CHAPTER 17

DESIGN FOR EQUIPMENT VIBRATIONS AND SEISMIC LOADINGS

17-1. Introduction.

a. Vibrations caused by steady state or transient loads may cause settlement of soils, excessive motions of foundations or structures, or discomfort or distress to personnel. Some basic design factors for dynamic loading are treated in this section. Design of a foundation system incorporates the equipment loading, subsurface material properties, and geometrical proportions in some analytical procedure.

b. Figure 17-1 shows some limiting values of vibration criteria for machines, structures, and personnel. On this diagram, vibration characteristics are described in terms of frequency and peak amplitudes of acceleration, velocity, or displacement. Values of frequency constitute the abscissa of the diagram and peak velocity is the ordinate. Values of peak displacement are read along one set of diagonal lines and labelled in displacement (inches), and peak acceleration values are read along the other set of diagonal lines and labelled in various amounts of g, the acceleration of gravity. The shaded zones in the upper right-hand corner indicate possible structural damage to walls by steady-state vibrations. For structural safety during blasting, limit peak velocity to 2.0 inches per second and peak acceleration to 0.10g for frequencies exceeding 3 cycles per second. These limits may occasionally have to be lowered to avoid being excessively annoying to people.

c. For equipment vibrations, limiting criteria consist of a maximum velocity of 1.0 inch per second up to a frequency of about 30 cycles per second and a peak acceleration of 0.15g above this frequency. However, this upper limit is for safety only, and specific criteria must be established for each installation. Usually, operating limits of equipment are based on velocity criteria; greater than 0.5 inch per second indicates extremely rough operation and machinery should be shut down; up to 0.10 inch per second occurs for smooth, well-balanced equipment; and less than 0.01 inch per second represents very smooth operation.

d. Figure 17-1 also includes peak velocity criteria for reaction of personnel to steady-state vibrations. Peak velocities greater than 0.1 inch per second are "troublesome to persons," and peak velocities of 0.01 inch per second are just "barely noticeable to persons." It is significant that persons and machines respond to equivalent levels of vibration. Furthermore, persons may notice vibrations that are about 1/100 of the value related to safety of structures.

17-2. Single degree of freedom, damped, forced systems.

a. Vibrations of foundation-soil systems can adequately be represented by simple mass-spring-dashpot systems. The model for this simple system consists of a concentrated mass, m, supported by a linear elastic spring with a spring constant, k, and a viscous damping unit (dashpot) having a damping constant, c. The system is excited by an external force, e.g., $Q = Q_o \sin(\omega t)$, in which Q_o is the amplitude of the exciting force, $\omega = 2\pi f_o$ is the angular frequency (radians per second) with f_o the exciting frequency (cycles per second), and t is time in seconds.

b. If the model is oriented as shown in the insert in figure 17-2(a), motions will occur in the vertical or z direction only, and the system has one degree of freedom (one coordinate direction (z) is needed to describe the motion). The magnitude of dynamic vertical motion, A_z, depends upon the magnitude of the external excitation, Q, the nature of Q_o, the frequency, f_o, and the system parameters m, c, and k. These parameters are customarily combined to describe the "natural frequency" as follows:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (17\text{-}1)$$

and the "damping ratio" as

$$D = \frac{c}{2\sqrt{km}} \qquad (17\text{-}2)$$

c. Figure 17-2(a) shows the dynamic response of the system when the amplitude of the exciting force, Q_o, is constant. The abscissa of the diagram is the dimensionless ratio of exciting frequency, f_o, divided by the natural frequency, f_n, in equation (17-1). The ordinate is the dynamic magnification factor, M_z, which is the ratio of A_z to the static displacement, $A_z = (Q_o/k)$. Different response curves correspond to different values of D.

d. Figure 17-2(b) is the dynamic response of the system when the exciting force is generated by a rotating mass, which develops:

$$Q_o = m_e(\bar{e})4\pi^2 f_o^2 \qquad (17\text{-}3)$$

where

- m_e = the total rotating mass
- e = the eccentricity
- f_o = the frequency of oscillation, cycles per second

e. The ordinate M_z. (fig 17-2(b)) relates the dynamic displacement, A_z, to m_e e/m. The peak value of the response curve is a function of the damping ratio and is given by the following expression:

$$M_{z(max)} \text{ or } M_z = \frac{1}{2D\sqrt{1-D^2}} \qquad (17\text{-}4)$$

For small values of D, this expression becomes 1/2D. These peak values occur at frequency ratios of

$$\frac{f_o}{f_n} = \sqrt{1-D^2} \qquad \text{(fig. 17-2a)}$$

or (17-5)

$$\frac{f_o}{f_n} = \frac{1}{\sqrt{1-2D^2}} \qquad \text{(fig. 17-2b)}$$

17-3. Foundations on elastic soils.

a. Foundations on elastic half-space. For very small deformations, assume soils to be elastic materials with properties as noted in paragraph 3-8. Therefore, theories describing the behavior of rigid foundations resting on the surface of a semi-infinite, homogeneous, isotropic elastic body have been found useful for study of the response of real footings on soils. The theoretical treatment involves a circular foundation of radius, r_o, on the surface of the ideal half-space. This foundation has six degrees of freedom: (1-3) translation in the vertical (z) or in either of two horizontal (x and y) directions; (4) torsional (yawing) rotation about the vertical (z) axis; or (5-6) rocking (pitching) rotation about either of the two horizontal (x and y) axes. These vibratory motions are illustrated in figure 17-3.

(1) A significant parameter in evaluating the dynamic response in each type of motion is the inertia reaction of the foundation. For translation, this is simply the *mass*, m = (W/g); whereas in the rotational

