CHAPTER 7

Flow Over Spillway

Rapidly varied flow has very pronounced curvature of the streamlines. The change in curvature may become so abrupt that the flow profile is virtually broken, resulting in a state of high turbulence; this is rapidly varied flow of discontinuous profile, of which the hydraulic jump is an example.

Characteristic features of rapidly varied flow are :

- 1. The curvature of the flow is so pronounced that the pressure distribution cannot be assumed to be hydrostatic.
- 2. The rapid variation in flow regime often takes place in a relatively short reach. Accordingly, the boundary friction., which would play a primary role in a gradually varied flow, is comparatively small and in most cases insignificant.
- 3. When rapidly varied flow occurs in a sudden-transition structure, the physical characteristics of the flow are basically fixed by the boundary geometry of the structure as well as by the state of the flow.
- 4. When rapid changes in water area occurs in rapidly varied flow, the velocity-distribution coefficients α and β are usually far greater than unity and cannot be accurately determined.
- 5. The separation zones, eddies, and rollers that may occur in rapidly varied flow tend to complicate the flow pattern and to distort the actual velocity distribution in the stream.

The Sharp - crested Weir.

The sharp-crested weir is not only a measuring device for open-channel flow but also the simplest form of overflow spillway. The characteristics of flow over a weir were recognized as the basis of design for the round-crested overflow spillway; that is, the profile of the spillway was determined in conformity with the shape of the lower surface of the flow nappe over a sharp-crested weir.

The shape of the flow nappe over a sharp-crested weir can be interpreted by the principle of the projectile. According to this principle, it is assumed that the horizontal velocity principle, it is assumed that the horizontal velocity component of the flow is constant and that the only force acting on the nappe is gravity. In time t, a particle of water

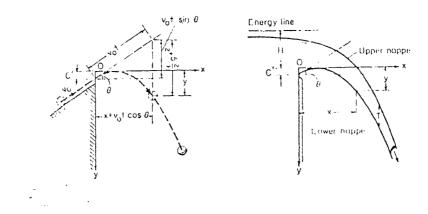


Fig. 1. Derivation of nappe profiles over sharp-crested weir by the principle of projectile

in the lower surface of the napple will travel a horizontal distance x from the face of the weir, equal to

$$x = v_o t \cos \theta \tag{1}$$

where

 v_o is the velocity at the point where x = 0 and

 θ is the angle of inclination of the velocity with the horizontal.

$$y = -v_0 t \sin \theta + \frac{1}{2} g t^2 + C$$
 (2)

where

C is the value of y at x=0, apparently, C is equal to the vertical distance between highest point of the nappe and the elevation of the crest.

$$\frac{y}{H} = A\left(\frac{x}{H}\right)^2 + B\frac{x}{H} + C \tag{3}$$

where

 $A = gH/2v_0^2 \cos^2 \theta$, $B = -\tan \theta$, and C = C'/H. Since the horizontal velocity component is constant, the vertical thickness of the nappe T-may be assumed constant. Adding a term D = T/H to the above equation, the general equation for the upper surface of the nappe is

$$\frac{y}{H} = A\left(\frac{x}{H}\right)^2 + B\frac{x}{H} + C + D \tag{4}$$

On the basis of data of numerous tests on nappe over a vertical sharp-crested. The following equations for the constants in the above equations have developed.

$$A = -0.425 + 0.25 \frac{h_{\nu}}{H} \tag{5}$$

B = 0.411 - 1.603
$$\frac{h_{\nu}}{H}$$
 - $\sqrt{1.568 \left(\frac{h_{\nu}}{H}\right)^2 - 0.892 \frac{h_{\nu}}{H} + 0.127}$ (6)

$$C = 0.150 - 0.45 \frac{h_{\nu}}{H} \tag{7}$$

$$D = 0.57 - 0.02 (10m)^{2} \exp(10m)$$
 (8)

where

 $m = h_v / H - 0.208$, and

h_v is the velocity head of the approach flow.

For high weirs, the velocity of approach is relatively small and can be ignored. Thus, the constants become A = -0.425, B = 0.055, C = 0.150, and D = 0.559. Experimental data have indicated that these equations are not valid when x / H is less than about 0.5 and that, for $h_v / H > 0.2$, additional data for verification are needed. For x / H < 0.5, the pressure within the nappe in the vicinity of the weir crest is actually above atmospheric because of the convergence of the streamlines. Consequently, forces other than gravity are acting on the nappe, which makes the principle of the projectile invalid.

