\sigma_{xk} = 4.78\,MPa$$ this is the allowable (service level) buckling strength for the LONG cylindrical shell

$$\frac{240}{80} \cdot \sigma_{xk} \cdot \sqrt{\frac{f_c}{24 \cdot MPa}} = 17.32\,MPa$$ maybe the true buckling load $$\sigma_{cr\_Spampinato} := \frac{240}{80} \cdot \sigma_{xk} \cdot \sqrt{\frac{f_c}{25 \cdot MPa}}$$

Remain under this value at the factored loads level

Short cylindrical shells

$L := 20 \cdot m$ rest as above

$$\sigma_{xk} := \frac{80 \cdot \frac{kgf}{cm^2}}{1 + \frac{L}{1100 \cdot t} \cdot \sqrt{\frac{r}{t}}}$$

$\sigma_{xk} = 2.89\,MPa$ this is the allowable (service level) buckling strength for the SHORT cylindrical shell

$\frac{240}{80} \cdot \sigma_{xk} \cdot \sqrt{\frac{f_c}{24 \cdot MPa}} = 10.47\,MPa$ maybe the true buckling load

Remain under this value at the factored loads level

2.c Cylindrical shells' buckling strength / Roark

reference is *Roark's Formulas for Stress and Strain*, Warren. C. Young p. 689.

Assuming a fully compressed cylinder to represent the case...

$r := 15 \cdot m$

$t := 12 \cdot cm$

$\sigma_{cr\_Roark} := 0.3 \cdot E_{cLT} \cdot \frac{t}{r}$ $\sigma_{cr\_Roark} = 14.27\,MPa$ Remain under this value at the factored loads level

2.d Cylindrical shells' buckling strength / Ugural

reference is *Stresses in Plates and Shells*, Ansel C.Ugural p. 467 and p. 471

Assuming a fully compressed cylinder to represent the case...

$r := 15 \cdot m$

$t := 12 \cdot cm$

Symmetrical Buckling

$\sigma_{cr} := 0.605 \cdot E_{cLT} \cdot \frac{t}{r}$ $\sigma_{cr} = 28.78\,MPa$

Unsymmetrical Buckling

more likely

$\psi := \frac{1}{16} \cdot \sqrt{\frac{r}{t}}$ $K := \left[1 - 0.901 \cdot \left(1 - e^{-\psi}\right)\right]$ $K = 0.55$

$\sigma_{cr\_Ugural} := 0.605 \cdot K \cdot E_{cLT} \cdot \frac{t}{r}$ $\sigma_{cr\_Ugural} = 15.74\,MPa$ Remain under this value at the factored loads level

2e. Cylinder shells' buckling / Pilkey

in p. 262

$L := 20 \cdot m$ $a := 6 \cdot m$ $b := 3 \cdot m$

$Z := \frac{L^2}{r \cdot t} \cdot \sqrt{1 - \nu^2}$ $Z = 217.73$ must be bigger than 50, so OK

$\eta := \frac{r}{t}$ $K_c := 0.22195 + \frac{29.7611}{\eta} - \frac{2322.08667}{\eta^2} + \frac{65832.1484}{\eta^3}$

$\sigma_{cr\_Pilkey} := K_c \cdot E_{cLT} \cdot \frac{t}{r}$ $\sigma_{cr\_Pilkey} = 16.42\,MPa$ Remain under this value at the factored loads level

2f. Long cylinder shells' buckling. Discussion

Mean Buckling strength: $\sigma_{cr\_cyl} := \frac{\sigma_{cr\_Spampinato} + \sigma_{cr\_Ugural} + \sigma_{cr\_Roark} + \sigma_{cr\_Pilkey}}{4}$

$\sigma_{cr\_cyl} = 15.85\,MPa$ Remain under this value at the factored loads level

- The preceding evaluation of critical pseudoelastic buckling strength have detected (Ugural) a symmetrical and an unsymmetrical buckling failure modes. The lower critical strength corresponds to the unsymmetrical mode, and the bigger to the symmetrical mode. The symmetrical failure mode happens at about twice the unsymmetrical strength, and seems to be the value reported by Haas. Since the unsymmetrical value must control design (even if a symmetrical failure remains possible) it seems the Haas/symmetrical buckling strength must be discarded.
- The Spampinato values are based in concretes of 24 MPa strength, so the buckling strength is corrected on modulus of deformation for consistency.
- The predicted value more accurate per the equal weight vote is that of Ugural, which is also the more modern text.