EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng Struct. Dyn. 2003; **32**:1117–1131 (DOI: 10.1002/eqe.265)

Application of non-linear multiple tuned mass dampers to suppress man-induced vibrations of a pedestrian bridge

Nakhorn Poovarodom[1,*,†], Sopak Kanchanosot[1] and Pennung Warnitchai[2]

[1] *Faculty of Engineering, Thammasat University, Pathumthani 12121, Thailand*
[2] *School of Civil Engineering, Asian Institute of Technology, Pathumthani 12120, Thailand*

SUMMARY

This paper presents an application of multiple tuned mass dampers (MTMDs) with non-linear damping devices to suppress man-induced vibrations of a 34 m long pedestrian bridge. The damping force generated by each of these damping devices is simply a drag force from liquid acting on an immersed section. The quadratic non-linear property of these devices was directly determined from free vibration tests of a simple laboratory set-up. Dynamic models of the bridge and pedestrian loads were constructed for numerical investigation based on field measurement data. The control effectiveness of non-linear MTMDs was examined along with its sensitivity against estimation errors in the bridge's natural frequency and magnitude of pedestrian load. The numerical results indicated that the optimum non-linear MTMD system was as effective and robust as its linear counterpart. Then, a six-unit non-linear MTMD system was designed, constructed, and installed on the bridge. Field measurements after the installation confirmed the effectiveness of non-linear MTMDs, and the measurement results were in good agreement with numerical predictions. After the installation, the average damping ratio of the bridge was raised from 0.005 to 0.036 and the maximum bridge accelerations measured during walking tests were reduced from about 0.80–1.30 $\mathrm{m\,s^{-2}}$ to 0.27–0.40 $\mathrm{m\,s^{-2}}$, which were within an acceptable range. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: multiple tuned mass dampers; non-linear damping; pedestrian bridge; man-induced vibration

INTRODUCTION

After the pioneering work on the optimal design of tuned mass dampers (TMDs) by Den Hartog in 1956 [1], there has been a considerable amount of subsequent research studies conducted on both the theory and practice of vibration control using TMDs. A recent extension of this original concept to the use of multiple tuned mass dampers (MTMDs), where the

* Correspondence to: Nakhorn Poovarodom, Department of Civil Engineering, Faculty of Engineering, Thammasat University, Pathumthani 12121, Thailand.
† E-mail: pnakhorn@engr.tu.ac.th

Contract/grant sponsor: Thailand Research Fund; contract/grant number: PDF/34/2541.

Received 15 February 2001
Accepted 25 September 2002

natural frequencies of TMDs are distributed over a frequency range, has shown a possibility to achieve superior control effectiveness and robustness compared to those of a single TMD of equal total mass [2, 3]. Several studies have been made to identify the optimal parameters for MTMDs [4, 5], and there have already been a few cases where TMDs were used to suppress vibrations in civil engineering structures [6].

So far, MTMDs have been generally considered to have linear dynamic properties, where each TMD unit can be modelled as a linear single-degree-of-freedom system consisting of a mass unit, a linear spring element, and a linear damper unit. Accordingly, the damping force generated by the damper unit must be in linear proportion to the relative velocity between the TMD's mass and the main structure. In practice, this linear damping force can be developed by the viscous force acting on the shear surface of a device immersed in high viscosity fluid. In this way, the viscosity of the fluid must be kept constant (or nearly constant) over the expected service life of the damper. This is sometimes difficult to achieve because viscosity is a fluid property that is quite sensitive to changes in temperature and, in many cases, can be affected by some chemical reactions of the fluid to the surrounding environment.

It has been shown that the vibration control effectiveness of MTMDs is quite stable over a wide range of their damping ratios, and the optimal damping ratios are much lower than that of a single TMD of equal total mass [3]. This indicates a possibility of using damping devices that are not necessarily linear, as long as their 'equivalent damping ratios' lie within such an effective range. Therefore, instead of using viscous shear force, it may be easier to use pressure-induced drag force, which is produced by dynamic fluid pressure acting on the surface of a moving immersed section. This type of drag force is not linear; its magnitude is in proportion to the square of the motion velocity, i.e. a quadratic non-linearity. The proportional constant depends on the fluid mass density and the geometry of the immersed section. Sufficient drag force can be easily obtained even with the use of low viscosity fluid such as water. Moreover, the non-linear characteristic of the drag force is quite stable because the fluid mass density, unlike the viscosity, is not sensitive to changes in temperature and other physical and chemical factors. Therefore, damping devices based on pressure-induced drag force can be constructed at relatively low cost, its characteristic can be maintained over a longer period of time, and its fluid can be easily replaced.