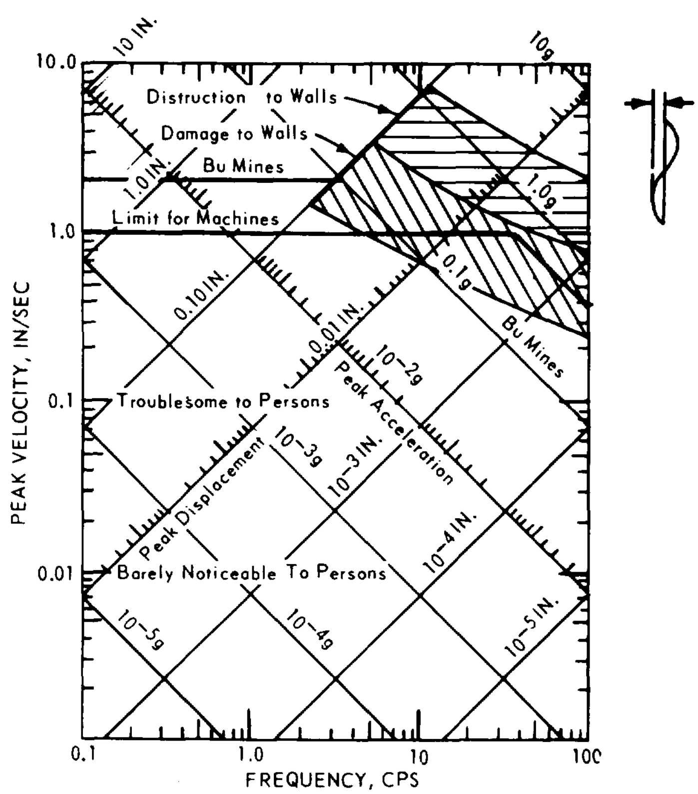

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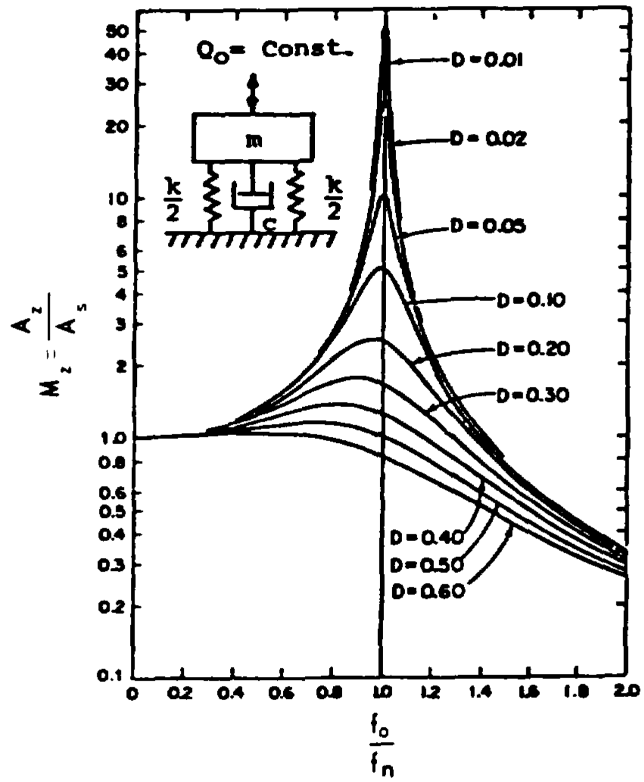
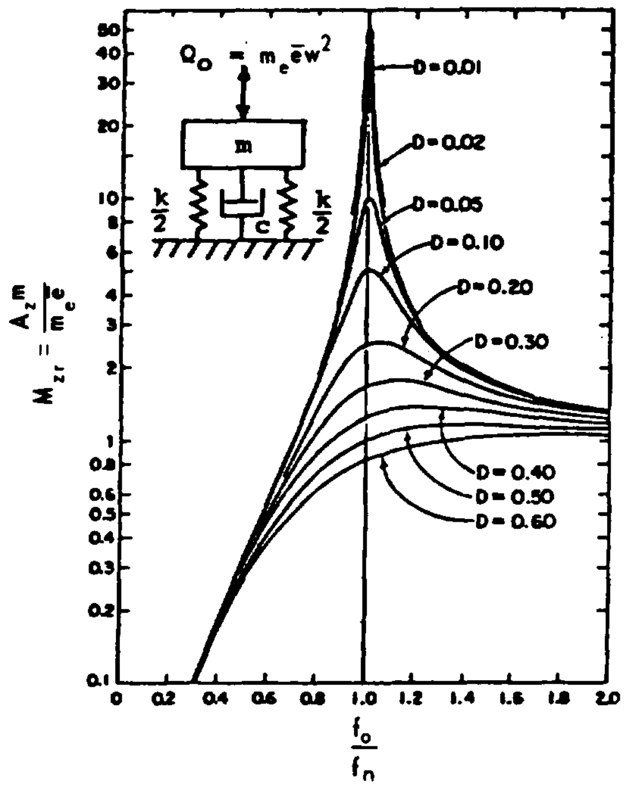
Figure 17-1. Response spectra for tibraton limits.



(a) CONST. FORCE AMPLITUDE EXCITATION

$Q = Q_o \sin \omega t$



(b) ROTATING MASS EXCITATION

$Q = m_e \bar{e} w^2 \sin \omega t$

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Figure 17-2. Response curves for the single-degree-of-freedom system with viscous damping.

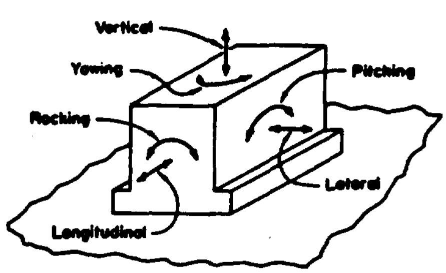

Figure 17-3. Six modes of vibration for a foundation.

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Figure 17-3. Six modes of vibration for a foundation.

modes of vibration, it is represented by the *mass moment of inertia* about the axis of rotation. For torsional oscillation about the vertical axis, it is designated as I_θ; whereas for rocking oscillation, it is I_ψ, (for rotation about the axis through a diameter of the base of the foundation). If the foundation is considered to be a right circular cylinder of radius r_o, height h, and unit weight y, expressions for the mass and mass moments of inertia are as follow:

$$m = \frac{\pi r_o^2 h y}{g} \qquad (17\text{-}6)$$

$$Is = \frac{\pi r_o^4 h y}{2g} \qquad (17\text{-}7)$$

$$I_\psi = \frac{\pi r_o^2 h y}{g} \left(\frac{r_o^2}{4} + \frac{h^2}{3} \right) \qquad (17\text{-}8)$$

(2) Theoretical solutions describe the motion magnification factors M, or ML, for example, in terms of a "mass ratio" B_z and a dimensionless frequency factor a_o Table 17-1 lists the mass ratios, damping ratios, and spring constants corresponding to vibrations of the rigid circular footing resting on the surface of an elastic semi-infinite body for each of the modes of vibration. Introduce these quantities into equations given in paragraph 17-2 to compute resonant frequencies and amplitudes of dynamic motions. The dimensionless frequency, ao, for all modes of vibration is given as follows:

$$a_o = \frac{2\pi f_o r_o}{V_s} = \omega r_o \sqrt{\frac{p}{G}} \qquad (17\text{-}9)$$

The shear velocity, V_s, in the soil is discussed in paragraph 17-5.

(3) Figure 17-4 shows the variation of the damping ratio, D, with the mass ratio, B, for the four modes of vibration. Note that D is significantly lower for the rocking mode than for the vertical or horizontal translational modes. Using the expression M = 1/(2D) for the amplitude magnification factor and the appropriate D, from figure 17-4, it is obvious that M, can become large. For example, if $B_\psi = 3$, then $D_\psi = 0.02$ and $M_\psi = 1/(2 \times 0.02) = 25$.

Table 17-1. Mass ratio, Damping Ration, and Spring Constant for Rigid Circular Footing on the Semi-Infinite Elastic Body

Mode of Vibration	Mass (or Inertia) Ratio, B_i	Damping Ratio D_i	Spring Constant k_i
Vertical	$B_z = \frac{(1 - v)}{4} \frac{m}{\rho r_o^3}$	$D_z = \frac{0.425}{\sqrt{B_z}}$	$k_z = \frac{4Gr_o}{1 - v}$
Sliding	$B_x = \frac{(7 - 8v)m}{32(1 - v)\rho r_o^3}$	$D_x = \frac{0.288}{\sqrt{B_x}}$	$k_x = \frac{32(1 - v)}{7 - 8v} Gr_o$
Rocking	$B_\psi = \frac{3(1 - v)}{8} \frac{I_\psi}{\rho r_o^5}$	$D_\psi = \frac{0.15}{(1 + B_\psi)\sqrt{B_\psi}}$	$k_\psi = \frac{8 Gr_o^3}{3(1 - v)}$
Torsional	$B_\theta = \frac{I_\theta}{\rho r_o^5}$	$D_\theta = \frac{0.50}{1 + 2B_\theta}$	$k_\theta = \frac{16}{3} Gr_o^3$

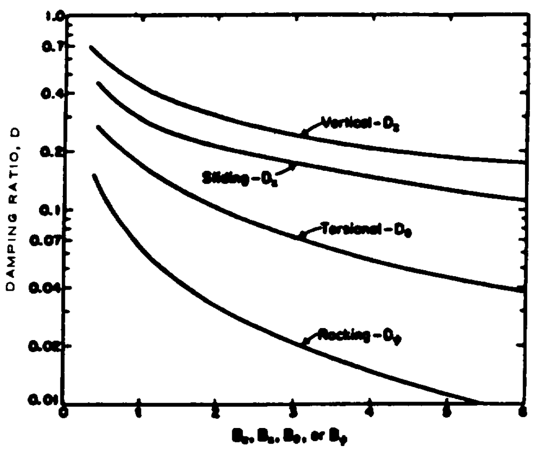

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Figure 17-4. Equivalent damping ratio for oscillation of rigid circular footing on elastic half-space.

b. Effects of shape of foundation. The theoretical solutions described above treated a rigid foundation with a circular contact surface bearing against the elastic half-space. However, foundations are usually rectangular in plan. Rectangular footings may be converted into an equivalent circular footing having a radius ro determined by the following expressions:

For translation in z- or x-directions:

$$r_o = \sqrt{\frac{4cd}{\pi}} \qquad (17\text{-}10)$$

For rocking:

$$r_o = \sqrt[4]{\frac{16cd^3}{3n}} \qquad (17\text{-}11)$$

For torsion:

$$r_o = \sqrt[4]{\frac{16cd(c^2 + d^2)}{6\pi}} \qquad (17\text{-}12)$$

In equations (17-10), (17-11), and (17-12), 2c is the *width* of the rectangular foundation (along the axis of rotation for rocking), and 2d is the *length* of the foundation (in the plane of rotation for rocking). Two values of r_o are obtained for rocking about both x and y axes.

c. Computations. Figure 17-5 presents examples of computations for vertical motions (Example 1) and rocking motions (Example 2).

d. Effect of embedment. Embedment of foundations a distance d below the soil surface may modify the dynamic response, depending upon the soil-foundation contact and the magnitude of d. If the soil shrinks away from the vertical faces of the embedded foundation, no beneficial effects of embedment may occur. If the basic evaluation of foundation response is based on a rigid circular footing (of radius r_o) at the surface, the effects of embedment will cause an *increase* in resonant frequency and a *decrease* in amplitude of motion. These changes are a function of the type of motion and the embedment ratio d/r_o.

