Technical Manual 1

Design of Monopole Bases

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Contents

Introduction

Organization

The following chapters will cover the following topics:

1. An historical perspective including the AISC approach to base plate design for building columns.
2. Classical methods for determining bolt forces and concrete stresses for grouted base plates.
3. Classical methods for determining bolt forces for ungrouted base plates.
4. Evaluation of various methods currently being used to determine base plate bending stresses for plain and stiffened plates

Historical Perspective

Monopoles have become increasingly popular for use in the telecommunication industry. The advantages include architectural attractiveness and a minimal use of land. Poles are of two general types, tapered polygonal poles and stepped pipe poles.

The tapered polygonal pole shown in Fig. 1-1, is custom manufactured to exact diameters required for the design. Each section is joined using telescoping lap joints.

Fig. 1- 1

Pipe poles are made from large diameter pipe sections and joined by external or internal flange connections as shown in Fig. 1-2.

Fig. 1- 2

While some poles may be directly buried into the earth, the most common method of attaching the pole to the foundation is with a base plate.

Base plates can be square with clustered anchor bolts as shown in Fig. 1-3 when overturning moments are relatively light.

Fig. 1- 3

The clear space below the leveling nut is not limited by the TIA-222 Standard; however, the ASCE Manual[1] suggests limiting the distance to two bolt diameters. AASHTO[2] limits this distance to one bolt diameter. AASHTO also recommends that the minimum base plate thickness be equal to the bolt diameter.

Base plates can also be polygonal or circular to accommodate a larger number of bolts. The plate may need to have gusset plates (stiffeners) in order to transfer forces due to axial and bending moment to the pole. A typical example is shown in Fig. 1-4.

Fig. 1- 4

Poles have been used in the power transmision field since the 1960's. Prior to that, poles were used almost exclusively for flags[3,4] and for highway structures[2]. In recent years, poles have become popular for both electric transmission towers and for telecommunication structures.

There is currently no industry standard for the design of pole base plates. Some state highway departments (New York) have developed their own methods, but no national standard exists. As such, the designer is left to arrive at appropriate methods based upon classical structural mechanics. While some testing has been done on smaller pole base plates used in highway construction (usually poles between 10 and 20 inches in diameter), no testing has been done on larger diameter pole base plates such as used in the telecommunication industry (poles 36 to 72 inches in diameter). Therefore, such design techniques may or may not be appropriate. Recent finite element studies[5,6,7] have indicated that current design practices used by pole manufacturers may be under-designed by 20 to 30%.

Although monopole failures are a relatively rare occurrence, a number of recent pole failures (see Fig. 1-5) have increased interest in manufacturer's design and manufacturing techniques.

Fig. 1- 5

The purpose of this report is to make available the various design techniques currently being used in the industry in the hope that more reliable methods of design may be developed in the future.

To begin, let us examine the traditional methods that have been developed for designing building columns subject to axial loads and moment.

AISC Method For Building Columns

Moment resisting base plates for building columns are covered in neither the AISC Specification nor the Manual of Steel Construction. Engineers must therefore refer to textbooks or technical papers for design methods although not all texts cover this topic.

Working stress methods for analyzing moment resistant base plate can be found in engineering texts(8,9,10,11,12). DeWolf(13) and Thambiratnam(14) compared methods of designing building columns to test data.

The AISC method for designing axially loaded base plates defines the critical section as being at .95 times the depth of the structural members for shapes of rectangular cross-section such as wide flanges and tubes. The critical section for pipe is defined at a location equal to .80 times the diameter of the pipe.

Two different approaches have been taken to determine the distribution of forces on the base plate, a flexible plate approach and a stiff plate approach.

Flexible Base Plate

Relatively flexible base plates are incapable of maintaining a linear strain distribution and the assumption is made that the compression force is centered beneath the compression flange of the column.

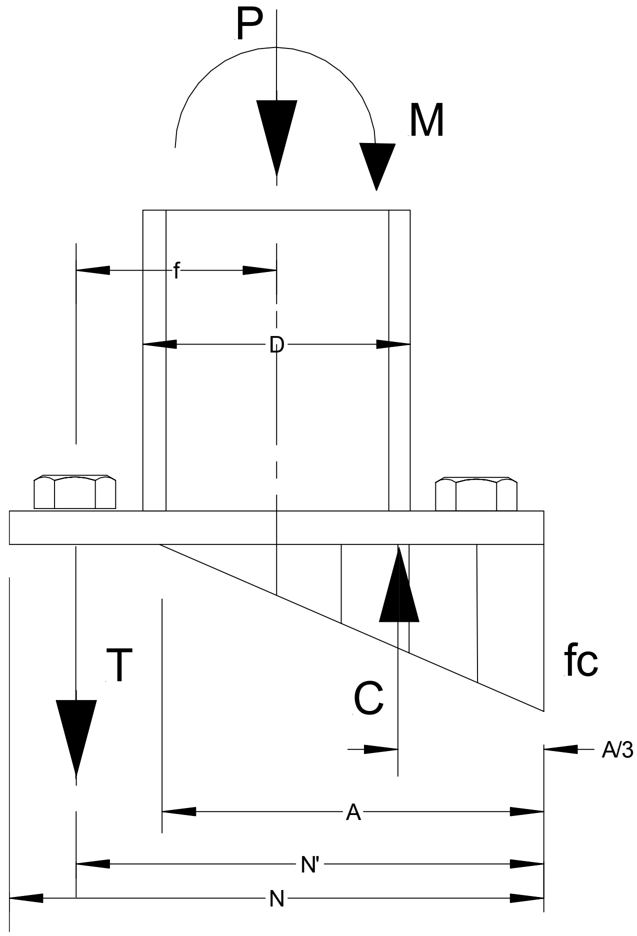


Fig. 1- 6 Flexible base plate force distribution

The following basic equations (B=plate width) define the static forces as shown in Fig 1-6.

$$T + P = C = \frac{f_c AB}{2}$$

$$T = \frac{M - \frac{PD}{2}}{f + \frac{D}{2}}$$

$$f_c = \frac{2(T + P)}{AB}$$

Example 1.1

Design a base plate for an axial load of 60 kips and a moment of 480 in-kips. Fy for the plate and anchor bolts is 36 ksi and f'c is 3 ksi. The structural member is an 8-inch wide flange and the base plate is 14x14. The bolts are 1.5″ from the edge.

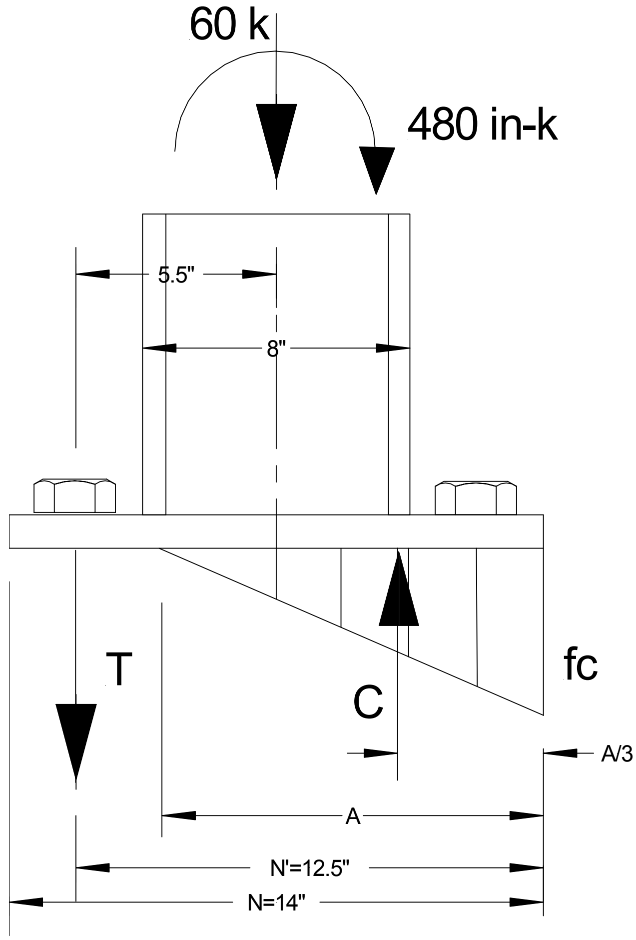


Fig. 1- 7

$$F_p = .35f'_c\sqrt{\frac{A_2}{A_p}} = .70f'_c = 2.1\,\text{ksi}$$

$$T = \frac{480 - \frac{60\times 8}{2}}{5.5 + \frac{8}{2}} = 25.26\,\text{kips} = 12.63\,\text{kips}/\text{bolt}$$

$$A = 3\times\frac{14-8}{2} = 9$$

$$C = T + P = 25.26 + 60 = 85.26$$

$$f_c = \frac{2(85.26)}{9\times 14} = 1.35\,\text{ksi} \le 2.1\,\text{ksi o.k.}$$

The thickness of the plate is determined by checking both the compression and tension sides. The critical section is at

$$b = \frac{14 - 0.95\times 8}{2} = 3.2\,\text{in}$$

$$f_{cpl} = \frac{9-3.2}{9}1.35 = 0.87\,\text{ksi}$$

$$M_{pl} = \frac{0.87\times 3.2^2}{2} + \frac{(1.35-0.87)3.2^2}{3} = 6.09\,\frac{\text{in-kip}}{\text{in}}$$

$$S_{reqd} = \frac{M}{F_b} = \frac{W\times t_{pl}^2}{6}, W = 1"$$

$$t_{pl} = \sqrt{\frac{6M}{F_b}}$$

$$F_b = .75F_y$$

$$t_{pl} = \sqrt{\frac{6\times 6.09}{.75\times 36}} = 1.16\,\text{in}$$

The critical width on the tension side is defined by AISC as shown in Fig 1-8.

