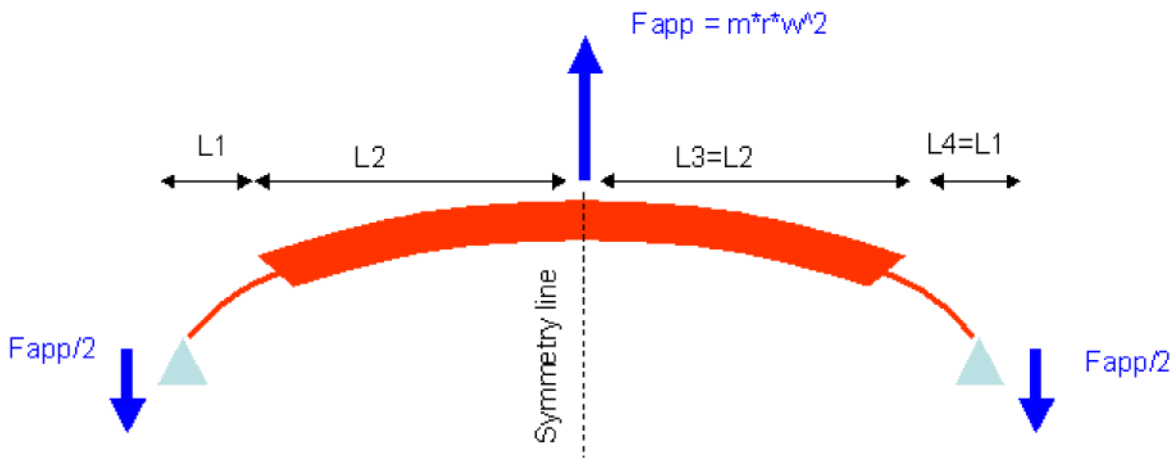
Objective: Analyse unbalance response assuming far below resonance -> neglect mass effects



Since the problem is symmetric about the center, we need only solve the left side.

Symbols
x = axial coordinate. x=0 at LHS
V(x) = Beam shear at position x (
M(x) = Beam moment at position x
T1(x), T(x) = Beam slope at position x in regions 1 and 2 (T stands for "Theta")
Y1(x), Y2(x) = Beam displacement at position x in regions 1 and 2
Subscripts 1 and 2 - refer to region 1 (thin(and region 2 (thick) of beam defined by L1 and L2
Neglect the small cavity at center of thick portion of beam - it does not contribute much to I
Es = Young's Modulus of steel
I = Diametral Moment of Inertia
CY1, CT1 - are integration constants associated with Y1(x), T1(x)
CY2, CT2 - are integration constants associated with Y2(x), T2(x)
Eq1, Eq2, Eq3, Eq4 - 4 boundary conditions used to solve 4 unknowns constants.
Fapp = applied Force = m*r*w^2 = unbalance force
w = radian speed = 2*pi*speed
All items in SI units

Note that bearing stiffness is assumed infinite. It does not affect the bending of the beam since the same force must be transferred through beam (under static analysis) regardless of bearing stiffness. If bearing stiffness is known, add displacement associated with bearing stiffness carrying Fapp/2

Initialize:

```
reset( )
```

Shear is constant (both regions)

```
V(x):=-Fapp/2
```

$$-\frac{\mathrm{Fapp}}{2}$$

```
M(x):=int(V(x),x);  // moment is 0 at LHS -> integration constant is 0
```

$$-\frac{\mathrm{Fapp}\,x}{2}$$

```
T1(x):=(1/Es/I1)*int(M(x),x)+CT1;
```

$$\mathrm{CT1}-\frac{\mathrm{Fapp}\,x^2}{4\,\mathrm{Es}\,\mathrm{I1}}$$

```
Y1(x):=int(T1(x),x)+CY1;
```

$$\mathrm{CY1}+\mathrm{CT1}\,x-\frac{\mathrm{Fapp}\,x^3}{12\,\mathrm{Es}\,\mathrm{I1}}$$

```
T2(x):=(1/Es/I2)*int(M(x),x)+CT2;
```

$$CT2 - \frac{Fapp\ x^2}{4\ Es\ I2}$$

```
Y2(x):=int(T2(x),x)+CY2;
```

$$CY2 + CT2\ x - \frac{Fapp\ x^3}{12\ Es\ I2}$$

List 4 boundary conditions (Eq1, Eq2,Eq3,Eq4)

```
Eq1:=0 = T2(x)|x=L1+L2
```

$$0 = CT2 - \frac{Fapp\ (L1 + L2)^2}{4\ Es\ I2}$$

```
Eq2:=0 = Y1(x)|x=0
```

$$0 = CY1$$

```
Eq3:= (Y1(x)|x=L1) = (Y2(x)|x=L1)
```

$$CY1 + CT1\ L1 - \frac{Fapp\ L1^3}{12\ Es\ I1} = CY2 + CT2\ L1 - \frac{Fapp\ L1^3}{12\ Es\ I2}$$

```
Eq4:=  (T1(x)|x=L1) = (T2(x)|x=L1)
```

$$CT1 - \frac{Fapp\ L1^2}{4\ Es\ I1} = CT2 - \frac{Fapp\ L1^2}{4\ Es\ I2}$$

Solve 4 above equations in 4 unknowns:

```
solution:=solve({Eq1,Eq2,Eq3,Eq4},{CY1,CY2,CT1,CT2}):
```