The above theory and equations apply only if the approach flow is subcritical. For supercritical flow, or F > 1, the napple profile becomes essentially a function of the Froude number rather than a function of the boundary geometry.

The discharge over sharp-crested weir can be expressed in the general form.

$$Q = CLH^{1.5} \tag{9}$$

where

C is the discharge coefficient.

L is the effective length of the weir crest, and

H is the measured head above the crest, excluding the velocity head.

$$L = L' - 0.1 NH$$
 (10)

where

L' is the measured length of the crest and

N is the number of contractions. For two end contractions, N=2. For one end contraction, N=1. When no contractions are present at the two ends, N=0.

According to a well-known weir formula of Rehbock the coefficient C in Eq is approximately

$$C = 3.27 + 0.40 \frac{H}{h} \tag{11}$$

where

h is height of weir.

This equation holds up to H/h greater than about 15, the weir becomes a sill, and the discharge is controlled by a critical section immediately upstream from the sill. The critical depth of the section is approximately equal to H + h. By the critical depth-discharge relationship, it can be shown that the coefficient C is

$$C = 5.68 \left(1 + \frac{H}{h} \right)^{1/2} \tag{12}$$

Experiments have shown that the coefficient C in Eq. (9) remains approximately constant for sharp-crested weirs under varying heads if the nappe is aerated.

Aeration of the Nappe

The overfalling nappe is considered aerated; that is, the upper and lower nappe surfaces are subject to full atmospheric pressure. Insufficient aeration below the nappe, however, usually occurs in overflow spillways and measuring weirs.

This reduction of pressure will cause undesirable effects, such as

- (1) increase in pressure difference on the spillway or weir itself,
- (2) change in the shape of the nappe for which the spillway crest is generally designed,
- (3) increase in discharge, sometimes accompanied by fluctuation or pulsation of the nappe, which may be very objectionable if the weir or spillway is used for measuring purposes, and
- (4) unstable performance of the hydraulic model.

On the basis of experimental studies on spillways with gates Hickox developed the following equation giving the quantity of air required for aeration in cubic feet per second per foot of length of weir:

$$q_a = \frac{5.68(CH)^{3.64}}{p^{1.14}} \tag{13}$$

where

- H is the measured head in ft over the top of the gate;
- p is the reduction of pressure in feet of water to be maintained beneath the nappe; and
- C is a coefficient depending on the ratio of the discharge beneath the gate to the discharge over the top of the gate.

The ratio is represented by a dimensionless value

$$\sigma = \frac{y\sqrt{H_u}}{H^{1.5}} \tag{14}$$

where-

y is the height of the opening below the gate in ft and H_c is the head on the center of the gate in ft.

For ungated weir or spillway, $\sigma = 0$.

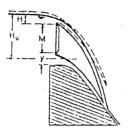


Fig. 2. Experimental setup for studying aeration below the nappe. The values C are as follows:

σ	0	0.5	1.0	1.5	2.0	2.5+
С	0.077	0.135	0.175	0.202	0.220	0.225

Intermediate values can be interpolated from a curve constructed with the above values.

Crest Shape of Overflow Spillways

In selecting a suitable profile, avoidance of negative pressures should be considered an objective, along with such other factors as maximum hydraulic efficiency, practicability, stability, and economy. On the basis of the Bureau of Reclamation data, the U.S. Army Corps of Engineers has developed several standard shapes at its Waterways Experiment Station. Such shapes designated as the WES standard spillway shapes, can be represented by the following equation:

$$X^{n} = KHd^{n-1}Y \tag{15}$$

Where

X and Y are coordinates of the crest profile with the origin at the highest point of the crest.

H_d is the design head excluding the velocity head of the approach flow, and

K and n are parameters depending on the slope of the upstream face. The values of K and n are given as follows:

Slope of upstream face	K	n
Vertical	2.000	1.850
3 on 1	1.936	1.836
3 on 2	1.939	1.810
3 on 3	1.873	1.776

For intermediate slopes, approximate values of K and n may be obtained by plotting the above values against the corresponding slopes and interpolating from the plot the required values for any given slope within the plotted range.

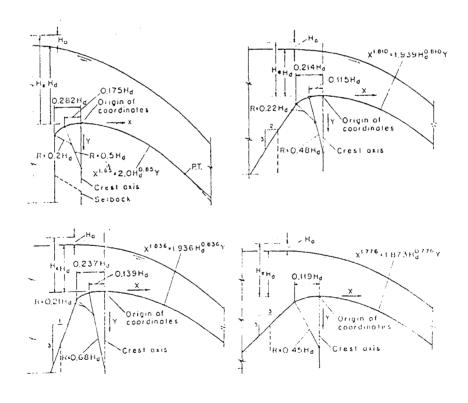


Fig. 3. The WES – standard spillway shapes.