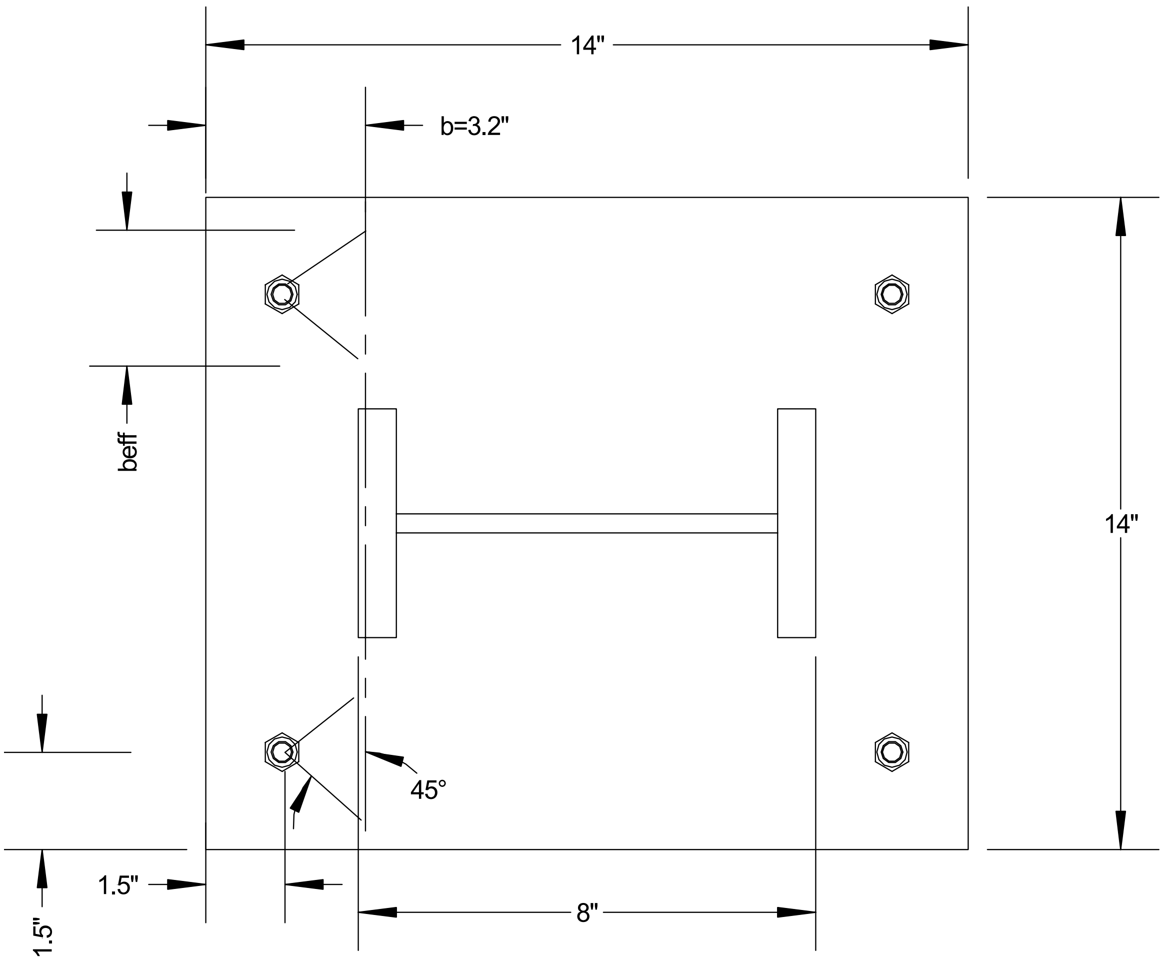


Fig 1-8

$$M_{pl} = 12.63 \times (3.2 - 1.5) = 21.47\,\text{in} - \text{kips}$$

$$3.2 - 1.5 = 1.7 > 1.5 \text{ use } 1.5$$

$$b_{eff} = 1.5 + (3.2 - 1.5) = 3.2\,\text{in}$$

$$t_{pl} = \sqrt{\frac{6 \times 21.47}{3.2 \times .75 \times 36}} = 1.22\,\text{in} \geq 1.16$$

Therefore the tension side controls.

Stiff Plate Approach

Thicker, stiffer, base plates are capable of approaching a linear strain distribution and the base plate is assumed to behave in a manner similar to reinforced concrete columns[8].

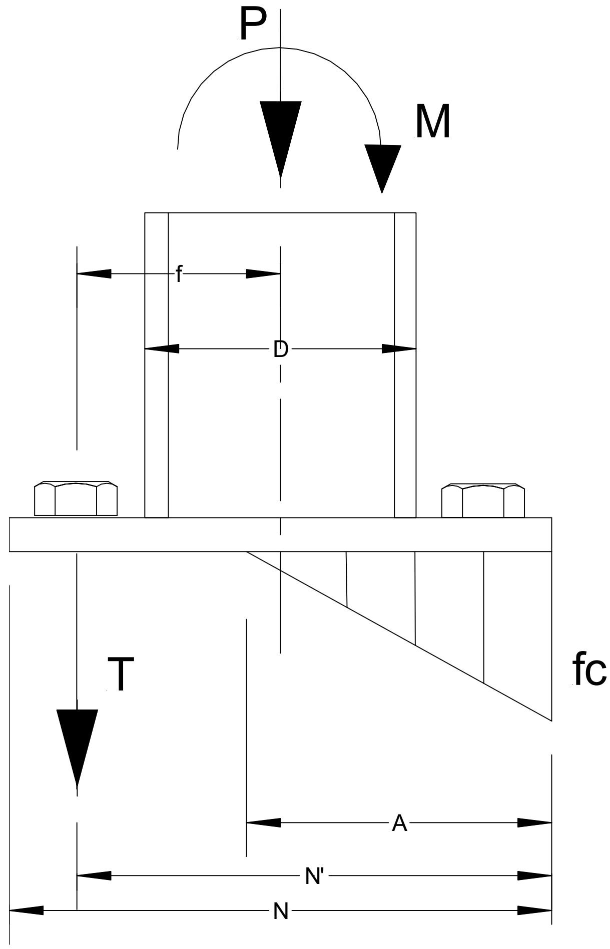


Fig 1- 9 Stiff base plate force distribution

The following basic equations define the static forces as shown in Fig 1-9.

$$P + T = C = \frac{f_c AB}{2}$$

where B=width of the base plate.

$$e = \frac{M}{P}$$

$$T = -P\left[\frac{\frac{N}{2} - \frac{A}{3} - e}{\frac{N}{2} - \frac{A}{3} + f}\right]$$

$$f_c = \frac{TA}{A_s n\left(\frac{N}{2} - A + f\right)}$$

$$P + T = \frac{TA^2B}{2A_s n\left(\frac{N}{2} - A + f\right)}$$

Defining

$$K_1 = 3\left(e - \frac{N}{2}\right)$$

$$K_2 = \frac{6nA_s}{B}(f + e)$$

$$K_3 = -K_2\left(\frac{N}{2} + f\right)$$

$$A^3 + K_1A^2 + K_2A + K_3 = 0$$

Example 1.2

Design the same plate of Example 1.1 using the stiff approach and assuming 1 ¼" diameter bolts.