For this reason, this paper presents a new concept of a simple damping device with such non-linear damping characteristic for a MTMD system. The introduced damping device is aimed to be more practical in terms of fabrication and maintenance than the conventional one. In this study, experiments were done in a simple laboratory set-up to investigate non-linear damping characteristics of the damping device first, then the model set-up was applied for constructing a non-linear MTMD system. Numerical analyses were carried out to identify optimal parameters and compare the control performances of the proposed non-linear MTMDs with those of linear MTMDs and a single TMD of equal total mass. Finally, the real application of the designed non-linear MTMDs was made on a pedestrian bridge in order to suppress the vertical vibration due to human walking.

NON-LINEAR DAMPER

As mentioned earlier, there are generally two types of forces acting on a moving section in fluid: viscous shear force and pressure-induced force. The viscous shear force results from

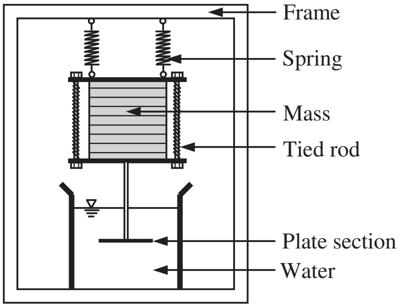

Figure 1. Schematic assembly of TMD unit.

integrating the fluid shear stresses throughout the entire surface, while the pressure-induced force results from integrating the fluid normal stresses over the same surface area. The magnitude of viscous shear force is in linear proportion to the velocity of the moving section, while the pressure-induced force is in linear proportion to the square of the velocity.

The moving section chosen in this study is a thin flat plate with its plane placed perpendicular to the direction of the motion as shown in Figure 1. With this configuration, the viscous shear force is minimized and can be neglected when compared with the pressure-induced force. Hence, the damping force generated by this device, denoted by f_D, is approximately equal to the pressure-induced force:

$$f_\mathrm{D} = \beta|\dot{u}|\dot{u} \tag{1}$$

where $\dot{u}$ is the velocity of the moving plate and β is a non-linear damping constant:

$$\beta = \tfrac{1}{2}\rho A C_\mathrm{d} \tag{2}$$

where ρ is the fluid mass density, A is the area of the plate, and C_d is a sectional constant known as drag coefficient.

If the plate moves at a constant velocity in fluid, C_d can be set equal to the drag coefficient of the plate section in steady flow, the value of which can be obtained from a large number of published experimental results in fluid dynamics. But in this case, the plate oscillates in the fluid, and the fluid flow around the oscillatory section is unsteady. It was considered that it would be more reliable to determine β, instead of C_d, directly from simple physical experiments.

A simple experimental model as shown in Figure 1 was set up for this purpose. The model was later used as a non-linear TMD unit. The model consists of a set of mass-springs with a specific frequency and a thin plate immersed in water. To evaluate β, free vibration tests were conducted, and the measured responses were compared with the response from numerical simulations of a single-degree-of-freedom system with the non-linear damping force model in Equation (1). The ratio of measured amplitudes of two adjacent cycles in terms of damping ratio, $(1/2\pi)\ln(A_r/A_{r+1})$, was plotted against the amplitude of the rth cycle, A_r, as shown in Figure 2. The result from the numerically simulated response with an assumed value of β is also presented in the figure as a solid line. The value of β that gave the smallest discrepancy

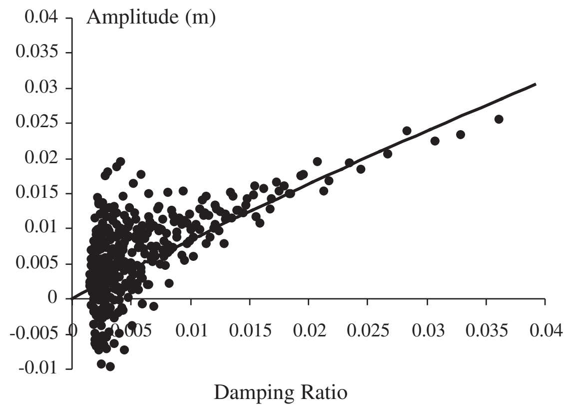

Figure 2. Amplitude-dependent characteristics of the quadratic non-linear damper.

between the results from the test and the simulation was taken as the non-linear damping constant to be used in the following analysis.

In the small amplitude range, the test result deviates from simulation by some degrees because of the effects of the Reynold's number [7] and some sources of experimental errors. However, in the large amplitude range, which is the working condition of TMDs at the desired performance, the discrepancies are so small that the reliable value of β can be determined. Apart from water, other types of fluid with different mass density and viscosity such as glycerin and silicone oil were also tested by following the same procedure. Their results were similar to the case of water, and the obtained non-linear damping constant β varied in linear proportion to the mass density as described by Equation (2). Water was finally selected because it is easy to procure and its cost is negligibly low.