The upstream face of the spillway crest may sometimes be designed to set back, as shown by the dashed lines in Fig. (3). The shape of the crest will not be affected materially

by such details, provided the modification begins with at least one-half the total head H_c vertically below the origin of coordinates. This is because the vertical velocities are small below this depth and the corresponding effect on nappe profile is negligible.

Discharge of the Overflow Spillway

The discharge over a spillway can be computed by an equation in the form of equation for spillways designed for the WES shapes, the equation is

$$O = CLH e^{1.5}$$
 (16)

where

 H_e is the total energy head on the crest in ft, including the velocity head in the approach channel. Model tests of the spillways have shown that the effect of the approach velocity is negligible when the height h of the spillway is greater than 1.33 H_d , where H_d is the design head excluding the approach velocity head. Under this condition and with the design head (that is, h/H_d greater than 1.33 and $H_e = H_d$, for the approach velocity head is negligible) the coefficient of discharge C has been found to be $C_d = 4.03$.

In low spillways with $h/H_d < 1.33$, the approach velocity will have appreciable effect upon the discharge or the discharge coefficient and, consequently, upon the nappe profile. A dimensionless plot based on the data of the Waterways Experiment Station can be used to show the effect of the approach velocity on the relationship between H_e/H_d and C/C_d for spillways designed for WES shapes having vertical upstream face. For spillways having sloping upstream face, the value of C can be corrected approximately for the effect of the upstream-face, slope by multiplying C by a correction factor obtained from the attached chart in Fig. (4).

Rating of Overflow Spillways

The spillway, must also operate under either lower or higher than the design head. For lower heads, the pressure on the crest will be above atmospheric but still less than hydrostatic. For higher heads, on the other hand, the pressure will be lower than atmospheric, and it may become so low that separation in flow will occur. Model

experiments indicate, however, that the design head may be safely exceeded by at least 50 %; beyond this, harmful cavitation may develop.

For spillways designed for WES shapes, the curves given in figure can be used to determine the coefficient of discharge for heads other than the design head.

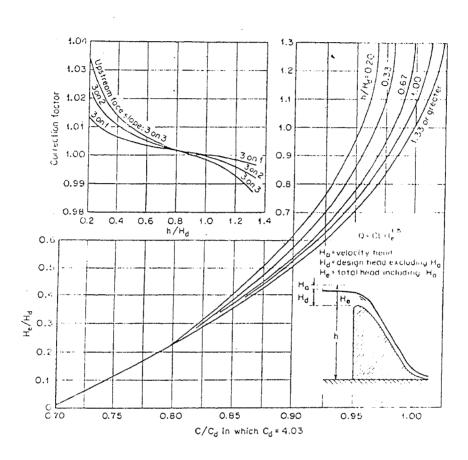


Fig. 4. Head-discharge relation for WES-standard spillway shapes.

For spillways designed for other shapes, has developed a universal curve showing the relationship between H_{c}/H_{D} and C/C_{D} . The term H_{D} is the design head including the approach velocity head, and C_{D} is the corresponding coefficient of discharge. The term H_{c} is the total head other than the design head, and C is the corresponding discharge coefficient. This curve is well supported by tests of some 50 overflow spillway crests of various shapes and operating conditions.

The dashed curve in Fig. (5), supported by data from 29 existing spillways, applies to spillways with overfall suppressed.

It is necessary to know the coefficient of discharge for the design head. If this coefficient is unknown, but the spillway shape is given, a method suggested by Buehler may be used. By this method, based on an equation derived by Brudenell, the coefficient of discharge is computed by the equation

$$C = 3.97 \left(\frac{H_e}{H_d}\right)^{0.12} \tag{17}$$

where

He is an operating head and

 H_D is the theoretical design head, including the approach velocity head, for a standard profile having a vertical upstream face. It should be noted that H_D is the theoretical design head of the standard profile for which the Brudenell equation was developed; therefore, it may not be equal to the actual design head used by Bradley or defined for other profiles.

The value of H_D may be obtained from a chart showing the standard profiles.

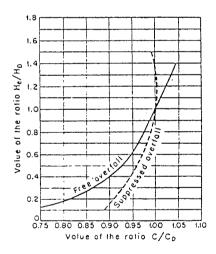


Fig. 5. Coefficient of discharge of overflow spillways for other than the design head.