$$F_p = .35 f'_c \sqrt{\frac{A_2}{A_p}} = .70 f'_c = 2.1\,\text{ksi}$$

$$e = \frac{480}{60} = 8\,\text{in}$$

$$n = \frac{29000}{57\sqrt{3000}} = 9.3$$

$$A_s = 2 \times .969 = 1.94$$

$$K_1 = 3\left(8 - \frac{14}{2}\right) = 3$$

$$K_2 = \frac{6 \times 9.3 \times 1.94}{14}(5.5 + 8) = 104$$

$$K_3 = -104\left(\frac{14}{2} + 5.5\right) = -1300$$

Solving for A by trial,

$$A = 7.3\,\text{in}$$

$$T = -60\left[\frac{7 - \frac{7.3}{3} - 8}{7 - \frac{7.3}{3} + 5.5}\right] = 20.5\,\text{kips} = 10.25\,\frac{\text{kips}}{\text{bolt}}$$

$$f_c = \frac{20.5 \times 7.3}{9.3 \times 1.94(7 - 7.3 + 5.5)} = 1.6\,\text{ksi} \le 2.1\,\text{ksi}$$

The thickness of the plate is determined by checking both the compression and tension sides. The critical section is at

$$b = \frac{14 - 0.95 \times 8}{2} = 3.2\,\text{in}$$

$$f_{cpl} = \frac{7.3 - 3.2}{7.3} 1.6 = 0.90\,\text{ksi}$$

$$M_{pl} = \frac{0.90 \times 3.2^2}{2} + \frac{(1.6 - 0.90)\,3.2^2}{3} = 7\,\frac{\text{in-kip}}{\text{in}}$$

$$t_{pl} = \sqrt{\frac{6 \times 7}{.75 \times 36}} = 1.25\,\text{in}$$

The critical width on the tension side is defined by AISC as shown in Fig 1-10.

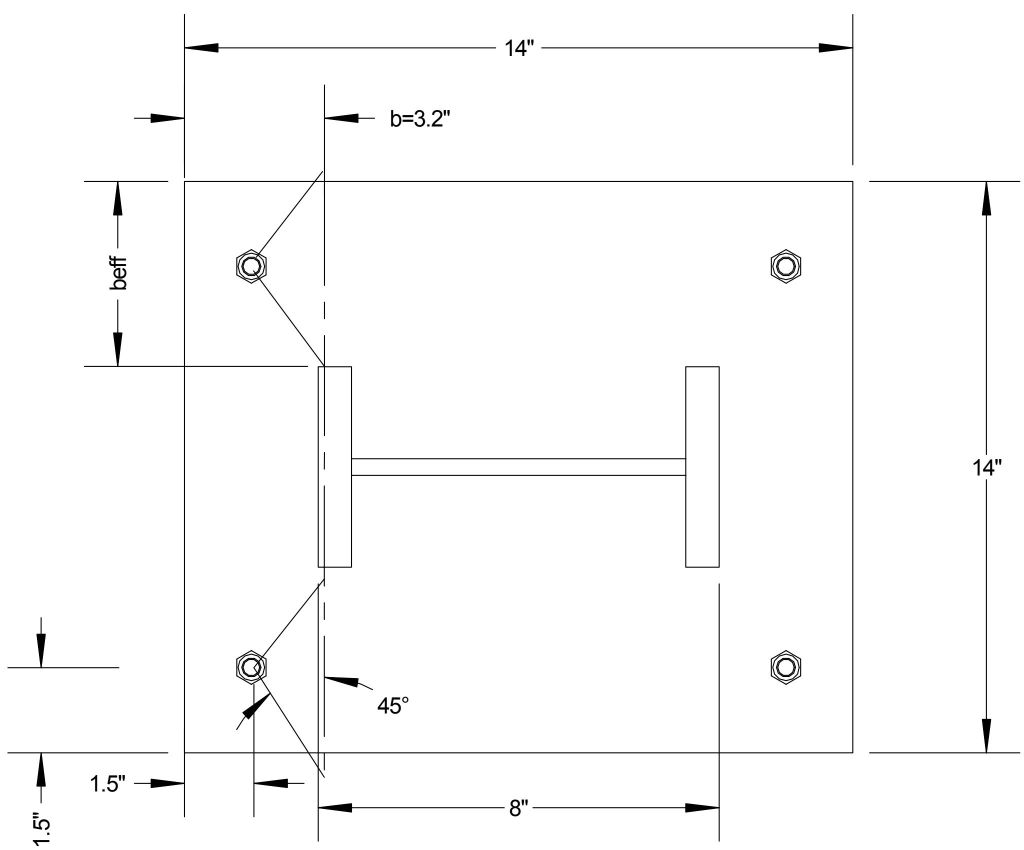


Fig 1-10

$$M_{pl} = 10.25 \times (3.2 - 1.5) = 17.43 \, \text{in} - \text{kips}$$

$$3.2 - 1.5 = 1.7 > 1.5 \, \text{in side clearanace use } 1.5 \text{ in}$$

$$b_{eff} = 1.5 + (3.2 - 1.5) = 3.2 \, \text{in}$$

$$t_{pl} = \sqrt{\frac{6 \times 17.43}{3.2 \times .75 \times 36}} = 1.10 \, \text{in} \leq 1.25 \, \text{in}$$

Therefore the compression side controls.

Example 1.3

Re-design Example 1.2 for an 8″ pipe column. A pipe would have the critical section defined as

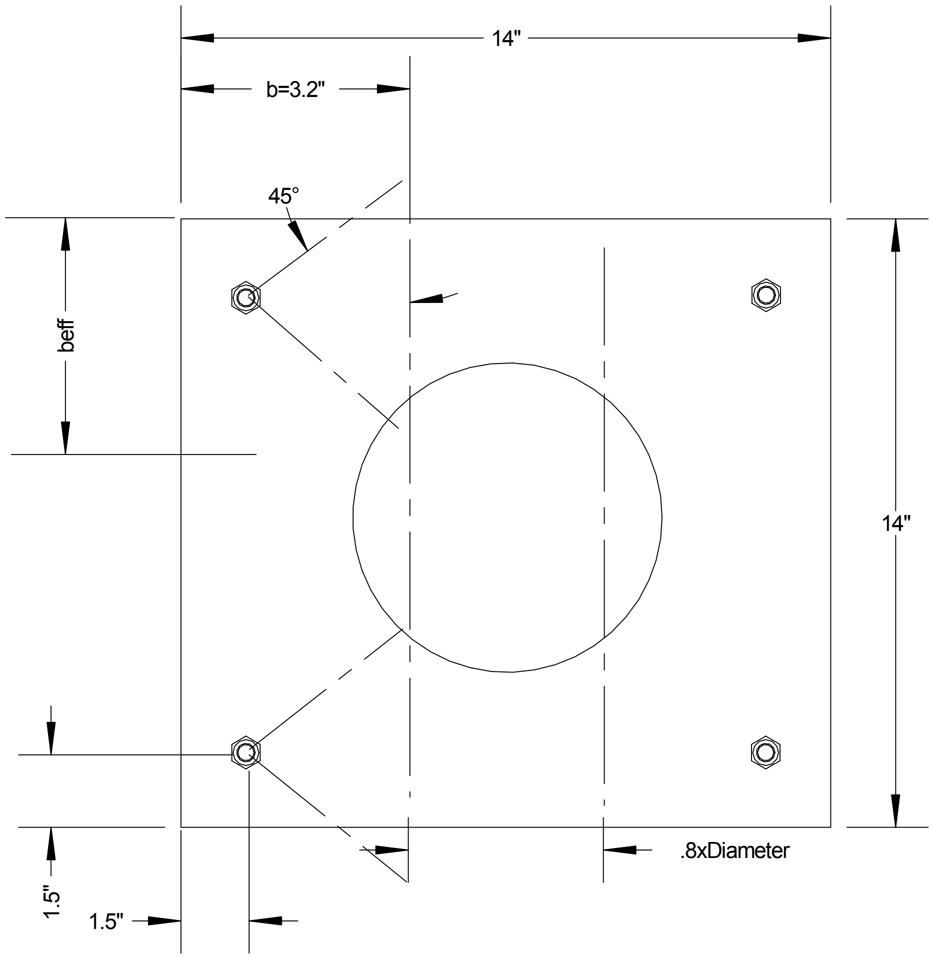


Fig 1-11

The analysis for bolt forces and concrete stress are identical to the wide flange base plate, however the distance to the critical section is now

$$b = \frac{14 - 0.80 \times 8}{2} = 3.8\,\text{in}$$

$$f_{cpl} = \frac{7.3 - 3.8}{7.3} 1.6 = 0.77\,\text{ksi}$$

$$M_{pl} = \frac{0.77 \times 3.8^2}{2} + \frac{(1.6 - 0.77)3.8^2}{3} = 9.55\,\frac{\text{in-kip}}{\text{in}}$$

$$t_{pl} = \sqrt{\frac{6 \times 9.55}{.75 \times 36}} = 1.46\,\text{in}$$

Checking the tension side

$$M_{pl} = 10.25 \times (3.8 - 1.5) = 23.58\,\text{in} - \text{kips}$$
$$3.8 - 1.5 = 2.3 > 1.5\,\text{in side clearance use}\,1.5\,\text{in}$$
$$b_{eff} = 1.5 + 2.3 = 3.8\,\text{in}$$
$$t_{pl} = \sqrt{\frac{6 \times 23.58}{3.8 \times .75 \times 36}} = 1.17\,\text{in} \le 1.46\,\text{in}$$

Therefore the compression side controls.

