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Survey Paper

Survey of Advanced Suspension Developments and Related Optimal Control Applications*

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Simple, mostly LQ-based optimal control concepts gave useful insight about performance potentials, bandwidth requirements, and optimal structure of advanced vehicle suspensions. The present paper reviews these optimal control applications and related practical developments.

Key Words—Optimal control; automotive control; active and semi-active vehicle suspensions; ride comfort.

Abstract—The paper surveys applications of optimal control techniques to the design of active suspensions, starting from simple quarter-car, 1D models, which are followed by their half-car, 2D, and full-car, 3D, counterparts. While the main emphasis is on Linear–Quadratic (LQ) optimal control and active suspensions, the paper also addresses a number of related subjects including semi-active suspensions; robust, adaptive and nonlinear control aspects and some of the important practical considerations. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

As is well known, conventional vehicle suspensions achieve the road-induced vibration isolation through passive means such as springs and dampers or shock absorbers. On the other hand, the active suspensions are characterized by a requirement that at least a portion of suspension force generation is provided through active power sources such as compressors, hydraulic pumps, etc. Practical applications of active suspensions have been facilitated by maturing of microprocessors and associated electronics, which in turn also influenced actuator and sensor developments. As a result of this matured infrastructure, the active suspensions have attracted considerable attention in the mid-1980s with the widely publicized demonstrations of the Lotus Esprit experimental vehicle. This car evolved through many years of R & D work. A January 21, 1987 New York Times article describes the vehicle capabilities as follows:

"Test drivers usually emerge from the car with their imagination in overdrive. 'The greatest single advance in car engineering since the war,' the British magazine Car declared on the cover of a recent issue. Car's editor, Steve Cropley, wrote that one could take the benefits of all other modern automobile developments, 'add them up and double the total—and you might come somewhere near the degree to which full active suspension improves a car'.

Doug Nye, a contributing editor of the American magazine Road & Track, said he was initially skeptical, but 'in fact it is amazing'. He said, 'I've been writing about cars for 24 years and honestly found it the most impressive thing I've ever tried'."

As always, these statements contain some hype and should be taken with a grain of salt. In particular, the above comments refer mainly to substantial improvements in vehicle handling under somewhat specialized and severe maneuvers. The corresponding improvement in ride comfort, which is probably more relevant to an average consumer, was not demonstrated then, and required further work. Nevertheless, the early demonstrations pointed to potentially "leap-frog" function improvements in vehicle ride and handling, with related impacts on overall vehicle controls and safety. This, in turn, spurred increased industry-wide activities, which eventually resulted in the introduction of the first narrow-bandwidth or "slow-acting" active production units from Nissan and Toyota.

Some of the potential benefits of active suspensions were predicted decades ago by the first pioneer researchers. Indeed, the optimal control techniques that were "launched" with Sputnik and used in the aerospace industry since the 1950s and 1960s, have also been applied to the study of active suspensions, starting from about the same period (Morman *et al.*, 1982). Early investigations (Bender *et al.*, 1967; Karnopp and Trikha, 1969) used the

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simplest possible, one-degree-of-freedom (DOF), quarter-car models. The resulting optimal suspension design problem fits naturally into the Linear–Quadratic-Gaussian (LQG), or equivalently, the Wiener–Hopf setting. It will be argued below that, despite their simplicity, these early contributions had a profound impact on the practical implementation some 15–20 years later, when the utilization of these theoretical results was made possible by the ever increasing capacity of microcomputers in the automotive industry.

It should be pointed out that the design and synthesis of active suspensions can be approached from many avenues: modal analysis (Williams and Wright, 1986); classical techniques, such as root locus, Bode, and Nichols plots (Thompson, 1970–1971; Karnopp, 1985; Williams, 1985); eigenvalue assignment (Michelberger *et al.*, 1990); sensitivity analysis (Redfield and Karnopp, 1989); regional pole constraints (Yedavalli and Liu, 1994); Youla parametrization and linear fractional transformation (Smith, 1995); velocity feedback (Lieh, 1996); model order reduction (Lohmann, 1995); simulation using multibody dynamics computer models (Orlandea and Chase, 1977; Grubel and Kortum, 1992; Schiehlen, 1992; Kortum and Sharp, 1993; Rill, 1994; Kortum *et al.*, 1994), design of experiments (Lamps and Ekert, 1993), and bondgraph modeling methodologies (Fritz *et al.*, 1994); optimization—e.g. nonlinear programming and multicriteria optimization (Foag and Grubel, 1987; Zaremba *et al.*, 1994; Mastinu, 1994) and optimal control (Bender *et al.*, 1967; Karnopp and Trikha, 1969; Hedrick *et al.*, 1974; Guenther and Leondes, 1977; Lu *et al.*, 1984); and, more recently, neural networks, fuzzy logic, evolution strategy and learning automata (Hampo and Marko, 1992; Yester and McFall, 1992; Feldkamp *et al.*, 1992; Nagai and Moran, 1993; Presser *et al.*, 1994; Marsh *et al.*, 1995; Kashani *et al.*, 1996). While each of these approaches can bring some useful perspectives and insight, the present paper will focus on the applications of optimal control techniques, and especially on the related work by the author and his colleagues at Ford. It will be demonstrated below that this constitutes a very natural approach revealing important global trends and insight such as the optimal system structure, performance potentials and trade-offs, and fundamental actuator and sensor requirements. The present survey is in part based on (Hrovat, 1993, 1994).

2. BACKGROUND

This section summarizes background material needed to formulate the optimal suspension design problem. This includes a brief introduction of an appropriate road input disturbance description, ride performance measure, and an appropriate definition of passive and active suspension actuators. A more detailed account of each of these important background topics is given in recent surveys (Hrovat, 1993, 1994); the present section summarizes only the main related aspects from these references.

Vehicle ride and handling is influenced by two main sets of disturbances. One is caused by road roughness, and the other by different forces and moments that originate in various inertial and aerodynamic loadings, as caused by braking, turning and wind gusts, for example. The load spectrum is typically bandwidth-limited to lower frequencies, and its effects can be relatively easily checked (mostly w.r.t. suspension deflections). The most relevant for ride studies are ground input disturbances caused by road roughness, which will also be the main focus of the present survey. Because of this, in a certain sense the ride optimization problem could be viewed as an equivalent optimal filtering problem where one would attempt to eliminate the negative effects of vibrations caused by road roughness. At the same time it would be desirable to pass the low frequency components of road inputs that are necessary to follow the terrain, while satisfying the given design and system imposed constraints.

There are many possible ways to analytically describe the road inputs, which can be classified as shock or vibration. Shocks are discrete events of relatively short duration and high intensity, as, e.g. caused by a pronounced bump or pothole on an otherwise smooth road. Vibrations, on the other hand, are characterized by prolonged and consistent excitations that are felt on, say, "rough" roads. Obviously, a well-designed suspension must perform adequately in a wide range of shock and vibration environments.

In the context of vibrations, the road roughness is typically specified as a random process of a given displacement power spectral density (p.s.d.). Typical measured spectral densities of various terrains from Sevin and Pilkey (1971) are reproduced in Fig. 1. An often used approximation of measured road displacement p.s.d.s for various terrains is given in the form

$$S(\Omega) = A\Omega^n, \tag{1}$$

where Ω is the spatial frequency, typically in units of "radians per length", and A and n are appropriate constants. The most commonly used case corresponds to $n \approx -2$. With this value the displacement spectra of equation (1) imply white-noise ground input velocity, which conveniently matches the well-known, standard Linear–Quadratic–Gaussian (LQG) assumptions for process noise.

The simple expression (1) approximates various roads with different degree of fidelity (Sevin and

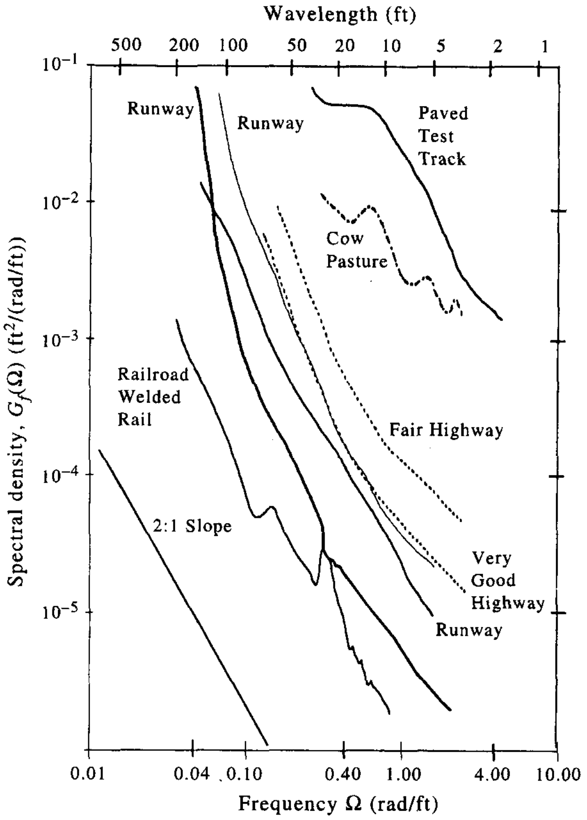


Fig. 1. Power spectral density of various terrains (Sevin and Pilkey, 1971).

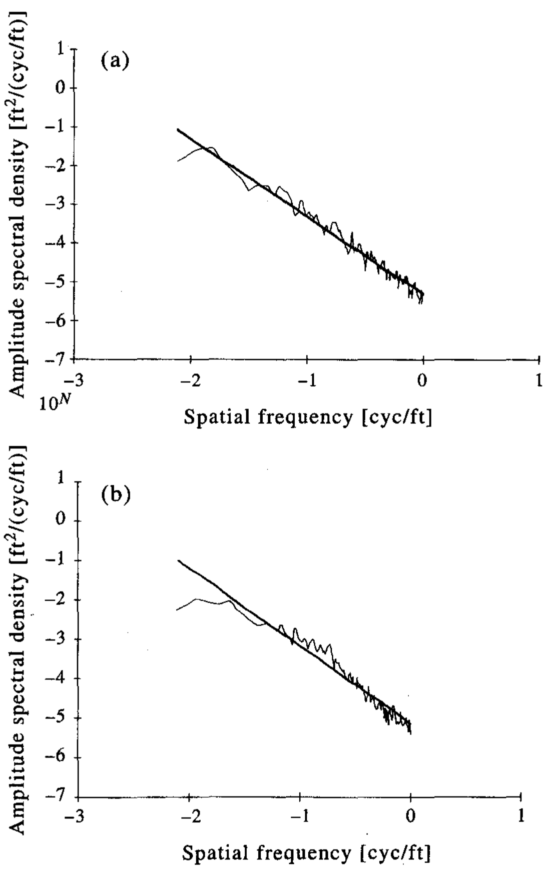


Fig. 2. (a) Comparison of model given by equation (2) with $n = -2.02$ and measured road roughness for "Rochester Road Section" as per Smith (1982). (b) Comparison of model given by (2) with $n = -1.99$ and measured road roughness for "Section between broken road and Rochester Road" as per Smith (1982).

Pilkey, 1971; Hullender *et al.*, 1972; Healey *et al.*, 1977; Hrovat, 1993). This is illustrated by comparing the ideal white-noise-in-velocity curves (straight lines in log–log scales) with the two different measured road sections reproduced from Smith (1982) in Figs. 2(a) and (b). It can be seen that the road from Fig. 2(a) fits the white-noise assumption quite well, whereas the comparative fit is less satisfactory in Fig. 2(b), especially for lower frequencies.

Other, more sophisticated, variants of road description can be found in Rotenberg (1972), Bormann (1978), Hac (1987) and Hrovat (1993). The extensions of these single-track models toward a spatial road surface description (Parkhilovskii, 1968; Rill, 1983; Hrovat, 1993) will be evoked in Section 5 which deals with a full-car, 3D vehicle model. For the most part the present paper will address simple, quarter-car models for which a simple white-noise road description is judged quite adequate to expose the main potential benefits and limitations of different suspensions: passive, active and semi-active (Hrovat, 1993).

To contain the above disturbances the design of ground vehicle suspensions is influenced by a number of often conflicting factors. Good vibration isolation is required to secure the occupants' ride comfort, whereas good road holding is important for vehicle handling, which in general leads to enhanced safety. Key design constraints are represented by maximum allowable relative displacements between the vehicle body and various unsprung mass components, including wheels, bump stops and protruding parts of the steering mechanism. Additional constraints are imposed by the overall system robustness, reliability, weight and cost requirements.

There are many different ways to quantify ride comfort. This includes a consideration of: root-mean-square (rms) values of vertical accelerations measured at vehicle floor or occupant's seat location; rms jerk—the derivative of the above acceleration; as well as more elaborate frequency-dependent measures (Konik *et al.*, 1992; Hrovat, 1993). Further refinements are possible with more comprehensive measures, where the interdependence between various modes of vibration, such as heave, pitch and roll, and many other factors has been considered (Stephens, 1977; Hrovat, 1993).

It was argued in Hrovat (1993) that for the present purpose of establishing the predominant characteristics and performance potentials of active suspensions, the simple rms acceleration measure is

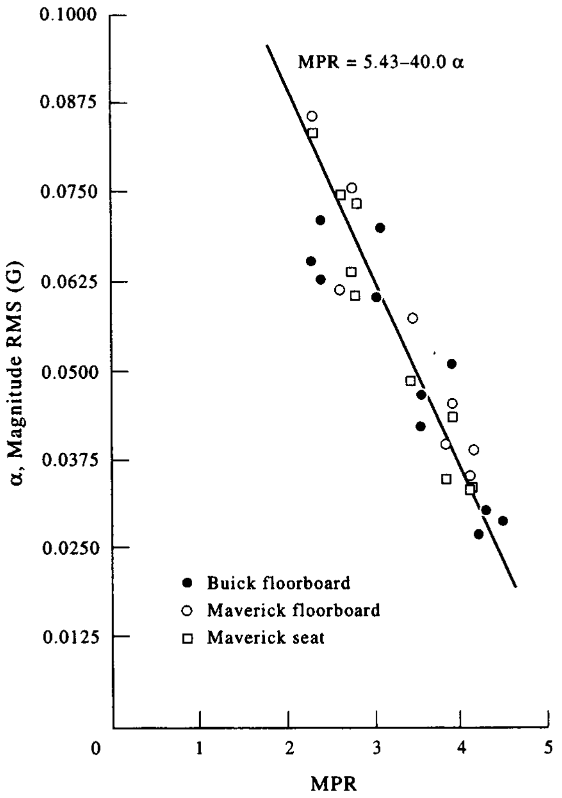


Fig. 3. Least squares curve fit to experimental rms accelerations versus mean personal ratings (Smith *et al.*, 1978).

quite adequate and will be primarily used throughout the paper. The adaptation of rms acceleration measure is based on the previous work by Smith *et al.* (1978). The authors conducted a field study involving two different automobiles, 78 passengers, and 18 different road sections to conclude that "excellent correlation was found to exist between the subjective ride ratings and simple root mean square acceleration measurements at either the vehicle floorboard or the passenger/seat interface". The main result from Smith *et al.* (1978) is reproduced in Fig. 3 which shows the correlation between the average or mean personal ratings and rms acceleration measured for combined vertical and lateral directions, where higher ratings imply better ride comfort. Similar correlations were obtained for vertical-only accelerations. Additional performance measures that take into account the suspension stroke limitations and tire handling constraints will be introduced in subsequent sections.

Different suspensions satisfy the vehicle ride and handling requirements to differing degrees. Although significant improvements can result from a designer's ingenuity, on the average, suspension performance is primarily influenced by the type or class of suspension used. Here one distinguishes, in an ascending order of improved performance, between passive, semi-active, and fully active suspensions. Passive suspensions are those found on most conventional vehicles. Roughly, they are characterized by the absence of external power sources, whereas the active suspensions require additional sources, such as compressors and pumps, to achieve superior ride and handling performance. The semi-active (or semi-passive) suspensions (Crosby and Karnopp, 1973) then fill the gap between their passive and active counterparts, since they essentially act as time-varying dampers and thus offer potentially significant performance improvements while requiring relatively small or negligible external power sources.

It is interesting that, although active and passive suspensions have been researched for decades, most of the studies neglected to clearly and precisely define the active/passive concepts. Because of this, the "active" suspensions are sometimes being referred to as the ones with "intelligence" in the form of computers and sensors. This however, may lead to misclassification of computer controlled shock absorbers as "active" devices.

More precise definitions of active and passive suspensions were given in Hrovat (1979) and Hrovat *et al.* (1980). These were based on similar definitions of passivity and passive operators as used in electrical networks and mathematics, for example (Anderson and Vongpanitlers, 1973). In the case of suspensions, the operator in question relates the suspension actuator force with the corresponding velocity across the actuator mounting points (Hrovat, 1993). For passive suspensions, the force–velocity relationship satisfies the mathematical definition of passivity, which in turn reflects power dissipation requirements characteristic of passive elements such as springs and shock absorbers. This approach was used to establish the passivity/activity of optimal control laws for simple quarter-car vehicle models (Hrovat *et al.*, 1980; Hrovat, 1982).

Before closing this section, it is appropriate to briefly review different vehicle models that are used for design of optimal suspension ride control algorithms. Such a design typically starts with simple quarter-car models which allow for only 1D, vertical or heave motion. Within this setting, the simplest possible, 1 DOF model consists of just the vehicle or sprung mass, which, as shown in Fig. 4, is supported by a suspension actuator placed between the vehicle and ground. The optimal suspenions for 1 DOF models will be discussed in Section 3.1.

For the models used in this work it is a common practice to assume that the vehicle is moving forward with a constant velocity V. This is often justified by the fact that in most of the instances when ride is a primary concern, the vehicle is travelling under constant or slowly varying speeds.

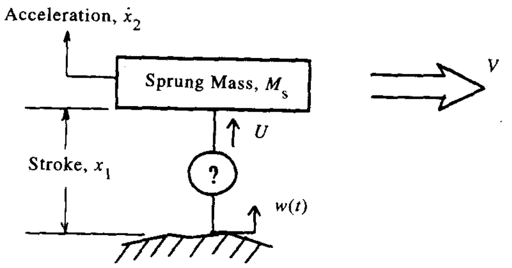


Fig. 4. 1 DOF model.