Upper Nappe Profile of Flow over Spillways

The WES shapes for high overflow spillways with vertical upstream face have been investigated, using model tests, by the U.S. Army Engineers Waterways Experiment Station. Figure 7 shows the shapes and coordinates X and Y of the upper nappe profile obtained from such tests for negligible approach velocity, for conditions with and without piers, and for three different head ratios. The term H_d is the design head, excluding the

velocity head, and H is the operating head other than the design head, also excluding the velocity head. Profiles for intermediate head ratios may be interpolated. Owing to the contraction effect of piers, a pronounced hump between X/H_d = -0.6 and 0 occurs on the

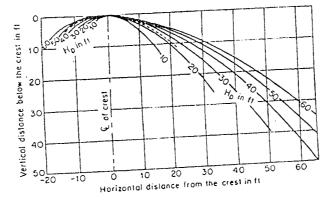


Fig. 6. Standard spillway shapes for different values of H_{D}

upper nappe profile along piers when the discharge is high. Upper napppe profiles for three gate bays adjacent to abutments are given in Fig. 8, showing the abutment effects on the nappe profiles.

It should be noted that the upper nappe surface is exposed to the atmosphere and, hence, subject to alteration due to wind and air currents and the absorption of surrounding air. As a result, the flow is aerated and the surface becomes wavy and unstable. The uppernappe-shape coordinates given in Figures 7 and 8 represent only the ideal cases, where air plays little or no part. The upper nappe surface for sloping upstream face should have a lower elevation than that for vertical upstream face. Hence, the given coordinates may also be used safely for spillways with sloping upstream face for which the actual data are not yet available.

Effect of Piers in Gated Spillways

Piers are needed to form the sides of the gates in gated spillways. The effect of the piers is to contract the flow and, hence, to alter the effective crest length of the spillways. The effective length of one bay of a gated spillway may be expressed as

$$L = L_0 - KNH_e$$
 (18)

where

L₀ is the clear span of the gate bay between piers;

K is the pier contraction coefficient;

N is the number of side contractions, equal to 2 for each gate bay; and

H_e is the total head on the crest *including* the velocity head. In computing the discharge through gated spillways, the effective length determined by the above equation should be used. The discharge coefficient, however, is assumed the same in both gated and ungated spillways.

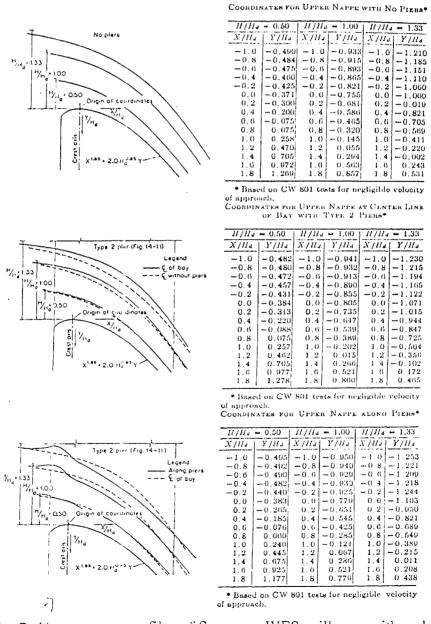


Fig. 7. Upper nappe profiles of flow over WES spillways with and without piers.