Grouted Base Plates

Process Equipment Design Method

Procedures for designing circular base plates with large openings (approaching the inner diameter of the shell) have been used for over 75 years. The most common use has been for chimneys and stacks used in the process equipment field. The method most often used was based upon the work of Taylor, Thompson, and Smulski[15]. The following description of the method is taken from Brownell and Young[16], Troitsky[17], Megyesy[18], and Bednard[19]. The method was developed for circular base plates; however the method can be conservatively used for polygonal plates and square base plates by using the diameter across the flats of the plate. The method is based upon reinforced concrete column theory using the Working Stress Design Method (WSD). Note that all of the grouted base plate techniques that follow assume that the eccentricity is sufficiently large enough to produce tension on a portion of the base plate.

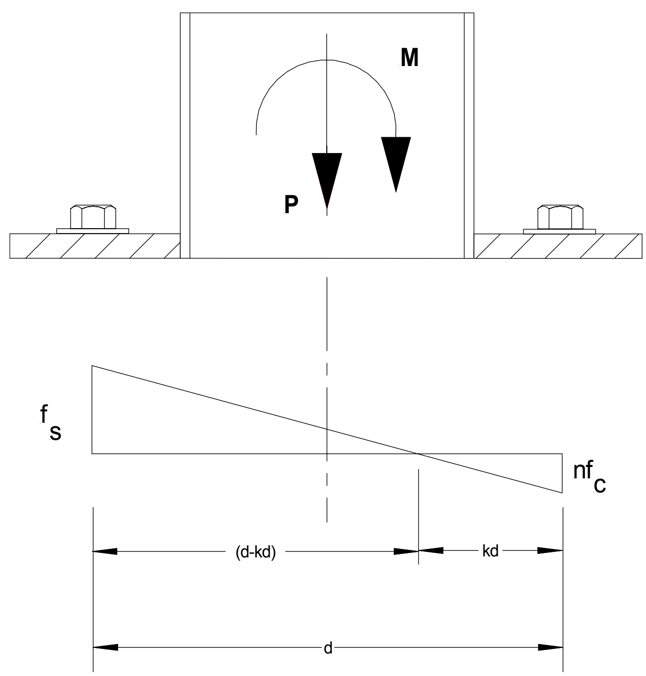


Fig. 2- 1

Defining the modular ratio as

$$n = \frac{E_s}{E_c}$$

Fig. 2-1 is a sketch representing the loading condition for the anchor bolts of the monopole base plate. The method assumes that the bolt circle is in the center of the bearing plate, which is typically the case when the shell of the pole runs through the base plate. The wind load and the dead weight load of the pole result in a tensile load on the upwind anchor bolts and a compressive load on the downwind anchor bolts. Denoting f_c as the compressive stress in the concrete, the induced compressive stress in the steel bolts is given by

$$f_s = nf_c$$

Considering the stress to be directly proportional to the distance from the neutral axis, a straight line may be drawn from f_s to nf_c as shown in Fig. 2-1. The neutral axis is located at a distance kd from the downwind side of the bearing plate and at a distance (d – kd) from the upwind side. This assumption is valid only if the base plate can be considered rigid.

By similar triangles

$$\frac{f_s}{(d - kd)} = \frac{nf_c}{kd}$$

therefore

$$k = \frac{nf_c}{nf_c + f_s} = \frac{1}{1 + \left(f_s / nf_c \right)}$$

The area of bolts can be expressed in terms of an equivalent ring of steel of thickness t having the same total cross-sectional area of steel as shown in Fig. 2-2. This assumption would only be valid if there are a sufficient number of bolts uniformly spaced around the bolt circle. Polygonal plates can be assumed to be a circular ring equal to the flat diameter of the polygon.

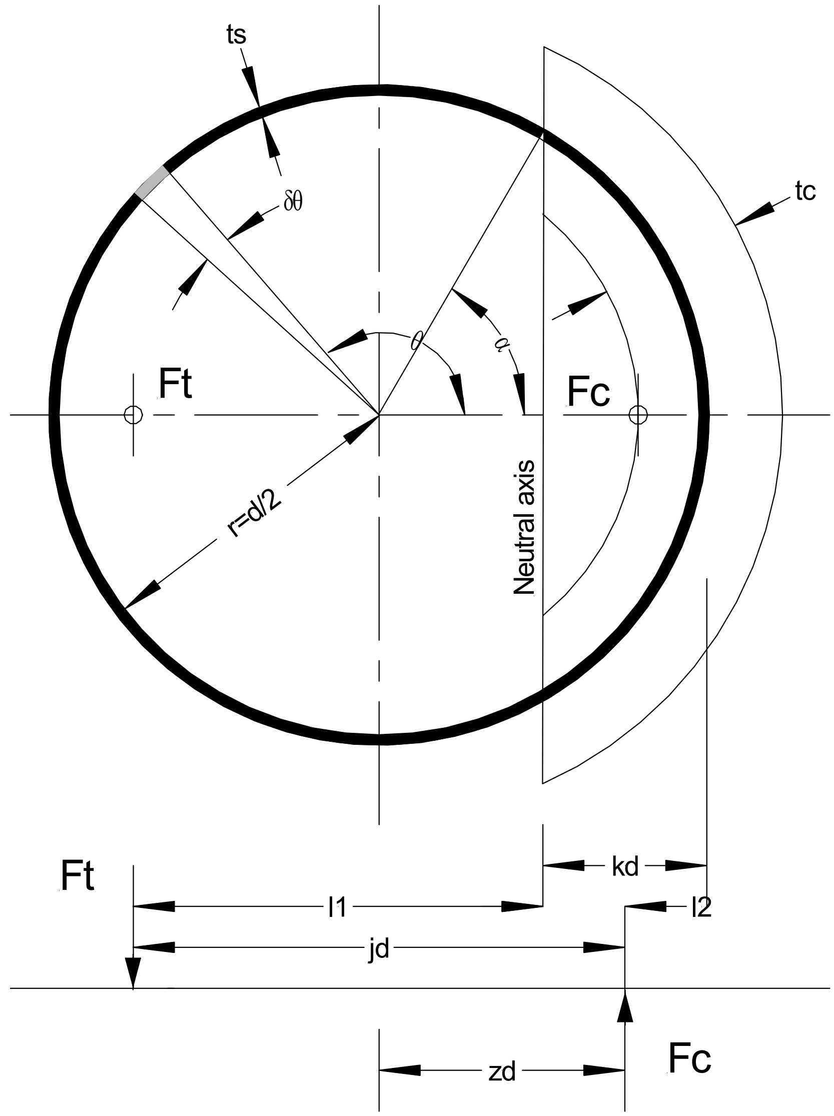


Fig. 2- 2

The location of the neutral axis from Fig. 2-2 can be defined in terms of the angle α.

$$\cos\alpha = \frac{d/2 - kd}{d/2} = 1 - 2k$$

And

$$\alpha = \cos^{-1}(1-2k)$$

A differential element of the steel ring is measured by dθ. The area of this element is

$$dA_s = t_s r d\theta$$

where

$$r = d/2$$

The distance from the neutral axis to this differential element is

$$r(\cos\alpha + \cos\theta)$$

Denoting the maximum steel stress as f_s, the stress in the element is

$$f_s' = f_s \frac{r(\cos\alpha + \cos\theta)}{r(1+\cos\alpha)}$$

The force in the element is therefore

$$dF_t = dA_s f_s' = f_s t_s r \frac{(\cos\alpha + \cos\theta)}{(1+\cos\alpha)}$$