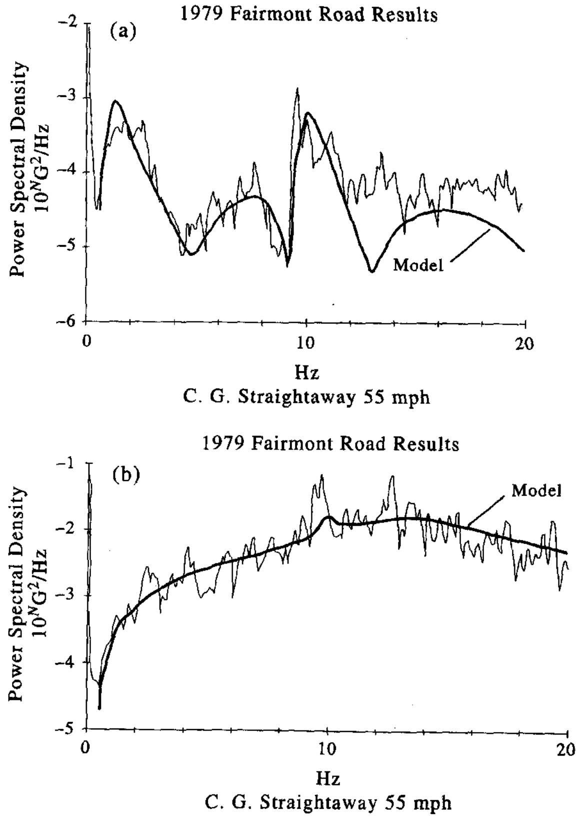


Fig. 5. (a) Comparison between measured and predicted CG acceleration p.s.d.s for a linear 6 DOF model as per Smith and Sigman (1981). (b) Comparison between measured and predicted unsprung mass acceleration p.s.d.s for a linear 6 DOF model as per Smith and Sigman (1981).

Under these conditions the road roughness disturbance appears as a vertical input proportional to V and the spatial slope of the road unevenness. It is in this context that one refers to the so-called "moving" ground.

Adding a wheel/tire subassembly (unsprung mass) to the above 1 DOF structure, leads to 2 DOF, quarter-car models. These are discussed in Section 3.2. The unsprung mass mode is often referred to as the "wheel-hop" mode, and is characterized by relatively light damping and a natural frequency between 8 and 12 Hz. The principal body mode or sprung mass mode, is typically around 1 Hz, with active suspensions leaning toward lower, 0.5 Hz-range values.

Typical half-car, 2D models include sprung mass pitch and heave or vertical modes, often augmented by front and rear unsprung-mass vertical dynamics. They are discussed in Section 4. The full-car, 3D models, on the other hand, include vehicle roll, as well as the pitch and heave modes and they are discussed in Section 5. These models consider only rigid body motions, and are based on lumped-parameter, linearized dynamics which is well-suited for applications of the LQG optimal control technique.

Most of the existing models of automotive ride dynamics are characterized by linear time-invariant differential equations. To what extent this assumption remains satisfied is best evaluated through actual vehicle tests. Some useful related results can be found in Healey *et al.* (1977), which are also discussed by Hrovat (1993).

Another set of validation tests was pursued by Smith and Sigman (1981), and Ruis and Borcherts (1981), where a linear 2D, single-track, 6 DOF vehicle model was used, consisting of sprung mass heave and pitch, front and rear unsprung mass bounce, and engine and passenger dynamics. The results in terms of measured and predicted C.G. acceleration p.s.d.s, and left front spindle (unsprung mass) acceleration p.s.d.s are reproduced in Figs. 5(a) and (b), respectively. The data were obtained with a 1979 Ford Fairmont traversing Ford's Dearborn Proving Ground straightaway at 55 mph. For the analysis, the measured road spectrum was approximated via appropriate exponential/log functions. Comparing the measured and predicted acceleration, it can be concluded that the agreement is in general very good for frequencies up to about 12 Hz, with the discrepancy at higher frequencies being again attributed to unmodeled, higher-frequency flexible modes, tire unbalance, and various nonlinearities.

Based on the above work, it can be concluded that linear, low-order models are in general adequate for optimal suspension control synthesis and analysis, which serves as a precursor necessary to establish the overall trends and R & D directions. Once a candidate suspension has been established in this way, further evaluations would typically proceed via more refined and complex models that would include flexible modes and various nonlinear effects. Final evaluation, of course, still implies the need for actual vehicle tests.

3. OPTIMAL SUSPENSIONS FOR QUARTER-CAR MODELS

Armed with the background information from the previous sections we will now review some of the many findings based on simple, quarter-car or "Unicycle", 1D vehicle models. These have been the subject of many investigations during the past 30 years. Here, it is the most appropriate to start with the simplest possible, 1 DOF models, which, while neglecting the unsprung masses, still yield some very useful results, forming the bounds for limiting optimal performance of more complex, higher degree-of-freedom models.

3.1. *One-DOF models*

In view of Section 2, the optimal suspension design problem for the 1 DOF model of Fig. 4 can be expressed as follows:

$$\text{minimize}\{\text{PI} = E(x_1^2 + ru^2)\} \tag{2}$$

subject to the state equations

$$\dot{x}_1 = x_2 - w, \tag{3}$$

$$\dot{x}_2 = u, \tag{4}$$

where the above expectation represents steady-state mean square values. The states x_1 and x_2 from Fig. 4 represent rattlespace (suspension deflection with respect to equilibrium) and vehicle vertical velocity with respect to an "inertial" ground—in practice, a low-frequency "smooth" component of road inputs. For now and in most of this paper, we assume that all the states are measurable and available for control. The rattlespace constraint is introduced in the performance index (2) to reflect a limited suspension stroke capability and thus prevent excessive suspension bottoming, which can lead to rapid deterioration of ride comfort and possible structural damage. As discussed in the previous section, the ground input velocity w is due to road roughness effects, and is typically modeled as a white-noise, Gaussian process specified via

$$E[w(t)] = 0, \tag{5}$$

$$\mathrm{E}[w(t_1)w(t_2)] = 2\pi W\,\delta(t_1 - t_2), \tag{6}$$

where $\delta(\cdot)$ stands for the impulse or Dirac delta function, and W is the two-sided road velocity power spectral density, equal to the product of the road roughness factor A and vehicle forward velocity V.

The optimal suspension design problem then consists of minimizing the performance index (2), which penalizes excessive suspension stroke and sprung mass acceleration u, subject to the state equations (3) and (4). The acceleration is a convenient control variable in the present setting, and it amounts to normalizing the original control force U from Fig. 4 with respect to the sprung mass m. The weighting parameter r in equation (2) acts as a "tuning knob", so that larger r results in smaller accelerations, i.e. improved ride comfort, and increased suspension stroke requirements.

It is interesting to observe that, although the above problem is in a standard LQ regulator (LQR) form, the first published solutions were obtained through optimal filtering based on the Wiener–Hopf approach (Bender, 1967a,b; Karnopp and Trikha, 1969; Bender *et al.*, 1967; Hullender *et al.*, 1972; Young and Wormley, 1973). The advantage of the LQ approach is that it also applies to linear, time-varying systems, and there are many well-developed numerical techniques for its solution in the most general multi-input–multi-output (MIMO) setting. Moreover, one can evoke the Certainty Equivalence Principle (CEP) (Sorensen, 1976), which states that the optimal controller for the original stochastic problem is the same as the optimal controller for an equivalent deterministic problem, obtained by replacing all random variables by their (minimal ms) expected values. In the present case, it can be shown (Hrovat and Hubbard, 1987) that the equivalent deterministic problem amounts to replacing the white-noise ground velocity by an equivalent step in ground displacement (perhaps the easiest way to see this is to think in terms of transfer functions with white-noise or impulse as ground velocity inputs). Alternately, the ground step can be replaced by an equivalent nonzero state (rattlespace) initial condition which occurs when dealing with optimal shock isolation during, for example, an airplane landing.

For the present second-order system, the LQ optimal control solution can be calculated analytically to yield the optimal control acceleration:

$$u_{\text{LQ}} = -r^{-1/2}x_1 - \sqrt{2}r^{-1/4}x_2. \tag{7}$$

The corresponding optimal trade-off curve is shown as a solid line in Fig. 6, where, as is the standard practice, both the rms acceleration and rattlespace have been normalized with respect to road/speed characteristics, so that $\tilde{x}_1 = x_{1,\text{rms}}/(2\pi AV)^{1/2}$. The normalization allows for more efficient representation of results, although it may occasionally be confusing. Note, in particular, that traveling on rough roads, and/or at high speeds, implies *smaller* normalized rattlespace (since the available rattlespace is fixed), and larger normalized accelerations. The solid straight line in Fig. 6 is parametrized by the weighting parameter r, and has a slope of -3 on the log–log scale, which implies that each 10% increase in allowed rattlespace results in 30% decrease in rms acceleration. Larger values of r result in softer suspensions, with smaller accelerations and larger rattlespace requirements.

Knowing that, under the assumptions in Section 2 and (Hrovat, 1993), all response variables are Gaussian, one can establish what the probability is of the suspension stroke remaining within certain bounds around the equilibrium. For example, the

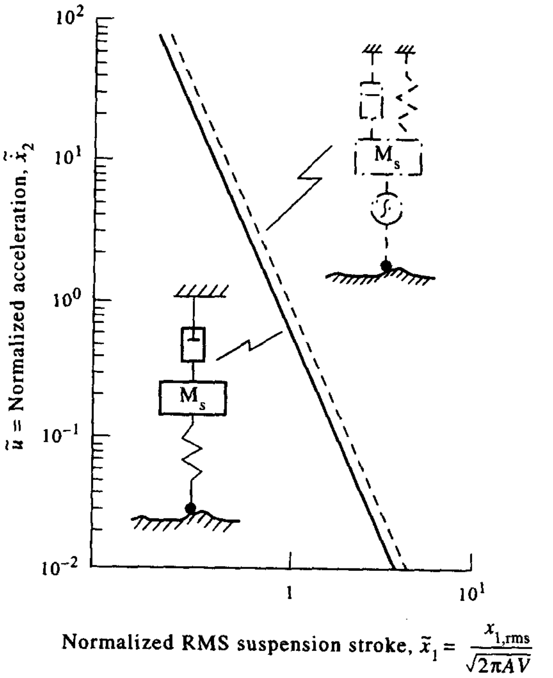


Fig. 6. Optimal performance trade-offs for 1 DOF model with PI_1 (full line) and PI_2 (dashed line).

variation of this variable will exceed three times the rms value during only 0.3% of ride time. It should also be remarked that, because of the Central Limit Theorem (Hamming, 1962), similar conclusions apply for non-Gaussian road sequences. This was also observed by a number of researchers (e.g. Healey, 1977; Hrovat, 1993) based on measured road data.

The optimal suspension structure can be deduced directly from the LQ problem formulation knowing that the LQ optimal control is a linear function of states. It consists of a linear spring placed between the vehicle body and the road, and a so-called "skyhook damper" placed between the vehicle sprung mass and an inertial, smooth ground. Since the latter is in practice not available from a moving vehicle, the actual suspension will have to be placed between the vehicle and the road which acts as a "moving ground". It can then be shown, using the previously discussed definition of passive systems, that the LQ-optimal suspension for the stochastic case is necessarily an active device, for any value of the weighting parameter r (Hrovat *et al.*, 1980).

Moreover, it follows from equation (7) that the optimal closed-loop system is oscillatory with the highly favorable damping ratio of $\sqrt{2}/2 \approx 0.7$. To put this in proper perspective, it should be pointed out that the corresponding ratios of typical conventional passive suspensions are around 0.2–0.3. As will be shown later, this range of value is close to optimal: smaller values would amplify the body resonance, whereas larger values would increase disturbance force transmissability from road through the damper. This is where the skyhook dampers fundamentally differ from their passive counterparts: by emulating a contact with the hypothetical, smooth, inertial ground, the optimal algorithm facilitates much larger damping rates leading to a well-damped body mode with no negative transmissibility-related side-effects. The net effect is a dramatic improvement in ride and handling for road/steering excitation frequencies around the body modes. For example, the dynamic sag due to body roll in turns can be two to three times smaller with the optimal active suspension of the same stiffness as its passive counterpart (Tseng and Hrovat, 1990a). Thus, even though the handling *per se* was not taken into account in the above optimization, the resulting optimal structure nevertheless offers substantial handling benefits.

An extension of the above elementary 1 DOF model was proposed in Hrovat and Hubbard (1981, 1987), where the sprung mass jerk, $\dot{u}$ (the derivative of acceleration u) was included as an additional comfort measure augmenting the PI of Eq. (2), which then becomes

$$PI_2 = E(x_1^2 + r_1 u^2 + r_2 \dot{u}^2). \tag{8}$$

The problem was transformed into a more standard, LQR form by defining the sprung mass acceleration u as an additional third state, and the sprung mass jerk $\dot{u}$ as the new control input. There are now two tuning parameters, r_1 and r_2, which weigh ms acceleration and jerk, respectively.

As shown in the above references, it is still possible to obtain the analytical solution for the resulting third-order LQR problem. The optimal structure in this case is shown with dashed lines in Fig. 6. In addition to a skyhook damper, this structure includes a skyhook spring and a load leveling device which operates on the integral of suspension deflection. This configuration in fact, has some desirable characteristics.

The load leveling secures zero steady offset in the case of external load disturbances caused by vehicle weight changes and inertia forces in turns or during braking, for example. Being fully integrated in the LQR synthesis process, this load leveling action can be relatively fast, with settling times of the order of 1–3 s (Hrovat and Hubbard, 1981; Hrovat and Hubbard, 1987), as opposed to more conventional slow-acting load levelers requiring dozens of seconds to settle. While the fast load leveling is facilitating load disturbance rejection and good tracking of low frequency components of the road, the skyhook damper and spring are minimizing transmissibility of road induced disturbances.

The corresponding optimal ride performance is represented by the dashed line in Fig. 6, which applies for the extreme case when $r_1 = 0$. This results in minimal possible jerk (and maximal acceleration) for a given rattlespace. Even in this extreme case there is a relatively modest (less than 11%) increase in rattlespace requirements for the same level of rms accelerations (Hrovat and Hubbard, 1981).

On the other hand, at the other extreme where $r_2 = 0$, for the "standard" 1 DOF case, the rms jerk is theoretically infinite. Taking into account the wheel-hop and other neglected dynamics, this implies that substantial reduction in jerk will be obtained with the new jerk-optimal structure. This in practice means that the high-frequency, NVH isolation of the proposed jerk-optimal suspension will be much superior to the one achievable with the standard structure.

The basis for this improved low-pass filtering is in the fact that the conventional spring in the standard configuration has been replaced by an integrator and a skyhook spring, both reducing the transmission at high frequencies. The improved high-frequency filtering in this case also leads to less stringent desired force bandwidth requirements. This is a favorable consideration for narrow bandwidth or "slow-acting" active suspension actuators, which hold the most promise for practical production implementations.

It is interesting that a similar, fast load-leveling structure with skyhook spring and damper was also obtained by Karnopp (1987), using a different approach based on judicious usage of root locus and other classical control techniques. The same reference also proposed a related practical hardware implementation suitable for narrow bandwidth actuators. This will be discussed further in Section 6.4.

In summary, the application of optimal control technique to a single, 1 DOF, quarter-car or "unicycle" model has resulted in fundamental insights about the optimal suspension structure and performance potentials. This includes the concept of a skyhook damper, which has found applications in most of the semi-active and active suspensions in production or under research and development. Finally, it should be observed that, although the simple model neglects the wheel-hop mode, the above principal isolation structure is still near optimal and, as needed, can be augmented by a damper in parallel with the active actuator. Typically, this configuration is often used in practice to control the 2 DOF case. It will be shown next, through a detailed discussion of the 2 DOF case, that this strategy is usually not far from optimal.

3.2. *Two DOF models*

A typical 2 DOF model structure including wheel-hop dynamics is shown in Fig. 7(a), along with its 1 DOF active [Fig. 7(b)] and 2 DOF passive [Fig. 7(c)] counterparts. This allows for inclusion of a handling measure in the performance index in the form

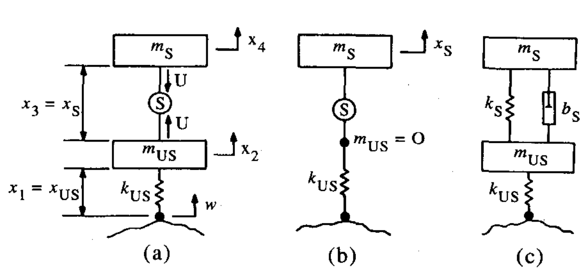


Fig. 7. (a) 2 DOF vehicle model with active suspension S; (b) corresponding limiting case when $m_{us} = 0$; (c) 2 DOF model with conventional passive suspension with sprung and unsprung masses m_s and m_{us}, and corresponding stiffnesses k_s and k_{us}.

$$\mathrm{PI}_3 = E(r_1 x_1^2 + r_2 x_3^2 + u^2), \tag{9}$$

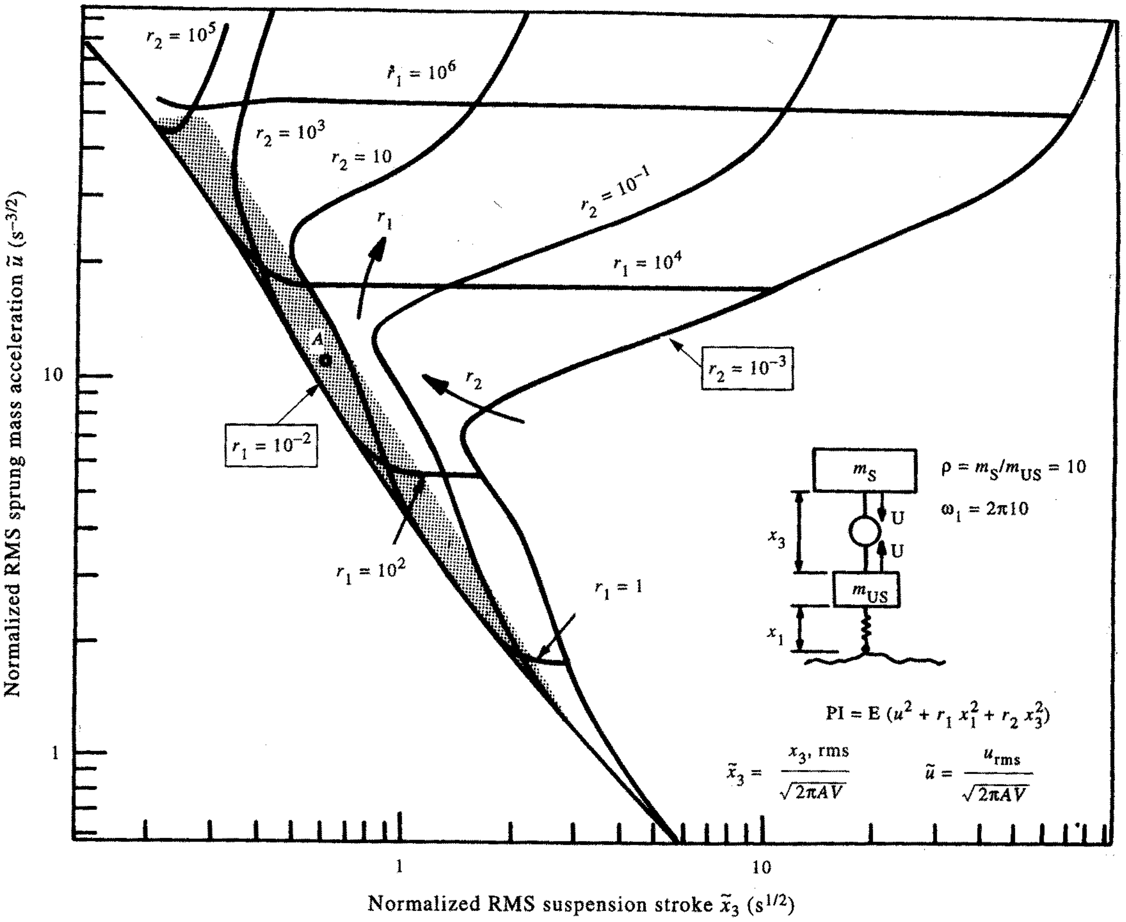
where x_1 and x_3 are the states representing tire and suspension deflection as per Fig. 7. The tire deflection is an indication of road holding ability, in the sense that smaller tire deflections (from a suitable equilibrium) lead to better vehicle handling characteristics (Lozia, 1991; Hrovat, 1993). Since this term in equation (9) represents an additional "soft" constraint, it is to be expected that the performance, in terms of rms acceleration and rattlespace variables, will somewhat deteriorate.