(1) For *vertical* vibrations, both analytical and experimental results indicate an increase in the static spring constant with an increase in embedment depth. Embedment of the circular footing a distance $d/r_o \leq 1.0$ produces an increase in the embedded spring constant $k_{zd'}$ which is greater than k_z (table 17-1) by $k_{zd}/k_z \cong (1 + 0.6\ d/r_o)$. An increase in damping also occurs, i.e., D_{zd}/D_z $(1 + 0.6\ d/r_o)$. These two approximate relations lead to an estimate of the reduction in amplitude of motion because of embedment from $A_{zd} / A_z = 1 / D_{zd} / D_z \times k_{zd}/k_z$). This amount of amplitude reduction *requires complete soil adhesion at the vertical face,* and test data have often indicated less effect of embedment. Test data indicate that the resonant frequency may be increased by a factor up to $(1 + 0.25\ d/r_o)$ because of embedment.

(2) The influence of embedment on *coupled rocking and sliding* vibrations depends on the ratio B_ω/B_x (table 17-1). For $B_\omega/B_x \cong 3.0$, the increase in natural frequency due to embedment may be as much as $(1 + 0.5\ d/r_o)$. The decrease in amplitude is stongly dependent upon the soil contact along the vertical face of the foundation, and each case should be evaluated on the basis of local soil and construction conditions.

e. Effect of finite thickness of elastic layer. Deposits of real soils are seldom homogeneous to significant depths; thus theoretical results based on the response of a semi-infinite elastic media must be used with caution. When soil layers are relatively thin, with respect to foundation dimensions, modifications to the theoretical half-space analyses must be included.

(1) Generally, the effect of a rigid layer underlying a single elastic layer of *thickness, H,* is to reduce the effective damping for a foundation vibrating at the upper surface of the elastic layer. This condition results from the reflection of wave energy from the rigid base back to the foundation and to the elastic medium surrounding the foundation. For *vertical* or *torsional* vibrations or a rigid circular foundation resting on the surface of the elastic layer, it has been established that a very large amplitude of resonant vibrations can occur if

$$\frac{V_s}{f_o} > 4H (17\text{-}13)$$

In equation (17-13), V, is the shear wave velocity in the elastic layer and fo is the frequency of footing vibrations. When the conditions of equation (17-4) occur, the *natural frequency* (equation (17-1)) becomes the important *design criterion* because at that frequen-

EXAMPLE 1

A. FOUNDATION FOR SINGLE - CYLINDER VERTICAL COMPRESSOR

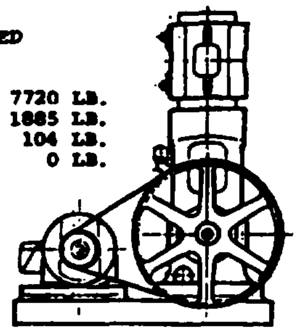
14" BORE , 9" STROKE

450 RPM OPERATING SPEED

UNBALANCED FORCES:

VERTICAL PRIMARY	= 7720 LB.
VERTICAL SECONDARY	= 1885 LB.
HORIZ. PRIMARY	= 104 LB.
HORIZ. SECONDARY	= 0 LB.

WT. MACHINE AND MOTOR = 10900 LB.



DESIGN CRITERION : SMOOTH OPERATION (LESS THAN 0.10 IN / SEC VELOCITY)

AT 450 RPM THIS REQUIRES A_z = 0.002 IN.

SOIL PROPERTIES: v_s = 680 FT / SEC

G = 11,000 LB / IN^2

γ = 110 LB / FT^3

ν = 0.33

SOLUTION :

FOR FIRST ESTIMATE OF FOUNDATION SIZE, DETERMINE STATIC SIZE FOR A_{zs} = 0.002 IN.

$$A_{zs} = \frac{Q_o}{k_z} = 0.002" = \frac{(1-\nu)Q_o}{4\,G\,r_o} = \frac{0.667(7720+1885)}{4 \times 11{,}000 \times r_o}$$

r_o = 72.8" = 6.07 FT FOR CIRCULAR FOUNDATION

THEN REQUIRED AREA $= \pi r_o^2$ = 115.6 FT^2

TRY 15'x8'x3' THICK FOUNDATION BLOCK

THEN A = 120 FT^2 , and r_o = 6.18 FT.

WT. FOUNDATION BLOCK = 54,000 LB.

WT. TOTAL = W = 64,900 LB.

FROM TABLE 17-1 :

$$B_z = \frac{(1-\nu)\,W}{4\,\gamma r_o^3} = \frac{0.67 \times 64900}{4 \times 110\,(6.18)^3} = 0.42$$

$$D_z = \frac{0.425}{\sqrt{B_z}} = 0.66$$

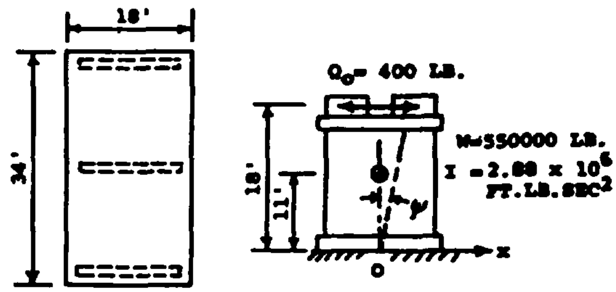
$M_z \approx 1.0$, THUS $A_z = A_{zs}$

$$A_{zs} = \frac{(1-\nu)\,Q_o}{4\,G\,r_o} = 0.00197" \text{ , FOR } r_o = 6.18 \text{ FT.}$$

THEREFORE, THE 15'x 8'x 3' THICK CONCRETE BLOCK FOUNDATION IS SATISFACTORY

EXAMPLE 2

B. MACHINE FOUNDATION SUBJECTED TO ROCKING VIBRATIONS



DESIGN CRITERION : 0.20 IN / SEC HORIZONTAL MOTION AT MACHINE CENTERLINE

AT 1300 RPM THIS LIMITS A_x TO 0.0015 IN FROM COMBINED ROCKING AND SLIDING.

(AT SLOWER SPEEDS THE ALLOWABLE A_x IS LARGER)

SOIL PROPERTIES: v_s = 770 FT / SEC

G = 14,000 LB / IN^2

γ = 110 LB / FT^3

ν = 0.33

HORIZONTAL TRANSLATION ONLY:

$$\text{EQUIVALENT } r_o = \sqrt{\frac{4cd}{\pi}} = \sqrt{\frac{18 \times 34}{\pi}} = 13.96 \text{ FT}$$

$$B_x = \frac{(7-8\nu)}{32(1-\nu)}\frac{m}{\rho r_o^3} = 0.37 \text{ , } \therefore M_x \approx 1.0$$

$$A_{xs} = \frac{Q_o}{k_x} = \frac{Q_o\,(7-8\nu)}{32\,(1-\nu)\,G\,r_o} = 0.00003 \text{ IN.}$$

HORIZONTAL TRANSLATION IS NEGLIGIBLE

ROCKING ABOUT POINT O

$$\text{EQUIVALENT } r_o = \sqrt[4]{\frac{16cd^3}{3\pi}} = \sqrt[4]{\frac{34\,(18)^3}{3\pi}} = 12.04 \text{ FT}$$

$$B_\psi = \frac{3(1-\nu)}{8}\,\frac{I}{\rho r_o^5} = \frac{3(0.67)}{8}\,\frac{2.88 \times 10^6}{\frac{110}{32.2}(12.04)^5} = 0.83$$

$$\text{THEN } D_\psi = \frac{0.15}{(1+B_\psi)\sqrt{B_\psi}} = 0.09 \text{ , AND FROM}$$

EQ.17-4, M_ψ = 5.6

THE STATIC MOMENT ABOUT O IS

T_s = 400 x 18 = 7200 FT.LB. , AND THE STATIC ANGULAR DEFLECTION IS

$$\psi_s = \frac{T_s}{k_\psi} = \frac{7200 \times 3(0.67)}{8(14000)\,144\,(12.04)^3} = \frac{0.51}{10^6} \text{ RAD}$$

THIS ROTATION WOULD PRODUCE A HORIZONTAL MOTION AT THE MACHINE CENTERLINE OF

$$A_{xs} = \psi_s h = \frac{0.51}{10^6}(18 \times 12) = 1.10 \times 10^{-4} \text{ IN.}$$

OR, THE DYNAMIC AMPLITUDE AT RESONANCE IS

$$A_x = M_\psi A_{xs} = 6.17 \times 10^{-4} \text{ IN.} < 0.0015 \text{ IN.}$$

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Figure 17-5. Examples of computations for vertical and rocking motions.

cy excessive dynamic motion will occur. To restrict the dynamic oscillation to slightly larger than the static displacement, the operating frequency should be maintained at one half, or less, of the natural frequency (fig 17-2).

