Analysis of Sympathetic Inrush Phenomena in Transformers Using Coupled Field-Circuit Approach

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Abstract—Sympathetic inrush current phenomenon occurs when a transformer is switched on in a power system network containing other transformers which are already energized. In this paper, the phenomenon of sympathetic inrush current is investigated using nonlinear-transient field-circuit coupled finite element formulation. The cases of the transformers connected in parallel and in series have been considered. The results obtained for a 31.5 MVA, three-phase transformer are presented and discussed in detail. This work also includes the investigation of parameters affecting the magnitude and duration of sympathetic inrush phenomenon, such as series resistance, switching-on angle, residual flux density and load conditions. These analyses provide a sound theoretical basis for a thorough understanding of the phenomenon.

Index Terms—Sympathetic inrush phenomenon, finite element method, field-circuit coupling, nonlinear transient.

I. INTRODUCTION

MAGNETIZING inrush current occurs during the energization of a single transformer connected to a power system network with no other transformers. However, the energization of a transformer connected to a network in the presence of other transformers, as shown in Fig. 1, which are already in operation, leads to the phenomenon of sympathetic inrush current. Although, severity of the inrush current is higher during single transformer energization, the sympathetic inrush current is of special importance due to its unusual characteristics. The inrush current in a transformer decays, usually, within a few cycles, but the sympathetic inrush current persists in the network for a relatively longer duration. This poses an additional threat to the reliability and security of the power system. It may lead to a false operation of transformer differential relays and may prolong temporary harmonic over-voltages. Finally, it may lead to an increase in the noise level of other transformers connected in the network [1].

The available literature includes numerous investigations on the inrush current phenomenon in power transformers and its impact on the design and operation of protection schemes [2–7]. Most of the approaches are based on the electrical equivalent circuit [5, 7–9] or magnetic equivalent circuit [10–13]. The sympathetic inrush current has also been analyzed in a few references [14–16]. However, the models reported do not provide sufficient accuracy in the case of asymmetrical or transient operations involving magnetic nonlinearities. During

the saturation of core, a large amount of flux is present in the air, which is not considered accurately in the equivalent network models. Also, it is difficult to take into account actual geometrical effects. In case of such phenomenon, flux distribution should be examined by solving the phenomenon electromagnetically to understand it exactly. The strong coupling between field and electric circuit, and the presence of nonlinear elements necessitate the use of direct solution approach.
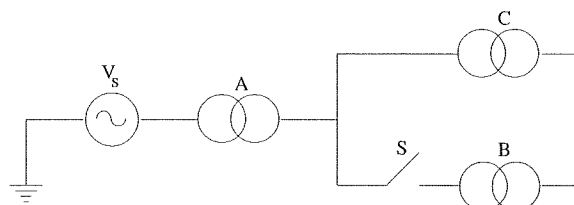

Fig. 1. Sympathetic interaction between transformers.

In this work, nonlinear-transient field-circuit coupled 2-D axisymmetric simulations are performed to investigate the sympathetic inrush phenomenon in parallel and series connected transformers. Coupled field-circuit analysis for the phenomenon has not been reported in the existing literature to the best of the authors' knowledge. Nonlinearity is taken into account by using the actual B-H curve of the core material. This paper describes a detailed modeling methodology and analysis of the sympathetic inrush phenomenon. The study of factors affecting the magnitude and duration of the sympathetic inrush current are also presented.

II. SYMPATHETIC INTERACTION BETWEEN TRANSFORMERS

A transformer already connected to supply system can experience unexpected saturation during the inrush transient of an incoming transformer. This saturation, which is established by an asymmetrical voltage drop across the system resistance caused by the inrush current in the incoming transformer, demands offset magnetizing currents of high magnitude in the already connected transformers. As shown in Fig. 1, when transformer B is switched on to the network where transformers A and C are already feeding loads, the transient inrush currents not only flow through B, but also through transformers A and C. This sharing of the transient inrush current is called as *sympathetic inrush phenomenon*.

The normal inrush current in the transformers decay, usually, within a few cycles, but the sympathetic inrush current persists in the circuit for a relatively longer duration [17].