The early research in this area led to conclusions about a relatively large amount of performance degradation, which was published in a somewhat pessimistic tone by a number of authors, probably first in Kawagoe (1983) and Kawagoe and Iguchi (1985). The authors claimed that, for the same level of tire and suspension deflection "compared to a passive system, optimal active control can reduce the rms vertical acceleration of a sprung mass by about 18%". This was a somewhat disappointing conclusion, since the reduction of only 18% may not even be perceived as a significant ride improvement by most drivers and vehicle occupants.

However, since the above finding was based on a single optimization point (one set of weights r_1 and r_2), it was of interest to see to what extent the above conclusion can be globalized. This prompted the related global study (Hrovat, 1984, 1987–1988), based on varying r_1 and r_2 throughout the range of values of practical significance. The solution of the corresponding fourth-order LQ problem, that was repeated hundreds of times was facilitated by the use of MATRIXx control systems CAD tool (Anonymous, 1983–1984).

The results of the global study are shown in Figs. 8(a) and (b) from Hrovat (1987–1988) in the form of "carpet" plots of normalized rms acceleration vs. suspension and tire deflections, respectively. Here, the ratio of sprung-to-unsprung mass was

(a)



(b)

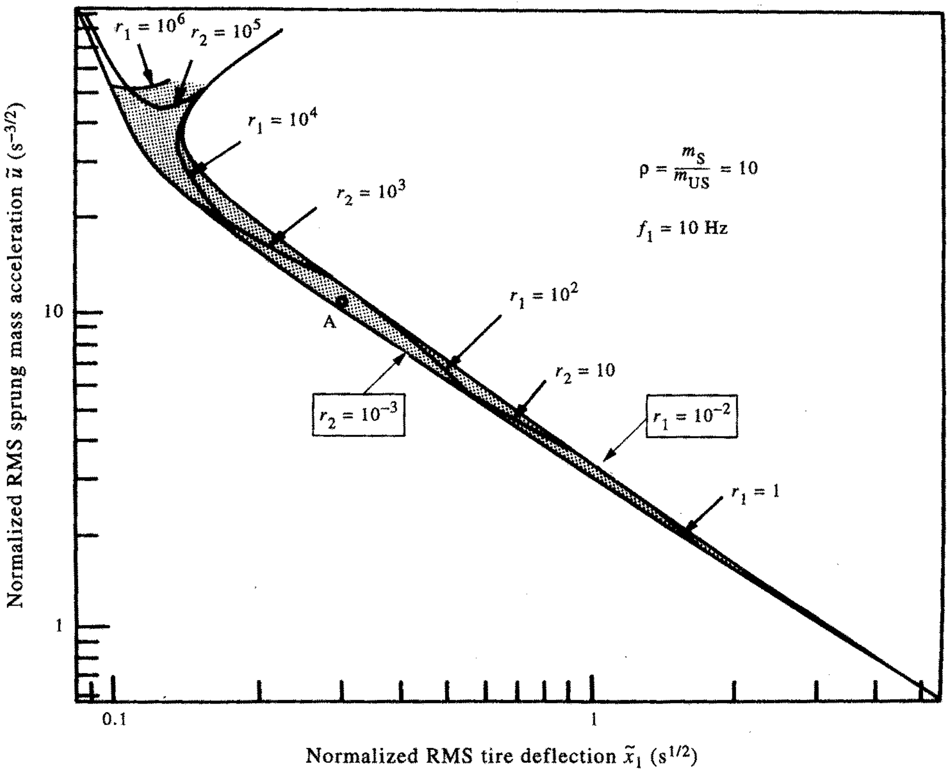


Fig. 8. (a) Optimal normalized r.m.s. acceleration versus normalized r.m.s. suspension stroke $\tilde{x}_3$ for 2 DOF model with $\omega_{us} = 2\pi \cdot 10$ rad s^{-1} and $\rho = 10$. (b) Optimal normalized r.m.s. acceleration versus normalized r.m.s. tire deflection $\tilde{x}_1$ for 2 DOF model with $\omega_{us} = 2\pi \cdot 10$ rad s^{-1} and $\rho = 10$ (Hrovat, 1987–1988).

$\rho = 10$, and the unsprung natural frequency was $\omega_{us} = 10 \times 2\pi$ rad s^{-1}. The plots are parameterized in terms of the PI weights r_1 and r_2, which act as "tuning" knobs controlling the optimal tradeoff between acceleration, suspension stroke (design constraint) and tire deflection (handling constraint).

For example, from Fig. 8(b) it follows that reducing r_1 while keeping r_2 constant results in increased tire deflection and decreased sprung mass acceleration. On the other hand, Fig. 8(a) shows that reduction of r_2 while keeping r_1 constant leads to increased suspension deflection and decreased accelerations. In the extremum case of $r_2 = 0$, the problem becomes a degenerate one where extremely large suspension deflections are possible since the suspension force is not a function of suspension deflection and thus cannot give static support for the vehicle weight. Indeed, Fig. 8 shows that there is no incentive to increase r_1/r_2 above ~ 1000, since then the acceleration and tire deflection remain practically constant, while the suspension deflection increases significantly. Similar problems do not occur if only $r_1 = 0$ and this case can be used for preliminary suspension design. In addition, the area of relatively large normalized accelerations (above say 50) in Figs. 8(a) and (b) corresponds to very large disturbance levels typical for off-road operations, whereas the area in the lower right corners is characteristic of very smooth roads and/or low vehicle speeds.

When these considerations are taken into account, it becomes evident that most optimal designs of practical interest for automotive applications will fall within the shaded area in Figs. 8(a) and (b). In particular, note that no control design, linear or non-linear, can fall below the $r_1 = 10^{-2} \approx 0$ line in Fig. 8a and $r_2 = 10^{-3} \approx 0$ line in Fig. 8(b). Typical usage of the global performance plots is best shown through an illustrative example.

Illustrative example. Assume that for a particular vehicle $\rho = 10$ and $\omega_{us} = 2\pi \cdot 10$ rad/s. The road is described by a road roughness coefficient $A = 1.6 \times 10^{-5}$ ft (4.9×10^{-6} m) which corresponds to a medium quality road. The vehicle is traversing this road at a speed of $V = 80$ ft s^{-1} (88.5 km h^{-1}). The tire deflection from equilibrium should be below 1 in (2.54 cm) 99.7% of the time so that $x_{1,\,\mathrm{r.m.s.}} < \frac{1}{3}$ in $= 0.847$ cm and $\tilde{x}_1 = x_{1,\,r.m.s.}/\sqrt{2\pi A V} < 0.3$ s$^{1/2}$. From Fig. 8(b) the minimum possible normalized rms acceleration is $\tilde{u} \approx 10x^{-3/2}$. Accepting $\tilde{u} = 10.9$ s$^{3/2}$ results in 3% g rms acceleration and tuning parameters $r_1 = 1100$ and $r_2 = 100$ shown as Design A. Using these values for r_1 and r_2 in Fig. 8a reveals a normalized rms secondary suspension deflection of $\tilde{x}_3 = 0.605$ s$^{1/2}$ which insures that the suspension deflection will remain within ± 2 in (5.08 cm) of the static value 99.7% of the time. With $r_1 = 1100$ and $r_2 = 100$ the optimal control gains are $k_1 = -6.084$, $k_2 = 0.548$, $k_3 = -10.0$, $k_4 = -4.438$, and the closed loop eigenvalues are $e_{1,2} = -2.20 \pm j2.26$ and $e_{3,4} = -2.75 \pm j62.9$. The first set of oscillatory eigenvalues corresponds to the vehicle heave mode with a natural frequency of 0.5 Hz and damping ratio of ~ 0.7. The second set corresponds to the wheel-hop mode with a resonant frequency of 10 Hz and relatively small damping ratio of 4.4%. Whether or not this small amount of damping is of consequence will depend on the particular hardware implementation, which may include a combination of active and passive means to improve closed-loop robustness. Also, in practice, an excessively soft suspension setting may require load-leveling to keep the suspension static or steady-state deflection close to zero.

With the help of the above carpet plots, it is now possible to put the conclusions from Kawagoe (1983), and Kawagoe and Iguchi (1985) into proper perspective. For this purpose consider Fig. 9 from Hrovat (1984), which displays normalized rms accelerations versus tire deflection for active and passive suspensions. For simplicity, the optimal active suspension performance is shown via the limiting curves $r_1 \approx 0$ and $r_2 \approx 0$ only, which are representative of the main trends. Here, the design points with lower accelerations and higher tire deflections are characterized by softer, lower frequency body modes and less damped wheel-hop modes.

A corresponding passive suspension performance for the structure shown in Fig. 7(c) is shown in Fig. 9 for comparison purposes. The passive performance trade-offs correspond to vehicles with heave mode resonant frequencies, f_2, between 1 and 1.5 Hz, and damping ratios, ζ, between 0.02 and 1. A typical passive suspension setting is shown as point P_1, where $f_2 = 1$ Hz and $\zeta = 0.3$. As can be seen from Fig. 9, this amount of damping is near optimal in terms of ride and handling compromise for the passive case. As mentioned earlier, this is within the range typically found in most of today's vehicles which have evolved through many iterations primarily based on intuition and experience.

The performance of an optimal active suspension with the same amount of wheel-hop is shown in Fig. 9 as point A_1, which approximates the setting chosen Kawagoe (1983). Comparing P_1 and A_1, it can be seen that the active suspension results in only 11% lower rms acceleration levels. Thus, it can be said that the conclusion from Kawagoe (1983) and Kawagoe and Iguchi (1985) certainly applies locally, in the vicinity of A_1 and P_1. However, it can also be seen that for some other road/speed operating conditions (to the right of A_1), there is a potential for substantial ride

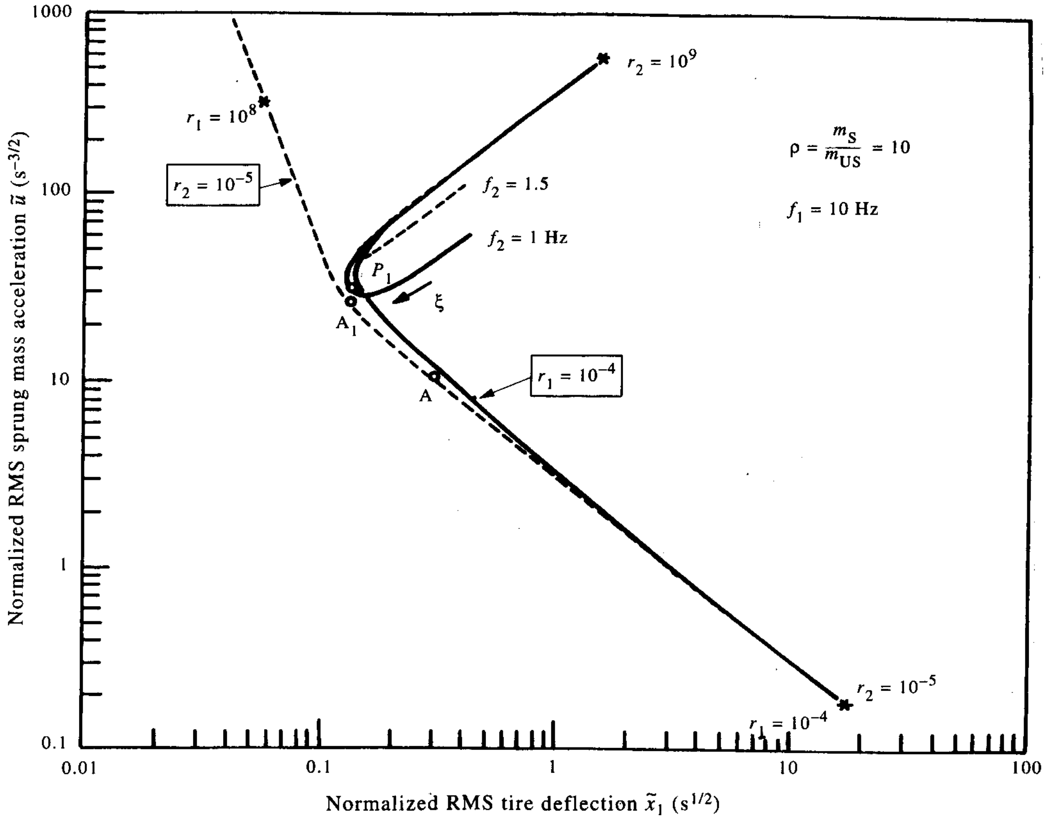


Fig. 9. Comparison between conventional passive suspension with $\rho = 10$, $\omega_{us} = 2\pi \cdot 10$ rad s^{-1}, $0 < \zeta < 1$, and optimal active suspension with $\rho = 0$ and $\omega_{us} = 2\pi \cdot 10$ rad s^{-1} (Hrovat, 1984, 1987–1988).

improvement for the same amount of wheel-hop. Indeed, as shown in Fig. 9, for passive suspensions almost any deviation in operating conditions from P_1 will result in degradation of performance. On the other hand, for active suspensions, either handling or ride can be improved by choosing the tuning parameters r_1 and r_2 such that the resulting operation settles either to the left or to the right of A_1, respectively. For example, the acceleration for design case A treated previously is 67% smaller than the one at P_1 in Fig. 9.

This indicates that, as pointed out by Hrovat (1984), the full advantage of active suspensions stems from possible *adaptive* tuning (or gain scheduling) of controller parameters, depending on the driving condition. For example, if the steering wheel position or lateral acceleration sensors indicate operation on a straight section of a road, where handling is less critical, then it may be possible to relax the wheel-hop constraint. The exact amount of wheel-hop that could be tolerated under different driving conditions should be predetermined through appropriate vehicle tests. Similar views regarding active suspensions were expressed in, for example (Hac, 1987; Sharp and Hassan, 1986), although this latter reference used a different road model and (dis)comfort metric.

A different view was taken in Chalasani (1986a), which was one of the first detailed, global optimal suspension studies to be published in the open literature. The study was based on a somewhat different performance index, which included unsprung mass velocity instead of tire deflection. Despite this, the overall approach and main results were similar in spirit to (Hrovat, 1984, 1987–1988). The main difference though, was in the interpretation of the results, since Chalasani (1986a) requires that the wheel-hop damping ratio be at least 0.2, which corresponds to a typical conventional passive suspension. This, in turn, severely limits the active suspension performance which led the author to conclude that, "Active suspensions based on linear, full-state-feedback control laws do not result in dramatic improvements in vibration isolation ... ". A 20% reduction in rms acceleration was quoted in Chalasani (1986a) as a typical example of performance improvement potentials. As shown before, such a constraint makes sense, and is indeed near optimal for passive suspensions, but can be unnecessarily conservative for their active counterparts.

At this stage it is appropriate to place the above 2 DOF, quarter-car results in proper perspective with respect to their 1 DOF counterparts. For this

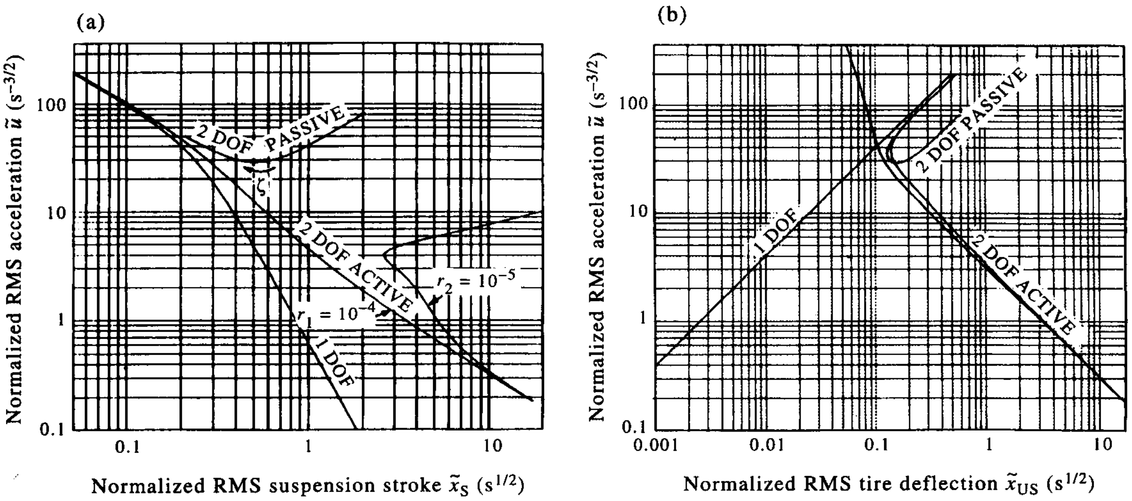


Fig. 10. Comparison between conventional passive suspension ($\omega_{us} = 2\pi$ rad s^{-1}, $\rho = 10$, $\omega_{us} = 2\pi \cdot 10$ rad s^{-1}, $0 < \zeta < 1$), optimal 2 DOF suspension ($\omega_{us} = 2\pi \cdot 10$ rad s^{-1}, $\rho = 10$), and optimal limiting case where $m_{us} = 0$: (a) u versus x_s; (b) $\tilde{u}$ versus $\tilde{x}_{us}$ (Hrovat, 1990).

purpose we will first introduce a new, modified 1 DOF model from Hrovat (1987–1988), which will facilitate comparisons with 2 DOF systems. The new 1 DOF model, shown in Fig. 7(b), corresponds to a special, limiting 2 DOF case, where the unsprung mass is reduced to zero. The optimal suspension control problem for this special case has been analytically solved with the help of appropriately defined states. The resulting optimal trade-off curves are shown in Fig. 10.

Note from Fig. 10(a) that for normalized rms suspension strokes $\tilde{x}_3$ above roughly 0.3, this new 1 DOF performance is similar to the traditional one. However, for $\tilde{x}_3$ below 0.3 the new 1 DOF curve is deflected toward smaller acceleration values. This is due to the gradual stiffening of the (secondary) suspension so that more and more of the overall deflection is now being taken by the (primary) tire spring. Of course, the overall deflection is still larger than in the traditional optimal 1 DOF case.