(2) The relative thickness (expressed by H/ro) also exerts an important influence on foundation response. If H/r_o is greater than about 8, the foundation on the elastic layer will have a dynamic response comparable to that for a foundation on the elastic half-space. For $H/r_o < 8$, geometrical damping is reduced, and the effective spring constant is increased. The values of spring constant, k, in table 17-1 are taken as reference values, and table 17-2 indicates the increase in spring constant associated with a decrease in thickness of the elastic layer. Values of the increase in spring constants for sliding and for rocking modes of vibration will tend to fall between those given for vertical and torsion for comparable H/r_o conditions.

f. Coupled modes of vibration. In general, vertical and torsional vibrations can occur independently without causing rocking or sliding motions of the foundation. To accomplish these uncoupled vibrations, the line of action of the vertical force must pass through the center of gravity of the mass and the resultant soil reaction, and the exciting torque and soil reaction torque must be symmetrical about the vertical axis of rotation. Also, the center of gravity of the foundation must lie on the vertical axis of torsion.

(1) When horizontal or overturning moments act on a block foundation, both horizontal (sliding) and rocking vibrations occur. The coupling between these motions depends on the height of the center of gravity of. the machine-foundation about the resultant soil reaction. Details of a coupled rocking and sliding analysis are given in the example in figure 17-6.

(2) A "lower bound" estimate of the first mode of coupled rocking and sliding vibrations can be obtained from the following:

$$\frac{1}{f_o^2} = \frac{1}{f_x^2} + \frac{1}{f_\psi^2} \qquad (17\text{-}14)$$

In equation (17-14), the resonant frequencies in the sliding x and rocking ψ motions can be determined by introducing values from table 17-1 into equations (17-1) and (17-5). (Note that equation (17-14) becomes less useful when D_x is greater than about 0.15). The first mode resonant frequency is usually most important from a design standpoint.

g. Examples. Figure 17-5, Example 1, illustrates a procedure for design of a foundation to support machine-producing vertical excitations. Figure 17-5, Example 2, describes the analysis of uncoupled horizontal and rocking motion for a particular foundation subjected to horizontal excitations. The design procedure of Example 1 is essentially an iterative analysis after approximate dimensions of the foundation have been established to restrict the static deflection to a value comparable to the design criterion.

(1) In figure 17-5, Example 1 shows that relatively high values of damping ratio D are developed for the vertical motion of the foundation, and Example 2 illustrates that the high damping restricts dynamic motions to values slightly larger than static displacement caused by the same force. For Example 2, establishing the static displacement at about the design limit value leads to satisfactory geometry of the foundation.

(2) Example 2 (fig 17-5) gives the foundation geometry, as well as the analysis needed to ascertain whether the design criterion is met. It is assumed that the 400-pound horizontal force is constant at all frequencies and that a simple superposition of the singledegree-of-freedom solutions for horizontal translation and rocking will be satisfactory. Because the horizontal displacement is negligible, the rocking motion dominates, with the angular rotation at resonance amounting to ($M_\psi \times \psi_s$) or $A_\psi = 5.6 \times 0.51 \times 10^{-6} = 2.85 \times 10^{-6}$ radians. By converting this motion to horizontal displacement at the machine center line, it is found that the design conditions are met.

(3) In figure 17-6, the foundation of Example 2 (fig. 17-5) is analyzed as a coupled system including both rocking and sliding. The response curve for angular rotation shows a peak motion of $A_\psi = 2.67 \times 10^{-6}$ radians, which is comparable to the value found by considering rocking alone. The coupled dynamic response of any rigid foundation, e.g., a radar tower, can

Table 17-2. Values of k_L/L for Elastic Layer (k from Table 17-1)

H/r_o	0.5	1.0	2.0	4.0	8.0	∞
Vertical	5.0	2.2	1.47	1.23	1.10	1.0
Torsion	--	1.07	1.02	1.009	--	1.0

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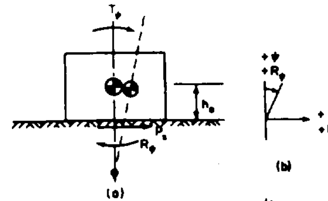


IN THE SKETCH REPRESENTING THE DYNAMIC MOTION OF THE FOUNDATION OF FIGURE 17-5, EXAMPLE 2, THE SUBSCRIPT "g" REFERS TO THE CENTER OF GRAVITY, AND "b" REFERS TO THE CENTER OF THE BASE.

$x_b = x_g - h_o\psi \qquad I_b = I_g + m\,h_o^2$

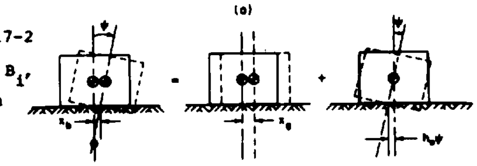
m = TOTAL MASS, AND I = MASS MOMENT OF INERTIA

$c_x = D_x\, 2\sqrt{k_x\, m} = \dfrac{18.4\ (1-\nu)}{(7-8\nu)} r_o^{\,2}\sqrt{\rho G}$

$c_\psi = D_\psi\, 2\sqrt{k_\psi\, I} = \dfrac{0.80\ r_o^4\ \sqrt{\rho G}}{(1-\nu)(1+B_\psi)}$

FROM EQUATION 17-2 AND VALUES FOR B_i, D_i, and k from TABLE 17-1

$P_x = -\,c_x\dot{x}_b - k_x x_b \quad , \quad R_\psi = -\,c_\psi\dot{\psi} - k_\psi\psi$



THE EQUATION OF EQUILIBRIUM FOR HORIZONTAL TRANSLATION IS

$$m\ddot{x}_g + c_x\dot{x}_b + k_x x_b = Q_x = m\ddot{x}_b + m\,h_o\ddot{\psi} + c_x\dot{x}_b + k_x x_b \qquad (a)$$

AND FOR ROTATION ABOUT THE CENTER OF GRAVITY IT IS

$$I_g\ddot{\psi} + c_\psi\dot{\psi} + k_\psi\psi - c_x h_o\dot{x}_b - k_x h_o x_b = T_\psi \quad \text{, OR}$$

$$I_b\ddot{\psi} - m\,h_o^2\ddot{\psi} + c_\psi\dot{\psi} + k_\psi\psi - c_x\dot{x}_b h_o - k_x x_b h_o = T_\psi \qquad (b)$$

LET

$$x_b = A_{x1}\sin\omega t + A_{x2}\cos\omega t = A_x\sin(\omega t - \alpha_x) \qquad (c)$$

$$\psi = A_{\psi1}\sin\omega t + A_{\psi2}\cos\omega t = A_\psi\sin(\omega t - \alpha_\psi)$$

$$Q_x = Q_o\sin\omega t$$

$$T_\psi = Q_o\,h\sin\omega t$$

NOTE: $A_x = \sqrt{A_{x1}^2 + A_{x2}^2}$; $\tan\alpha_x = \dfrac{-A_{x2}}{A_{x1}}$

$A_\psi = \sqrt{A_{\psi1}^2 + A_{\psi2}^2}$; $\tan\alpha_\psi = -A_{\psi2}/A_{\psi1}$

INTRODUCING THE EXPRESSIONS (c) INTO EQUATIONS. (a) AND (b) GIVE FOUR EQUATIONS WITH FOUR UNKNOWNS (A_{x1} , A_{x2} , $A_{\psi1}$, $A_{\psi2}$) , FOR EACH CHOSEN VALUE OF ω ($\omega = 2\pi$-FREQUENCY). THUS A COMPUTER SOLUTION IS NEEDED. THE GRAPH BELOW SHOWS THE ROCKING RESPONSE CURVE FOR THE FOUNDATION (SEE SKETCH ABOVE AND FIGURE 17-5). THE PARAMETERS NEEDED FOR THE SOLUTION ARE NOTED BELOW.

Q_o = 400 LB. (FREQUENCY INDEPENDENT)*

h = 18 FT. , h_o = 11 FT.