MODEL FORMULATION FOR STRUCTURE-MTMD SYSTEMS

A model of a single degree-of-freedom structure attached by n TMD units as shown in Figure 3 is considered. This model can be used for examining the effectiveness of MTMDs in suppressing a specific vibration mode of the main structure. The equations of motion of the system can be formulated in a matrix form as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{p} \tag{3}$$

where

$$\mathbf{u} = [u_S, u_1, u_2, \ldots, u_n]^T \tag{4}$$

and

$$\mathbf{p} = [p(t), 0, 0, \ldots, 0]^T \tag{5}$$

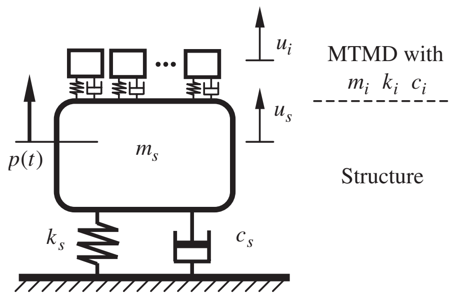

Figure 3. Structure with MTMDs.

$(\dot{\ })$ and $(\ddot{\ })$ signify the first- and second-order partial derivatives with respect to time t, u is the displacement, and $p(t)$ is the external force. The subscript ‘S’ indicates that the quantity belongs to the main structure, while the subscript ‘i’ is for the ith TMD, where $i = 1, 2, \ldots, n$. The mass matrix, $\mathbf{M}$, and the stiffness matrix, $\mathbf{K}$, can be derived as

$$\mathbf{M} = \begin{bmatrix} m_S & 0 & 0 & \ldots & 0 \\ 0 & m_1 & 0 & \ldots & 0 \\ 0 & 0 & m_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & m_n \end{bmatrix} \tag{6}$$

$$\mathbf{K} = \begin{bmatrix} k_S + \sum_{i=1}^{n} k_i & -k_1 & -k_2 & \ldots & -k_n \\ -k_1 & k_1 & 0 & \ldots & 0 \\ -k_2 & 0 & k_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -k_n & 0 & 0 & \ldots & k_n \end{bmatrix} \tag{7}$$

For MTMDs with linear damping properties, the damping matrix, $\mathbf{C}$, is given by

$$\mathbf{C} = \begin{bmatrix} c_S + \sum_{i=1}^{n} c_i & -c_1 & -c_2 & \ldots & -c_n \\ -c_1 & c_1 & 0 & \ldots & 0 \\ -c_2 & 0 & c_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -c_n & 0 & 0 & \ldots & c_n \end{bmatrix} \tag{8}$$

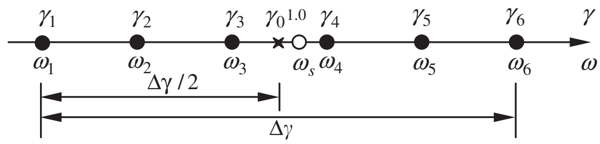

Figure 4. The distribution of TMDs' frequency.

Alternatively, for MTMDs with quadratic non-linear damping devices, the damping matrix becomes

$$\mathbf{C}(\dot{u}) = \begin{bmatrix} c_S + \sum_{i=1}^{n} \beta_i|\dot{u}_i - \dot{u}_S| & -\beta_1|\dot{u}_1 - \dot{u}_S| & -\beta_2|\dot{u}_2 - \dot{u}_S| & \dots & -\beta_n|\dot{u}_n - \dot{u}_S| \\ -\beta_1|\dot{u}_1 - \dot{u}_S| & \beta_1|\dot{u}_1 - \dot{u}_S| & 0 & \dots & 0 \\ -\beta_2|\dot{u}_2 - \dot{u}_S| & 0 & \beta_2|\dot{u}_2 - \dot{u}_S| & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ -\beta_n|\dot{u}_n - \dot{u}_S| & 0 & 0 & \dots & \beta_n|\dot{u}_n - \dot{u}_S| \end{bmatrix} \tag{9}$$

where m, k and c are mass, stiffness and linear damping coefficients of each degree-of-freedom, respectively.

The matrix equation of Equation (3) can be solved numerically to obtain the response of the system. The numerical integration in this study employs the linear acceleration assumption with Wilson-θ modification [8]. The time step for the integration was set to 2% of the natural period of the main structure in order to reduce the artificial damping and period elongation effects from the numerical scheme.

To facilitate the following investigation on the performance of MTMDs, non-dimension parameters of the ith TMD are defined as mass ratio, μ_i, tuning ratio, γ_i, and damping ratio (for linear TMD), ξ_i.

$$\mu_i = \frac{m_i}{m_S}; \quad \gamma_i = \frac{\omega_i}{\omega_S}; \quad \xi_i = \frac{c_i}{2m_i\omega_i} \tag{10}$$

where $\omega = \sqrt{k/m}$ is the circular natural frequency. In the case of non-linear MTMDs, the parameter β_i is used directly in the analysis.