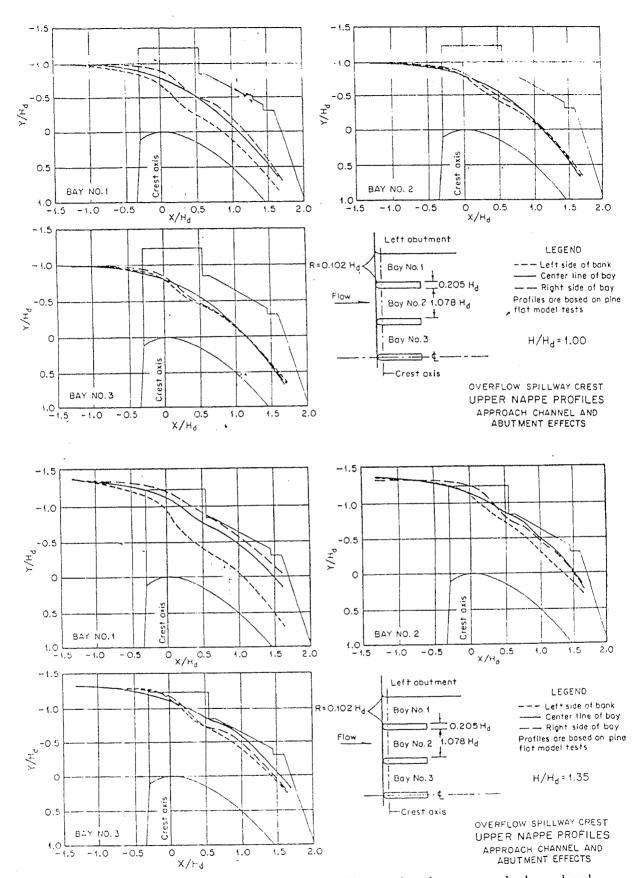


Fig. 8 Upper nappe profiles of flow over WES spillways showing approach channel and abutment effects.

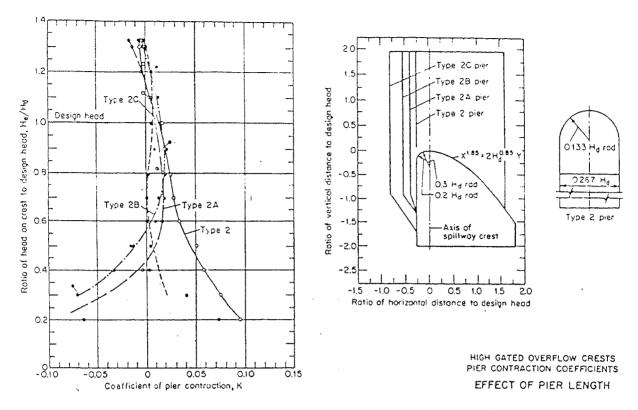


Fig. 9. Coefficient of contraction for the round-nose pier in high dams

The pier contraction coefficient varies mainly with the shape and position of the pier nose, the head condition, the approach depth of flow, and the operation of the adjacent gates. The approximate K value given by Creager and Justin ranges from 0.1 for thick, blunt noses to 0.04 for thin or pointed noses and is 0.035 for round noses. These values apply to piers having a thickness equal to about one-third the head on the crest when all gates are open. When one gate is open and the adjacent gates are closed, these values become roughly 2.5 times larger.

A round-nose pier is recommended for general use with high heads. The K value for the round-nose pier plotted against the ratio of H_e/H_d with variable distances upstream from the crest is shown in Fig (9). The effect of other nose shape on the contraction coefficient is shown in Fig. (10). The height of the test spillways was 6.67H_d, which had negligible velocity of approach. Under the testing conditions, these data are applicable to high spillways and the condition that the adjacent gates are open. For low spillways with appreciable approach velocity, the pier contraction coefficient for the round-nose pier with various approach depths is shown in Fig. (11). In the absence of adequate data, pier

contraction coefficients for other nose shapes for low spillways may be obtained by proportioning from the data for high spillways (i.e., from Fig 10)

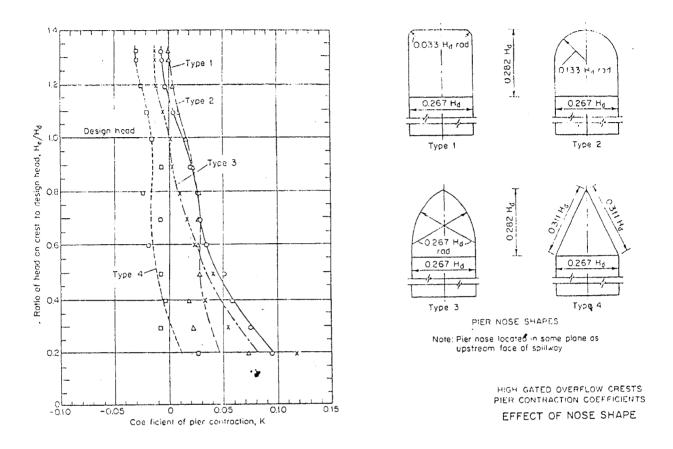


Fig. 10. Coefficient of contraction for piers various shapes in high dams with the nose located in the same plane

Pressure on Overflow Spillways

As the spillway must be operated under heads other than the design head, the pressure will increase under the lower heads and decrease under the higher heads.

Assuming a two-dimensional irrotational flow, the pressure on the spillway crest may be accurately determined analytically by a numerical method, graphically by flow-net analysis, or instrumentally by an electronic analogy. More exact determination of the pressure, however, will depend upon model tests.