$m = \dfrac{550{,}000}{32.2} = 17{,}080$ LB SEC^2 / FT.

$I_b = 2.88 \times 10^6$ FT LB SEC^2

r_o = 13.96 FT (SLIDING)

r_o = 12.04 FT (ROCKING)

$k_x = 1.39 \times 10^8$ LB / FT

$k_\psi = 1.41 \times 10^{10}$ FT LB / RAD

$c_x = 1.45 \times 10^6$ LB SEC / FT

$c_\psi = 3.62 \times 10^7$ FT LB SEC/RAD

* FOR ROTATING MASS MACHINE TYPE EXCITATION, WE WOULD INTRODUCE

$Q_o = m_e\,\bar{e}\,\omega^2$

m_e = eccentric mass

$\bar{e}$ = eccentric radius

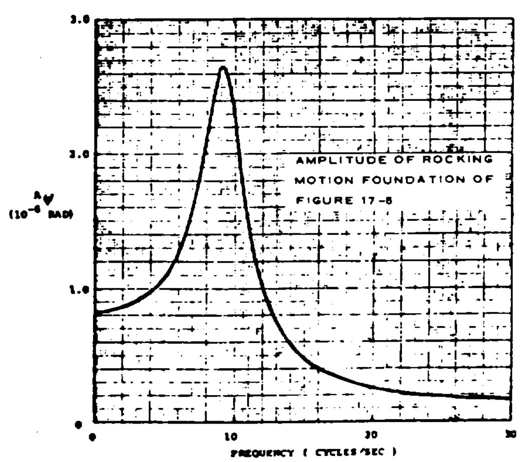


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Figure 17-6. Coupled rocking and sliding motion of foundation.

be evaluated by the procedure illustrated in figure 17-6.

17-4. Wave transmission, attenuation, and isolation. Vibrations are transmitted through soils by stress waves. For most engineering analyses, the soil may be treated as an ideal homogeneous, isotropic elastic material to determine the characteristics of the stress waves.

a. Half-space. Two types of body waves may be transmitted in an ideal half-space, compression (P-) waves and shear (S-) waves; at the surface of the halfspace, a third wave known as the Rayleigh (R-) wave or surface wave will be transmitted. The characteristics that distinguish these three waves are velocity, wavefront geometry, radiation damping, and particle motion. Figure 17-7 shows the characteristics of these waves as they are generated by a circular footing undergoing vertical vibration on the surface of an ideal half-space with is μ = 0.25. The distance from the footing to each wave in figure 17-7 is drawn in proportion to the velocity of each wave. The wave velocities can be computed from the following:

ρ

P-wave velocity:

$$v_c = \sqrt{\frac{\lambda + 2G}{p}} \qquad (17\text{-}15)$$

S-wave velocity:

$$v_s = \sqrt{\frac{G}{p}} \qquad (17\text{-}16)$$

R-wave velocity:

$$v_R = Kv_s \qquad (17\text{-}17)$$

where

$\lambda = \dfrac{2\mu G}{1-2\mu}$ and G are Lame's constants; $G = \dfrac{E}{2(1 + j)}$

$p = \underline{y/G}$ = mass density of soil

y = moist or saturated unit weight

K = constant, depending on Poisson's ratio $0.87 \leq K \leq 0.98$ for $0 \leq \; \leq 0.5$

(1) The P- and S-waves propagate radially outward from the source along hemispherical wave fronts, while the R-wave propagates outward along a cylindrical wave front. All waves encounter an increasingly larger volume of material as they travel outward, thus decreasing in energy density with distance. This decrease in energy density and its accompanying decrease in displacement amplitude is called *geometrical damping* or radiation damping.

(2) The particle motions are as follows: for the P-wave, a push-pull motion in the radial direction; for the S-wave, a transverse motion normal to the radial direction; and for the R-wave, a complex motion, which varies with depth and which occurs in a vertical plane containing a radius. At the surface, R-wave particle motion describes a retrograde ellipse. The shaded zones along the wave fronts in figure 17-7 represent the relative particle amplitude as a function of inclination from vertical.

b. Layered media.

(1) In a layered medium, the energy transmitted by a body wave splits into four waves at the interface between layers. Two waves are reflected back into the first medium, and two waves are transmitted or refracted into the second medium. The amplitudes and directions of all waves can be evaluated if the properties of both media and the incident angle are known. If a layer containing a lower modulus overlies a layer with a higher modulus within the half-space, another surface wave, known as a Love wave, will occur. This wave is a horizontally oriented S-wave whose velocity is between the S-wave velocity of the layer and of the underlying medium.

(2) The decay or attenuation of stress waves occurs for two reasons: geometric or radiation damping, and material or hysteretic damping. An equation including both types of damping is the following:

$$A_2 = A_1 \left(\frac{r_1}{r_2}\right)^C \exp[-\alpha (r_2 - r_1)] \qquad (17\text{-}18)$$

where

A_2 = desired amplitude at distance r_2

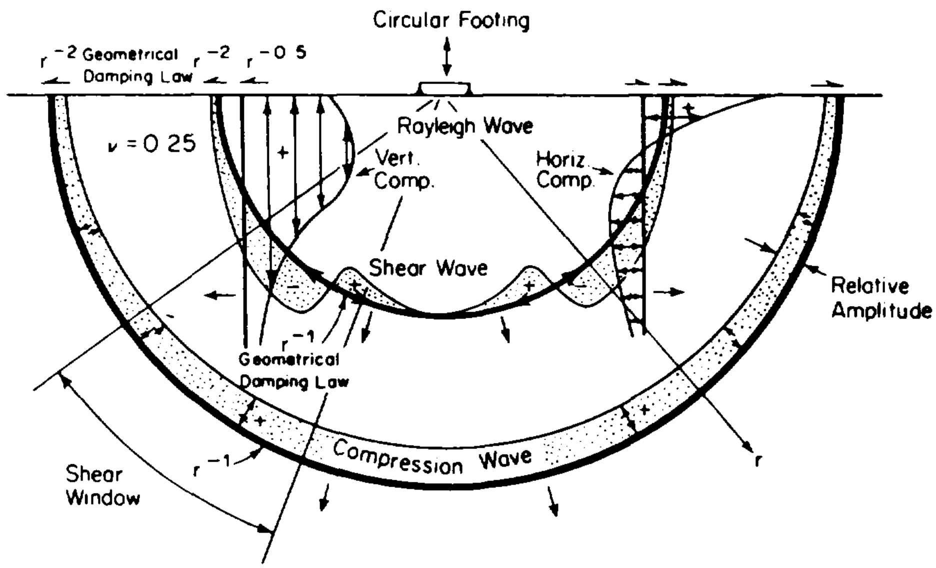
A, = known or measured amplitude at radial distance r, from vibration source

C = constant, which describes geometrical damping
= 1 for body (P- or S-) waves
= 0.5 for surface or R-waves

α = coefficient of attenuation, which describes material damping (values in table 17-3)

c. Isolation. The isolation of certain structures or zones from the effects of vibration may sometimes be necessary. In some instances, isolation can be accomplished by locating the site at a large distance from the vibration source. The required distance, r_2, is calculated.from equation (17-18). In other situations, isolation may be accomplished by wave barriers. The most effective barriers are open or void zones like trenches or rows of cylindrical holes. Somewhat less effective barriers are solid or fluid-filled trenches or holes. An effective barrier must be proportioned so that its depth is at least two-thirds the wavelength of the incoming wave. The thickness of the barrier in the direction of wave travel can be as thin as practical for construction considerations. The length of the barrier perpendicular to the direction of wave travel will depend upon the size of the zone to be isolated but should be no shorter than two times the maximum plan dimension of the structure or one wavelength, whichever is greater.

17-5. Evaluation of S-wave velocity In soils. The key parameter in a dynamic analysis of a



(Courtesy of F. E. Richart, Jr., J. R. Hall, Jr., and R. D. Woods. Vibrations of Soils and Foundations, 1970, p 91. Reprinted by permission of Prentice-Hall, Inc., Englewood Cliffs, N. J.)

Figure 17-7. Distribution of displacement waves from a circular footing on the elastic half-space.

soil-foundation system is the shear modulus, G. The shear modulus can be determined in the laboratory or estimated by empirical equations. The value of G can also be computed by the field-measured S-wave velocity and equation (17-16).

a. Modulus at low strain levels. The shear modulus and damping for machine vibration problems correspond to low shear-strain amplitudes of the order of 1 to 3 x 10^{-4} percent. These properties may be determined from field measurements of the seismic wave velocity through soil or from special cyclic laboratory tests.

b. Field wave velocity tests. S-wave velocity tests are preferably made in the field. Measurements are obtained by inducing a low-level seismic excitation at one location and measuring directly the time required for the induced S-wave to travel between the excitation and pickup unit. Common tests, such as uphole, downhole, or crosshole propagation, are described in geotechnical engineering literature.