Also displayed in Fig. 10 are the corresponding limiting performance curves for a typical 2 DOF passive and optimal active suspensions. It can be seen that, while the optimal active suspensions for 2 DOF systems can still significantly outperform passive ones, they fall short of the optimal 1 DOF performance. For example, at a normalized rms suspension stroke of $0.6s^{1/2}$, the 1 DOF case results in a normalized rms acceleration of $3s^{-3/2}$, whereas the 2 DOF results in $10.9s^{-3/2}$, a 263% increase. The main reason for this deterioration stems from the conflicting requirements imposed on the active actuator of Fig. 7(a): it should simultaneously provide small sprung mass acceleration (for comfortable ride), and a considerable amount of unsprung mass damping needed to reduce the wheel-hop (for good handling). It is easier to satisfy these conflicting requirements when the unsprung mass becomes smaller, which gives an additional incentive for reducing the unsprung weight through the use of, e.g., aluminum wheels and lightweight, composite materials. The best performance within the present single-actuator 2 DOF structure of Fig. 7(a) is then obtained when $m_{us} = 0$, as shown by Figs. 10(a) and (b).

Recognizing that the optimal control has already achieved maximum possible improvements for a given *structure* and unsprung mass, the comparison between one- and two-DOF trade-offs points to the need for *fundamental structural/design* changes that will lead to increased wheel-hop damping, without an associated penalty on ride comfort. This prompted the author in early 1984 to consider the usage of dynamic vibration absorbers, (cf. Hrovat, 1993 and references therein), attached to the unsprung mass as shown in Fig. 11.

Dynamic absorbers are often used to contain pronounced, lightly damped oscillations. Typical applications of dynamic absorbers (DA) range from electric hair clippers (Den Harton, 1956) to tuned mass dampers for structural control of tall buildings (Hrovat *et al.*, 1983). In automotive applications, dynamic absorbers have often been used, sometimes as afterthoughts, to reduce (unexpected) vibrations. However, apparently there has been only one widespread, production application of dynamic absorbers with the aim of improving ride comfort. This is the case of the well-known French subcompact Citroen 2 CV, first introduced in 1949. The 2 CV was legendary for having a superior ride for a car of its size.

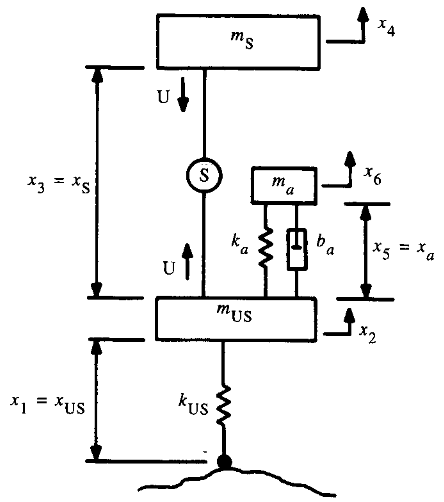


Fig. 11. 2 DOF vehicle model with dynamic absorber.

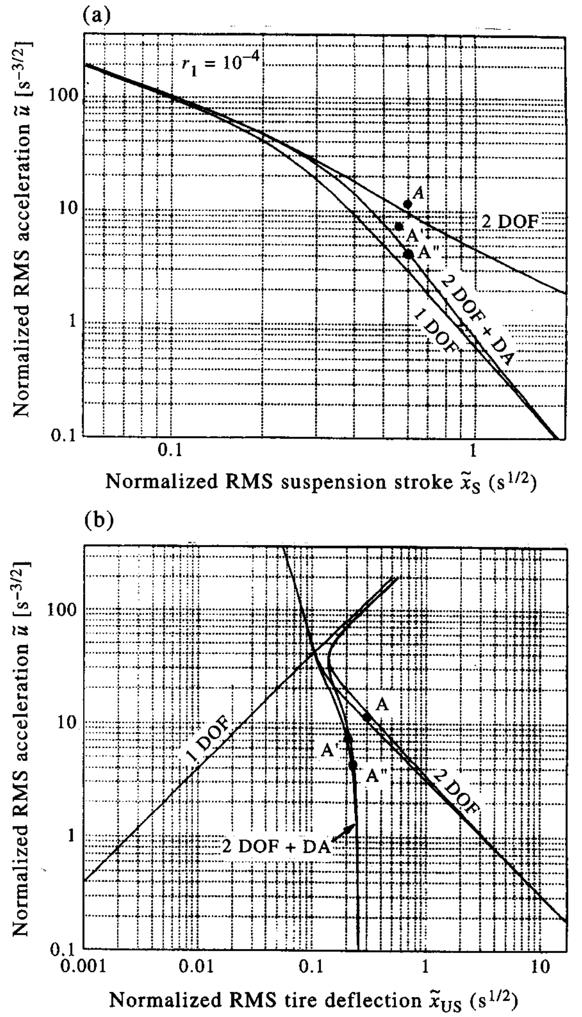


Fig. 12. Comparison between optimal 2 DOF suspension ($\omega_{us} = 2\pi \cdot 10$ rad s^{-1}, $\rho = 10$), a suboptimal 2 DOF + DA suspension ($\omega_a = \omega_{us}$, $\rho_a = \rho$, $\zeta_a = 0.2$), and optimal limiting case when $m_{us} = 0$, and $r_1 = 10^{-4}$ and $r_2 = 10^{-5}$ (Hrovat, 1990).

In the area of analysis, the pioneering work by Thompson (1970–1971) was probably the first to consider dynamic absorbers in the context of (active) vehicle suspensions. A global study of the potential benefits of DA was reported in Hrovat (1987, 1990). The resulting plots of limiting performance trade-offs are shown in Fig. 12, where the traces marked "2 DOF + DA" correspond to optimal 2 DOF controller gains with an appended DA with the mass equal to 10% of the unsprung mass, natural frequency $\omega_a = \omega_{us} = 2\pi \cdot 10\,r\,s^{-1}$, and a damping ratio $\zeta = 0.2$.

Comparing the 2 DOF + DA case with the optimal 2 DOF and 1 DOF trade-offs in Fig. 12(a), it can be seen that the DA can contribute to substantial improvements in performance, especially for $\tilde{x}_s > 0.2\ s^{1/2}$. As an example, point A represents the aforementioned 2 DOF performance (cf. Figs. 8 and 9), with an unsprung mass damping ratio of only 4.4%. The addition of a DA results in a performance at A′ in Fig. 12, where acceleration, rattlespace and tire deflection have all been reduced, while at the same time the wheel-hop damping ratio increased to 16%. Further substantial reduction in acceleration results for point A″, which has the same suspension stroke requirements and lower tire deflection than A. This clearly illustrates that the addition of DA has significant potential for improved ride and handling performance of active suspensions. Their main drawback is the added weight and packaging requirements, which represent a real design and development challenge that must be resolved prior to their possible widespread usage.

Further performance benefits are possible with the help of various forms of active DA's and unsprung mass actuators as proposed by Hrovat (1987, 1990, 1993). At this point, it is appropriate to ask what is the best possible quarter-car structure that would result in an absolute minimum for performance index PI_3 of equation (9). For this purpose, consider the suspension configuration from Hrovat (1990), shown in Fig. 13(a). Unlike the conventional 2 DOF structure from Fig. 7(a), the proposed configuration is not constrained by the requirement of equal sprung and unsprung forces, U_s and U_{us}. This implies that such a two-control configuration is very flexible and powerful. For example, the unsprung control force U_{us} can be used to alter both the effective unsprung mass and tire stiffness without negatively affecting the sprung mass acceleration which is controlled via the sprung mass force U_s.

As shown by Hrovat (1990), the above inquiry into the best possible 2 DOF quarter-car performance leads to a singular optimal control problem characterized by a lack of penalty on the unsprung actuator force [cf. PI_3, equation (9)]. This is known as a "partially cheap optimal control problem"

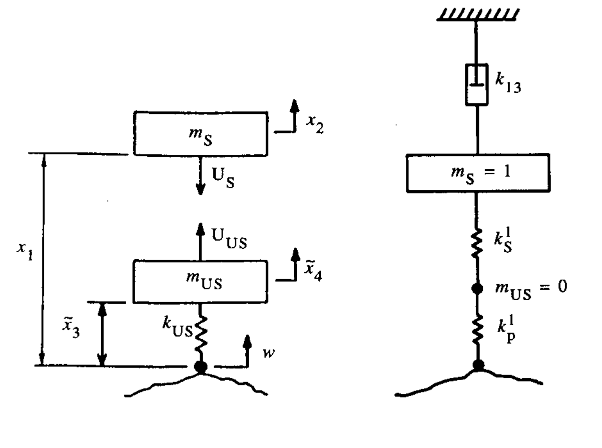


Fig. 13. (a) 2 DOF vehicle model with active suspension capable of producing independent forces U_s and U_{us}; (b) singular optimal incremental suspension structure (Hrovat, 1990).

(Kokotovic, 1984). In this context the cheap, unsprung control primarily serves as a structure optimizer: it can effectively reduce or eliminate the unsprung inertia and add or substitute additional inertias and compliances as needed to minimize PI_3.

The solution to the above optimization problem was developed in (Hrovat, 1987, 1990) by transforming the above cheap control problem to an equivalent singular perturbation problem (Kokotovic, 1984), using a blend of physical reasoning and a special scaling introduced in Saberi and Sannuti (1987). The resulting optimal structure is shown in Fig. 13(b). With the hindsight from (Hrovat, 1990), this simple (yet another special 1 DOF optimal) structure can be explained as follows. The sprung mass control U_s is used to contain the slow body-related mode in accordance with an appropriately modified, "standard" 1 DOF optimal control law with the damping ratio of 0.7. The cheap and fast control U_{us} accomplishes two tasks. In effect, it completely eliminates the unsprung mass, and adjusts the primary and secondary incremental stiffnesses so that the unsprung motion can instantaneously adapt to the best possible configuration for a given set of performance weights, r_1 and r_2.

Because of the presence of two independent control forces, U_s and U_{us}, the new 1 DOF optimal structure of Fig. 13(b) differs substantially from the previously discussed 1 DOF configuration of Fig. 7(b), since the latter cannot alter the primary suspension stiffness. As such the new 1 DOF optimal structure serves as a metric of the best possible PI_3 performance that can be obtained with 2 DOF, quarter-car models. For an illustrative example introduced in Hrovat (1990), this limiting case resulted in normalized rms sprung mass acceleration of only $1.17\ s^{-3/2}$, with normalized rms tire and suspension deflections of 0.23 and $0.6\ s^{1/2}$, respectively (to fully appreciate this performance, the reader should consider Fig. 12). This example illustrates that further substantial ride and handling improvements are possible with high-performance, active and semi-active unsprung mass actuators. While in some cases this superior performance may not be practical due to packaging, weight and cost constraints, it serves as an absolute reference against which any practical implementation can be compared.

As pointed out in Chalasani (1986a), a large portion of the benefits of an active suspension is obtained from the slow, body-motion-related mode. This can be seen from the optimal system eigenvalues and the optimal skyhook-like structure, which, to a degree, contributes to improved high-frequency isolation. The dominant, slow-mode eigenvalues for the optimal suspensions are characterized by substantially more damping than their conventional, passive counterparts. The net effect can also be nicely seen from frequency response plots, as contained for example in Kawagoe (1983), Kawagoe and Iguchi (1985), and Chalasani (1986a). The latter reference also revealed that, unlike their passive counterparts, the LQ-optimal suspensions can result in a non-zero DC gain between the rattlespace and road velocity input. However, in practice, this may not be too detrimental, since one would typically use high-pass filters to eliminate the very low frequency components of the ground input signals resulting from hills and prolonged road grades.

Additional interesting aspects pertinent to active suspensions—such as different invariant properties and related ride and handling effects—have been revealed in papers by Yue *et al.* (1988), Hedrick and Butsuen (1990), and more recently by Smith (1995). In particular, Hedrick and Butsuen (1990) have shown, using the simple quarter-car model example, "that only one of three transfer functions of interest (acceleration, suspension deflection and tyre deflection) can be independently specified".

Regarding the possible improvements around the fast, wheel-hop mode, it was argued in Yue *et al.* (1988) that these improvement potentials are relatively small due to the presence of an "invariant point" in the sprung mass acceleration transfer function. More precisely, it was shown that, under the assumption of zero tire damping, the magnitude of the transfer function between body acceleration and road velocity at the wheel-hop frequency remains constant, independent of the control force. However, Levitt and Zorka (1991) have shown that even a small amount of tire damping could alter the above invariance and lead to a significant change in body acceleration around the wheel-hop frequency.

The results of a global numerical study (Hrovat, 1988) demonstrated that even a tire damping ratio of only 2% can substantially improve the shape of the limiting optimal curves, which become much closer to the optimal 1 DOF curves in Fig. 12. It was also shown there that even in this case, the dynamic absorber still offers tangible benefits, which are, of course, somewhat less dramatic than before. Since in practice the tire damping can vary from a few percent at low speeds to near zero at high speeds, one should have the above results in mind when evaluating different passive and active suspension concepts. Nevertheless, the above invariant properties offer useful insight and guidance relevant for many vehicle operation modes, especially for cruising at higher speeds.

It should also be noted that, due to a natural separation between the slow, body mode and the fast wheel-hop mode, it is possible to use singular perturbations theory to partition the problem into an equivalent slow 1 DOF system and a fast, wheel-hop associated subsystem (Salman *et al.*, 1990; Krtolica *et al.*, 1990). This again creates a link with and further justification for 1 DOF studies.

Finally, one could parallel the 1 DOF case and introduce jerk as an additional ride term in PI_3, equation (9). This was done for 2 DOF, quarter-car systems by Rutledge (1990), Rutledge *et al.* (1996), and Tseng and Hrovat (1990b), where numerous global optimal trade-off maps have been developed. Similar 2 DOF variants of jerk-optimal 1 DOF investigations can be found in Thompson and Davis (1988), and ElMadany (1990a), where in addition to using the derivative control constraint, the authors relied on additional integrators for state estimation purposes. Further extensions of the fast load-leveling concept, based on Linear–Quadratic-Integral (LQI) control, are given in (ElMadany, 1990b; Edwards *et al.*, 1996). As in the more conventional 2 DOF case of PI_3, the jerk-augmented performance can be further improved via DA's and other structural means as investigated by Thompson and Davis (1989), and on a global scale by Tseng and Hrovat (1990c).

In summary, the optimal suspensions for 2 DOF quarter-car models without tire damping result in significant loss of performance with respect to their 1 DOF counterparts. Nevertheless, such an optimal 2 DOF system can still substantially outperform the conventional suspensions due to the *inherent adaptive* capabilities of active suspensions. Further improvements are possible through a reduction of the unsprung mass and through structural changes aimed at containing the wheel-hop in a manner that would not negatively impact the quality of the isolation of the sprung mass vibration.

4. OPTIMAL SUSPENSIONS FOR HALF-CAR MODELS

There are a number of publications (e.g. Thompson, 1979; Hac, 1986) investigating optimal and suboptimal ride characteristics of half-car, 2D vehicle models, which include both sprung mass heave and pitch modes as shown in Fig. 14. Most of the past work was based on numerical analysis which was used to evaluate optimal active suspension performance at a (usually) single operational point. This was extended by Krtolica and Hrovat (1992), who developed a global analytical solution for the optimal suspension control problem minimizing

$$PI_4 = E(r_1\ddot{z}^2 + r_2\ddot{\Theta}^2 + r_3z_f^2 + r_4z_r^2) \qquad (10)$$

subject to 2 DOF, fourth-order system state equations describing vehicle heave and pitch dynamics as per Fig. 14. In equation (10) the first two terms penalize excessive heave and pitch accelerations, $\ddot{z}$ and $\ddot{\Theta}$, respectively, while the last two terms penalize excessive front and rear suspension deflections, z_f and z_r.

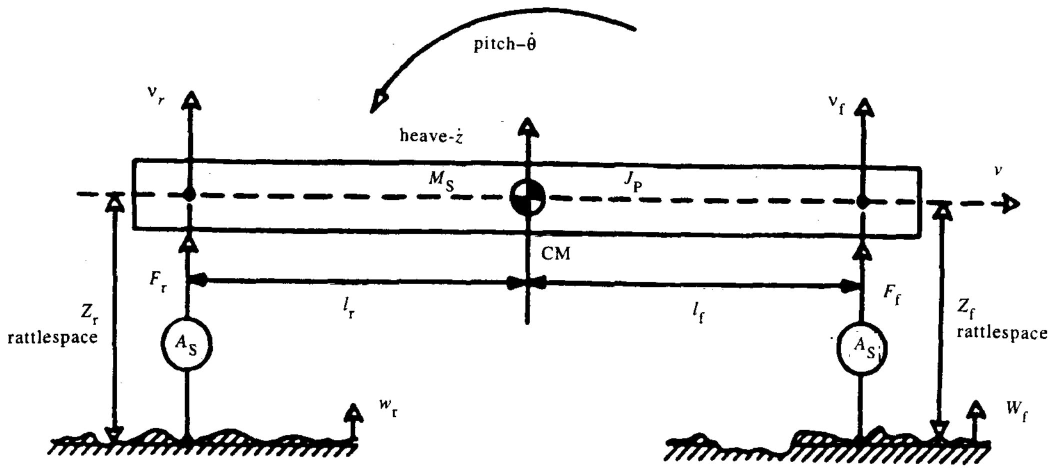


Fig. 14. Half-car 2 DOF model.

As a byproduct of the analytical solution, it was possible to establish a number of properties characterizing the resulting LQ-optimal suspension. For example, as in the simple 1 DOF quarter-car case discussed in Section 3.1, the optimal half-car suspension is also characterized by a favorable damping ratio of 0.7. The above reference by Krtolica and Hrovat (1992) also contains necessary and sufficient conditions for decoupling of the original 2 DOF, half-car problem into two 1 DOF, quarter-car subproblems. These conditions are

$$M_s \cdot l_f \cdot l_r = J_p, \tag{11}$$

$$r_1 \cdot l_f \cdot l_r = r_2, \tag{12}$$

where M_s and J_p are vehicle mass and the pitch moment of inertia about the center of mass (CM), and l_f and l_r are the front and rear distances from the CM (cf. Fig. 14).

The first condition, equation (11), depends on vehicle physical parameters and is approximately (within 20%) satisfied by most vehicles. The second condition, equation (12), can be satisfied by appropriately choosing the weighting parameters r_1 and r_2, which often results in a reasonable compromise between the pitch and heave aspects of a ride.

The same conditions also apply for the 4 DOF, half-car model which includes unsprung masses as per Fig. 15(a). The result is two decoupled 2 DOF, quarter-car models shown in Fig. 15(b). Note that the 2D, half-car model introduces one qualitatively new aspect—consideration of the front wheels as a convenient mechanism to preview the road ahead of the rear wheels. This anticipatory information can further improve the ride/handling trade-offs, above and beyond the optimal quarter-car trade-offs of Figs. 8(a) and (b).

