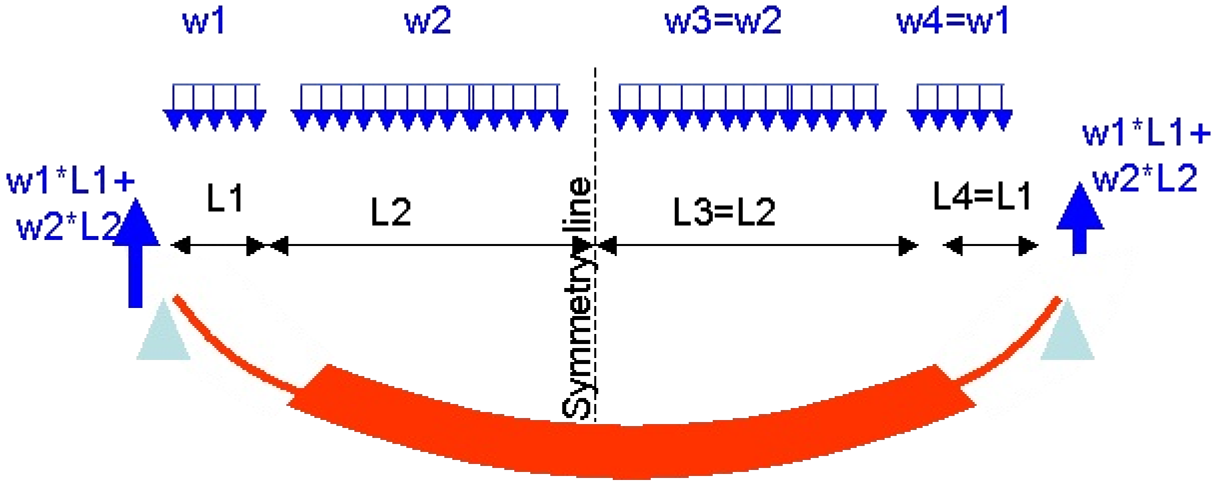
Objective: Analyse static deflection from gravity



Since the problem is symmetric about the center, we need only solve the left side.

Symbols
x = axial coordinate. x=0 at LHS
w1, w2 = weight per length in regions 1 and 2
V1(x),V2(x) = Beam shear at position x in regions 1 and 2
M1(x), M2(x) = Beam moment at position x in regions 1 and 2
T1(x), T(x) = Beam slope at position x in regions 1 and 2 (T stands for "Theta")
Y1(x), Y2(x) = Beam displacement at position x in regions 1 and 2
Subscripts 1 and 2 - refer to region 1 (thin(and region 2 (thick) of beam defined by L1 and L2
Es = Young's Modulus of steel
I = Diametral Moment of Inertia
CV2 - integration constant associated with V2(x)
CM2 - integration constant associated with M2(x)
CT1, CT2 - integration constants associated with T1(x), T2(x)
CY1, CY2 - integration constant associated with Y2(x)
A1, A2 = area in sections 1 and 2

Eq1, Eq2, Eq3, Eq4, Eq5, Eq6 - 6 boundary conditions used to solve 6 unknowns constants.
All items in SI units

Initialize:

```
reset()
```

Shear is starts at (w1*L1+w2*L2) and decreases with slope w:

```
V1(x):=-w1*x+(w1*L1+w2*L2) // value at x=0 is w1*L1+w2*L2
```

$$\mathrm{L1}\ \mathrm{w1} + \mathrm{L2}\ \mathrm{w2} - \mathrm{w1}\ x$$

```
V2(x):=-w2*x+CV2
```

$$\mathrm{CV2} - \mathrm{w2}\ x$$

```
M1(x):=int(V1(x),x);   // moment is 0 at LHS -> integration constant is 0
```

$$x\,(\mathrm{L1}\ \mathrm{w1} + \mathrm{L2}\ \mathrm{w2}) - \frac{\mathrm{w1}\ x^2}{2}$$

```
M2(x):=int(V2(x),x)+CM2
```

$$-\frac{w2\,x^2}{2}+CV2\,x+CM2$$

```
T1(x):=(1/Es/I1)*int(M1(x),x)+CT1
```

$$CT1-\frac{\frac{w1\,x^3}{6}-x^2\left(\frac{L1\,w1}{2}+\frac{L2\,w2}{2}\right)}{Es\,I1}$$

```
T2(x):=(1/Es/I2)*int(M2(x),x)+CT2
```

$$CT2+\frac{-\frac{w2\,x^3}{6}+\frac{CV2\,x^2}{2}+CM2\,x}{Es\,I2}$$

```
Y1(x):=int(T1(x),x)+CY1; //CY1=0 since Y1=0 @ x=0
```

$$CY1+CT1\,x+\frac{x\,(4\,L1\,w1\,x^2-w1\,x^3+4\,L2\,w2\,x^2)}{24\,Es\,I1}$$

```
Y2(x):=int(T2(x),x)+CY2;
```

$$CY2+CT2\,x+\frac{CM2\,x^2}{2\,Es\,I2}+\frac{CV2\,x^3}{6\,Es\,I2}-\frac{w2\,x^4}{24\,Es\,I2}$$