Owing to some practical constraints, the scope of investigation is limited to cases where

1. The number of TMD units is six.
2. All non-linear damping constants, β_i for $i = 1, 2, \dots, n$, are identical.
3. All linear damping ratios, ξ_i for $i = 1, 2, \dots, n$, are identical.
4. All TMD units have identical spring stiffness.
5. The masses of TMD units are different from each other, but the total mass ratio, $\sum_{i=1}^{6} \mu_i$, is 0.01.
6. The masses are set such that the natural frequencies of TMD units are distributed with equal spacing in the neighbourhood of the structural natural frequency. The distribution band of frequency ratios of MTMDs is $\Delta\gamma$. The central frequency ratio, γ_0 defined as $(\gamma_1 + \gamma_6)/2$, expresses the offset of the centre of $\Delta\gamma$ from 1.0 as shown in Figure 4.

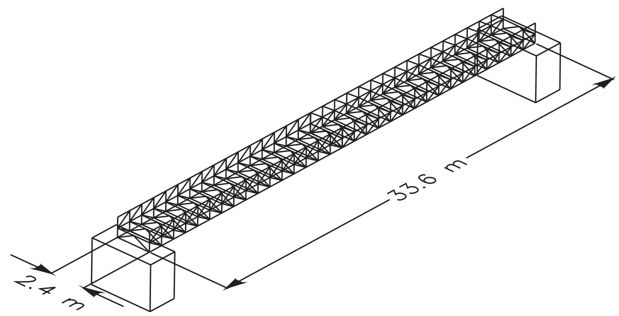

Figure 5. Steel frame alignment of the Jatujak pedestrian bridge.

NUMERICAL INVESTIGATION FOR A PEDESTRIAN BRIDGE PROBLEM

An application of MTMDs to suppress man-induced vibrations of a pedestrian bridge was numerically investigated. The investigation employed the structure-MTMD model formulated in the previous section. To ensure that the model will truly represent the vibration problem, important physical and dynamic properties of the bridge, as well as key characteristics of human walking loads, were directly identified from field measurements.

The pedestrian bridge selected in this study is located at Jatujak weekend market in Bangkok, Thailand. The bridge was found to vibrate strongly in the vertical direction, and the vibration caused alarm to pedestrians. The bridge structure is a steel truss simply supported on rigid concrete piers as shown in Figure 5. The span length is 33.6 m and the width is 2.4 m. Its mass and effective flexural rigidity are approximately uniform over the entire span. The mass per unit length is about $1100\,\mathrm{kgm}^{-1}$. The bridge can be modelled as a uniform simple span beam, and its dynamic response can be sufficiently represented by the fundamental vibration mode with a half-cycle sine shape. Therefore, the corresponding first modal mass, m_S, is about 18 500 kg.

During the time when there were no pedestrians, free vibration tests were carried out, and the vertical acceleration response at the mid-span of the bridge was measured and recorded by three piezoresistive accelerometers and a 12-bits portable data acquisition system. The fundamental frequency computed from the records was found to be 2.00 Hz and the damping ratio was about 0.005.

Characteristics of the pedestrian loads were identified from direct observation conducted during peak time on a weekend. From this study, the average pacing rate and forward velocity of pedestrians were estimated to be 1.83 Hz and $1.11\,\mathrm{ms}^{-1}$, respectively. The arrival time of pedestrians was found to be random. The number of pedestrians arriving on the bridge during each 15 s time interval was counted and the statistical distribution appeared to resemble a Poisson distribution with an occurrence rate of about 5 pedestrians per $15\,\mathrm{s}^{-1}$ interval as shown in Figure 6. From the average number of pedestrians arriving, walking speed and bridge span length, the total number of pedestrians simultaneously on the bridge, n_p, is about 10.

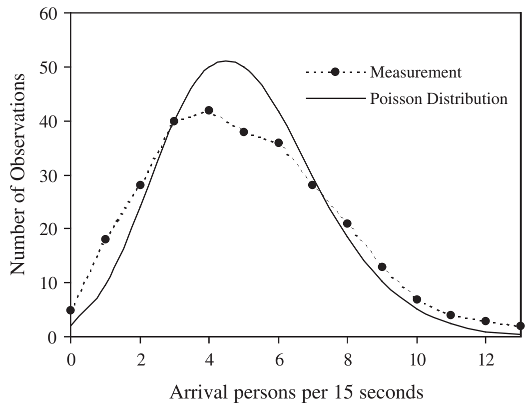

Figure 6. Distribution of arrival probability of pedestrians.

The model of the vertical dynamic force, $p(t)$, in Equation (5) of one walking pedestrian can be mathematically described by a Fourier series [9]:

$$p(t) = \sum_{j=1}^{N} W\alpha_j \sin(2\pi j f_\mathrm{p} t - \phi_j) \tag{11}$$

where W is the weight of a pedestrian which is assumed to be 700 N in this study, α_j is the Fourier coefficient of the jth harmonic, f_p is the pacing rate in footfalls s^{-1} or Hz, ϕ_j is the phase lag of the jth harmonic relative to the first harmonic and N is the total number of contributing harmonics.