The pressure distributions on a spillway crest with and without piers under three different head ratios, based on CW 801 tests of WES shapes, are shown in dimensionless plots (Fig. 12). Pressures for intermediate head ratios can be obtained by interpolation.)

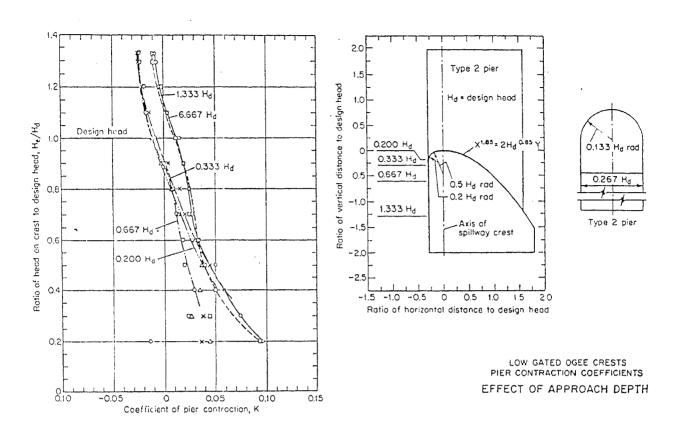


Fig. 11. Coefficient of contraction for the round-nose pier in low dams

The pressure reduction on the upstream face of a vertical weir has been determined both theoretically and experimentally by Harris. On the basis of CW 801 tests for ungated WES-shape crests of vertical upstream face and of U.S. Bureau of Reclamation tests of pressure on a charp-crested weir under the design head, the resultant of the reduced pressure is found to be approximately 12.9H_d lb per unit length of the spillway, acting horizontally at a distance of 0.161 H_d ft below the top of the crest.

Flow at the Toc of Overflow Spillways

The theoretical velocity of flow at the toe of an overflow spillway (Fig. 14) may be computed by

$$V_1 = \sqrt{2g(Z + H_g - y_1)}$$
 (19)

where

Z is the fall, or vertical distance in ft from the upstream reservoir level to the floor at the toe;

Ha is the upstream approach velocity head; and

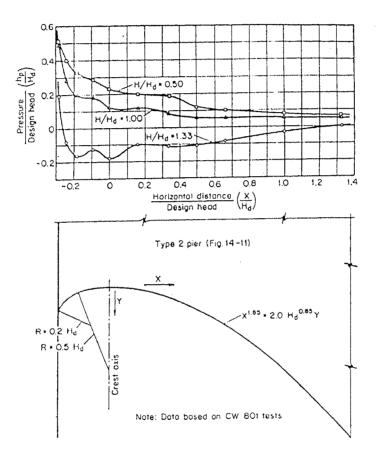


Fig. 12 Crest pressures on WES high over flow spillways (a) No piers

 y_1 is the depth of flow at the toe. The energy loss involved in the flow over the spillway, the actual velocity is always less than the theoretical value.

On the basis of experience, theoretical analysis, and a limited amount of experimental information obtained from prototype tests on Shasta and Grand Coulee dams, the U.S. Bureau of Reclamation prepared a chart (Fig. 13) to show the actual velocity at the toe of spillways under various heads, falls, slopes from 1 on 0.6 to 1 on 0.8, and the condition of average surface roughness. It is felt that this chart is sufficiently accurate for

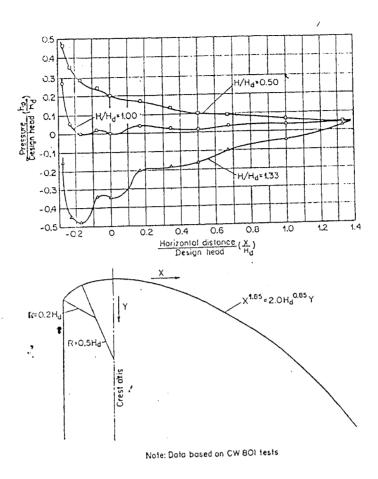


Fig. 12 Crest pressures on WES high over flow spillways (continued) (b) Along centerline of pier bay

preliminary-design purposes, although it can be refined by additional experimental information which may become available in the future.

Experiments by Bauer indicate that friction losses in accelerating the flow down the face of a spillway may be considerably less than the normal friction loss in flow with well-developed turbulence. Therefore, the friction loss is not significant on steep slopes, but it would become important if the slope were small. For this reason, the chart in Fig. 13 is not applicable to slopes flatter than 1 on 0.6.