The insight into this behavior can be obtained by comparing Fig. 2(a), which shows the inrush current in an isolated transformer connected to the power system network, with Fig. 2(b), which shows both the inrush current and the sympathetic inrush current in the incoming and the already energized transformer respectively. Longer duration of the sympathetic inrush current may mal-operate differential relays. It may also prolong harmonic over-voltages in the system and increase noise level in the already connected transformers.

We have followed the convention of representing the inrush current in blue and the sympathetic inrush current in red in all the figures in this paper.
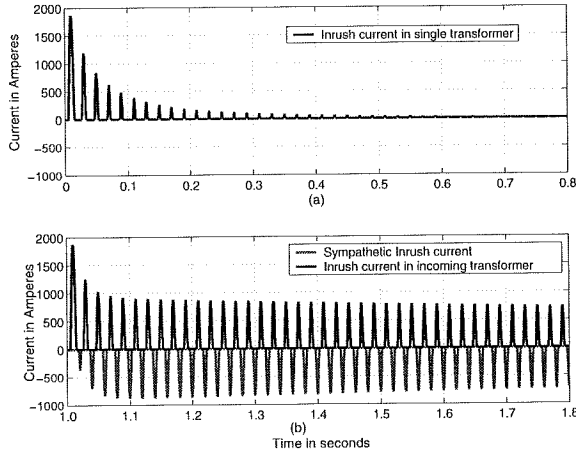

Fig. 2. Typical waveforms of (a) inrush and (b) sympathetic inrush current.

III. COUPLED FIELD-CIRCUIT FORMULATION

A. Electromagnetic Model

For a 2-D axisymmetric magnetic analysis, consider the classical Poisson's equation [18]:

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{1}{\mu}\frac{\partial A_R}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{r}\frac{1}{\mu}\frac{\partial A_R}{\partial z}\right) = -J_\phi \quad (1)$$

where A_R $(= r\,A_\phi)$ and J_ϕ denote the magnetic vector potential and the free or the source current density respectively, and μ is the material permeability.

It is assumed that the windings are made up of thin stranded conductors. These thin stranded conductors can be modeled as carrying uniform current densities, i.e., the induction term in the magnetic field equations representing eddy currents is neglected. Assuming that each turn carries I amperes of current, the magnitude of current density in a winding is:

$$J_\phi = d\frac{nI}{S_c} \quad (2)$$

where n and S_c are the number of turns and the cross-sectional area of the winding respectively, and d is the polarity (+1 or -1) to represent the forward or the return path in ϕ-direction. After space discretization of Eq. (1) using Galerkin method [19], we have,

$$[K]\{A_R\} + [D]\{I\} = 0 \quad (3)$$

where

$$[K] = \sum_\Omega \frac{1}{r}\frac{1}{\mu}\iint_{\Delta_e}\left(\frac{\partial N_e^T}{\partial r}\frac{\partial N_e}{\partial r} + \frac{\partial N_e^T}{\partial z}\frac{\partial N_e}{\partial z}\right)dr\,dz$$

$$[D] = \sum_\Omega \frac{n}{S_c}\iint_{\Delta_e} N_e^T dr\,dz$$

where N_e and Δ_e are the element shape function and the element area respectively, and Ω indicates the domain under consideration.

B. External Circuit Equations

The matrix form of the external circuit equations can be written as:

$$[U] = \left\{\frac{d\Phi}{dt}\right\} + [R]\{I\} + [L]\left\{\frac{dI}{dt}\right\} \quad (4)$$

If flux linkages are expressed in terms of the magnetic vector potential, the matrix form of the external circuit equations can be written as:

$$[U] = [G]\left\{\frac{dA_R}{dt}\right\} + [R]\{I\} + [L]\left\{\frac{dI}{dt}\right\} \quad (5)$$

where U is the vector of input voltages, $[L\,]$ and $[R\,]$ are the matrices of leakage inductances and winding resistances respectively, and G is a matrix similar to D and it depends on the geometrical features of the windings.