Table 17-3. Attenuation Coefficients for Earth Materials

Materials		a (1/ft) @ 50 HZ[a]
Sand	Loose, fine	0.06
	Dense, fine	0.02
Clay	Silty (loess)	0.06
	Dense, dry	0.003
Rock	Weathered volcanic	0.02
	Competent marble	0.00004

a α is a function of frequency. For other frequencies, f, compute $\alpha_f = (f/50) \times \alpha_{50}$

(1) A problem in using seismic methods to obtain elastic properties is that any induced elastic pulse (blast, impact, etc.) develops three wave types previously discussed, i.e., P-, S-, and R-waves. Because the velocity of all seismic waves is hundreds of feet per second and the pickup unit detects all three wave pulses plus any random noise, considerable expertise is required to differentiate between the time of arrival of the wave of interest and the other waves. The R-wave is usually easier to identify (being slower, it arrives last; traveling near the surface, it contains more relative energy). Because R- and S-wave velocities are relatively close, the velocity of the R-wave is frequently used in computations for elastic properties.

(2) Because amplitudes in seismic survey are very small, the computed shear and Young's moduli are considerably larger than those obtained from conventional laboratory compression tests.

(3) The shear modulus, G, may be calculated from the S- (approximately the R-wave) wave velocity as follows:

$$G = \rho V_s^2 \qquad (17\text{-}19)$$

where

ρ = y/32.2 = mass density of soil using wet or total unit weight

V_s = S-wave velocity (or R-wave), feet per second

This equation is independent of Poisson's ratio. The V_s value is taken as representative to a depth of approximately one-half wavelength. Alternatively, the shear modulus can be computed from the P-wave velocity and Poisson's ratio from:

$$G = \frac{p(1 - 2\mu)V_p^2}{2(1-\mu)} \qquad (17\text{-}20)$$

The use of this equation is somewhat limited because the velocity of a P-wave in water is approximately 5000 feet per second (approximately the velocity in many soils) and Poisson's ratio must be estimated. For saturated or near saturated soils, μ - 0.5. The theoretical variation of the ratio V_s/V_p with μ is shown in figure 17-8.

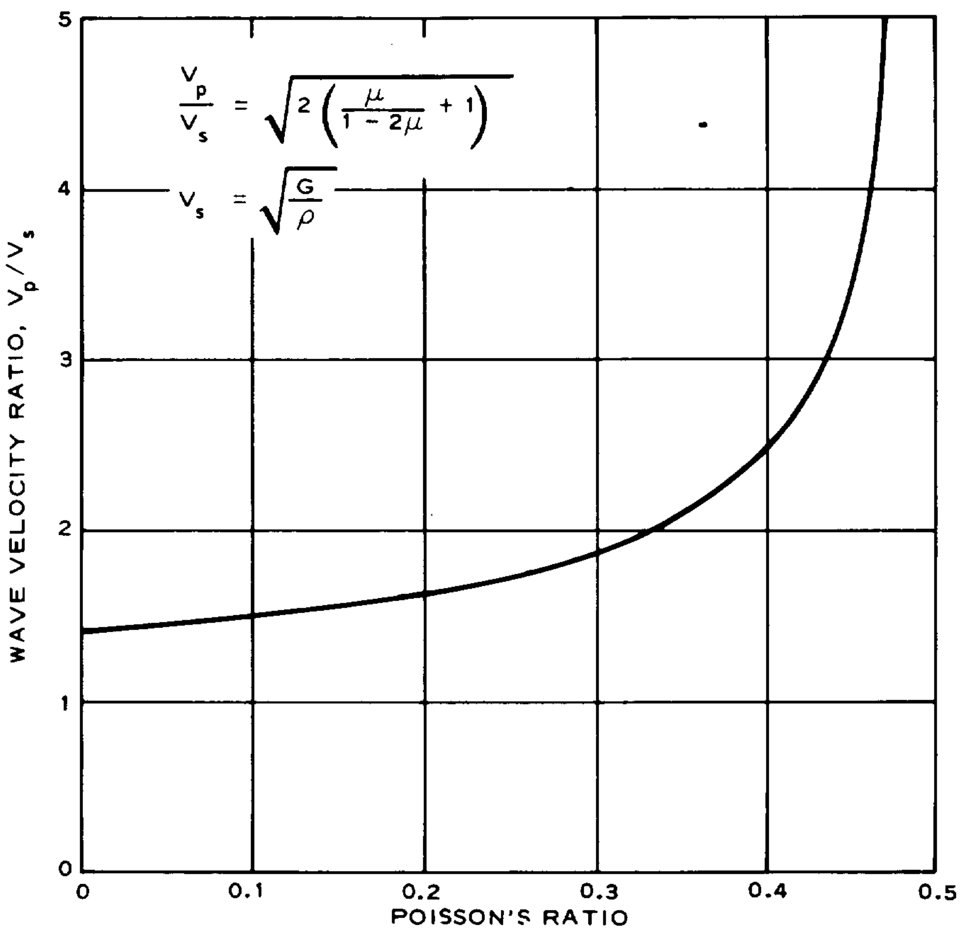


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Figure 17-8. Theoretical relation between shear velocity ratio Vp/Vs and Poisson's ratio.

c. Laboratory measurement of dynamic stress-strain properties. Low shear-strain amplitude, i.e. less than 10^{-2} percent, shear modulus data may be obtained from laboratory tests and usually involve applying some type of high-frequency forced vibration to a cylindrical sample of soil and measuring an appropriate response. Some types of tests allow the intensity level of the forced vibration to be varied, thus yielding moduli at different shear strains.

(1) High strain-level excitation, i.e. 0.01 to 1.0 percent, may be achieved by low-frequency, cyclic loading triaxial compression tests on soil samples. The modulus, damping, and strain level for a particular test are calculated directly from the sample response data. The usual assumption for calculating the modulus and damping from forced cyclic loading tests on laboratory samples is that at any cyclic strain amplitude the soil behaves as a linear elastic, viscous, damped material. A typical set of results may take the form of a hysteresis loop as shown in figure 17-9. Either shear or normal stress cyclic excitation may be used. The shear modulus is calculated from the slope of the peak-to-peak secant line. The damping is computed from the area of the hysteresis loop, and the strain level is taken as the single-amplitude (one-half the peak-to-peak amplitude or origin to peak value) cyclic strain for the condition during that cycle of the test. Note that the equations for modulus and damping shown in figure 17-9 assume the soil behaves as an equivalent elastic viscous, dampened material, which is linear within the range of strain amplitude specified. This assumption is usually made in most soil dynamics analyses because of the low-vibration amplitudes involved. If the cyclic hysteresis loops are obtained from triaxial test specimens, the resulting modulus will be the stress-strain modulus, E. If the tests involve simple shear or torsion shear such that shear stresses and strains are measured, the resulting modulus will be the shear modulus, G. In either case, the same equations apply.

(2) The shear modulus, G, can be computed from the stress strain modulus and Poisson's ratio as follows:

$$G = \frac{E}{2(1+\mu)} \qquad (17\text{-}21)$$

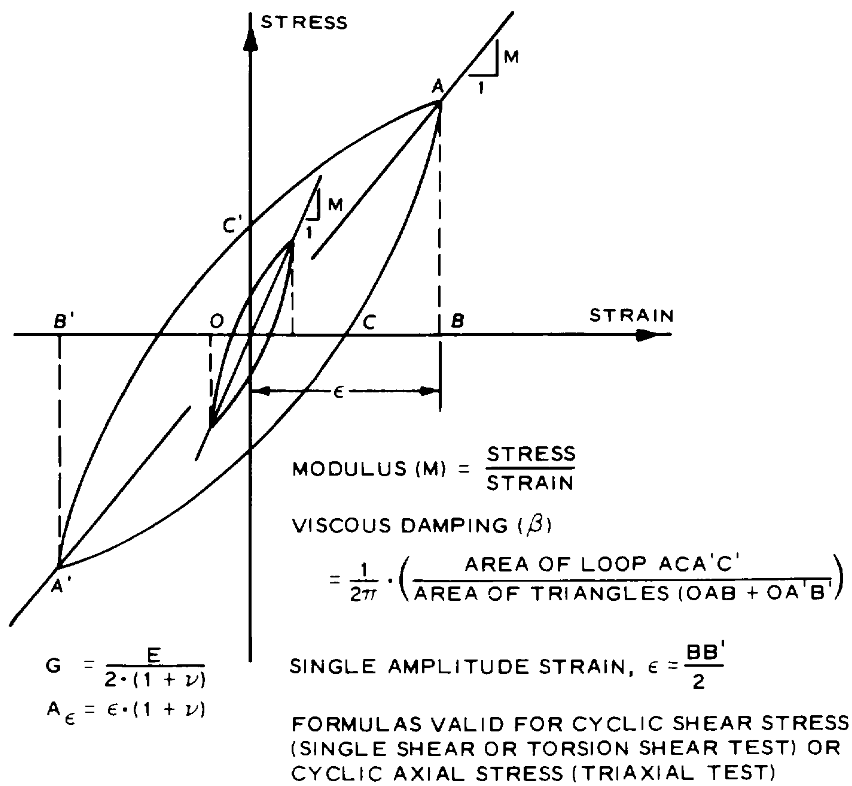

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Figure 17-9. Idealized cyclic stress-strain loop for soil.