Global performance potentials with preview applied to simple 1 DOF quarter-car models were investigated in Bender (1967b), and Tomizuka (1976). Hrovat (1991a,b) introduces equivalent delayed-system formulation to develop similar results for 2 DOF, quarter-car, discrete-time models applicable to the configuration of Fig. 15(b). The related global performance plots are reproduced in Fig. 16, where the solid lines represent the limiting curves without the preview. The optimal performance curves with preview times t_r up to 1 s are shown as dashed lines.

From Fig. 16(a) it can be seen that for normalized suspension deflections below about $0.6\ s^{1/2}$, preview information of up to 1 s can reduce the accelerations by approximately 50 to 70%. Similarly, Fig. 16(b) shows that even a relatively small amount of preview of only 50 ms, can significantly reduce the tire deflection for normalized accelerations above circa $15\ s^{-3/2}$. For example, at normalized accelerations of $20\ s^{-3/2}$, the tire deflection can be reduced by about 30% with 50 ms of preview. Since in practice the most useful previews will probably be limited to below about 0.3 s, it can be concluded that preview information will probably be most useful for normalized accelerations above approximately $10\ s^{-3/2}$.

Moreover, one can now use the above plots to adapt different control gains for the front and rear suspensions, while satisfying tire and suspension deflection constraints. This is illustrated in Fig. 16(a) with the help of design points C_f and C_r, for the front and rear control settings. Here both suspension units are subjected to the same stroke constraint, while the rear unit can benefit from 0.11 s of preview, which corresponds to a vehicle with a wheelbase of 2.75 m traversing a fair-to-medium road at a speed of 88.5 kph (further details about this example can be found in Hrovat, 1991a). The result is a considerable improvement in both heave and pitch aspects of the ride. Obviously, if a suitable sensor were available, then the performance could be further improved with the road preview ahead of the front wheels. Corresponding additional benefits could be again estimated with the help of Fig. 16.

As in the quarter-car case, it is now possible to consider jerk-optimal suspensions for half-car models (Rutledge, 1990), and, also, the usage of singular perturbation theory (Kokotovic, 1984) to exploit the two-time scale separation between body and wheel-hop modes (Salman *et al.*, 1990; Ono *et al.*, 1994). In addition, with the half-car structure it is possible to exploit the spatial separation between the front and rear of the vehicle. This was first done in Salman *et al.* (1990), where the authors essentially neglected the cross-coupling between the front and rear performance index terms corresponding to the fast, wheel-hop dynamics. The resulting hierarchical control structure consisted of a slow mode similar to the one from Fig. 14 [with a set of tire springs as per Fig. 7(b)], and a decoupled set of fast control modes used to contain the front and rear wheel-hops. However, as shown by Krtolica *et al.* (1990), for the particular data used in Salman *et al.* (1990) the decoupling conditions (11) and (12) are almost satisfied anyway, so that even a simple set of two fully-decoupled front and rear quarter-car models yields a performance close to optimal! It is only when strongly coupled data are used that one can start to appreciate the proposed spatial and temporal separation.

Recently, there was an increase in interest for optimal preview applications to advanced suspension design. A comprehensive analysis of the 2D case in the continuous-time domain was performed by Hac (1992b); the corresponding analytical solutions were similar to their limiting

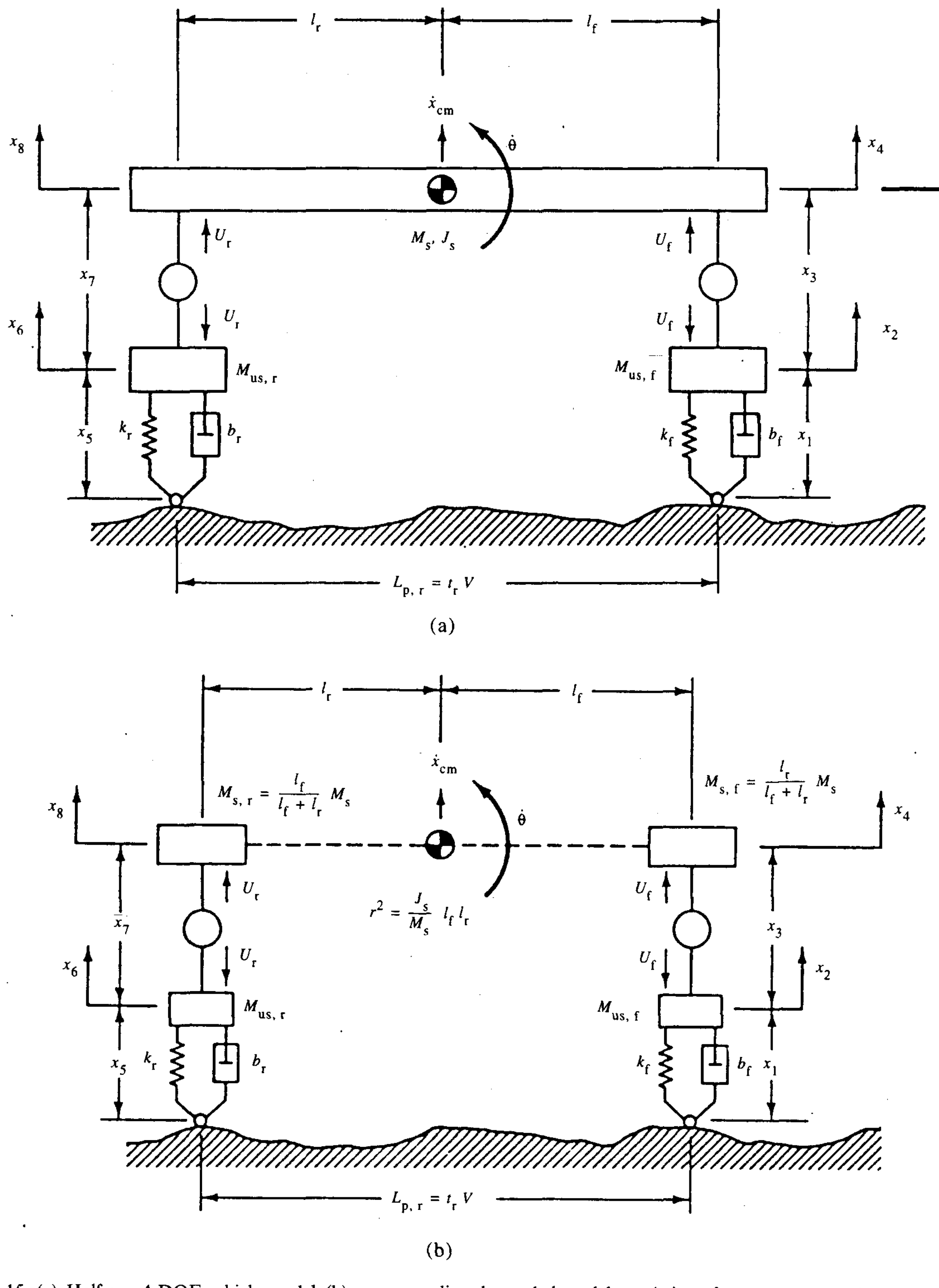


Fig. 15. (a) Half-car, 4 DOF vehicle model; (b) corresponding decoupled model consisting of two quarter-car 2 DOF subsystems.

discrete-time counterparts from (Hrovat, 1991a). The performance potentials of front wheel preview have been investigated in (Araki *et al.*, 1994), and those of the slow-active suspensions in (Sharp and Pilbeam, 1994; Pilbeam and Sharp, 1996), where it was concluded that, "0.033 s of preview is of little use, although 0.1 s is very useful". The effectiveness of preview for advanced suspensions with explicit consideration of jerk as a ride comfort measure, has been investigated in (Youn, 1996). An attempt to apply the preview to a nonlinear 2D truck model has been discussed in (Huisman *et al.*, 1994), along with the corresponding power requirements. Moran *et al.* (1996) use H_∞ optimization to derive preview controls; whereas Peng and Ma (1996) use game-theoretic concepts to consider control

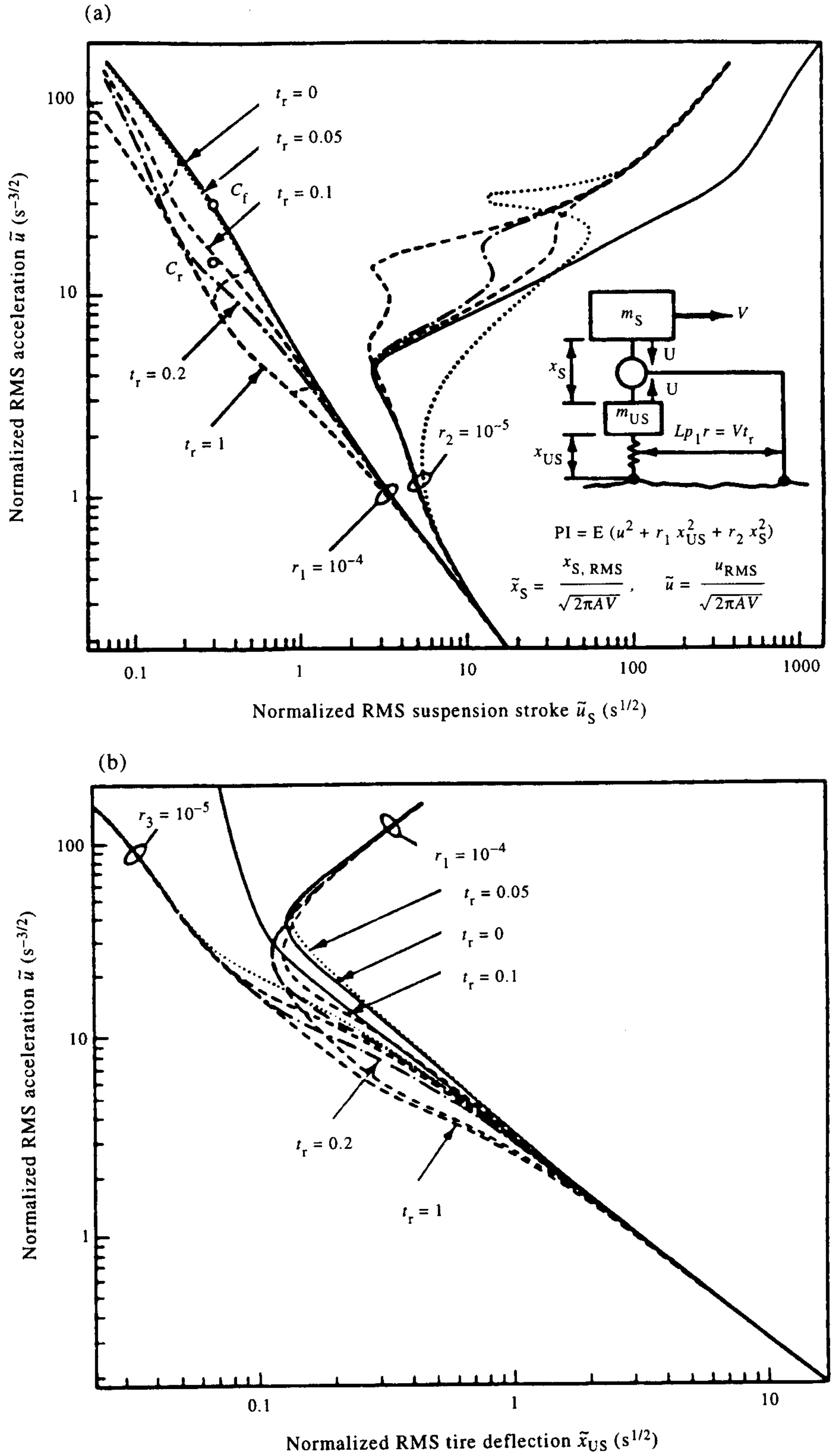


Fig. 16. Optimal suspension performance for 2 DOF model with different preview times t_r, and $\omega_{us} = 2\pi \cdot 10\ \mathrm{rad\,s^{-1}}$, $\rho = 10$: (a) $\tilde{u}$ versus $\tilde{x}_s$ boundaries; (b) $\tilde{u}$ versus $\tilde{x}_{us}$ boundaries (Hrovat, 1991a).

algorithm robustness w.r.t. worst-case scenarios, with and without road or disturbance preview. Finally, an elegant analytical application of decentralized control concepts to active vehicle suspensions with preview is given in (Hac, 1995).

5. OPTIMAL SUSPENSIONS FOR FULL-CAR MODELS

In this section we will briefly survey some past and recent full-car, 3D results, and place them in

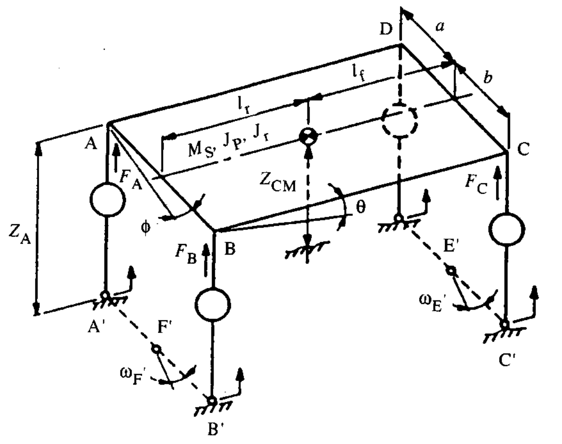


Fig. 17. 3D vehicle model schematic.

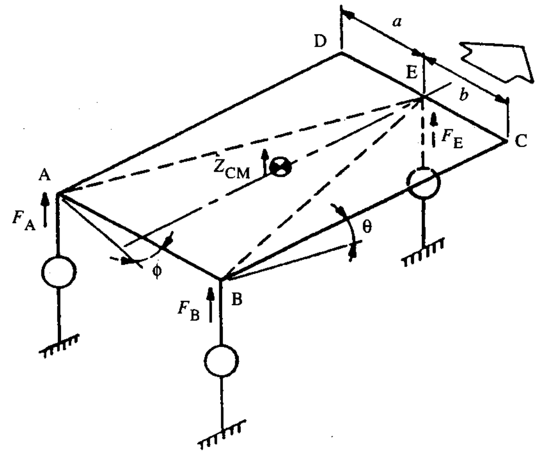


Fig. 18. Equivalent "tricycle" configuration.

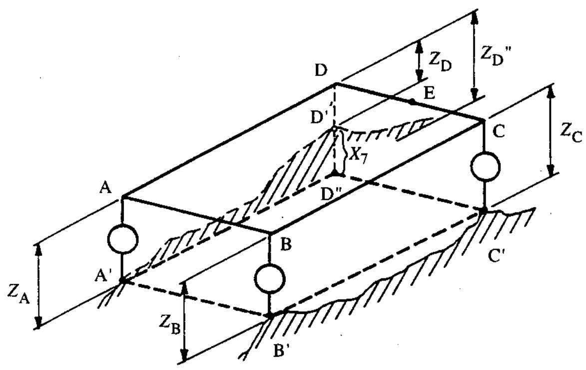


Fig. 19. Introduction of additional state $x_7 = x_8$.

a proper perspective with respect to previously discussed half-car and quarter-car counterparts. Some of the earlier investigations of the optimal 3D suspensions can be found in Barak (1985), Barak and Hrovat (1988), and Chalasani (1986b), where numerical solutions of the corresponding LQ problem were obtained for a few specific operational road/speed conditions. On the other hand, recent complete analytical solution of the 2D case (Krtolica and Hrovat, 1992) facilitated similar analytical extensions (Hrovat, 1991b) toward the full 3D structure shown in Fig. 17. The corresponding optimization problem was formulated by mimicking the basic quarter-car and half-car performance requirements, equations (8) and (9), so that the full-car performance index now becomes

$$\mathrm{PI}_5 = E(q_A z_A^2 + q_B z_B^2 + q_C z_C^2 + q_D z_D^2 + r_1 \ddot{z}_{cm}^2 + r_2 \ddot{\Theta}^2 + r_3 \ddot{\Phi}_2), \qquad (13)$$

where the first four terms penalize excessive steady-state suspension deflection at the four corners (cf. Fig. 17), while the last three serve as a measure of ride discomfort with respect to vehicle heave, pitch and roll acceleration, respectively.

The analytical solution of the above seventh-order system was found with the help of a two-layered partitioning (Hrovat, 1991b). In the first step, the original seventh-order system was partitioned into a sixth-order, tricycle structure shown in Fig. 18 and an additional "odd" state. The second step constituted further partitioning of the sixth-order Riccati equation, corresponding to the above tricycle model, into two third-order subsystems, which could then be solved analytically. The analytical solution of the sixth-order system was next merged with the additional seventh state shown in Fig. 19. This additional state is necessary to reflect the fact that the road elevations are not necessarily coplanar at the four tire-road interfaces.

The solution to the seventh-order LQ problem was completed by transforming the above seven states into a more practical set of states: three sprung mass velocities for heave, pitch and roll modes, and four corner suspension deflections. Once the analytical solution to the full-car optimal suspension problem was found, it was possible to establish a number of related properties characterizing the optimal structure. For example, it was shown that all three optimal body modes have the highly desirable damping ratio of 0.7 which is achieved via "skyhook" damping. This holds for any set of PI_5 weights in equation (13), and can be extended to even more general performance indices.