C. Nonlinear-Transient Field-Circuit Coupled Model

From (3) and (5), the field-circuit coupled global system of equations can be written as:

$$\begin{bmatrix} 0 & 0 \\ G & L \end{bmatrix}\left\{\begin{array}{c} \dot{A}_R \\ \dot{I} \end{array}\right\} + \begin{bmatrix} K & D \\ 0 & R \end{bmatrix}\left\{\begin{array}{c} A_R \\ I \end{array}\right\} = \left\{\begin{array}{c} 0 \\ U \end{array}\right\} \quad (6)$$

In this system of equations, the unknowns are the nodal values of the vector potential (A_R) and the currents in the external circuits (I). To solve this time dependent system of equations, a numerical integration scheme such as Euler Backward or Crank-Nicholson algorithm can be used. The nonlinearities in the modeled system are taken into account by using the Newton-Raphson iterative procedure. The resulting system of equations is,

$$\begin{bmatrix} \beta(K^n + K_{nl}^n) & \beta D \\ \frac{G}{\Delta t} & \beta R + \frac{L}{\Delta t} \end{bmatrix}\left\{\begin{array}{c} \Delta A_R \\ \Delta I \end{array}\right\}_{t+\Delta t}^{n+1}$$

$$= \left\{\begin{array}{c} 0 \\ \beta V_{t+\Delta t} + (1-\beta)V_t \end{array}\right\}$$

$$+ \begin{bmatrix} -(1-\beta)K & -(1-\beta)D \\ \frac{G}{\Delta t} & -(1-\beta)R + \frac{L}{\Delta t} \end{bmatrix}\left\{\begin{array}{c} A_R \\ I \end{array}\right\}_t$$

$$- \begin{bmatrix} \beta(K^n) & \beta D \\ \frac{G}{\Delta t} & \beta R + \frac{L}{\Delta t} \end{bmatrix}\left\{\begin{array}{c} A_R \\ I \end{array}\right\}_{t+\Delta t}^{n} \quad (7)$$

where the term K_{nl}^n accounts for the nonlinearity of the core material. The value of β determines the nature of the time-stepping scheme [19].

IV. COMPUTATIONAL METHODOLOGY

A 31.5 MVA, 132/33 kV, star/delta, three-phase transformer is analyzed to investigate the sympathetic inrush phenomenon. The design details and name-plate data of the transformer under consideration are given in Table I. The 2-D axisymmetric field-circuit coupled model is developed based on the methodology described in section III. The transformer is analyzed on a per-phase basis with excitation of LV winding. Three-phase analysis is also done subsequently to compare the inrush currents in all the three phases.

TABLE I

THREE-PHASE THREE-LIMB TRANSFORMER DATA

Rated LV current	318.2 A
Rated HV current	137.78 A
Number LV turns	433
Number HV turns	1000
Percentage impedance	14.33
Core steel type	M3H

Fig. 3 shows the circuit connection diagram used in the simulation. It consists of two field domains corresponding to the two identical transformers, which are modeled using FEM. The resistances R_1 and R_2 correspond to the source and the transmission line resistances respectively. The equations of the field domains are then coupled with the electrical circuit equations of the associated power system network (as described in section III). The advantage with using the FEM based model is that the actual geometry and material properties are considered, which increases the accuracy of the obtained results as compared with other models, which are based on equivalent circuit and reluctance network approaches. Strongly coupled field-circuit model considers the dynamics of the circuit and field parts accurately. In this work, 2-D axisymmetric model is used because it is sufficiently accurate, and also transient nonlinear problems require the simulation to run for a longer time, thereby, making 3-D models computationally expensive.
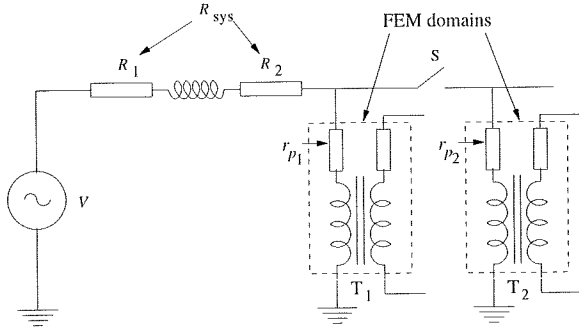

Fig. 3. Electrical circuit connection for sympathetic inrush current analysis.