The total tensile force is therefore

$$\begin{aligned} F_t &= 2\int_\alpha^\pi dF_t = 2 f_s t_s r \int_\alpha^\pi \frac{(\cos\alpha + \cos\theta)}{(1+\cos\alpha)} d\theta \\ &= f_s t_s r \left[\frac{2}{(1+\cos\alpha)} ((\pi - \alpha)\cos\alpha + \sin\alpha) \right] \\ &= f_s t_s r C_t \end{aligned}$$

where

$$C_t = \left[\frac{2}{(1+\cos\alpha)} ((\pi - \alpha)\cos\alpha + \sin\alpha) \right]$$

The c.g. of the tensile force, l_1, can be located by first determining the moment on the tension side and then dividing by F_t. The moment of the differential element is

$$\begin{aligned} dM_t &= dF_t r(\cos\alpha + \cos\theta) \\ &= f_s t_s r \left[\frac{(\cos\alpha + \cos\theta)}{1+\cos\alpha} r(\cos\alpha + \cos\theta) \right] d\theta \\ &= f_s t_s r \left[\frac{(\cos\alpha + \cos\theta)^2}{(1+\cos\alpha)} \right] d\theta \end{aligned}$$

Integrating

$$\begin{aligned} M_t &= 2 f_s t_s r^2 \int_\alpha^\pi \frac{(\cos\alpha + \cos\theta)^2}{(1+\cos\alpha)} d\theta \\ &= 2 f_s t_s r^2 \left[\frac{(\pi - \alpha)\cos^2\alpha + \frac{3}{2}(\sin\alpha\cos\alpha) + \frac{1}{2}(\pi - \alpha)}{1+\cos\alpha} \right] \end{aligned}$$

Dividing

$$l_1 = \frac{M_t}{F_t} = r\left[\frac{(\pi-\alpha)\cos^2\alpha + \frac{3}{2}(\sin\alpha\cos\alpha) + \frac{1}{2}(\pi-\alpha)}{(\pi-\alpha)\cos\alpha + \sin\alpha}\right]$$

The values of α, C_t and l_1 are constants for a given value of k.

Values for the compression area follow a similar formulation. The width of the compression ring is defined as $t_c = b_{pl} - t_s$. The differential element for a compression ring of thickness t_c is

$$dA_c = t_c r d\theta$$

The distance from the neutral axis to this element is

$$r(\cos\theta - \cos\alpha)$$

The maximum distance from the neutral axis is

$$r(1-\cos\alpha)$$

The element stress is directly proportional to the distance from the neutral axis

$$f_c' = f_c \frac{r(\cos\theta - \cos\alpha)}{r(1-\cos\alpha)}$$

The compressive stress in the steel is

$$f_{sc}' = nf_c \frac{(\cos\theta - \cos\alpha)}{(1-\cos\alpha)}$$

The force is determined by multiplying by the element areas

$$dF_c = f_c' dA_c = f_c t_c r\left[\frac{\cos\theta - \cos\alpha}{1-\cos\alpha}\right]d\theta$$

$$dF_{sc} = nf_{sc}' dA_s = nf_c t_s r\left[\frac{\cos\theta - \cos\alpha}{1-\cos\alpha}\right]d\theta$$

The total is therefore

$$dF_{ctot} = (t_c + nt_s) r f_c \left[\frac{\cos\theta - \cos\alpha}{1-\cos\alpha}\right]d\theta$$

Integrating

$$F_c = (t_c + nt_s) r f_c 2\int_0^\alpha \frac{\cos\theta - \cos\alpha}{1-\cos\alpha} d\theta$$

$$F_c = (t_c + nt_s) r f_c \left[\frac{2(\sin\alpha - \alpha\cos\alpha)}{1-\cos\alpha}\right]$$

$$F_c = (t_c + nt_s) r f_c C_c$$

Conversely

$$f_c = \frac{F_c}{(t_c + nt_s)rC_c}$$

Where

$$C_c = 2\left[\frac{\sin\alpha - \alpha\cos\alpha}{1-\cos\alpha}\right]$$

The c.g. of the compressive force, l_2, can be located by first determining the moment on the compression side and then dividing by F_c. The moment of the differential element is

$$\begin{aligned} dM_c &= dF_c r(\cos\theta - \cos\alpha) \\ &= (t_c + nt_s)r^2 f_c \frac{(\cos\theta - \cos\alpha)^2}{(1-\cos\alpha)} d\theta \end{aligned}$$

Integrating

$$\begin{aligned} M_c &= (t_c + nt_s) f_c r^2 2\int_0^\alpha \frac{(\cos\theta - \cos\alpha)^2}{1-\cos\alpha} d\theta \\ &= (t_c + nt_s) f_c r^2 2\left[\frac{\cos^2\alpha - \frac{3}{2}(\sin\alpha\cos\alpha) + \frac{1}{2}\alpha}{1-\cos\alpha}\right] \end{aligned}$$

Dividing

$$l_2 = \frac{M_c}{F_c} = r\left[\frac{\alpha\cos^2\alpha - \frac{3}{2}(\sin\alpha\cos\alpha) + \frac{1}{2}\alpha}{\sin\alpha - \alpha\cos\alpha}\right]$$

The total distance between the forces F_t and F_c is equal to l_1+l_2. Defining the dimensionless ratio j as

$$\begin{aligned} j &= \frac{l_1 + l_2}{d} \\ &= \tfrac{1}{2}\left[\frac{(\pi-\alpha)\cos^2\alpha + \frac{1}{2}(\pi-\alpha) + \frac{3}{2}\sin\alpha\cos\alpha}{(\pi-\alpha)\cos\alpha + \sin\alpha}\right] + \tfrac{1}{2}\left[\frac{\frac{1}{2}\alpha - \frac{3}{2}\sin\alpha\cos\alpha + \alpha\cos^2\alpha}{\sin\alpha - \alpha\cos\alpha}\right] \end{aligned}$$

The distance from the neutral axis to the centerline of the pole is

$$\frac{d}{2}\cos\alpha$$

and the distance zd is equal to

$$zd = l_2 + \frac{d}{2}\cos\alpha$$

The quantity, z is therefore

$$z = \tfrac{1}{2}\left[\cos\alpha + \left(\frac{\frac{1}{2}\alpha - \frac{3}{2}\sin\alpha\cos\alpha + \alpha\cos^2\alpha}{\sin\alpha - \alpha\cos\alpha}\right)\right]$$

The values of C_c, l_2, j and z are constants for a given value of k.

The two equilibrium conditions that must be met are

$$M_{wind} - Pzd - F_t jd = 0$$

$$F_t = \frac{M_{wind} - Pzd}{jd}$$

$$F_t + P - F_c = 0$$

$$F_c = F_t + P$$

Substituting

$$f_s = 2\pi \left[\frac{M_{wind} - Pzd}{C_t jdA_s} \right]$$

$$= \frac{F_t}{t_s r C_t}$$

$$f_c = \frac{kdf_s}{n(d - kd)}$$

The peak bearing stress, at the outer edge of the base plate is

$$f_{c\max} = f_c \left(\frac{2kd + b_{pl}}{2kd} \right)$$

Where b_{pl} is the width of the bearing plate.

Example 2.1

Analyze a grouted base plate for a 55″ diameter pole that runs through the base plate. The bolt circle is 64″ with (20) 2-1/4″ bolts. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips). The thickness of the base plate is 2-1/4″. The outer diameter of the base plate is 73″. The concrete has a strength of 3000 psi.

