

Considering two halves of different rigidity ( $S_1 > S_2$ ) and under the hypothesis they are not connected, we have variation of the radius which are different for the two elements.

**Same is for the length variation.**

With the elements joined together to reinstate the congruence of the displacement we have to consider that forces and moments raise up at the interface

This statement allow us to start with investigation for local stress in these cases.

Consider now simply the deformations. We have:

Circumferential deformation ( $\epsilon_t$ ) and Meridional deformation ( $\epsilon_a$ ) for a thin cylinder are:

$$\epsilon_t = 1/E * (\sigma_t - \mu \sigma_a) \quad (1) \quad ; \quad \epsilon_a = 1/E * (\sigma_a - \mu \sigma_t) \quad (2)$$

It can be demonstrate that local actions associated to circumferential deformation have a very small effect in respect of the cylinder stability.

Actions and local stresses associated with the meridional deformation are even lower.

Let us to understand how it globally acts the meridional deformation in case of a slender geometry:

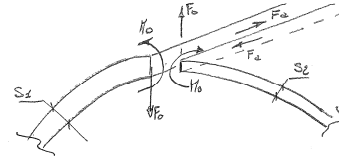
For any cylinder with elastic behavior we have:

$$\sigma_a = P \cdot D / 4S; \quad \sigma_t = P \cdot D / 2S$$

From (2) we have:

$$\epsilon_a = 1/E * (\sigma_a - \mu \sigma_t) = 1/E * (P \cdot D / 2S) * (1/2 - \mu)$$

D = average diameter



Length variation results:

$$\Delta L = \epsilon_a \cdot L = (P \cdot D \cdot L) / (2 \cdot E \cdot S) * (1/2 - \mu)$$

$$\text{With } thk=S_1 \quad \Delta L_1 = \epsilon_a \cdot L = (P \cdot D \cdot L) / (2 \cdot E \cdot S_1) * (1/2 - \mu)$$

$$\text{With } thk=S_2 \quad \Delta L_2 = \epsilon_a \cdot L = (P \cdot D \cdot L) / (2 \cdot E \cdot S_2) * (1/2 - \mu)$$

Final difference in length is :

$$\Delta L_{1,2} = (P \cdot D \cdot L) / (2 \cdot E) [1/S_1 - 1/S_2] * (1/2 - \mu)$$

$$\alpha = S_1 / S_2$$

$$\Delta L_{1,2} = (P \cdot D \cdot L) / (2 \cdot E \cdot S_1) * [1 - \alpha] * (1/2 - \mu)$$

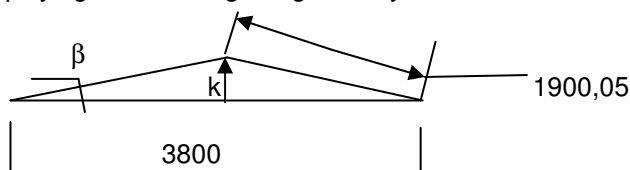
Let complete with some figure

S1=	16 [mm]	P=	4,2 [Mpa]
S2=	8 [mm]	D=	200 [mm]
L=	3800 [mm]	$\mu$ =	0,3
E=	184.000 [N/mm <sup>2</sup> ]		

$$\Delta L_{1,2} = ((4,2 \cdot 200 \cdot 3800) / (2 \cdot 184.000 \cdot 16)) / (1 - 2) * (0,5 - 0,3) = -0,11 \text{ [mm]}$$

Simplifying with a triangular geometry to accommodate the  $\Delta L_{1,2}$

(parabolic displacement should be the real one)



$$\beta = \arccos(1900 / 1900,05) = 0,4^\circ$$

$$k = 1900,05 \cdot \sin \beta = 13,8 \text{ mm}$$

Small difference for  $\Delta L$  could lead to important deflection at middle span with consequence on interface points when present, (as in the case I'm dealing with) which in case of very rigid piping spool coming to the header, might results in flange leakage and or overstress.

Piping stress analisys must take into accout the diferential nozzles displacements by imposing the displacement itself to the piping interface points.  
The new set of values for the nozzles loads is to be used to check for the integrity of the equipment.  
Nipples rotation in following deflection may induce a bending moments on the fin tubes.

Let me propose one other way for interpreting:

The internal system of forces caused by the pressure will be balanced by a deformation of the body which, when axial symmetry is respected, will be symmetric.  
In case of a different material distribution not respecting the axial symmetry the deformation should be in reason of the position of the neutral axis for said material distribution.

Considering the header some sentence more:

Any opening like nipple attachement cause local effects.

Vicinity of gross discontinuity may results in superposition of local effects.

Local condition in general can't generate global effects.

Superposition of a number of local effects may cause a global effect.

The holes and nipples rows cause effects that are depending upon the holes distance the thicknes ratio in between body and nipples thk

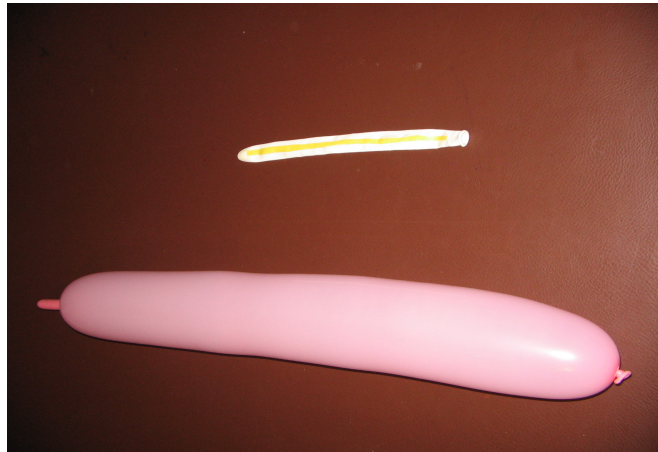
The presence of the nipples it cause local stiffening that reduce deformatio capability for the material surrounding the holes. A carefull check can be done by FEA only or experimental analisys

Here a practical representation  
(a little bit naif...)

The balloon represent the cylinder under pressure which, with constant thickness, is straight (pink balloon)

I assume:

the balloon can be considered as elastic and isotropic with very small variation of the thickness



When on the balloon (the white one) is attached a strip of the same rubber (yellow) by one side than the balloon bends being double thk in a portion of its body.



Note:

After a couple of trial I decided that the glue was not significantly affecting the behavior