Since the Fourier coefficient of the first harmonic, α_1, is much higher than those of other harmonics (α_1 is about 0.4 while α_j for $j>1$ is about 0.1 or less [9]) and the first harmonic frequency is in the neighbourhood of the fundamental frequency of structure, the loading series in Equation (11) can be simplified to be only the first harmonic.

The loading model of a pedestrian moving across the bridge must include the influence of position of load that changes with time. This influence modifies the steady state harmonic form of Equation (11). However, in this case, the bridge is quite long so that the estimated number of cycles of the harmonic pacing force that is required to approximately attain the steady state response is reached when a pedestrian approaches the mid-span of the bridge. For the purpose of estimating the maximum bridge response, the moving load from one pedestrian could be approximately represented by a stationary harmonic load applying at the mid-span of the bridge.

The aggregate effect of more than one pedestrian with the assumed Poisson arrival can be determined by superimposing stochastically the vibration induced by a single pedestrian [10]. The enhancement factor for multiplying the vibration effect caused by a single pedestrian is approximately equal to the square root of the number of pedestrians simultaneously present on the bridge, which is 10 pedestrians in this study. Since the system considered is linear, an equivalent effect could be obtained by multiplying the enhancement factor to the load instead of the response. The final model of load can be written for the study of the MTMDs'

Table I. The optimal TMDs parameter and steady state response ($\mu = 0.01$).

Case	Frequency	Damping	Max. DMF	Max. u_i (cm)
I. Optimal linear TMD	$\gamma = 0.99$	$\xi = 0.060$	12.71	3.02
II. Optimal linear MTMDs	$\gamma_0 = 0.994,\ \Delta\gamma = 0.12$	$\xi_i = 0.021$	10.61	7.60
III. Optimal non-linear MTMDs	$\gamma_0 = 0.995,\ \Delta\gamma = 0.12$	$c = 20\ \mathrm{N/(ms^{-2})}$	11.81	8.12
IV. Selected non-linear MTMDs	$\gamma_0 = 0.990,\ \Delta\gamma = 0.13$	$c = 50\ \mathrm{N/(ms^{-2})}$	13.14	5.44

Table II. Selected non-linear MTMDs parameters for real application (Case IV).

TMD number	Mass (kg)	Stiffness (N/m)	Natural frequency (Hz)	Plate section (cm × cm)	c ($\mathrm{N/(ms^{-2})}$)
1	35.6	4810	1.85	(10×10)	50
2	33.6	4810	1.90	(10×10)	50
3	31.9	4810	1.95	(10×10)	50
4	30.3	4810	2.01	(10×10)	50
5	28.7	4810	2.06	(10×10)	50
6	27.3	4810	2.11	(10×10)	50

performance as

$$p(t) = W\alpha_1 \sqrt{n_p}\ \sin(2\pi f_p t) \tag{12}$$

The governing equation of the system as in Equation (3), under the loading condition as in Equation (12), can be numerically integrated to investigate the performance of a single linear TMD, linear MTMDs and non-linear MTMDs in the range of normal pacing frequency, $1.6\ \mathrm{Hz} \leqslant f_p \leqslant 2.4\ \mathrm{Hz}$. For each of these cases, the frequency tuning and damping quantity that yield the most effective control performance (i.e. optimal condition), were identified numerically. The optimal condition in this case is defined as the condition where the maximum value of the dynamic magnification factor (DMF) (the ratio of dynamic response over its static response) of the main structure is minimum.

The optimal parameters, the maximum DMF and the maximum displacement of TMDs and MTMDs are shown in Table I. TMDs and MTMDs categories are referred herein as Case I for optimal linear single TMD, Case II for optimal linear MTMDs and Case III for optimal non-linear MTMDs. In practice, apart from the control effectiveness, the robustness of MTMDs against estimation error in natural frequency and some other important practical considerations, such as the allowable stroke length, will have to be taken into account during the design. Therefore, an additional case (Case IV) of non-linear MTMDs with a slightly wider frequency ratio band and relatively higher damping was selected where MTMDs' performances are also included in Table I and details of their parameters are shown in Table II.

DMF curves for all cases, including the system without TMDs, are plotted against the pacing frequency normalized by the structural natural frequency as shown in Figure 7. In Case I, the optimal linear TMD is effective, however, because a single unit is required, the system requires large mass and damper unit, but the TMD's response is small. The optimal linear MTMDs in Case II are more effective and the units are more compact but their responses are