V. RESULTS AND DISCUSSION

A. Parallel Connected Transformers

Parallel configuration of two transformers is shown in Fig. 3. The switch S is closed after one second to allow initial circuit transient currents to reach their respective steady state values. For the configuration of the parallel-connected transformers, the influence of the factors such as system resistance, switching-on angle, residual magnetism of incoming transformer and load current, is investigated.

- **Effect of system resistance:** Inrush current in transformer T_2 and sympathetic inrush current in transformer T_1 are shown in Fig. 4(a), Fig. 4(b) and Fig. 4(c) for system resistances of 1 Ω, 5 Ω and 20 Ω respectively. R_{sys} reduces the magnitudes of the initial peaks of the inrush current in T_2 considerably. The sympathetic inrush current experienced by the already energized transformer T_1 is due to the coupling between the two transformers on account of the asymmetrical voltage drop in the system resistance R_{sys} of the transmission line feeding them. Hence, the higher the R_{sys}, the higher is the sympathetic inrush [17] as evident from the figure. However, the magnitude of R_{sys} has very little effect on the duration of the sympathetic inrush current in T_1. From Fig. 4, it can be observed that the magnitude of the current at around 1.8 s is almost same for all values of R_{sys}.
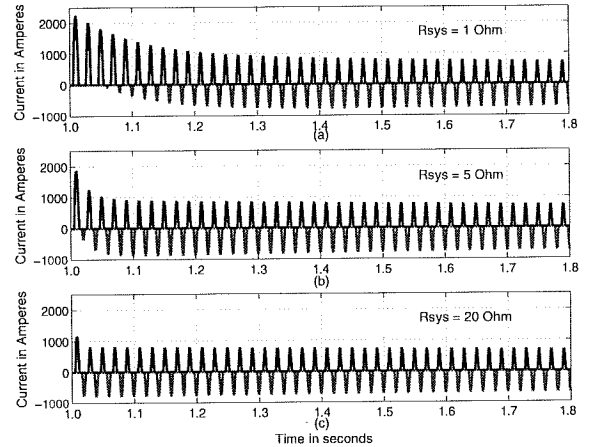

Fig. 4. Effect of system resistance (R_g) on sympathetic inrush current phenomenon.

- **Effect of switching-on angle:** Fig. 5 shows the effect of switching-on angle on the magnitude of the sympathetic inrush current. Similar to the single transformer case, the magnitude of the sympathetic inrush current reduces with an increase in the switching-on angle up to 90°, and then increases in the negative direction up to 180°. The magnitudes of the inrush currents are minimum when the switching-on angle is 90° (because the inductive reactance of the transformer is much larger than the resistance). Figs. 5(a), 5(b) and 5(c) show the sympathetic inrush current patterns for switching angles of 36°, 90° and 180° respectively.
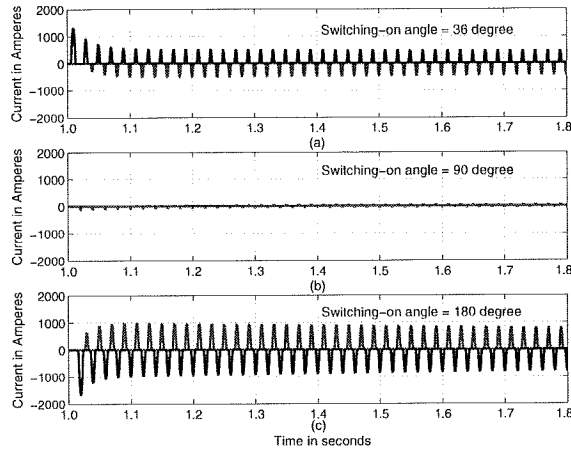
Fig. 5. Effect of switching-on angle on sympathetic inrush current phenomenon.