List 6 boundary conditions (Eq1, Eq2,Eq3,Eq4,Eq5, Eq6)

```
Eq1:= (T2(x)|x=L1+L2) = 0 //0 slope @ rhs
```

$$CT2+\frac{CM2\,(L1+L2)-\frac{w2\,(L1+L2)^3}{6}+\frac{CV2\,(L1+L2)^2}{2}}{Es\,I2}=0$$

```
Eq2:=(Y1(x)|x=0)=0
```

$$CY1=0$$

```
Eq3:= (Y1(x)|x=L1) = (Y2(x)|x=L1) // continuity of displacement @x=L1
```

$$CY1+CT1\,L1+\frac{L1\,(3\,w1\,L1^3+4\,L2\,w2\,L1^2)}{24\,Es\,I1}=CY2+CT2\,L1+\frac{CM2\,L1^2}{2\,Es\,I2}+\frac{CV2\,L1^3}{6\,Es\,I2}-\frac{L1^4\,w2}{24\,Es\,I2}$$

```
Eq4:=  (T1(x)|x=L1) = (T2(x)|x=L1) // continuity of slope @x=L1
```

$$CT1-\frac{\frac{L1^3\,w1}{6}-L1^2\left(\frac{L1\,w1}{2}+\frac{L2\,w2}{2}\right)}{Es\,I1}=CT2+\frac{-\frac{w2\,L1^3}{6}+\frac{CV2\,L1^2}{2}+CM2\,L1}{Es\,I2}$$

```
Eq5:=(M1(x)|x=L1)=(M2(x)|x=L1) // continuity of moment @x=L1
```

$$L1\,(L1\,w1+L2\,w2)-\frac{L1^2\,w1}{2}=-\frac{w2\,L1^2}{2}+CV2\,L1+CM2$$

```
Eq6:=(V1(x)|x=L1)=(V2(x)|x=L1) // continuity of shear @x=L1
```

$$L2\,w2=CV2-L1\,w2$$

Solve 6 above equations in 6 unknowns (CY2, CT1, CT2, CM2, CV2, CY1)

```
solution:=solve({Eq1,Eq2,Eq3,Eq4,Eq5,Eq6},{CY2, CT1, CT2, CM2, CV2,CY1}):
```

Display solution results for unknown integration constants:

```
CY2:=CY2|solution
```

$$-\frac{I1\,L1^4\,w2-6\,I1\,L1^4\,w1+5\,I2\,L1^4\,w1-8\,I1\,L1^3\,L2\,w2+8\,I2\,L1^3\,L2\,w2}{24\,Es\,I1\,I2}$$

```
CT1:=CT1|solution
```

$$-\frac{2\ I2\ L1^3\ w1 + 2\ I1\ L2^3\ w2 + 3\ I1\ L1^2\ L2\ w1 + 6\ I1\ L1\ L2^2\ w2 + 3\ I2\ L1^2\ L2\ w2}{6\ Es\ I1\ I2}$$

```
CT2:=CT2|solution
```

$$-\frac{3\ L1^3\ w1 - L1^3\ w2 + 2\ L2^3\ w2 + 3\ L1^2\ L2\ w1 + 6\ L1\ L2^2\ w2 + 3\ L1^2\ L2\ w2}{6\ Es\ I2}$$

```
CM2:=CM2|solution
```

$$\frac{L1^2\ w1}{2} - \frac{L1^2\ w2}{2}$$

```
CV2:=CV2|solution
```

$$L1\ w2 + L2\ w2$$

```
CY1:=CY1|solution
```

$$0$$

Now we have the solution for displacement. We just need to find the numerical values and plug them in. The remainder of this file does that.

```
rho:=7750
```

$$7750$$

```
A1:=float(PI*0.0738^2/4)
```

$$0.004277623973$$

```
w1:=rho*A1*9.8
```

$$324.8855408$$

```
A2:=float(PI*0.2423^2/4)
```

$$0.04611016854$$

```
w2:=rho*A2*9.8
```

$$3502.067301$$

```
L1:=0.1801
```

$$0.1801$$

```
L2:=0.80060045
```

$$0.80060045$$

```
Es:=2.3E11
```

$$2.3\ 10^{11}$$

```
I1:=float( PI * 0.0738^4/64)
```

$$0.000001456113894$$

```
I2:=float(PI*0.24230673^4/64)
```

$$0.0001692122521$$

Here are the numerical values of the constants:

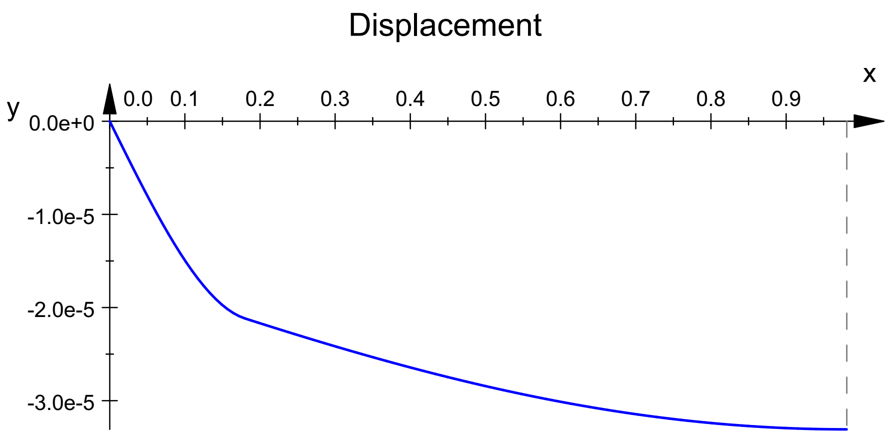
```
float(CY2), float(CT1),float(CT2),float(CM2),float(CV2)
```

$$-0.00001637595037,\ -0.0001635500671,\ -0.00002699286464,\ -51.52754967,\ 3434.478978$$

```
Y(x):=piecewise([x<L1,Y1(x)],[x<=L1+L2,Y2(x)])
```

$$\begin{cases} \mathrm{CY1} + \mathrm{CT1}\,x + \dfrac{x\left(4\,\mathrm{L1}\,\mathrm{w1}\,x^2 - \mathrm{w1}\,x^3 + 4\,\mathrm{L2}\,\mathrm{w2}\,x^2\right)}{24\,\mathrm{Es}\,\mathrm{I1}} & \text{if } x < 0.1801 \\ \mathrm{CY2} + \mathrm{CT2}\,x + \dfrac{\mathrm{CM2}\,x^2}{2\,\mathrm{Es}\,\mathrm{I2}} + \dfrac{\mathrm{CV2}\,x^3}{6\,\mathrm{Es}\,\mathrm{I2}} - \dfrac{\mathrm{w2}\,x^4}{24\,\mathrm{Es}\,\mathrm{I2}} & \text{if } x \le 0.98070045 \end{cases}$$

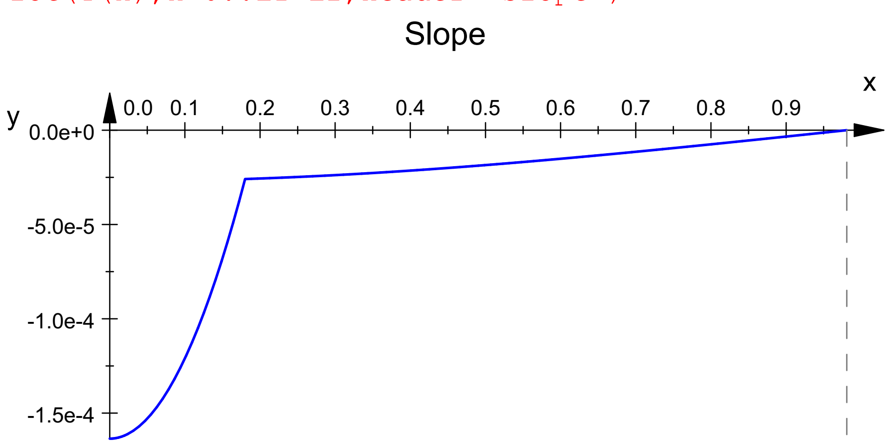
```
plot(Y(x),x=0..L1+L2,Header="Displacement")
```




```
T(x):=piecewise([x<L1,T1(x)],[x<=L1+L2,T2(x)]);
```

$$\begin{cases} \mathrm{CT1} - \dfrac{\dfrac{\mathrm{w1}\,x^3}{6} - x^2\left(\dfrac{\mathrm{L1}\,\mathrm{w1}}{2} + \dfrac{\mathrm{L2}\,\mathrm{w2}}{2}\right)}{\mathrm{Es}\,\mathrm{I1}} & \text{if } x < 0.1801 \\ \mathrm{CT2} + \dfrac{-\dfrac{\mathrm{w2}\,x^3}{6} + \dfrac{\mathrm{CV2}\,x^2}{2} + \mathrm{CM2}\,x}{\mathrm{Es}\,\mathrm{I2}} & \text{if } x \le 0.98070045 \end{cases}$$