$$n = \frac{E_s}{E_c} = \frac{29000}{57\sqrt{3000}} = 9.29$$

$$A_s = 20(3.25) = 65\,\text{in}^2$$

$$r = \frac{64}{2} = 32\,\text{in}$$

$$t_s = \frac{65}{64\pi} = .323\,\text{in}$$

$$t_c = \frac{73-55}{2} - .323 = 8.677$$

Assuming a trial value of k of 0.30…

$$\alpha = \cos^{-1}(1-2k) = \cos^{-1}(.4) = 1.1593\,\text{rad}$$

$$C_c = 2\left[\frac{(\sin(1.1593) - 1.1593\cos(1.1593))}{1-\cos(1.1593)}\right] = 1.5093$$

$$C_t = \frac{2}{1+\cos(1.1593)}\left[(\pi - 1.1593)\cos(1.1593) + \sin(1.1593)\right] = 2.442$$

$$z = \tfrac{1}{2}\left[\cos(1.1593) + \left(\frac{\tfrac{1}{2}1.1593 - \tfrac{3}{2}\sin(1.1593)\cos(1.1593) + 1.1593\cos(1.1593)^2}{\sin(1.1593) - 1.1593\cos(1.1593)}\right)\right] = .4376$$

$$j = \tfrac{1}{2}\left[\frac{(\pi-1.1593)\cos^2(1.1593) + \tfrac{1}{2}(\pi-1.1593) + \tfrac{3}{2}\sin(1.1593)\cos(1.1593)}{(\pi-1.1593)\cos(1.1593) + \sin(1.1593)}\right] +$$
$$\tfrac{1}{2}\left[\frac{\tfrac{1}{2}1.1593 - \tfrac{3}{2}\sin(1.1593)\cos(1.1593) + 1.1593\cos^2(1.1593)}{\sin(1.1593) - 1.1593\cos(1.1593)}\right] = .781$$

$$F_t = \frac{42780 - 46\times.4376\times64}{.781\times64} = 830\,\text{kips}$$

$$F_c = 830 + 46 = 876\,\text{kips}$$

$$f_s = \frac{830}{.323 \times 32 \times 2.442} = 32.88\,\text{ksi}$$

$$f_c = \frac{876}{(8.677 + 9.29 \times .323)\,32 \times 1.5093} = 1.55\,\text{ksi}$$

$$k_{calc} = \frac{1}{1 + \dfrac{32.88}{9.29 \times 1.55}} = .305$$

Calculate a new value for k,

$$k = \frac{.3 + .305}{2} = .3025$$

The next loop

$$\alpha = 1.165$$
$$C_c = 1.516$$
$$C_t = 2.4366$$
$$z = .437$$
$$j = .781$$
$$F_t = 829.8$$
$$F_c = 875.8$$
$$f_s = 32.92$$
$$f_c = 1.546$$
$$k_{calc} = .303$$

Which is sufficiently accurate to proceed to final calculations

$$P_{bolt} = f_s A_{bolt} = 32.92 \times 3.25 = 107\,\text{kips}$$

$$f_{c\max} = \frac{1.546 \times 2 \times .3025 \times 64 + \left(\dfrac{73 - 55}{2}\right)}{2 \times .3025 \times 64} = 1.78\,\text{ksi} \le .7 \times 3 \times 1.33 = 2.8\ \text{ksi o.k.}$$

The maximum bending moment in the plate is actually due to a trapezoidal stress distribution, however most process equipment designers use the average stress at the bolt circle for ease of computation. The stress on the compression side is

$$M_{max} = \frac{f_c l^2}{2} = \frac{1.546 \times 9^2}{2} = 62.6\,\frac{in - kip}{in}$$

$$f_{max} = \frac{6M}{bt^2} = \frac{6 \times 62.6}{1 \times 2.25^2} = 74.2\,ksi \geq .75 \times 36 \times 1.33 = 36\,ksi\ n.g.$$

The stress on the tension side is

$$M_{max} = Pa = 107 \times \frac{64 - 55}{2} = 481.5\,in - kip$$

$$f_{max} = \frac{6 \times 481.5}{\frac{55\pi}{20} 2.25^2} = 66\,ksi \geq .75 \times 36 \times 1.33 = 36\,ksi\ n.g.$$

Lutz Modification

One of the major assumptions in the Process Equipment Method is that the center of pressure is coincident with the steel bolt ring. Should the compression ring extend inside the pole and the bolt circle is not centered on the outstanding leg of the base plate or, based upon the relative stiffness of the base plate, it is deemed that the center of compression will be at or near the pole diameter, then the results will be in error.

Lutz[20] extended the theory to account for the fact that the compression ring may have a different diameter than the steel tension ring.

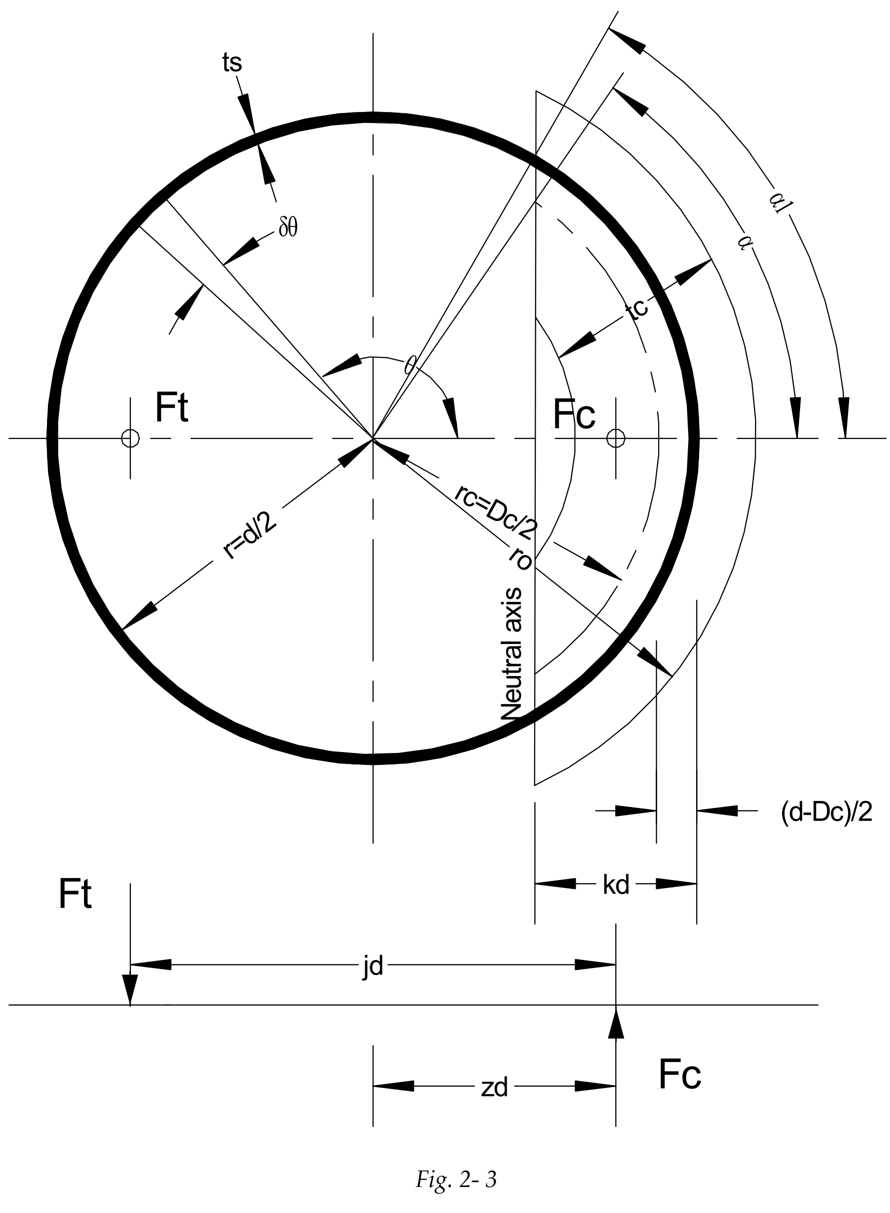


Fig. 2- 3

Lutz observed that a simple correction using two values of k would yield the correct result.

$$k_t = \frac{1 - r_c/r}{2} + k\frac{r_c}{r}$$

$$"jd" = \left(j - \tfrac{1}{2}\right)d + r_c$$

$$F_t = \frac{M_{wind} - Pzd}{\left(j - \tfrac{1}{2}\right)d + r_c}$$

$$f_c = \frac{F_c}{\left(t_c + nt_s\dfrac{r}{r_c}\right)r_c C_c}$$

$$k = \frac{\tfrac{1}{2}\left(1 + \dfrac{r}{r_c}\right)}{1 + \left(f_s / nf_c\right)}$$

The peak bearing stress, at the outer edge of the base plate, with radius r_o, is

$$f_{c\max} = f_c\left(\frac{2kd + r_o - r_c}{2kd}\right)$$

The value C_c is calculated using k while the values for C_t, z and j are then calculated using the value of k_t. Note that when $r_c = r$ then $k_t = k$ and the results are identical to the Process Equipment Method.