- **Effect of residual magnetism:** Similar to the single transformer case, residual magnetism of the incoming transformer plays a key role in the sympathetic inrush phenomenon. Fig. 6 shows the inrush current in T_2 and sympathetic inrush current in T_1 at different residual magnetization conditions. It can be seen that the magnitudes of these currents largely depend on the polarity of the residual flux density. If the transformer is switched-on at angle zero with positively increasing voltage wave, positive polarity of flux density leads to higher values of inrush currents and vice-versa.
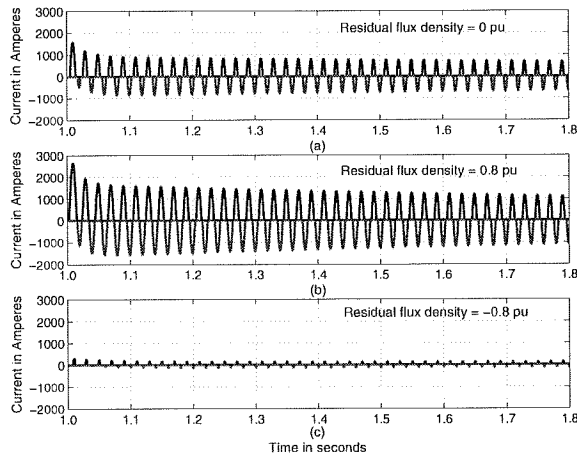

Fig. 6. Effect of residual flux on sympathetic inrush phenomena.

- **Effect of load current:** When the transformer is switched on to a heavy load with power factor close to unity, the peak values of both inrush and sympathetic inrush currents are slightly smaller as compared to the no-load case (see Fig. 7). The peak of inrush current in T_2 as well as the sympathetic inrush current in T_1 slightly increase with reduction in power factor as evident from Fig. 8, which shows these currents for different power factors of the load. Thus, the load current of the switched

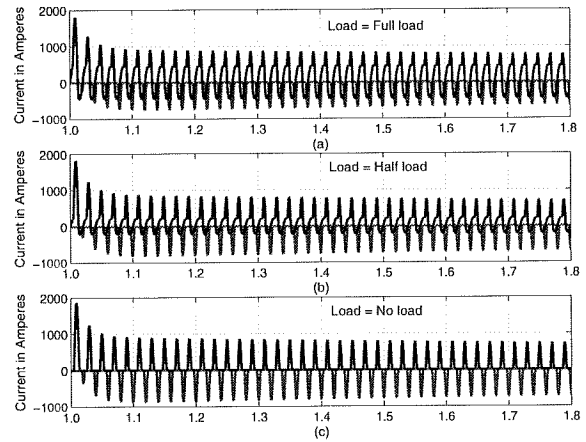transformer produces small effect on the magnitude and duration of the sympathetic inrush current.

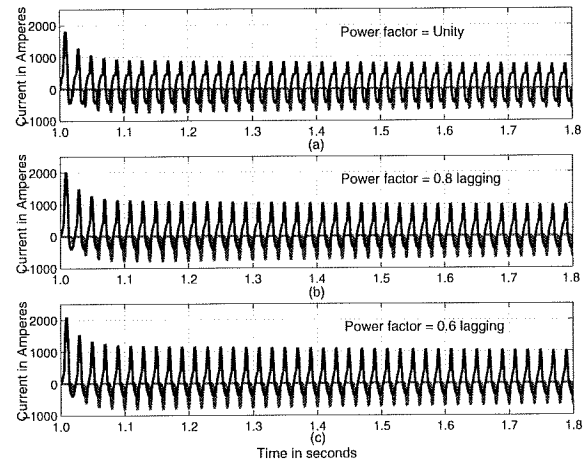Fig. 7. Effect of load current on sympathetic inrush phenomena.

Fig. 8. Effect of load power factor on sympathetic inrush phenomena.

B. Series Connected Transformers

In the case of transformers connected in series, the primary LV winding of the first transformer (33/132 kV) is connected to supply, and its secondary winding feeds the HV winding of the transformer (132/33 kV) being energized. The nameplate details of both the transformers are same as in Table I. Fig. 9 shows T_1, the already connected transformer, and T_2, the transformer that is being energized by closing the switch S. The currents in the primaries of transformers T_2 and T_1 are shown in Fig. 10(a) and Fig. 10(b) respectively. The positive half cycles of the primary current of T_1 correspond to the inrush current of T_2 referred to the primary of T_1 since the two transformers are series connected. The negative half cycles of this current correspond to the sympathetic inrush phenomenon.