At the end of the sloping spillway surface, the flow changes its direction rather abruptly and thereby produces appreciable centrifugal pressures. In order to create a smooth transition of the flow and to prevent the impact of the falling water from scouring the foundation, the surface at the spillway toe is usually designed as a curved bucket. To be

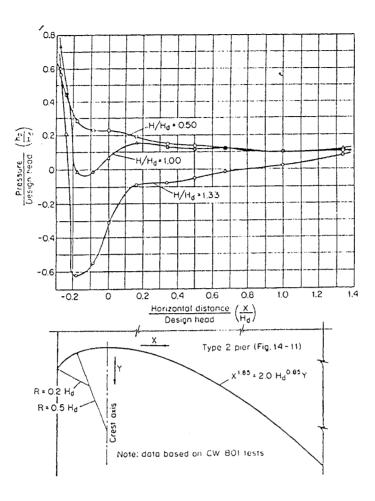


Fig. 12 Crest pressures on WES high over flow spillways (continued) (c) Along piers.

thoroughly effective the bucket should be tangent to the foundation or nearly so. The radius R of the bucket, measured in feet, may be estimated approximately by the following empirical formula:

$$R = 10^{-(v+6.4H+10)/(3.6H+64)}$$
 (20)

where

V is the velocity in ips of the flow at the toe and

H is the head in ft, excluding approach velocity head, on the spillway crest.

Submerged Overflow Spillways

Spillways and weirs are said to be submerged when the tail water is higher than the crest. The flow is classified into four distinct types according to the flow condition prevalent on the downstream apron: (1) supercritical flow, (2) subcritical flow involving hydraulfe jump, (3) flow accompanied by a drowned jump with diving jet, and (4) flow approaching complete submergence.

Submergence of spillway or weir will reduce the coefficient of discharge of the corresponding unsubmerged flow.

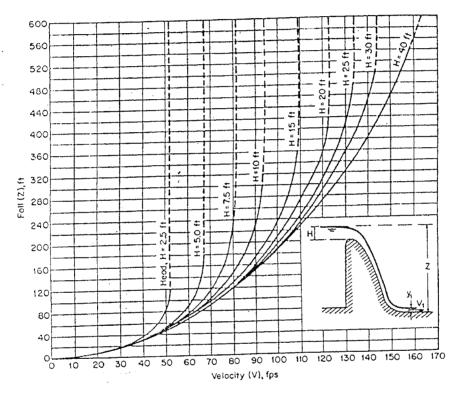


Fig. 13 Curves for determining velocity at the toe of spillways with slopes 1 on 0.6 to 0.8

The Bureau of Reclamation's test results on this reduction, expressed in percentage of the discharge coefficient for unsubmerged flow Fig. 4, have been presented in a chart for the four types of flow mentioned above. This chart in a slightly modified form Fig. 14, was further checked against other data by the U.S. Army Engineers Waterways Experiment Station. The chart is also applicable to the determination of coefficient for WES shapes under submerged conditions.

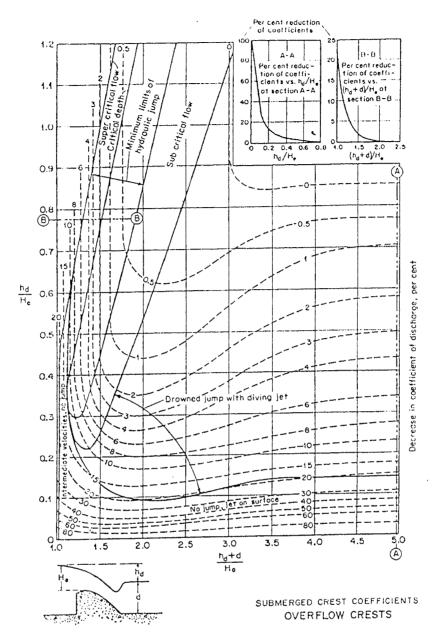


Fig 14. Decrease in discharge coefficient for submerged overflow spillways.

In the chart Fig. 14, h_d is the drop from the upper pool to the tail-water elevation, H_e is the total head above the crest, and d is the tailwater depth. The general pattern of the curves shows that, for low ratios $(h_d + d) / H_e$, the flow is of type 1, or supercritical, and that the reduction in coefficient is affected essentially by this ratio and is practically independent of h_d / H_e . The cross section BB in the upper right-hand corner of the chart shows the variation of $(h_d + d) / H_e$ at $h_d / H_e = 0.78$. For large values of $(h_d + d) / H_e$, on the other hand, the reduction in coefficient is affected essentially by the ratio h_d / H_e . Under this condition, for values of h_d / H_e less than 0.10, the flow is of type 4, the jet is on the surface,

and no jump occurs. For values of h_d/H_e greater than 0.10, the flow is of type 3, or accompanied by a drowned jump with diving jet. The cross section AA shows the variations of h_d/H_e at $(h_d+d)/H_e$ near 5.0. Sub-critical flow, or flow of type 2, occurs in the region indicated on the chart. Other regions for transitional flow conditions are also shown.