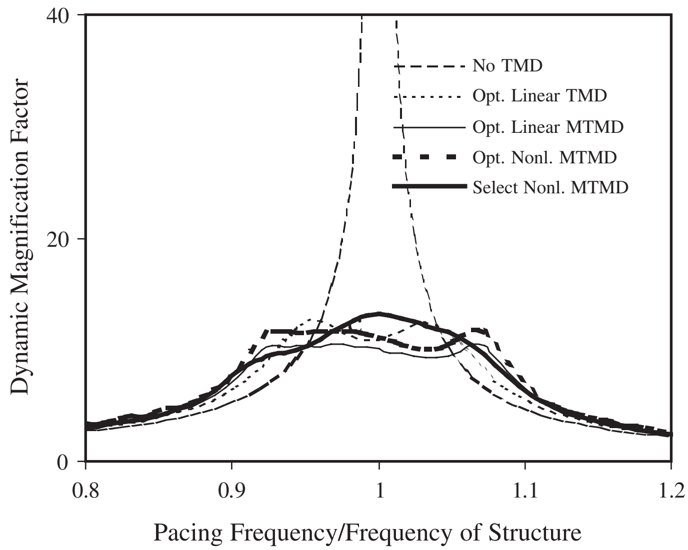

Figure 7. The dynamic magnification factor of system for varying excitation frequency.

relatively large. It is noticed that the values of the optimal frequency band conform to those from the previous study [3] and the robustness should be further investigated. The optimal non-linear MTMDs in Case III are slightly less effective than for Case II but they are still better than in Case I. However, the problem is that the TMDs' responses are even higher than those in Case II which may cause difficulty in real practice. The selected non-linear MTMDs in Case IV are slightly less effective than in the other cases. However, the TMDs' responses are small and, therefore, desirable because higher damping than the optimal value was selected.

In real situations, the performance of MTMDs can be significantly deteriorated if structural parameters used in the optimal designs contain some errors. Especially, due to the tuning requirements, the error in natural frequency of the main structure would be a critical factor. Therefore, in the following discussion, the robustness of TMDs' control effectiveness against estimation error in structural natural frequency was studied and the results are shown in Figure 8. The abscissa is the error from the estimated ω_S while the ordinate is the maximum DMF. By widening the total frequency ratio band from the optimal one, the selected non-linear MTMDs show some extent of improvement on robustness of the system when there is error in ω_S estimation.

Another investigation on the change of the effectiveness of the non-linear MTMDs when there is variation in the intensity of pedestrian loads was also studied. From Equation (9), it is clear that the system damping is generally dependent on the level of response amplitude. Therefore, under the condition where the excitation level differs from that which was used in the design stage, the corresponding performance of non-linear MTMDs may deteriorate from the optimal one. To investigate this effect, the variation of force magnitude is considered by multiplying the force $p(t)$ in Equation (12) by a scale factor ranging from 0.5 to 2.0. The equations of motion in Equation (3) with this modified excitation level were then numerically

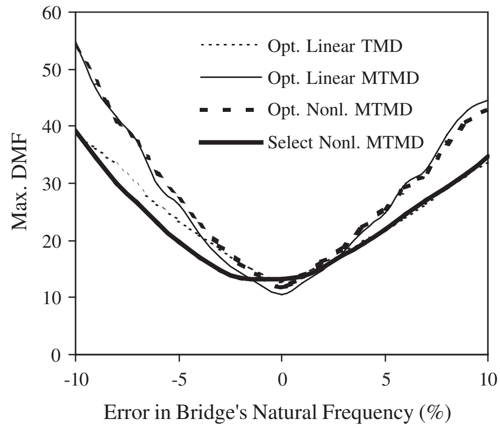

Figure 8. Robustness of TMDs against estimation error in structural natural frequency.

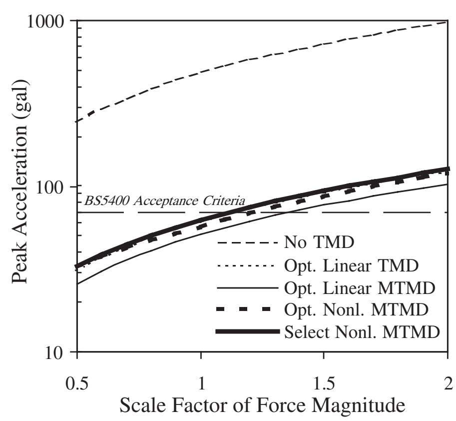

Figure 9. Acceleration amplitude of structure under various force intensity.

integrated again. In the case of no error in ω_S estimation, the abscissa in Figure 9 is the scale factor where the ordinate is the peak acceleration of the bridge. The results show that there is no significant deterioration of the non-linear MTMDs' performance. It is noted here that from British Standard BS5400, the upper bound acceleration amplitude for the pedestrian bridge with 2 Hz natural frequency is about 0.7 ms^{-2} [11]. Although this limit is stated for a bridge excited by one pedestrian and no allowance is made for multiple random arrivals of pedestrians, the condition is included in Figure 9 for only approximate comparison. The vibration amplitudes of the structure with MTMDs under service conditions can be successfully reduced in comparison with the human acceptance criteria and better MTMDs performance can be achieved further by increasing mass ratio.

Figure 10. Non-linear MTMDs on a pedestrian bridge.

FIELD EXPERIMENTS

The non-linear MTMDs with the selected parameters in Table II were assembled and tested for its dynamic properties, then they were installed at the mid-span of the bridge, 3 units at each side, as shown in Figure 10. Free vibration and forced vibration tests by a single pedestrian were carried out in order to evaluate the performance of the non-linear MTMDs.