C. Sympathetic Inrush in Three Phase Transformers

A three-phase model has also been developed, which represents the actual condition at site. Fig. 11 shows the normal
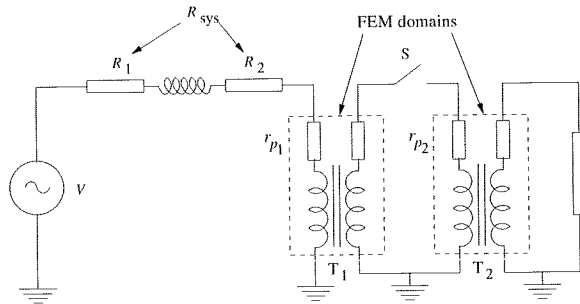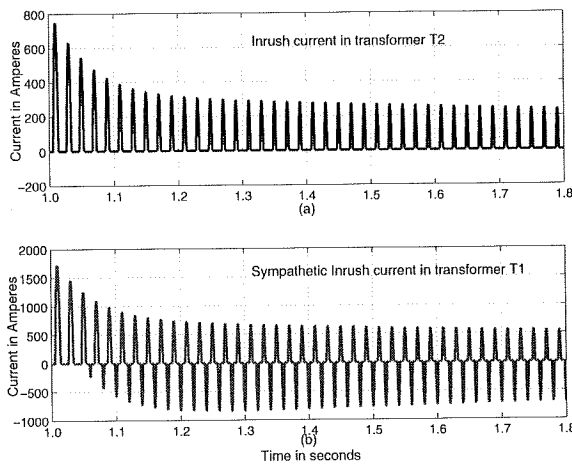
Fig. 9.   Series connected transformers.

Fig. 10.   Sympathetic inrush current phenomenon in series connected transformers.

inrush current and the sympathetic inrush current for the configuration of parallel connected transformers. All the three-phase currents in both the transformers are marked.
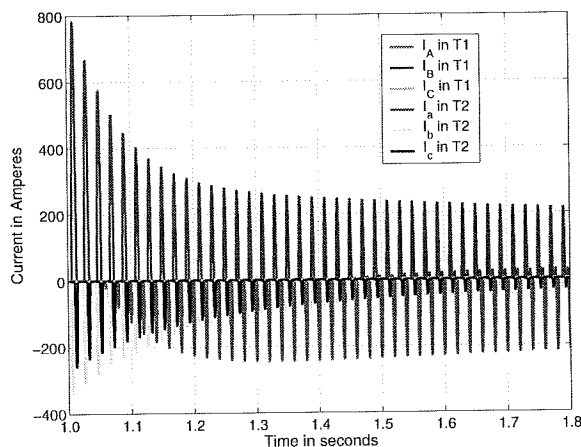

Fig. 11.   Inrush current phenomenon in three-phase transformers.

VI. CONCLUSION

In this paper, we have presented a detailed formulation and modeling for the analysis of sympathetic inrush phenomenon

for the configuration of parallel and series connected transformers. Two 31.5 MVA, three-phase, 132/33 kV, transformers are modeled using FEM. The FEM field equations of the transformers are then solved simultaneously with the circuit equations of the power system network. The analysis is first done on a per-phase basis. Subsequently, three-phase simulation results are also reported.

It is observed that the sympathetic inrush current persists in the network for a much longer duration than the inrush current for the singly connected transformer. The parameters affecting the magnitude and duration of the current, such as series resistance, switching-on angle, residual flux density and load conditions are discussed in detail. It is observed that even though an increase in the system resistance increases the magnitude of the sympathetic inrush current appreciably, there is little effect on its duration. The change in switching-on angle, and the magnitude and direction of the residual flux density can cause significant variations in the phenomenon of sympathetic inrush currents. Moreover, the magnitude and power factor of the load currents have only a very small effect on the magnitude as well as duration of the sympathetic inrush currents.

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