It was also shown that under the following mild conditions, the original 3D problem can be decoupled into simpler pitch + heave and roll subproblems (see Fig. 20),

$$a = b, \qquad q_A = q_B, \qquad q_C = q_D, \qquad (14)$$

where a and b are the lateral distances from the CM in Fig. 17. It is also assumed that the ground roughness has been modeled according to a spatial road surface description from Parkhilovskii (1968)

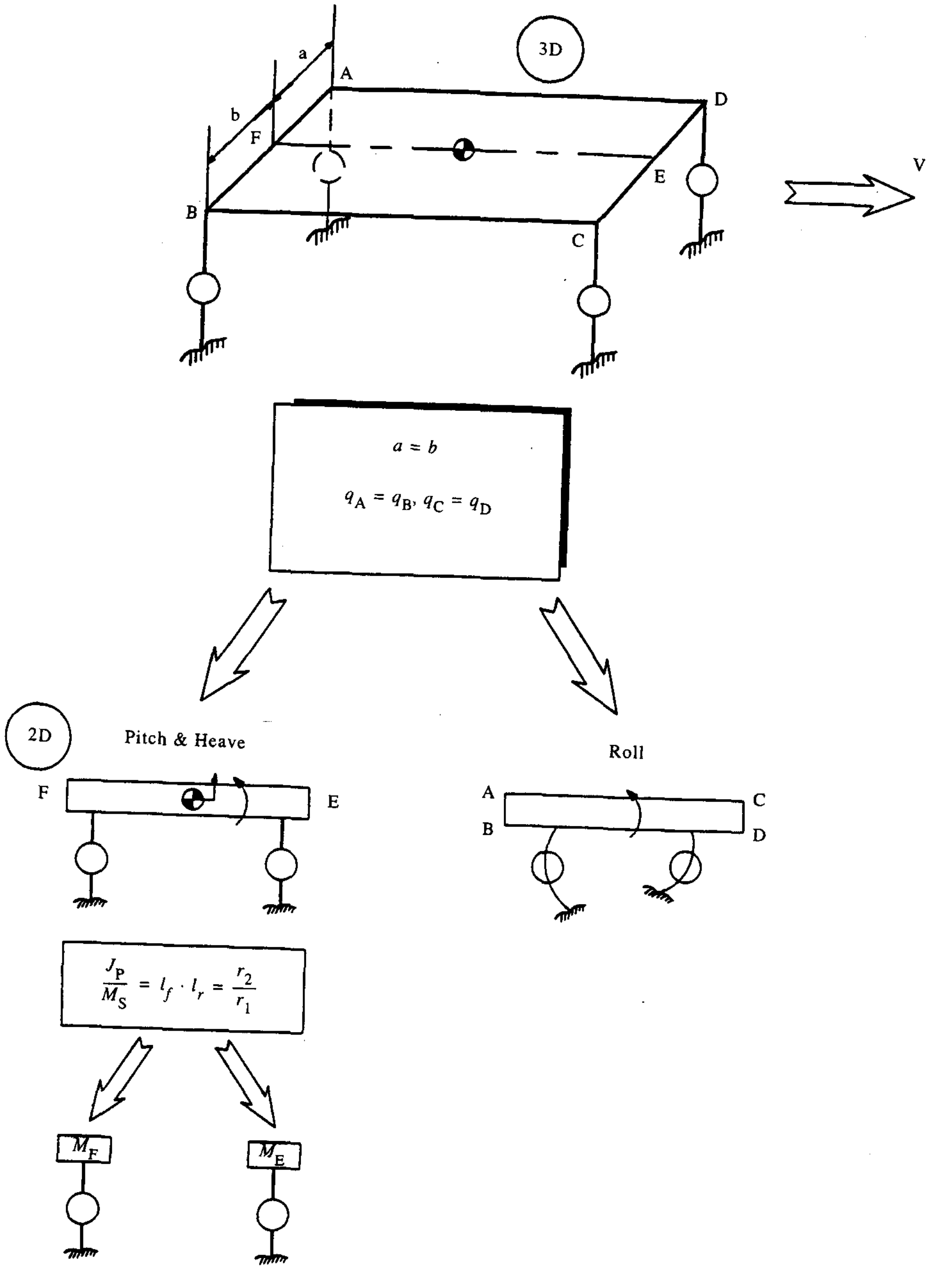


Fig. 20. Front–rear-roll decomposition.

and Rill (1983), which, as discussed in Hrovat (1993) conveniently facilitates the above decoupling. Moreover, if the previously established conditions (11) and (12) hold, then the pitch + heave dynamics can be further decoupled into front and rear quarter-car submodels as shown in Fig. 20. In this way the original 3D problem is transformed to the much simpler, quarter-car models for which there is a wealth of results available, as discussed in previous sections. It should also be pointed out that, similar to the 2D case, it is now possible to consider 3D two-time scale and hierarchical control schemes, effects of flexible modes, preview, and different jerk-optimal PI augmentations.

6. RELATED TOPICS

The objective of this section is to briefly discuss developments in important areas which are directly or indirectly related to the main theme of the present paper. A full survey of the related topics would typically require a separate paper for each, something that is obviously beyond the scope here. However, it is hoped that, while not complete, the following sections will help put the subject of optimal suspension controls in a proper perspective with respect to these related developments, and direct the interested reader to appropriate references where further information and leads can be found.

6.1. *Semi-active suspensions*

The semi-active (SA) suspension concept was introduced in the early 1970's (Crosby and Karnopp, 1973; Karnopp and Crosby, 1974; Karnopp *et al.*, 1974) in the form of a variable, controllable damping. In this section we will somewhat loosely refer to "semi-active" as any essentially passive suspension concept, where a relatively small amount of external energy is used to improve the performance.

Since their introduction, semi-active dampers have been considered for diverse applications ranging from trains (Klinger *et al.*, 1976), to tractors and other off-road vehicles (Roley, 1975), to high-speed air-cushion vehicles (Margolis *et al.*, 1975; Margolis and Hrovat, 1976) and military tanks (Miller and Nobles, 1988). Large scale, production applications of some very elementary SA concepts started in the early 1980's, with the introduction of the first variable damping shock absorbers. These variable dampers typically changed the damping from soft to firm and vice versa, through a manual or slow adaptive control, and as such represented a very rudimentary form of the SA damping. The corresponding ride and handling performance improvements were minimal and indeed often imperceptible, although some more recent and matured implementations show promise, especially in adapting to increased handling demands.

A somewhat more sophisticated and more demanding approach (in terms of implementation) is the so-called "on-off" SA strategy, which was first proposed by Margolis *et al.* (1975). Simply stated, it switches the damper off, i.e. creates a low or near-zero force, whenever the sprung and unsprung masses move in the same direction and the unsprung mass has a larger velocity. In any other situation the damper is set to "on" state, which produces large damper forces. The main idea of such a strategy was to reduce the sprung mass acceleration and motion, which was successfully demonstrated in early simulation and experimental studies (Margolis *et al.*, 1975; Margolis and Hrovat, 1976; Hrovat and Margolis, 1981; Krasnicki, 1981).

During recent years there has been considerable interest in the on-off SA concept in the industry. As a result, there appeared further improvements and refinements of the concept (Miller, 1988; Ivers and Miller, 1989; Crawford, 1988; Decker and Schramm, 1990), which seem to offer some tangible ride and handling benefits. Consequently, this is emerging as the first bona fide, "full-blooded" SA device to see production (Emura *et al.*, 1994; Higashiyama *et al.*, 1994; Nakayama *et al.*, 1996).

The original continuously-variable SA policy (Crosby and Karnopp, 1973; Karnopp and Crosby, 1974; Karnopp *et al.*, 1974) represents a next step up in sophistication and in practical implementation complexity. It requires that the SA actuator continuously reproduce the (1 DOF LQ-optimal) skyhook damping force whenever this is possible in view of the passivity constraint. When this is not possible, the damper is simply turned off (here care should be exercised to avoid discontinuous-in-force switchings which can lead to excessive jerk). The continuously-variable SA policy was subsequently extended to 2 DOF, quarter-car and more complex models, with simultaneous possible limiting of the damper maximum force leading to so-called "clipped" SA control (Siebenhaar, 1975; Roley, 1975; Hrovat, 1979; Hrovat *et al.*, 1988; Butsuen and Hedrick, 1989). An experimental, laboratory development of a continuously variable, skyhook SA damper has been recently summarized by Kitching *et al.* (1996).

The question of the optimal SA damping policy was addressed in Hrovat (1979), Hrovat *et al.* (1980, 1988), and subsequently in Butsuen and Hedrick (1989) where it was concluded that the clipped SA policy is also optimal for quarter-car models. Unfortunately, this conclusion was not entirely correct (cf. e.g. Tseng and Hedrick, 1993), although, in practice, the clipped SA policy may often be very close to being optimal. Both linear and bilinear system dynamics (Hrovat, 1979; Kimbrough, 1986; Hrovat *et al.*, 1988) were considered, with the SA damping parameter acting as a bilinear control variable limited to nonnegative values due to the passivity constraint. The approach in (Hrovat, 1979; Hrovat *et al.*, 1988) was also used to prove that under some relatively mild conditions, the solution to the related nonlinear, stochastic optimal control problem exists, and, more importantly, is given in a feedback form. An approximate, suboptimal solution was then attempted using open-loop, anticipatory optimization and subsequent regression to obtain corresponding (sub)optimal feedback controls. The preliminary numerical results indicated that such an (sub)optimal approach will not significantly outperform the clipped SA control. While these early results were obtained in the late 1970's using, by current standards, rather minuscule computational means, the available results to date seem to confirm these early findings (see, e.g. Tseng and Hedrick, 1993).

Another approach toward improved SA performance was introduced by Kimbrough (1986), based on the Lyapunov equation. Further improvements of the SA concept were also investigated through hardware modifications and extensions (Ivers and Miller, 1991; Wallentowitz and Konik, 1991). Usage of Electrorheological (ER) and magnetorheological (MR) fluids has been proposed as a hardware alternative for the more conventional, variable-orifice hydraulic dampers (Pinkos *et al.*,

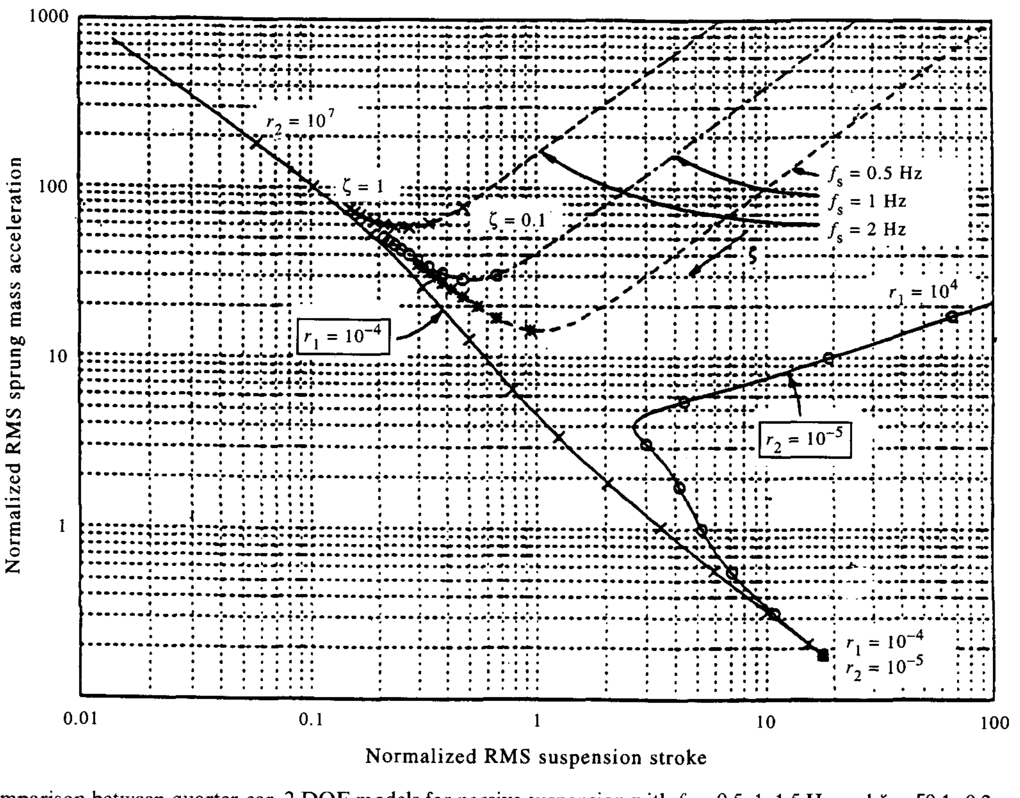


Fig. 21. Comparison between quarter-car, 2 DOF models for passive suspension with $f_s = 0.5$, 1, 1.5 Hz and $\zeta = [0.1, 0.2, \ldots, 1]$, and active suspension with $r_1 = 10^{-4}$ and $r_2 = 10^{-5}$ (Tseng and Hrovat, 1989).

1994; Sturk *et al.*, 1995). From a practical standpoint, the MR concept appears more promising for suspension and other vibration isolation applications (such as, e.g. engine mounts), since it can operate on vehicle battery voltage, whereas the ER concept is based on high voltage electric fields. There were also attempts to design the SA controllers using genetic algorithm (Yeh *et al.*, 1994), internal model control (Park and Koo, 1994), dynamic programming (Muijderman *et al.*, 1994), frequency-selective skyhook damping (Konik *et al.*, 1996), neural networks (Moran *et al.*, 1994), a sliding mode controller (Isobe *et al.*, 1996), and preview control (Kawagoe, 1983; Kawagoe and Iguchi, 1985; Hac and Youn, 1992; Jezequel and Roberti, 1992; Huisman *et al.*, 1992a, b; Kim and Yoon, 1995). Actual production hardware and/or close-to-production hardware implementations are described in (Emura *et al.*, 1994; Higashiyama *et al.*, 1994; Irmscher *et al.*, 1994; Konik *et al.*, 1996; Nakayama *et al.*, 1996).

Hubbard and Margolis (1976) introduced the concept of an incrementally semi-active spring, realized by means of on and off switching of additional gas spring accumulators. This is a promising practical complement to SA damping, since, based on discussions in Section 3.2, both the suspension damping and stiffness should change to optimally adapt to different road/speed conditions. This can also be seen in Fig. 21 from Tseng and Hrovat (1989) which illustrates the performance of adaptive "passive" suspensions with the sprung mass natural frequency varying from 0.5 to 2 Hz, and the corresponding damping ratio ζ in the range from 0 to 1. Clearly, the suspension stiffness has a significant influence on ride quality so that, similar to the fully active case, softer suspensions lead to more comfortable rides at the expense of increased suspension stroke requirements. Fig. 21 also shows the corresponding optimal active boundaries [cf. Figs. 8(a) and (b)]. It can be seen that even such an idealized adaptive "passive" suspension (which in practice still may require compressors or pumps) cannot fully match the performance of the optimal active counterpart.

A concept of "ground-hook" semi-active control has recently been introduced by Novak and Valasek (1996) to minimize the amplitude of dynamic road-tire force, or equivalently, tire deflection. Additional aspects of SA suspensions, associated control and estimation algorithms and various hardware improvements and refinements can be found in Margolis (1982a,b), Rakheja and Sankar (1985), Redfield (1990), Karnopp (1990), Hac (1992a), Yi and Hedrick (1995), Hedrick *et al.* (1994), Irmscher *et al.* (1994), Redlich and Wallentowitz (1994), Irmscher and Hees (1996), and especially in related surveys by Ivers and Miller (1991), Wallentowitz and Konik (1991), and Elbeheiry *et al.* (1995).

6.2. *State estimation and system robustness*

The underlying assumption in the preceding exposition has been that all the states are exactly known, which then gave us the best possible performance. In practice, some of the states may not be easily accessible. For the simple, quarter-car model, typical measurements are sprung mass acceleration and suspension deflection, whereas the tire deflection is difficult to measure. Fortunately, the latter can often be neglected in the feedback control law without significant loss of performance, although there are operations, such as the stiffer suspension settings at the upper left portions of Figs. 8(a) and (b), when this cannot be done, necessitating a full-state feedback, most likely based on a full-state estimation (Hrovat, 1987, 1987–1988).

Based on the above measurements it is possible to estimate the full state vector using, for example, the Kalman filter in the context of the Linear–Quadratic-Gaussian (LQG) methodology (Alexandridis and Weber, 1984; Yue *et al.*, 1988; Ulsoy and Hrovat, 1990; Ulsoy *et al.*, 1994). While such systems may reduce sensor requirements, it is important to remember that they typically lead to a degradation in performance. Indeed, for some realistic examples of systems using a single, rattle space deflection measurement as in Yue *et al.* (1988), it was shown by Ulsoy *et al.* (1994) that such LQG policy can increase the LQ cost by more than 80%. However, the rattle space deflection based controller would result in a zero DC gain for the corresponding rattlespace-to-ground input transfer function.

In addition, the LQG controller is in general less robust with respect to modeling errors. It was shown by Ulsoy *et al.* (1994) that, for a "typical" set of control gains, the gain margin of the suspension stroke-based LQG regulator can be only 0.2 dB, with the phase margin of 18°. The corresponding LQ margins were ∞ dB and 100°. It should be pointed out that for a "softer" gain setting, even the full-state LQ regulator resulted in a small gain margin of only 1.5 dB, and a phase margin of 33°. This is in part the consequence of a particular quarter-car structure which included a parallel passive spring to (partially) support the vehicle. This additional spring resulted in non-zero weightings on cross-terms between states and control in the performance index, for which the "standard" LQ guarantees of ∞ gain margin and at least 60° of phase margin do not apply.

The above work indicates that it is important to consider closed-loop system robustness with respect to unmeasured states, structural changes (e.g. the introduction of a parallel supporting spring), as well as parametric uncertainties and, possibly to a lesser degree, uncertainties due to sensor and actuator dynamics. In the case of partial state measurements, it is possible to recover system robustness (assuming adequate LQ characteristics) through stochastic stability robustness analysis and (partial) Loop Transfer Recovery (Ray, 1991; Ulsoy and Hrovat, 1990; Palkovics and Venhovens, 1992; Ulsoy *et al.*, 1994). To address various forms of uncertainties, a number of recently developed and computerized robust control techniques can be considered, especially promising being the ones based on frequency domain analysis (Kiriczi and Kashani, 1990; Yamashita *et al.*, 1990; DeJager, 1991; Moran and Nagai, 1992).

In particular, reference DeJager (1991) investigated potential applications of H_∞ optimization for active suspension design. It was found that, based on a performance mimicking equation (9), the resulting controller introduced too much damping in the wheel-hop mode, since the H_∞ technique tends to minimize the peaks in the frequency response curves. As might have been expected from the previous discussions, excessive damping will in general lead to deterioration of ride comfort performance.

Similarly, a relatively large amount of wheel-hop damping can also be seen in Kiriczi and Kashani (1990). To avoid this issue, Yamashita *et al.* (1990) neglect the tire deflection in the performance formulation, justifying it by the already large inherent damping that was present in their quarter-car laboratory setup. The results show considerable reduction in ISO-2631 weighted sprung mass acceleration in the range between 3 and 8 Hz. However, there were no data given for rms acceleration, and tire and suspension deflections. In view of these issues it is appropriate to consider a mixed H_2/H_∞ metric; one way to combine the desirable robustness properties characteristic for the H_∞-based techniques, has been proposed by Tran and Hrovat (1992a).

Due to the practical importance of robust suspension design, which also has an impact on system fault tolerance, it is expected that work in this area will intensify in the coming years. This will be facilitated by continuing progress in the development of robust control theories and associated CAD tools for linear, nonlinear and adaptive control system design.

6.3. *Nonlinear controls*

So far we have primarily focused on optimal control methods applicable to linear vehicle models which are of course only approximately valid since the real vehicle system is necessarily nonlinear. For many operations, however, the linear system approximation is appropriate. This is also supported by experimental validations as discussed in Section 2. However, there are situations which amplify the nonlinear effects. One example is created by discrete-event disturbances, such as single bumps or potholes, which can cause suspension bottoming—a highly nonlinear phenomenon. Another example is the always present dry friction, which may cause the suspension to lock on very smooth roads. The vehicle then essentially rides on tires.

To treat the nonlinear effects of suspension bottoming and the like, Gordon *et al.* (1991) propose to use an approach which is similar in spirit to the optimal semi-active approach (Hrovat *et al.*, 1988), discussed in Section 6.1. As was the case with this latter reference, the method from Gordon *et al.* (1991) is computationally intensive, especially for higher-order vehicle models. However, with continuous improvements in computer processing power this will be less of an issue in the future. Encouraged by their favorable results with discrete disturbance responses, Gordon *et al.* (1991) argue for superiority of nonlinear over LQ-adaptive suspension algorithms. However, as shown in Tran and Hrovat (1992b), under certain conditions one can equally well present the reverse argument, especially if the full potentials of the adaptive strategies are taken into account.