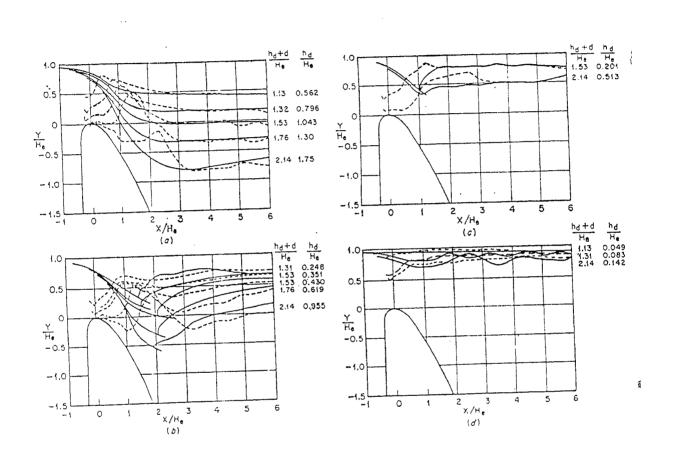


Fig. 15 Typical pressure (dashed lines) and surface (slid lines) profiles for flow over submerged overflow dams. (a) Supercritical flow; (b) flow involving hydraulic jump; (c) flow with a drowned hydraulic jump; (d) flow approaching complete submergence.

The typical pressure and surface profiles for submerged spillway flow are shown for different values of $(h_d+d)/H_e$ and h_d/H_e for four types of flow Fig. 15.

Example

Determine the crest elevation and the shape of an overflow-spillway section having a vertical upstream face and a crest length of 250 ft. The design discharge is 75,000 cfs. The upstream water surface at design discharge is at El. 1,000.0 and the average channel floor is at EI. 880.0 Fig 16.

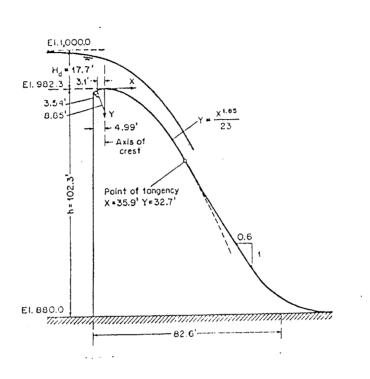


Fig. 16 Design of an overflow-spillway section.

Solution

Assume high overflow spillway. Therefore, the effect of approach velocity is negligible and $C_d = 4.03$

$$Q = CLH_e^{1.5}$$

$$H_e^{1.5} = \frac{75,000}{4.03 \times 25}$$

$$= 74.4$$

$$H_e = 17.3 \text{ ft}$$

again

$$Q = AV$$

 $V_a = \frac{Q}{A}$ Therefore, approach velocity

$$= \frac{75,000}{250 \times 120}$$

$$= 2.5 \text{ ft/s}$$

$$H_a = \frac{V_u^2}{2g}$$

$$= \frac{2.5^2}{2 \times 32.2}$$

= 0.1 ft

the approach velocity head,

The design head, $H_d = H_c - H_a$ = 17.8 - 0.1 = 17.7 ft

The height of the dam ,
$$h = 120 - 17.7$$

= 102.3 ft > 1.33 H_d

hence the effect of approach velocity is negligible

The cress elevation is at 1,000 - 17.7 = 982.3 ft

$$X^{n} = KH_{d}^{n-1}Y$$

For vertical upstream face

$$K = 2 & n = 1.85$$

$$X^{1.35} = 2H_d^{0.85}Y$$

$$X^{1.35} = 2 \times 17.7^{0.851}Y$$

$$Y = \frac{X^{1.85}}{2 \times 17.7^{0.85}} = \frac{X^{1.85}}{23}$$

Assume slope at point as

$$\frac{dY}{dX} = \frac{1}{23} \times 1.85 \times X^{0.85} = \frac{1}{0.6}$$

$$X^{0.85} = 20.72$$

$$X = 35.9 \text{ ft} & & Y = 32.7 \text{ ft}$$

Coordinates of the shape computed by the above equation are plotted as shown in Fig. (16)