Free vibration tests

A number of free vibration tests of the bridge with and without non-linear MTMDs were conducted to determine the average damping ratio. The statistical average results within the expected vibration amplitude range show that, with non-linear MTMDs, the structural damping ratio was 0.036 compared with 0.005 obtained without MTMDs at the same amplitude range. Figure 11 shows the free vibration response at mid-span of the bridge. The results from the field tests, Figure 11(a), are in accordance with the results from numerical simulation as shown in Figure 11(b).

Forced vibration tests

Forced vibration tests were carried out by a 780 N weight pedestrian walking across the bridge with pacing frequency of about 2 Hz. In order to clearly investigate the vibrations, the testing pedestrian walked with exaggerated motion in the vertical direction in order to induce higher dynamic vertical force than a normal walking characteristic. After several practices, this testing pedestrian was able to produce almost the same walking pattern. The acceleration at the mid-span was measured and compared with the response from the assumed model of

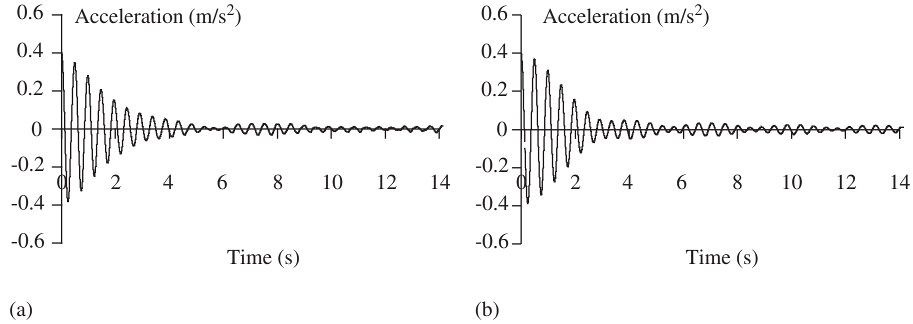

Figure 11. Free vibration record at mid-span of the bridge with MTMDs; (a) field measurement, (b) numerical simulation.

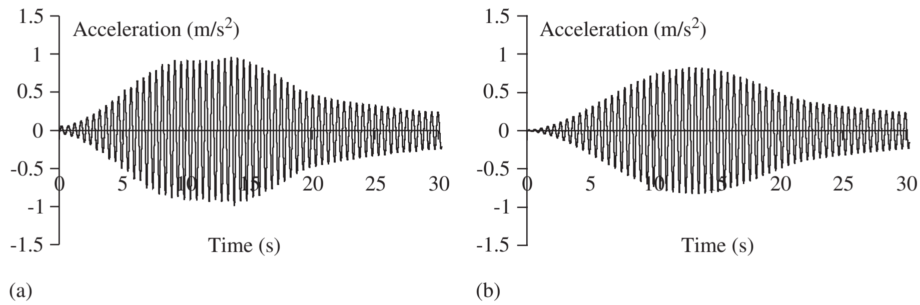

Figure 12. Response at mid-span of the bridge without MTMDs caused by a single pedestrian; (a) field measurement, (b) numerical simulation.

modal force for one pedestrian, $p^*(t)$ as

$$p^*(t) = \alpha_1^* W \sin(2\pi f_p^* t) \sin\left(\frac{\pi v_s t}{L}\right) \tag{13}$$

The sine function in the second term results from the pedestrian position moving along the bridge. L is the bridge span length and v_s is the velocity of the pedestrian which was kept identical for all tests. Since the walking test was not natural, α_1^* and f_1^*, which are the coefficients for the first harmonic and the pacing frequency of this walking test condition, respectively, were estimated by fitting the results from numerical simulation to field measurement. The obtained values were $\alpha_1^* = 0.75$ and $f_p^* = 1.95$ Hz. The time history responses at the mid-span of the bridge without MTMDs are shown in Figure 12(a) for field measurement and Figure 12(b) for numerical simulation. For the bridge with non-linear MTMDs, the field measurement acceleration at mid-span of the bridge and the corresponding acceleration of the third TMD are shown in Figure 13(a) and (b), respectively, while those for numerical simulation are shown in Figure 14(a) and (b), respectively. The results from field measurement clearly indicate the effectiveness of non-linear MTMDs and these are in good agreement with the numerical simulation of the system model. After repeating a number

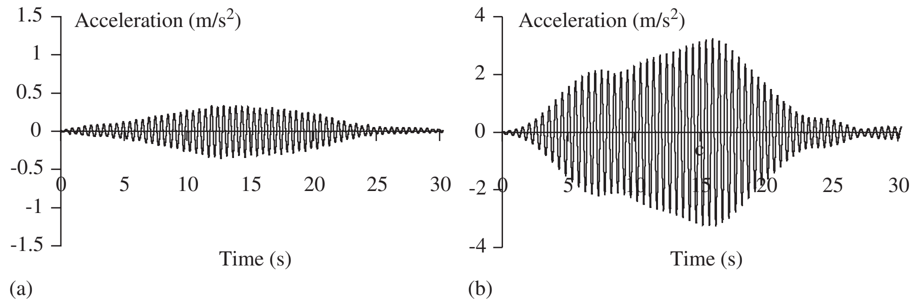

Figure 13. Field measurement acceleration caused by a single pedestrian; (a) at mid-span of the bridge with MTMDs, (b) at the third TMD.