Instead of competiting alternatives, the two approaches can be viewed as complementing ingredients of an overall optimized suspension strategy. In this context, the LQ-optimal adaptive or gain scheduling controls would be used for most driving situations with nonlinear policies being activated during excessive discrete-event disturbances and other special circumstances. An approach that would naturally blend the two was proposed by Hrovat (1989), and is under investigation (Ricci *et al.*, 1996).

We conclude that, as a result of the substantial ongoing theoretical advances in the areas of nonlinear and adaptive controls (cf. e.g. Astrom and Wittenmark, 1989; Slotine and Li, 1991), it is expected that the future will bring increased applications of these and similar techniques in advanced suspension design. Some early results in these directions have been mentioned above; additional attempts can be found in Cheok *et al.* (1985), Sunwoo *et al.* (1990), Blankenship *et al.* (1993), Tran and Hrovat (1993), Alleyne and Hedrick (1995) and others.

6.4. *Practical considerations*

For practical implementation of the above optimal control suspension strategies, it is sometimes preferable to simplify the optimal strategies thus leading to simpler software implementations at the expense of hopefully only slightly degraded performance. For example, in the case of quarter-car models, the full-state LQ-optimal regulator can be simplified to a three-term controller structure consisting of secondary suspension spring and damper (for body support and wheel-hop damping), and a skyhook-like damper for improved sprung-mass damping and isolation. This is a suboptimal policy since it neglects tire deflection feedback. In many cases (but not always!) the effects of such an approximation are insignificant.

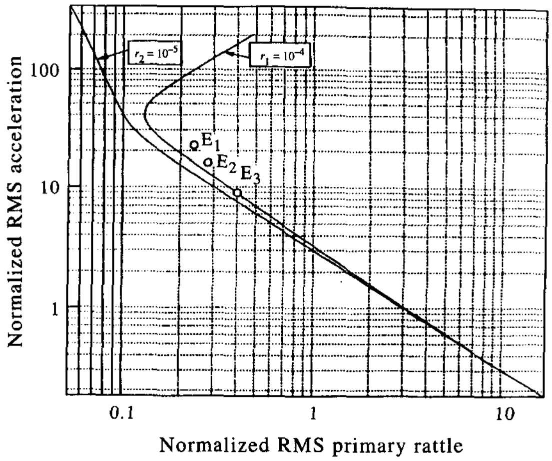


Fig. 22. Optimal performance boundaries for $r_1 = 10^{-4}$ and $r_2 = 10^{-5}$ and experimental results E_1, E_2, E_3, for a quarter-car setup with an active suspension ($\omega_{us} = 2\pi \cdot 12$ rad s^{-1}, $\rho = 12.5$) (Hrovat, 1987–1988).

A similar approximation was used in quarter-car laboratory experiments with a wide-bandwidth electro-hydraulic suspension leading to the results shown in Fig. 22, where control gains have been adapted to the assumed varying road conditions (Hrovat, 1987–1988). The comparison with superimposed optimal 2 DOF curves shows that, although not optimal, the experimental results follow the desired trends and are relatively close to the best possible performance. This at the same time gives credence to the underlying optimal analysis and proposed adaptive concepts.

It should be pointed out that even a limited amount of "real-life", experimental experience can be extremely important to appreciate the effects of often neglected, but in practice potentially crucial phenomena such as dry friction and actuator bandwidth and fidelity. Indeed, researchers and engineers dealing with the related hardware quickly learn of the omnipresence of dry or Coulomb friction. The presence of dry friction is probably the main reason why some of the early prototype and production units show less ride improvement than would be expected based on the analysis. To this end recent increased emphasis on experimental corroboration, as seen for example from Ivers and Miller (1991), Rajamani and Hedrick (1991), Yamaguchi *et al.* (1992), Alleyne *et al.* (1992), and Ono *et al.* (1994), is a welcome step in the right direction.

Even a relatively small amount of dry friction can substantially degrade ride performance. For example, for a 3000 lb vehicle with a per-corner weight of 750 lb, a corresponding actuator dry friction of only 25 lb can contribute to a respectable

3.3% g rms sprung mass acceleration. To see this, assume that the commanded acceleration (or corresponding force) is zero, that the actuator bandwidth is below the wheel hop frequency, and that the wheel-hop dynamic motion is sufficiently agile to "break" the friction (if it were not, then by the previous discussion the suspension would lock and result in the undesirable situation of a "ride-on-tires").

Another important aspect of practical significance for active suspensions is the issue of actuator dynamics. The first level of optimization analyses presented thus far serves to establish performance potentials of an idealized active suspension actuator, capable of delivering any force, infinitely fast. Since the main objective of such studies was to establish a *global* snapshot of the maximum possible benefits, it was natural to consider such an idealized and simplified actuator. However, in practice, obtaining high-fidelity actuation is a very challenging task, requiring a full understanding of underlying system dynamics and a mastery of mechatronic hardware design skills (Hrovat and Hubbard, 1987).

Once the upper bounds of vehicle system performance have been established, the next level of analysis would typically include (hardware specific) actuator dynamics (Alleyne and Hedrick, 1995), especially actuator bandwidth limitations and requirements. More precisely, this typically refers to actuator force generation bandwidth for all possible motions across the actuator mounting points. One distinguishes between narrow/low-bandwidth or slow-active, and broad/high-bandwidth suspensions similar to the prototypes developed by Lotus and other suppliers and car manufacturers. The former attempt to control only the relatively slow body mode and as such their bandwidth is typically below 3–5 Hz.

On the other hand, the broad-bandwidth suspensions attempt to control both the body and wheel-hop modes, and thus their bandwidth should be typically above 15–20 Hz.

One way to control the bandwidth is by introduction of additional compliances in the form of gas-spring accumulators possibly augmented with (SA) damping. This has a positive side-effect of providing a high-frequency, NVH isolation. Additional insight into related structure and dynamics is given by Karnopp (1987), where typical narrow-bandwidth suspension configurations were analyzed with the help of bond graph modeling methodology applied to simple, quarter-car vehicle models shown in Fig. 23. A generic suspension configuration from Fig. 23(a) includes a (possibly variable) hydraulic damping valve, hydro-pneumatic accumulator, and two oil flow sources, Q_A and Q_B, controlled by appropriate electro-hydraulic valves, for example. The equivalent, all mechanical structure can then be derived with the help of a bond graph from Fig. 23(b) with a transformer ratio set to one. The resulting structure is shown in Fig. 23(c). Note that the hydraulic damping and pneumatic compliance are in effect acting geometrically in parallel with each other.

In practice, only one of the two flow sources will be present. One example when only V_A is present is shown in Fig. 23(d). This configuration is typical of air suspensions actuators. On the other hand, a possible hardware structure for the case when only V_B is present, is shown in Fig. 23(e), which is similar to a typical hydraulic suspension unit. Further potentially important extensions of the above generic configurations may include a consideration of hydraulic inertias, valve/solenoid dynamics, spool resistances and other related effects, which can be elegantly incorporated with the help of the bond graph modeling technique.

An example of narrow-bandwidth suspension hardware is shown in Fig. 24(a) based on Akatsu *et al.* (1990). It includes: (1) an accumulator (for bandwidth limitation); (2) a mechanical, supporting spring (although this may be absent in some implementations); and (3) a relatively inexpensive pressure control valve. Structurally, this hardware is similar to the configuration from Fig. 23(e), with the single flow source V_B. Note the single-acting actuator configuration which is simple and inexpensive. It does, however, limit the system controllability on the rebound stroke, which is in good part determined by the spring/accumulator settings. While typical, the configuration of Fig. 24(a) is not the only one possible and future narrow-bandwidth suspensions may profit from the added flexibility of double-acting actuators and judicious choice of accumulators and parallel springs.

Another hardware alternative would result by replacing the above pressure-control valve in Fig. 24(a), with an appropriate flow-control valve. A schematic of a typical proportional, pressure-compensated flow control valve (Merritt, 1967) is shown in Fig. 24(b). It consists of a pressure control valve augmented by a solenoid-controlled flow metering orifice. Due to a particular interconnection between the two, the valve spool will maintain (at the steady state) a constant pressure drop ΔP across the metering orifice, which is preset to a desired value through appropriate solenoid control voltage. This, in turn, will result in a desired output flow, independent of input and output pressure variations.

In the context of narrow-bandwidth suspension actuators, a possible application of the above flow-control valve, can be rationalized as follows. Any force production with such an actuator will be accompanied with corresponding displacements

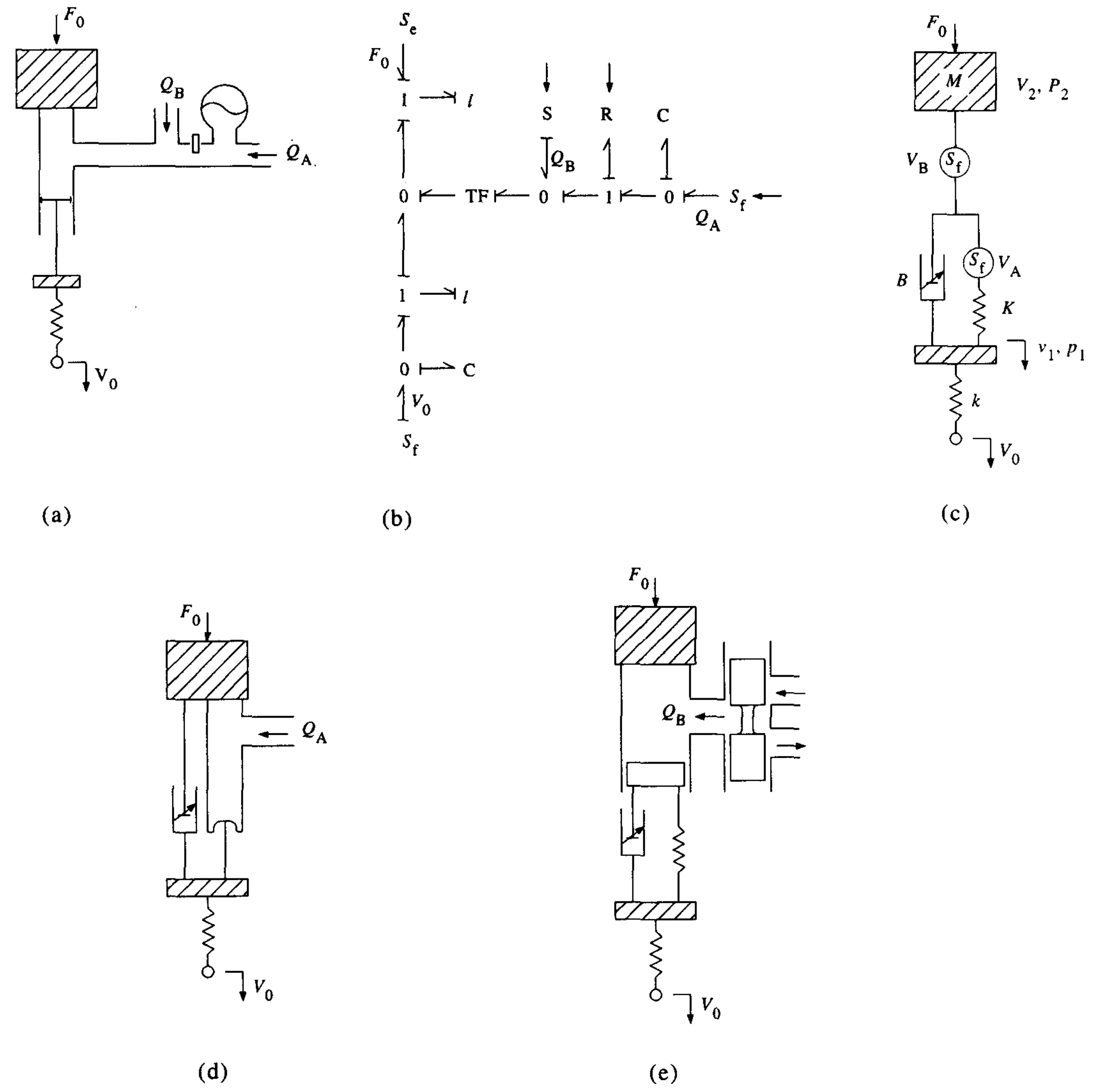


Fig. 23. Narrow-bandwidth suspension configurations: (a) generic structure; (b) corresponding bond graph; (c) equivalent mechanical system; (d) typical structure with $Q_B \equiv 0$; (e) typical structure with $Q_A \equiv 0$ (based on Karnopp, 1987).

and velocities of the in-series compliance [cf. Fig. 23(e) and Fig. 24]. Thus, in this case, instead of a force control (which is a more common and more direct approach), one could consider the related flow- or velocity-control problem. However, due to the hardware simplicity, the more direct approach via the pressure control valve seems to be preferred in practice, and is the only one seen in actual production implementations thus far. Additional information about the narrow-bandwidth active suspensions, including important design/structural considerations, performance potentials, and power requirements, can be found in (Karnopp, 1987; Sharp and Hassan, 1987; Hrovat, 1987; Williams et al., 1993).

Barring possible exceptions due to extraordinarily clever designs (or lack thereof), it is in general to be expected that the broad-bandwidth systems will be more difficult to implement. Also, according to Hillebrecht *et al.* (1992) since this type of suspension must contain the high-frequency road-induced vibrations very rapidly, it is expected that "the cost of the hardware is therefore extremely high" (in reality, due to a finite suspension bandwidth, there will always be a need for in-series compliance, which is currently provided via bushings and mounts). Moreover, the same reference anticipates the corresponding fuel consumption to rise significantly. On the other hand one could also argue that there is a potential for increased fuel consumption of narrow-bandwidth suspensions due to increased pump flow requirement caused by more compliant actuator structure, which can also be more sensitive to changes in suspension spring rate (Goran and Smith, 1996).

If the more complex broad-bandwidth approach is to be justified, it must show clear prospects for functional improvements above and beyond what can be expected from its narrow-bandwidth counterpart. The influence of actuator bandwidth

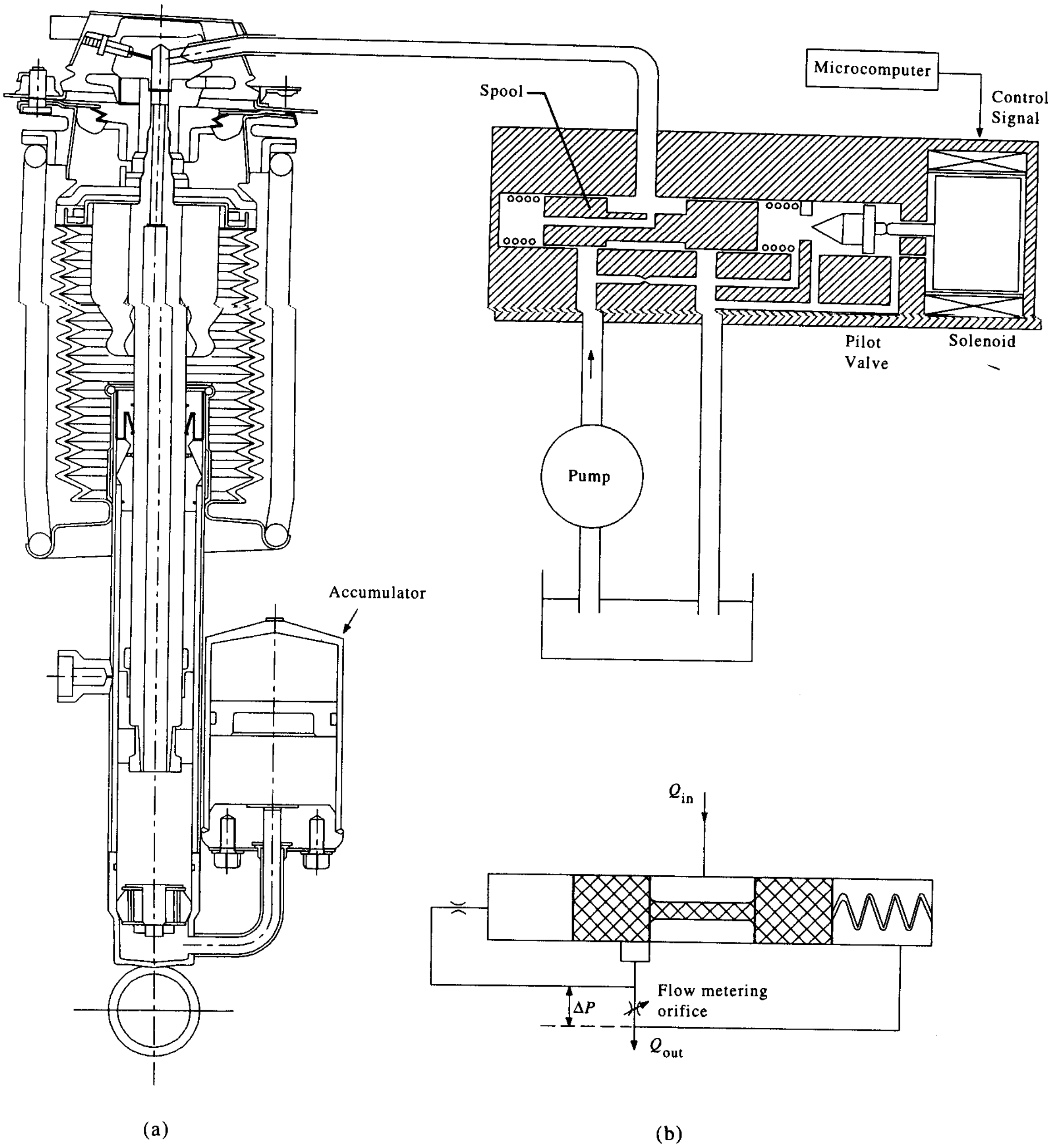


Fig. 24. Schematic diagram of Nissan Infinity Q45a Electrohydraulic Pressure Control System (based on Akatsu *et al.*, 1990).

on basic system performance is shown in Fig. 25 from Tseng and Hrovat (1991), based on Hrovat (1987) where the actuator force production dynamics has been approximated via a simple, first-order, low-pass filter of a specified bandwidth. From Fig. 25, it can be seen that the performance of the 5 Hz bandwidth actuator is already very close to the limiting curves corresponding to the ideal, infinitely-fast actuators. This is in accordance with the previous observation (Chalasani, 1986a,b) that, based on the 2 DOF quarter-car model, most of the ride benefits can be expected around the slow, body mode. Since the latter is typically around 1–2 Hz, an actuator with a 5 Hz bandwidth would appear well-positioned to achieve most of the benefits. The approximate analysis of Fig. 25 supports these expectations and offers a justification for the usage of narrow-bandwidth actuators.