The shear strain amplitude, A_E, may be computed from the axial strain amplitude, $_E$,and Poisson's ratio as follows:

$A_E = E(1 + \mu)$ (17-22)

For the special case of saturated soils, Poisson's ratio is 0.5, which leads to the following:

$G = E/3$

$A_E = 1.5_E$

d. Correlations.

(1) Empirical correlations from many sets of data have provided several approximate methods for estimating the S-wave velocity and shear modulus for soils corresponding to low-strain excitation. For many undisturbed cohesive soils and sands:

G =1230(21973 - e)^2 (OCR)" (o)0.5 (pounds 1 + per square inch) (17-23)

where

e = void ratio

η = empirical constant, which depends on the PI of cohesive soils (table 17-4). For sands, PI = 0 and $\eta = 0$, so OCR term reduces to 1.0. For clays, the maximum value is $\eta = 0.5$ for PI $\geq$ 100.

$\sigma_0 = 1/3\ (\sigma_1 + \sigma_2 + \sigma_3)$ = mean normal effective stress, pounds per square inch

(2) For sands and gravels, calculate the low-strain shear modulus as follows:

$G = 1000(K_2)(\sigma_0)^{0.5}$ (pounds per square foot) (17-24)

where

K_2 =empirical constant (table 17-5)

=90 to 190 for dense sand, gravel, and cobbles with little clay

σ_0 = mean normal effective stress as in equation (17-23) (but in units of pounds per square foot)

(3) For cohesive soils as clays and peat, the shear modulus is related to S_u as follows:

$G = K_2 s_u$ (17-25)

For clays, K_2 ranges from 1500 to 3000. For peats, K_2 ranges from 150 to 160 (limited data base).

(4) In the laboratory, the shear modulus of soil increases with time even when all other variables are held constant. The rate of increase in the shear modulus is approximately linear as a function of the log of time after an initial period of about 1000 minutes. The change in shear modulus, ΔG, divided by the shear modulus at 1000 minutes, G_{1000}, is called the normalized secondary increase. The normalized secondary increases range from nearly zero percent per log cycle for coarse sands to more than 20 percent per log for sensitive clays. For good correlation between laboratory and field measurements of shear modulus, the age of the in situ deposit must be considered, and a secondary time correction applies to the laboratory data.

e. Damping in low strain levels. Critical damping is defined as

$c_c = 2\sqrt{km}$ (17-26)

where k is the spring constant of vibrating mass and m represents mass undergoing vibration (W/g). Viscous damping of all soils at low strain-level excitation is generally less than about 0.01 percent of critical damping for most soils or:

$D = c/c, \leq 0.05$ (17-27)

It is important to note that this equation refers only to material damping, and not to energy loss by radiation away from a vibrating foundation, which may also be conveniently expressed in terms of equivalent viscous damping. Radiation damping in machine vibration problems is a function of the geometry of the problem rather than of the physical properties of the soil.

Table 17-4. Values of Constant r Used with Equation (17-23) to Estimate Cyclic Shear Modulus at Low Strains

Plasticity Index	K
0	0
20	0.18
40	0.30
60	0.41
80	0.48
$\geq$100	0.50

(Courtesy of 0. Hardin and P. Drnevich. "Shear Modulus and Damping in Soils: Design Equations and Curves," Journal., Soil Mechanics and Foundations Division. Vol 98. No. SM7. 1972, pp 667-692. Reprinted by permission of American Society of Civil Engineers, New York.)

Table 17-5. Values of Constant K_2 Used with Equation (17-24) to Estimate Cyclic Shear Modulus at Low Strains for Sands

e	K_2	D_r(%)
0.4	70	90
0.5	60	75
0.6	51	60
0.7	45	45
0.8	39	40
0.9	33	30

(Courtesy of H. B. Seed and L M. Idriss, "Simplified Procedures for Evaluating Liquefaction Potential," Journal, Soil Mechanics and Foundations Division Vol 97, No. SM9. 1971, pp 1249-1273. Reprinted by permission of American Society of Civil Engineers, New York.)

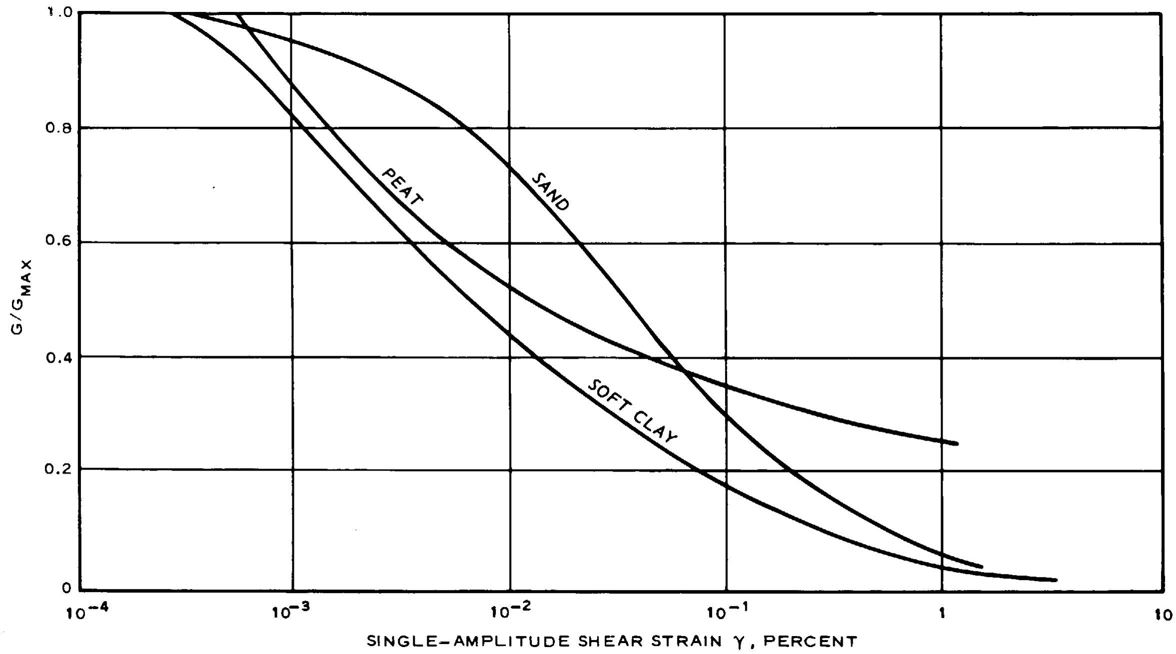
f. Modulus and damping at high strain levels. The effect of increasingly higher strain levels is to reduce the modulus (fig 17-10) and increase the damping of the soil (fig 17-11). Shear modulus and damping values at high strains are used mainly in computer programs for analyzing the seismic response of soil under earthquake loading conditions. The various empirical relations for modulus and damping pertain to sands and soft, normally consolidated clays at low-to-medium effective confining pressures, in the range of about 100 feet or overburden. Stiff overconsolidated clays and all soils at high effective confining pressure exhibit lower values of damping and higher values of modulus, especially at high strain levels. As a maximum, the modulus and damping values for stiff or strong soils at very high effective confining pressures correspond to values pertaining to crystalline or shale-type rock.