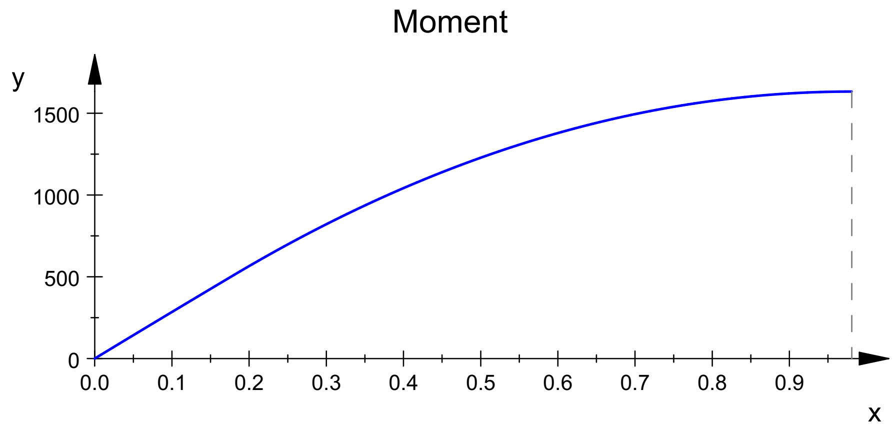
```
plot(T(x),x=0..L1+L2,Header="Slope")
```




```
M(x):=piecewise([x<L1,M1(x)],[x<=L1+L2,M2(x)]);
```

$$\begin{cases} x\,(\mathrm{L1}\,\mathrm{w1} + \mathrm{L2}\,\mathrm{w2}) - \dfrac{\mathrm{w1}\,x^2}{2} & \text{if } x < 0.1801 \\ -\dfrac{\mathrm{w2}\,x^2}{2} + \mathrm{CV2}\,x + \mathrm{CM2} & \text{if } x \le 0.98070045 \end{cases}$$

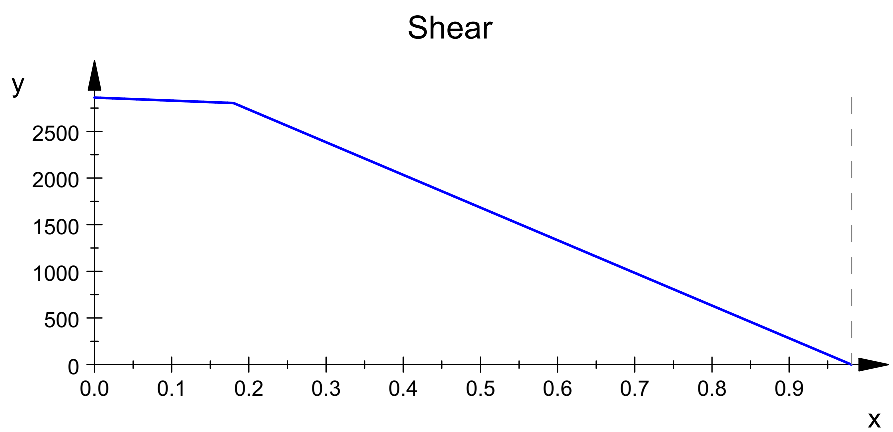
```
plot(M(x),x=0..L1+L2,Header="Moment")
```




```
V(x):=piecewise([x<L1,V1(x)],[x<=L1+L2,V2(x)]);
```

$$\begin{cases} \mathrm{L1\ w1} + \mathrm{L2\ w2} - \mathrm{w1}\, x & \text{if } x < 0.1801 \\ \mathrm{CV2} - \mathrm{w2}\, x & \text{if } x \leq 0.98070045 \end{cases}$$

```
plot(V(x),x=0..L1+L2,Header="Shear")
```



Do order-of-magnitude sanity check by computing natural frequency from static deflection using SDOF model

```
StaticDeflection:=Y2(x)|x=L1+L2
```

$$-0.00003308006744$$

```
Fnat:=float(sqrt(9.8/abs(StaticDeflection))/2/PI)
```

$$86.62634213$$

This is slightly less than calculated natural frequency of ~95hz. Since only the center of the beam moves 33 microns, it makes sense that the "effective" static deflection should be lower, which would give a higher natural frequency calculated from static deflection, closer to matching other natural frequency calc