It should be re-emphasized that the above is only an approximate analysis, because in practice, beyond certain frequencies, any suspension performance is dominated by relevant passive components. Also, as pointed out by Hrovat and Hubbard (1987) the actual active suspension force production within the desired bandwidth and road disturbance constraints is a challenging topic in its own right, that may require original and ingenious design solutions. One possible approach may include

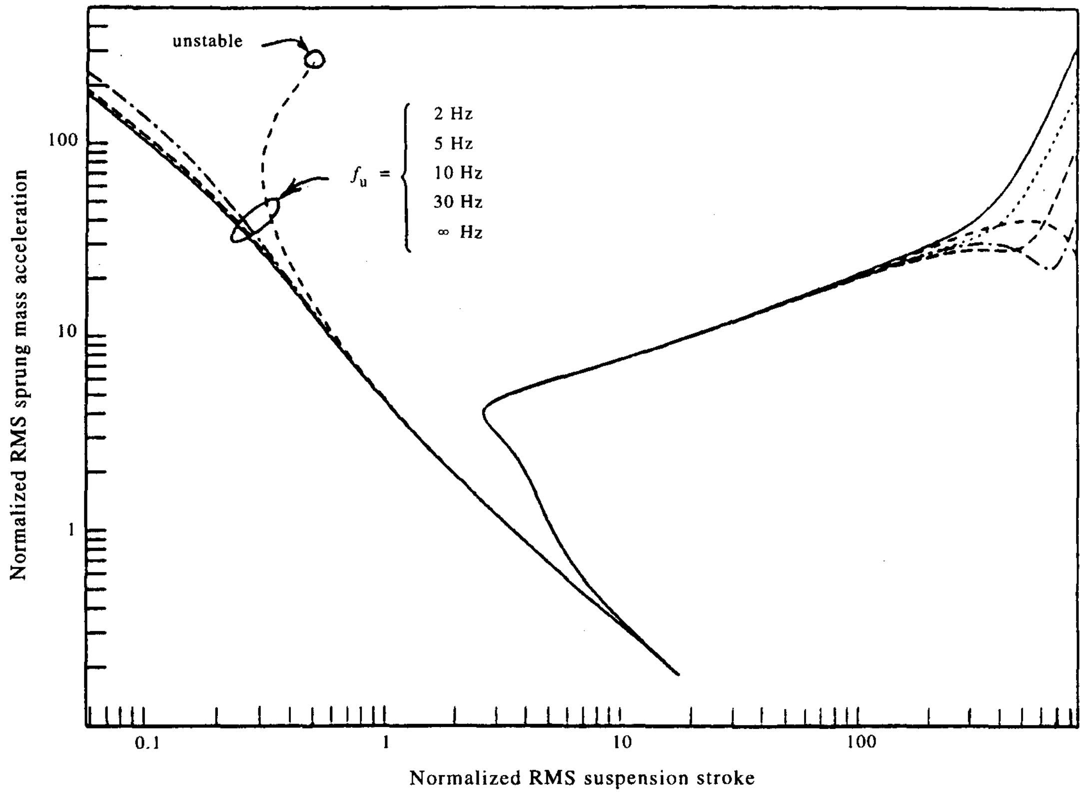


Fig. 25. Active suspension performance with varying actuator bandwidth f_u (Tseng and Hrovat, 1991).

the usage of sturdy, polymer-based tug-and-twist actuators proposed by Paynter (1988, 1996).

Another interesting design alternative was proposed in (van der Knaap, 1989; Knaap and Pacejka, 1993), and analyzed/evaluated in Venhovens *et al.* (1992) and Knaap *et al.* (1994). Conceptually, the authors propose to control the overall suspension force via the mechanism shown in Fig. 26. The suspension mechanism force F_C is controlled by changing the adjustable lever distance "b" in Fig. 26. This concept was realized in practice through a clever, albeit somewhat involved 3D contraption comprised of a cone mechanism with one end of the spring S from Fig. 26 attached to the vehicle sprung mass at point A, and the other to an adjustable crank mechanism regulated by an electrical servomechanism (Venhovens *et al.*, 1992). The crank rotation causes the spring to rotate along the surface of an imaginary cone, with the length of the spring remaining theoretically constant (assuming no ground-induced motion). The net effect is that the force F_C from Fig. 26 is now continuously controlled through the servo-crank mechanism rotations, with ideally negligible power consumption.

To date, the authors reported good success with the attitude control during cornering and braking. However, although theoretically zero or very low power requirements were expected based on idealized calculations, in practical implementation the power consumption was still significant, and amounted to an average consumption of circa 500 W, with peaks of 2.5 kW. This could be attributed to a particular design implementation (friction, etc.), as well as the difficulties associated with the construction of an ideal, modulated energy device. Also, limited ride tests to date show relatively modest, only about 10% improvements in the rms accelerations, which are primarily concentrated around the dominant, sprung mass oscillation mode. Since the above concept imbeds the characteristics of a variable spring, semi-active suspension, it is expected to have significant practical ride improvement potentials, which may be materialized through further developments and design changes.

The present status in industry as seen from the recent production entries, confirms the above statements. Indeed, although there are several production examples of electronically controlled load leveling and damping (cf. e.g. Chance, 1984; Soltis, 1987), the only active units in production are all of the narrow-bandwidth type, and can be seen, on the Nissan Infinity Q45a and the Toyota Celica. This is despite the fact that the development of broad-bandwidth units had a "head-start" advantage through, for example, Lotus' work as demonstrated in the early 1980s. It is interesting that the (early) Lotus effort did not seem to use skyhook damping

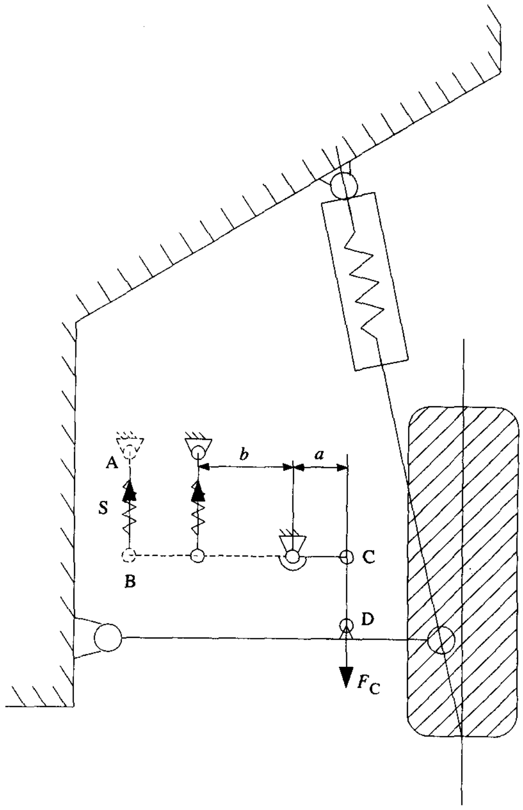


Fig. 26. Delft suspension mechanism with variable lever "b" (based on Venhovens *et al.*, 1992).

(Thompson and Davis, 1991), which may be part of the reason for the lack of corresponding ride improvements. It is also to be observed that, strictly speaking, some "wide bandwidth" actuators may not qualify as such according to the above definition in terms of force production capability, especially around the wheel-hop frequency. Consequently, to establish the wide bandwidth capability of a given actuator, one must first verify that its force frequency response is flat up to the cutoff (3 db) frequency, for different command force levels, and different actuator mounting point motions. However, in practice one seldom sees data demonstrating that this test has been satisfied for the candidate wideband suspensions.

Nevertheless, the pioneers from Lotus deserve full credit for ushering in the trend toward automotive applications of active suspensions. Also, the jury is still out as far as what the "ultimate" active suspension should be, and it is anticipated that the distinction between narrow- and broad-bandwidth suspensions will gradually diminish as the overall suspension ("in-series") compliance becomes an important control variable. The potential advantages of broad-bandwidth suspensions may be expected in the areas of (adaptive) Coulomb friction reduction, new/improved functions, improved handling, and eventually safety, through rapid preview action, for example. As shown in Fig. 16(b), as little as a 50 ms of preview can lead to significant reduction in tire deflection. It is anticipated that, since most of this improvement comes from better wheel-hop control, the broad-bandwidth actuators may be called for here. More precise bandwidth requirements should be the subject of future studies extending the efforts that lead to Fig. 25. Also, the establishing of the extent of the benefits from a reduced deflection magnitude will require further analytical and actual vehicle test investigations. Fast-acting suspension units will be very useful for such and similar studies, where the desired bandwidth can be gradually altered through interchangeable passive modules. A potentially attractive, electro-mechanical implementation of a practical broad-bandwidth actuator has been proposed in (Davis and Patil, 1992; Miller *et al.*, 1992). The proposed Electrical Active Suspension (EAS) actuator consisted of a ball-nut combination, which was used to transform the rotational power from a highly efficient permanent magnet electromotor into a corresponding suspension force and translational velocity. The main advantages of such a concept are in its inherent regenerative capabilities and relatively high bandwidth potentials.

As far as the performance of recent production units is concerned, there are somewhat mixed impressions. While there appear to be some tangible improvements in handling, as seen through reduced roll and pitch during cornering and braking (Akatsu *et al.*, 1990; Goto *et al.*, 1990), the ride improvements are less certain. Some of the reasons for this may be in the relatively high levels of inherent system damping (Akatsu *et al.*, 1990), inadequate inner-loop force generation capability, and the usage of non-optimal and non-adaptive control algorithms. There is also a lack of rms data, which would facilitate more objective and complete assessments based on the above analysis. The limited available rms acceleration data (Akatsu *et al.*, 1990), while encouraging, indicate somewhat modest, about 25% improvements although no corresponding tire and suspension deflection data were given. This level of improvements may not be sufficient to justify relatively high introductory costs of active suspensions (Konik *et al.*, 1992; Schreffler, 1994).

The above cost and complexity issues have been partially offset through the recently introduced active and semi-active roll-control suspensions. It is well known that a good ride comfort can be achieved with softly-tuned passive or semi-active suspension, with corresponding soft roll bar settings. However, this will result in large pitch and roll motions during more aggressive braking and cornering, respectively. The active (or semi-active) roll control is then used to satisfy these conflicting constraints. In practice, the concept is typically realized by inserting a small hydraulic jack and a gas spring (an accumulator) with a control solenoid, all placed in series with the anti-roll bar. The anti-roll bar now can be as stiff as in saloon race cars, without negatively impacting the straight line ride since the solenoid connecting the hydraulic jack and the accumulator is kept open, thus lowering the effective stiffness of the roll-bar–accumulator combination.

During turns, however, the solenoid is closed, so that the full stiffness of the beefed-up anti-roll bar acts to prevent excessive vehicle roll. In the case of extreme turns, the anti-roll action can be further amplified by directing a high pressure oil into the appropriate hydraulic jack cylinder. A production implementation of this concept can be found in 1995 Citroen Xantia Activa (Anonymous, 1995; Ney, 1996), where it effectively augments the already established ride capabilities of their Hydractive suspension (Carbonaro, 1990).

Similar roll-control benefits were discussed in (Reusing *et al.*, 1992), where increased anti-roll moments have been achieved with the help of a hydraulic rotary actuator inserted around the middle of the stabilizer bar. Additional analytical and experimental aspects of roll control have been addressed in many recent papers. For example, an analysis of roll control during a cornering maneuver with a narrow-bandwidth suspension was performed in (Shuttlewood *et al.*, 1992), and the achievable roll response performance with active suspension actuators is presented in (Dorling and Cibon, 1996). An application of active roll stabilization for commercial trucks is described by Kusahara *et al.* (1994). The effectivness of roll moment distribution control with active suspensions has been investigated through analysis and computer simulation (Abe, 1992; Cooke *et al.*, 1996), as well as actual vehicle tests (Williams and Haddad, 1995).

In summary, even though the first generation of active suspensions clearly leaves some room for improvements, it also provides valuable experience to be used for the design and development of more mature and improved, next generation products. Some of the above-mentioned alternatives and preliminary data from (Inagaki *et al.*, 1992) seem promising in this regard.

6.5. *Other aspects*

So far the main focus was on applications dealing with models limited to rigid body dynamics only. This includes vehicle heave, pitch, and roll modes. A consideration of some flexible mode effects can be found in Hac (1986), Nagai and Sawada (1987), and Patten *et al.* (1990), for example. Moreover, these models were almost exclusively given for time-invariant systems, which implies constant or near constant, cruising vehicle speeds. This is a mode of operation most relevant for the ride dynamics. For the cases where vehicle speed variation is important, an extension toward a standard LQG, time-varying setting can be found in (Hammond and Harrison, 1981; Harrison, 1991).

It should also be pointed out that the surveyed work primarily focuses on the secondary suspensions. Additional benefits in improved ride comfort and reduced tire loads can be achieved by controlling/modifying the primary suspensions, e.g. with softer tires (Redlich and Wallentowitz, 1994) or with additional tire damping (Bachrach and Rivin, 1983). Further benefits can also be achieved through proper tire "posture", by actively changing suspension geometry (camber, caster), as was recently proposed in the form of the prototype "biodynamic suspension" (Mitamura, 1994). The benefit potentials of active cab and seat isolation for heavy commercial vehicles are addressed in (Alexandridis and Weber, 1984) and (Stein and Ballo, 1991; Ballo, 1995), respectively.

It is believed that the above optimal control and robustness results provide a solid base for fault tolerant control design, which typically constitutes the next important step in developing a product. This includes the generation of suitable dynamic models needed for system diagnostics and failure accommodation strategies. While all these aspects are extremely important and must be addressed prior to production, they are beyond the scope of the present paper. The interested reader is referred to ongoing research publications in this dynamic field (e.g. DeBenito and Eckert, 1990).

In addition to fault analysis and accommodation, the areas of fuzzy logic and neural network control applications can also benefit from the above optimal control findings. This includes the choice of proper input variables and overall control policy which, at least to a first degree, may mimic the LQ-optimal logic. Due to their practical and intuitive appeal, these techniques, and especially fuzzy logic, may be an efficient "digestive" mechanism to bring the more structured control approaches "home"—in the production car suspensions, power trains, brakes, and other devices.

The emphasis in the present paper was on optimal control applications to the design of advanced automotive suspensions, although many conclusions apply in general to other transportation areas. For more specific surveys of results pertinent to commercial vehicle applications (Gohring *et al.*, 1992; Vohringer *et al.*, 1994); rail vehicle dynamics (Goodall *et al.*, 1981; Williams, 1981; Goodall and Kortum, 1983); magnetically-suspended vehicles (Gottzein *et al.*, 1977; Goodall and Williams, 1984; Nagai and Tanaka, 1992); off-road and military vehicles (Horton and Crolla, 1986); race cars (Metz and Maddock, 1986; Milliken and Milliken, 1995); surface effect ships (Sorensen and Egeland, 1995) and other vehicles, the interested reader should consult the cited papers and references therein.

An integrated treatment of active suspension performance in the broader context of active vibration isolation, as well as the rapidly expanding area of automotive electronics and control, can be found in Fuller *et al.* (1996), Tomizuka and Hedrick (1995) and Jurgen (1995). An introduction to the subject of advanced suspensions, and suspension systems and vehicle dynamics in general, can be found in a number of related books e.g. Bastow (1990), Wong (1993), Gillespie (1994). Earlier advanced suspension surveys from the perspective of European and Japanese researchers, can be found in Sharp and Crolla (1987) and Nagai (1993), respectively. A classified bibliography of more than 500 papers related to the design and analysis of advanced ground vehicle suspension systems was compiled by Elbeheiry *et al.* (1995).

It should be mentioned that active suspensions can be integrated with other advanced vehicle subsystems into more effective, overall interactive vehicle control systems. This in turn will lead to superior and new functions that will impact many aspects of vehicle performance and safety (Yokoya *et al.*, 1990; Matsuo *et al.*, 1990; Yonekawa *et al.*, 1991; Ono *et al.*, 1992; Sato *et al.*, 1992; Mitamura, 1994; Potter *et al.*, 1996). Most of these references typically combine active or semi-active suspensions with four-wheel-steer (4WS), anti-lock braking (ABS) and four-wheel-drive (4WD) to achieve superior performance, handling control, ride comfort and overall vehicle safety during acceleration/braking on straight-line driving, as well as in turns on dry or slippery roads. Further major potential functional improvements, especially in the area of active safety and accident prevention, are anticipated by combining different advanced suspension concepts with recently introduced Vehicle Dynamic/Stability Control or Interactive Vehicle Dynamic control (VDC/FDR, VSC, IVD) schemes (e.g. Van Zanten *et al.*, 1994). Finally, while the present survey is obviously limited to terrestrial applications only, those interested for suspension and mobility issues in other (e.g. lunar) environments should consult Wallace and Rao (1993) and references therein.

7. CONCLUDING REMARKS

Based on the two or three decades of analytical developments, it can be concluded that each of the above discussed vehicle models with increasing complexity had a useful evolutionary role and contributed to subsequent advancements. For example, the analysis based on a simple 1 DOF, quarter-car model established that active suspensions have a potential to substantially improve ride and handling when compared with their conventional, passive counterparts. The resulting optimal structure gave birth to the concepts of skyhook damping and fast load leveling which are now being developed toward actual, large-scale production applications.

The fundamental insight gained from this simple model created a solid foundation for the next step, i.e. investigation of 2 DOF, quarter-car models which include wheel-hop dynamics. This introduced an additional handling-related constraint with associated loss of performance when compared with the 1 DOF case. Much of this loss can be recovered via appropriate structural changes such as the introduction of dynamic (vibration) absorbers and reduction of unsprung masses.

Once a good understanding of active suspension capabilities based upon the 1D quarter-car models has been obtained, the next natural step is to consider 2D models that include pitch and heave modes. The additional dimension creates opportunities for preview control through front wheels. This can lead to further significant improvements in ride and handling. Finally, it is believed that a comprehensive consideration of full-car, 3D models in conjunction with other active vehicle control schemes will bring further distinct functional and safety-related benefits. A more precise quantification of the latter is the subject of ongoing research efforts.

In closing, it should be reemphasized that the above somewhat idealized optimal control approach is not a panacea, and it does not contain all the answers and micro details that are needed for a practical implementation of advanced suspension concepts. For example, most of the results are based under the assumption of an ideal, infinite bandwidth force-production actuator. In reality, the latter will always have a limited bandwidth, which will also be sensitive to the road-induced disturbances (Hrovat and Hubbard, 1987). Additional factors influencing practical implementations include considerations of: sensor performance, reliability and cost; overall system power consumption, weight, cost and packaging requirements (Goran *et al.*, 1992); and more precise overall ride and handling implications to be assessed through tests and more detailed vehicle models (Orlandea and Chase, 1977; Wilson and Bachrach, 1984; Sayers and Mousseau, 1990).

One could easily be overwhelmed by the above numerous considerations (which could be further obscured by sometimes too optimistic advertisements about "omnipotent, latest-and-greatest" control techniques) and become "lost in a jungle of details". It is precisely for this reason that the above relatively simple optimal control path is advocated: by predicting the main trends in terms of performance, suspension structure, and bandwidth, the approach serves as a beacon through the "jungle" avoiding the quicksand of endless and rudderless trial-and-error iterations.

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