Example 2.2

Analyze a grouted base plate with an inner diameter of 48″ and an outer diameter of 73″. The bolt circle is 64″ with (20) 2-1/4″ bolts. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips). The thickness of the base plate is 2-1/4″. The concrete has a strength of 3000 psi.

$$n = \frac{E_s}{E_c} = \frac{29000}{57\sqrt{3000}} = 9.29$$

$$A_s = 20(3.25) = 65\,\text{in}^2$$

$$r = \frac{64}{2} = 32\,\text{in}$$

$$r_c = \frac{(73+48)/2}{2} = 30.25\,\text{in}$$

$$t_s = \frac{65}{64\pi} = .323\,\text{in}$$

$$t_c = \frac{73-48}{2} - .323 = 12.177$$

Assuming a trial value of k of 0.2773…

$$\alpha = \cos^{-1}(1-2\times.2773) = 1.109\,\text{rad}$$

$$C_c = 2\left[\frac{(\sin(1.109) - 1.109\cos(1.109))}{1-\cos(1.109)}\right] = 1.447$$

$$k_t = \frac{1-30.25/32}{2} + .2773\frac{30.25}{32} = .2895$$

$$\alpha_1 = \cos^{-1}(1-2\times.2895) = 1.1362\,\text{rad}$$

$$C_t = \frac{2}{1+\cos(1.1362)}\left[(\pi - 1.1362)\cos(1.1362) + \sin(1.1362)\right] = 2.4482$$

$$z = \tfrac{1}{2}\left[\cos(1.1362) + \left(\frac{\tfrac{1}{2}1.1362 - \tfrac{3}{2}\sin(1.1362)\cos(1.1362) + 1.1362\cos(1.1362)^2}{\sin(1.1362) - 1.1362\cos(1.1362)}\right)\right] = .4399$$

$$j = \tfrac{1}{2}\left[\frac{(\pi - 1.1362)\cos^2(1.1362) + \tfrac{1}{2}(\pi - 1.1362) + \tfrac{3}{2}\sin(1.1362)\cos(1.1362)}{(\pi - 1.1362)\cos(1.1362) + \sin(1.1362)}\right] +$$

$$\tfrac{1}{2}\left[\frac{\tfrac{1}{2}1.1362 - \tfrac{3}{2}\sin(1.1362)\cos(1.1362) + 1.1362\cos^2(1.1362)}{\sin(1.1362) - 1.1362\cos(1.1362)}\right] = .7807$$

$$F_t = \frac{42780 - 46 \times .4399 \times 64}{(.7807 - .5)64 + 30.25} = 860.42\,\text{kips}$$

$$F_c = 860.42 + 46 = 906.42\,\text{kips}$$

$$f_s = \frac{860.42}{.323 \times 32 \times 2.4482} = 33.97\,\text{ksi}$$

$$f_c = \frac{906.42}{\left(12.177 + 9.29 \times .323\dfrac{32}{30.25}\right)30.25 \times 1.447} = 1.349\,\text{ksi}$$

$$k_{calc} = \frac{\tfrac{1}{2}\left(1 + \dfrac{32}{30.25}\right)}{1 + \dfrac{33.77}{9.29 \times 1.349}} = .277$$

Which is sufficiently accurate to proceed to final calculations

$$P_{bolt} = f_s A_{bolt} = 33.97 \times 3.25 = 110.4\,\text{kips}$$

$$f_{c\max} = \frac{1.349 \times 2 \times .2773 \times 32 + 36.5 - 30.25}{2 \times .2773 \times 32} = 1.70\,\text{ksi}$$

Complete Square Base Plate – Load On Diagonal

The Process Equipment Method assumes that the bolts can be idealized as a ring of steel and that the compression area can be idealized as a ring with the center of pressure at the center of the ring. Often base plates are encountered that have as few as four bolts. These cases can result in significant errors when calculating pressures and bolt forces. Base plates that extend inside the pole will also have a triangular stress distribution that may differ from the assumed ring theory.

Horn[21], developed a method for square base plates with a circular opening which takes into account individual bolt locations. Circular or polygonal plates can be analyzed using the Complete Circular Base Plate Method discussed in the next section.

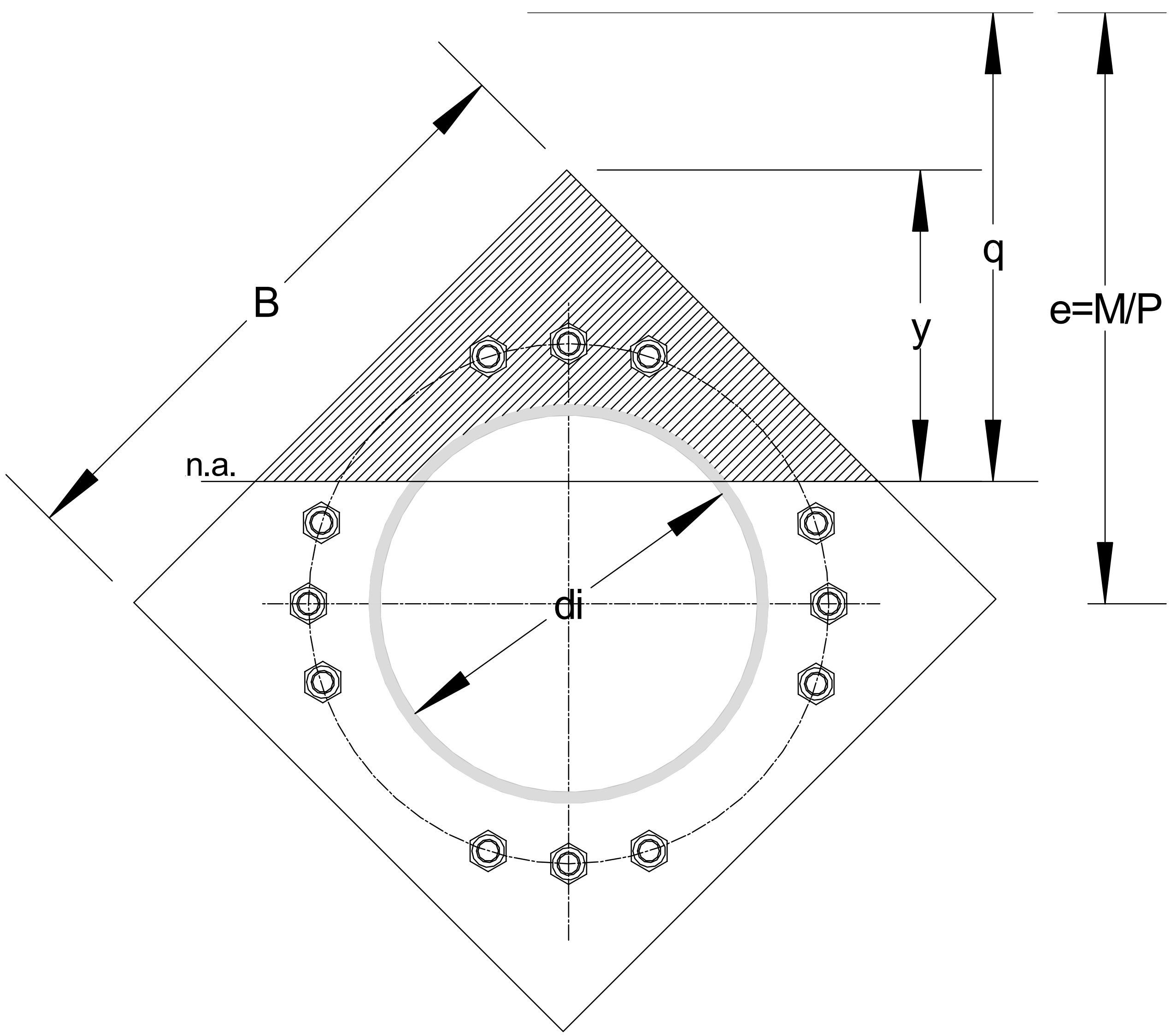


Fig. 2- 4

The method is valid when there is net tension on the section, a condition that occurs when

$$e > \frac{\frac{B^4}{12} - \frac{\pi d_i^4}{64}}{.707B\left(B^2 - \frac{\pi d_i^2}{4}\right)}$$

The method will make use of transformed areas of the bolts. Bolts that are in the tension zone will have their areas transformed by

$$A_{bolt} = nA_b$$

Bolts within the compression zone will have their areas transformed by

$$A_{bolt} = (n-1)A_b$$

The method will calculate moment of inertia of the areas and the moment areas about the "e" line . The moment of inertia divided by the moment area will then be the location of the neutral axis. The process iterates on the value of q until it is determined to a sufficient degree of accuracy.