Display solution:

```
solution[1][1];
```

$$CT1 = \frac{Fapp\ I2\ L1^2 + 2\ Fapp\ I1\ L1\ L2 + Fapp\ I1\ L2^2}{4\ Es\ I1\ I2}$$

```
solution[1][2];
```

$$CT2 = \frac{Fapp\ L1^2 + 2\ Fapp\ L1\ L2 + Fapp\ L2^2}{4\ Es\ I2}$$

```
solution[1][4];
```

$$CY2 = -\frac{Fapp\ I1\ L1^3 - Fapp\ I2\ L1^3}{6\ Es\ I1\ I2}$$

```
solution[1][4];
```

$$CY2 = -\frac{Fapp\ I1\ L1^3 - Fapp\ I2\ L1^3}{6\ Es\ I1\ I2}$$

```
Y1(x):=subs(Y1(x),solution[1]);
```

$$\frac{x\ (Fapp\ I2\ L1^2 + 2\ Fapp\ I1\ L1\ L2 + Fapp\ I1\ L2^2)}{4\ Es\ I1\ I2} - \frac{Fapp\ x^3}{12\ Es\ I1}$$

```
Y2(x):=subs(Y2(x),solution[1]);
```

$$\frac{x\ (Fapp\ L1^2 + 2\ Fapp\ L1\ L2 + Fapp\ L2^2)}{4\ Es\ I2} - \frac{Fapp\ x^3}{12\ Es\ I2} - \frac{Fapp\ I1\ L1^3 - Fapp\ I2\ L1^3}{6\ Es\ I1\ I2}$$

Now we have the solution for displacement. We just need to find the numerical values and plug them in. The remainder of this file does that.

```
w:=float(2*PI*50)
```

$$314.1592654$$

```
mr:=1
```

1

```
Fapp:=mr*w^2
```

98696.04401

```
L1:=0.1801
```

0.1801

```
L2:=0.80060045
```

0.80060045

```
Es:=2.3E11
```

$2.3\ 10^{11}$

```
I1:=float( PI * 0.0738^4/64)
```

0.000001456113894

```
I2:=float(PI*0.24230673^4/64)
// neglect small portion missing in middle... not significant
```

0.0001692122521

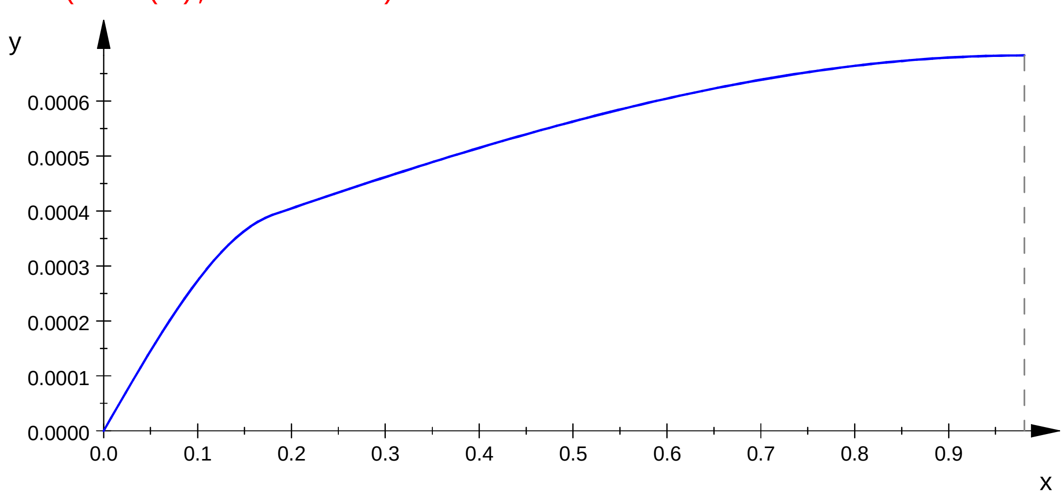
```
Y1(x)
```

$$\frac{x\,(\mathrm{Fapp\ I2\ L1}^2 + 2\ \mathrm{Fapp\ I1\ L1\ L2} + \mathrm{Fapp\ I1\ L2}^2)}{4\ \mathrm{Es\ I1\ I2}} - \frac{\mathrm{Fapp}\ x^3}{12\ \mathrm{Es\ I1}}$$

```
Ytot(x):=piecewise([x<L1,Y1(x)],[x<=L1+L2,Y2(x)]);
```

$$\begin{cases} \dfrac{x\,(\mathrm{Fapp\ I2\ L1}^2+2\ \mathrm{Fapp\ I1\ L1\ L2}+\mathrm{Fapp\ I1\ L2}^2)}{4\ \mathrm{Es\ I1\ I2}} - \dfrac{\mathrm{Fapp}\ x^3}{12\ \mathrm{Es\ I1}} & \text{if } x < 0.1801 \\ \dfrac{x\,(\mathrm{Fapp\ L1}^2+2\ \mathrm{Fapp\ L1\ L2}+\mathrm{Fapp\ L2}^2)}{4\ \mathrm{Es\ I2}} - \dfrac{\mathrm{Fapp}\ x^3}{12\ \mathrm{Es\ I2}} - \dfrac{\mathrm{Fapp\ I1\ L1}^3-\mathrm{Fapp\ I2\ L1}^3}{6\ \mathrm{Es\ I1\ I2}} & \text{if } x \le 0.98070045 \end{cases}$$

```
plot(Ytot(x),x=0..L1+L2)
```




```
Ytot(x)|x=(L1+L2)
```

0.0006831103676