17-6. Settlement and liquefaction.

a. Settlement. Repeated shearing strains of cohesionless soils cause particle rearrangements. When the particles move into a more compact position, settlement occurs. The amount of settlement depends on the initial density of the soil, the thickness of the stratum, and the intensity and number of repetitions of the shearing strains. Generally, cohesionless soils with relative densities (D_r) greater than about 75 percent should not develop settlements. However, under 10^6 or 10^7 repetitions of dynamic loading, even dense sands may develop settlements amounting to 1 to 2 percent of the layer thickness. To minimize settlements that might occur under sustained dynamic loadings, the soil beneath and around the foundation may be precompacted during the construction process by vibroflotation, multiple blasting, pile driving, or vibrating rollers acting at the surface. The idea is to subject the soil to a more severe dynamic loading condition during construction than it will sustain throughout the design operation.

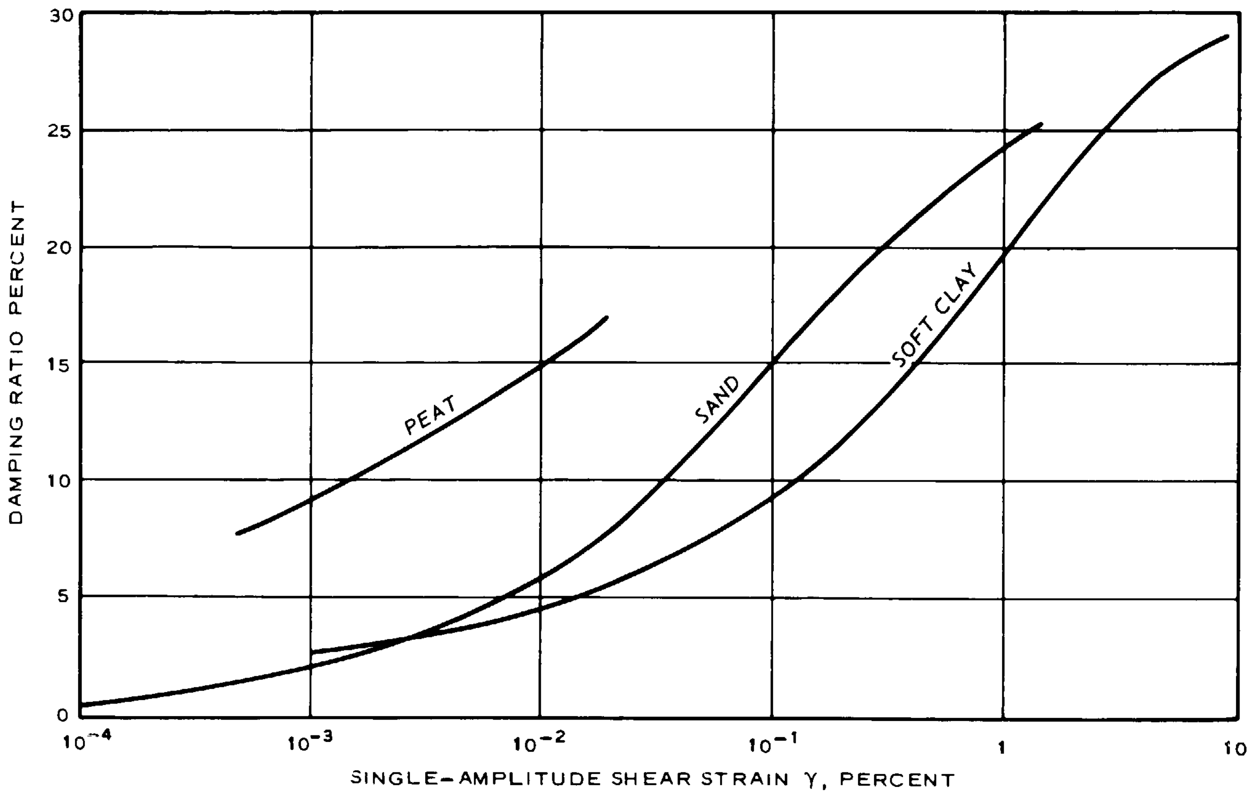
b. Liquefaction of sands. The shearing strength of saturated cohesionless soils depends upon the effective stress acting between particles. When external forces cause the pore volume of a cohesionless soil to reduce the amount V, pore water pressures are increased during the time required to drain a volume V of water from the soil element. Consequently, pore pressure increases depend upon the time rate of change in pore volume and the drainage conditions (permeability and available drainage paths). When conditions permit the pore pressure, u, to build up to a value equal to the total stress, σ_n, on the failure plane, the shear strength is reduced to near zero and the mixture of soil grains and water behaves as a liquid. This condition is true liquefaction, in which the soil has little or no shearing strength and will flow as a liquid. Liquefaction or flow failure of sands involves a substantial loss of shearing strength for a sufficient length of time that large deformations of soil masses occur by flow as a heavy liquid.

c. Liquefaction due to seismic activity. Soil deposits that have a history of serious liquefaction problems during earthquakes include alluvial sand, aeolian sands and silts, beach sands, reclaimed land, and hydraulic fills. During initial field investigations, observations that suggest possible liquefaction problems in seismic areas include low penetration resistance; artesian heads or excess pore pressures; persistent inability to retain granular soils in sampling tubes; and any clean, fine, uniform sand below the groundwater table. The liquefaction potential of such soils for structures in seismic areas should be addressed unless they meet one of the criteria in table 17-6. In the event that



(Courtesy of H. B. Seed and L M. Idriss, "Simplified Procedures for Evaluating Liquefaction Potential," Journal, Soil Mechanics and Foundations Division Vol 97, No. SM9. 1971, pp 1249-1273. Reprinted by permission of American Society of Civil Engineers, New York.)

Figure 17-10. Variation of shear modulus with cyclic strain amplitude; G_{max} = G at E = 1 to 3 x 10^{-4} percent; scatter in data up to about ± 0.1 on vertical scale.



(Courtesy of H. B Seed and 1. M Idrisv. "Simplified Procedure for Evaluating Soil Liquefaction Potential." Journal, Soul Mechanics and Foundations Division. Vol 97, No. SM9. 1971, pp 1249-1273. Reprinted by permission of the American Society, of Civil Engineers. Newt York.)

Figure 17-11. Variation of viscous damping with cyclic strain amplitude, data scatter up to about ±50 percent of average dumping values shown for any strain.

Table 17-6. Criteria for Excluding Need for Detailed Liquefaction Analyses

1. CL, CH, SC, or GC soils.
2. GW or GP soils or materials consisting of cobbles, boulders, uniform rock fill, which have free-draining boundaries that are large enough to preclude the development of excess pore pressures.
3. SP, SW, or SM soils which have average relative density equal to or greater than 85 percent, provided that the minimum relative density is not less than 80 percent.
4. ML or SM soils in which the dry density is equal to or greater than 95 percent of the modified Proctor (CE 55) density.
5. Soils of pre-Holocene age, with natural overconsolidation ratio equal to or greater than 16 <u>and</u> with relative density greater than 70 percent.
6. Soils located above the highest potential groundwater table.
7. Sands in which the "N" value is greater than three times the depth in feet, or greater than 75; provided that 75 percent of the values meet this criterion, that the minimum "N" value is not less than one times the depth in feet, that there are no consistent patterns of low values in definable zones or layers, and that the maximum particle size is not greater than 1 in. Large gravel particles may affect "N" values so that the results of the SPT are not reliable.
8. Soils in which the shear wave velocity is equal to or greater than 2000 fps. Geophysical survey data and site geology should be reviewed in detail to verify that the possibility of included zones of lowvelocity is precluded.
9. Soils that, in undrained cyclic triaxial tests, under isotropically consolidated, stress-controlled conditions, and with cyclic stress ratios equal to or greater than 0.45, reach 50 cycles or more with peak-to-peak cyclic strains not greater than 5 percent; provided that methods of specimen preparation and testing conform to specified guidelines.

Note: The criteria given above do not include a provision for exclusion of soils on the basis of grain-size distribution, and in general, grain-size distribution alone cannot be used to conclude that soils will not liquefy. Under adverse conditions nonplastic soils with a very wide range of grain sizes may be subject to liquefaction.

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none of the criteria is met and a more favorable site cannot be located, the material in question should be removed, remedial treatment applied as described in chapter 16, or a detailed study and analysis should be conducted to determine if liquefaction will occur.

17-7. Seismic effects on foundations.

Ground motions from earthquakes cause motions of foundations by introducing forces at the foundation-soil contact zone. Methods for estimating ground motions and their effects on the design of foundation elements are discussed in TM 5-809-10 / AFM 88-3, Chapter 13.