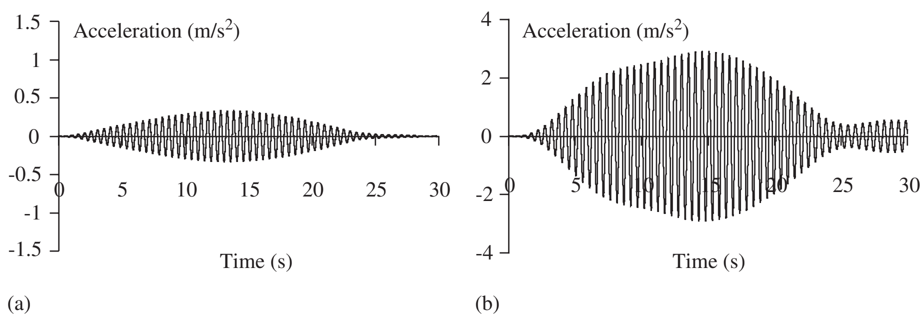

Figure 14. Numerical simulation of acceleration caused by a single pedestrian; (a) at mid-span of the bridge with MTMDs, (b) at the third TMD.

of walking tests, the results show that the maximum accelerations at mid-span of the bridge without MTMDs were in the range of 0.80–1.30 ms^{-2}. After the installation of non-linear MTMDs, the accelerations were reduced to 0.27–0.40 ms^{-2}, which were within the acceptance criteria [11].

CONCLUSIONS

From the outstanding features of MTMDs such as the control performance being quite stable over a wide range of their damping ratios, and the optimal damping ratios are small, a new concept of a practical damping device for MTMDs system is presented in this paper. The damping force is quadratic non-linear generated from drag force acting on a thin plate immersed in water. The mechanism of this damper is simple so that it can be constructed at a relatively low cost. Its characteristic can be maintained over a longer period of time, and water can be easily replaced. The results from the numerical study revealed that their effectiveness and robustness were similar to those of the linear MTMDs. The validity was checked by the real installation of the non-linear MTMDs on a pedestrian bridge to control man-induced vibration. The tested results were in good agreement with the results from the numerical study and they show satisfactory performance in vibration reduction. From a practical point of view and its effective performance, the proposed damper may give an alternative solution for a simple MTMDs' application to suppress vibration problems.

ACKNOWLEDGEMENTS

This study was financially supported by a research grant from the Thailand Research Fund, research number PDF/34/2541. The source of support is gratefully acknowledged. Appreciation is expressed to the students at Thammasat University who helped in the experiments of this research.

REFERENCES

1. Den Hartog JP. *Mechanical Vibrations*. McGraw Hill: New York, 1956; 93–106.
2. Xu K, Igusa T. Dynamic characteristics of multiple substructures with closely spaced frequencies. *Earthquake Engineering and Structural Dynamics* 1992; **21**:1059–1070.
3. Yamaguchi H, Harnpornchai N. Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillations. *Earthquake Engineering and Structural Dynamics* 1993; **22**:51–62.
4. Warnitchai P, Poovarodom N. A new approach in optimal design of multiple tuned mass dampers. *Proceedings of the 4th East Asia-Pacific Conference* (EASEC), 1993; 1931–1936.
5. Abe M, Fujino Y. Dynamic characterization of multiple tuned mass dampers and some design formulas. *Earthquake Engineering and Structural Dynamics* 1994; **23**:813–835.
6. Soong TT, Dargush GF. *Passive Energy Dissipation Systems in Structural Engineering*. Wiley: Chichester, 1997; 227–279.
7. Janna WS. *Introduction to Fluid Mechanics*. PWS Publishing: Boston, 1993; 311.
8. Clough RW, Penzien J. *Dynamics of Structures* (2nd edn). McGraw Hill: New York, 1993; 330–332.
9. Bachmann H, Ammann W. Vibrations in Structures Induced by Man and Machines. *Structural Engineering Document No.* 3e, International Association for Bridge and Structural Engineering, 1987; 24–25.
10. Matsumoto Y, Nishioka T, Shiojiri H, Matsuzaki K. Dynamic design of footbridges. *Proceedings P*-17/78; 1–15, International Association for Bridge and Structural Engineering 1978.
11. British Standards Institution, *Vibration Serviceability Requirements for Foot and Cycle Track Bridges*. BS5400, Part 2, Appendix C, 1978.