Defining

$$r_i = \frac{d_i}{2}$$
$$y = .707B - e + q$$
$$n = \frac{29000}{57\sqrt{f_c'}}$$
$$f_c = \frac{Py_{max}}{qA_T - Q_T}$$

$$A_T = A_1 + A_2 + A_c + \sum_{comp}(n-1)A_b + \sum_{tens} nA_b$$

The area of the triangular wedge is defined as

$$A_1 = \frac{y \times 2y}{2} = y^2$$

Should the neutral axis (n.a.) fall below the diagonal of the plate, two wedge areas will have to be subtracted

$$y_2 = y - .707B$$
$$A_2 = 0 \text{ if } y_2 \le 0$$
$$= -2y_2^2 \text{ if } y_2 > 0$$

Should the neutral axis fall below the top of the circular opening, the area of the semi-circle must be deducted

$$l = r_i - e + q$$

$$y_0 = \max(r_i - l, -r_i)$$

$$A_c = -\frac{r_i^2 \pi}{2} + y_0 \sqrt{r_i^2 - y_0^2} + r_i^2 \sin^{-1}\left(\frac{y_0}{r_i}\right), l > 0$$

The moment area is defined as

$$Q_T = Q_1 + Q_2 + Q_c + Q_{bolts}$$

$$Q_1 = A_1 \left(q - \frac{y}{3}\right)$$

$$Q_2 = A_2 \left(q - \frac{y_2}{3}\right)$$

$$y_{1c} = 0, \text{if } l < 0$$

$$= \frac{-2\left(r_i^2 - y_0^2\right)^{1.5}}{3A_c}$$

$$Q_c = A_c \left(e - y_{1c}\right)$$

$$Q_{bolts} = \sum_{j=1}^{j=ntens} nA_b \left(e - y_j\right) + \sum_{j=1}^{j=ncomp} (n-1) A_b \left(e - y_j\right)$$

The moment of inertia of the areas is defined as

$$I_T = I_1 + I_2 + I_c + I_{bolts}$$

$$I_1 = A_1 \left(\frac{y^2}{18} + \left[q - \frac{y}{3}\right]^2\right)$$

$$I_2 = A_2 \left(\frac{y_2^2}{18} + \left[q - \frac{y_2}{3}\right]^2\right)$$

$$I_c = -\left[\frac{\pi r_i^4}{8} + \frac{y_0 \sqrt{\left(r_i^2 - y_0^2\right)^3}}{2} - \frac{r_i^2 y_0 \sqrt{r_i^2 - y_0^2}}{4} - \frac{r_i^4 \sin^{-1}\left(\frac{y_0}{r_i}\right)}{4}\right] - A_c y_{1c}^2 + A_c \left(e - y_{1c}\right)^2$$

$$I_{bolts} = \sum \left[\frac{A_{bolt}^2}{4\pi} + A_{bolt} \left(e - y_j\right)^2\right]$$

$$q = \frac{I_T}{Q_T}$$

Modification for Clipped Corners

When the corners of a square base plate are clipped as shown in Fig. 2-5, there are "negative" areas that must be removed from the analysis.

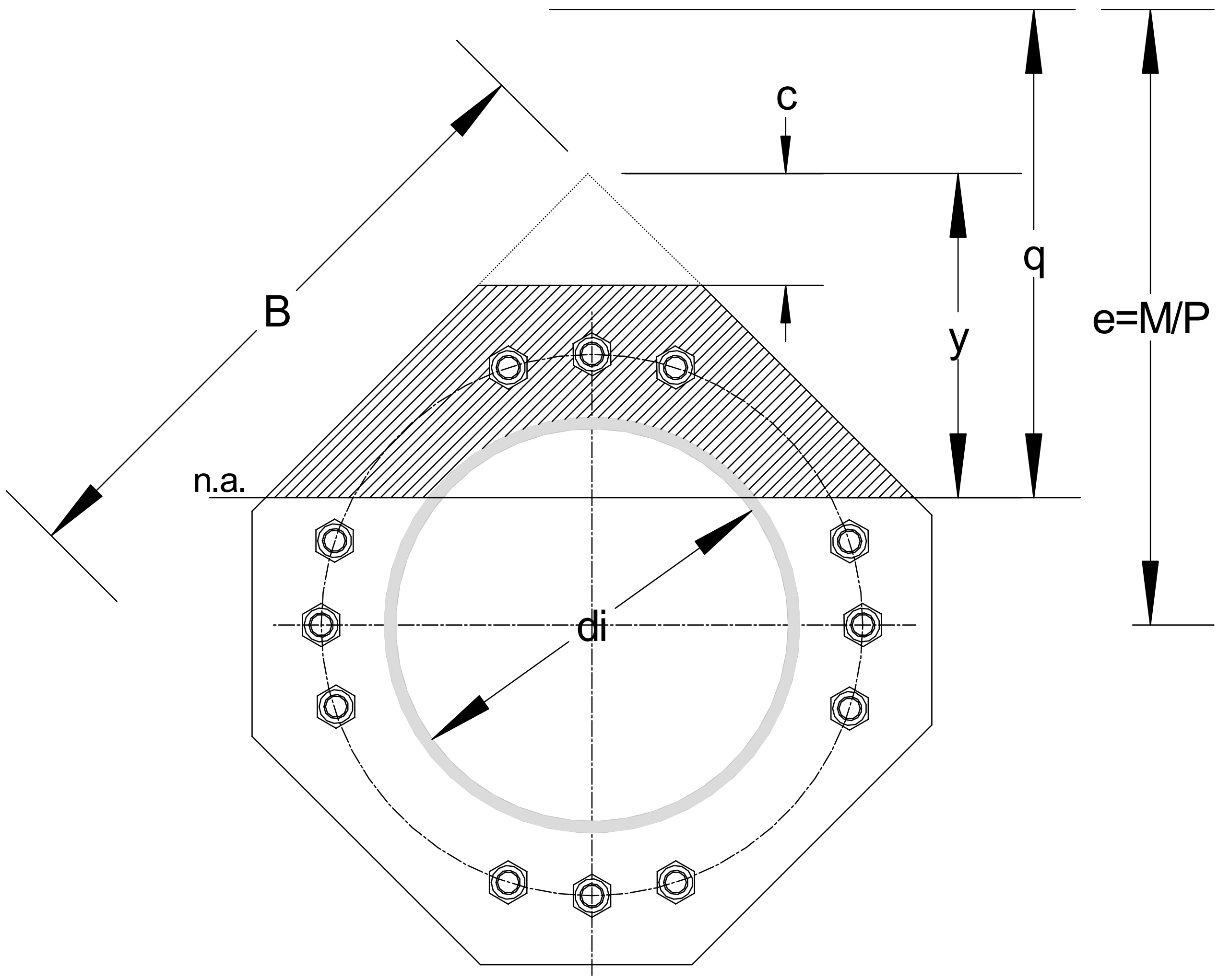


Fig. 2- 5

The previous equations may be used with the addition of the following terms

$$A_3 = -c^2$$

$$l_3 = q - y + \frac{2c}{3}$$

$$Q_3 = A_3 l_3$$

$$I_3 = A_3\left(\frac{c^2}{18} + l_3^2\right)$$

$$\begin{aligned} y_4 &= 0 \text{ if } y \le .707B - c \\ &= \min\left(y - .707B + c, 2c\right) \end{aligned}$$

$$A_4 = -y_4^2$$

$$l_4 = e - c + \frac{2y_4}{3}$$

$$Q_4 = A_4 l_4$$

$$I_4 = A_4\left(\frac{y_4^2}{18} + l_4^2\right)$$

$$\begin{aligned} y_5 &= 0 \text{ if } y \le .707B \\ &= \min\left(y - .707B, c\right) \end{aligned}$$

$$A_5 = y_5^2$$

$$l_5 = e + \frac{y_5}{3}$$

$$Q_5 = A_5 l_5$$

$$I_5 = A_5\left(\frac{y_5^2}{18} + l_5^2\right)$$

$$A_T = A_1 + A_2 + A_3 + A_4 + A_5 + A_c + \sum\nolimits_{comp}(n-1)A_b + \sum\nolimits_{tens} nA_b$$

$$Q_T = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_c + Q_{bolts}$$

$$I_T = I_1 + I_2 + I_3 + I_4 + I_5 + I_c + I_{bolts}$$

$$f_c = \frac{P\left(y - c\right)}{qA_t - Q_t}$$

Example 2.3

Analyze the bolt forces on a grouted square base plate loaded on the diagonal and a bolt circle of 64″ and (12) 2-1/4″ bolts spaced at 4.5″. The plate is 68″ square and the inside of the base plate has a diameter of 58″. The plate is 2.25″ thick, A36 plate. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips). The concrete strength is f'_c=3000 psi.

$$e = \frac{M}{P} = \frac{42780}{46} = 930\,\text{in}$$

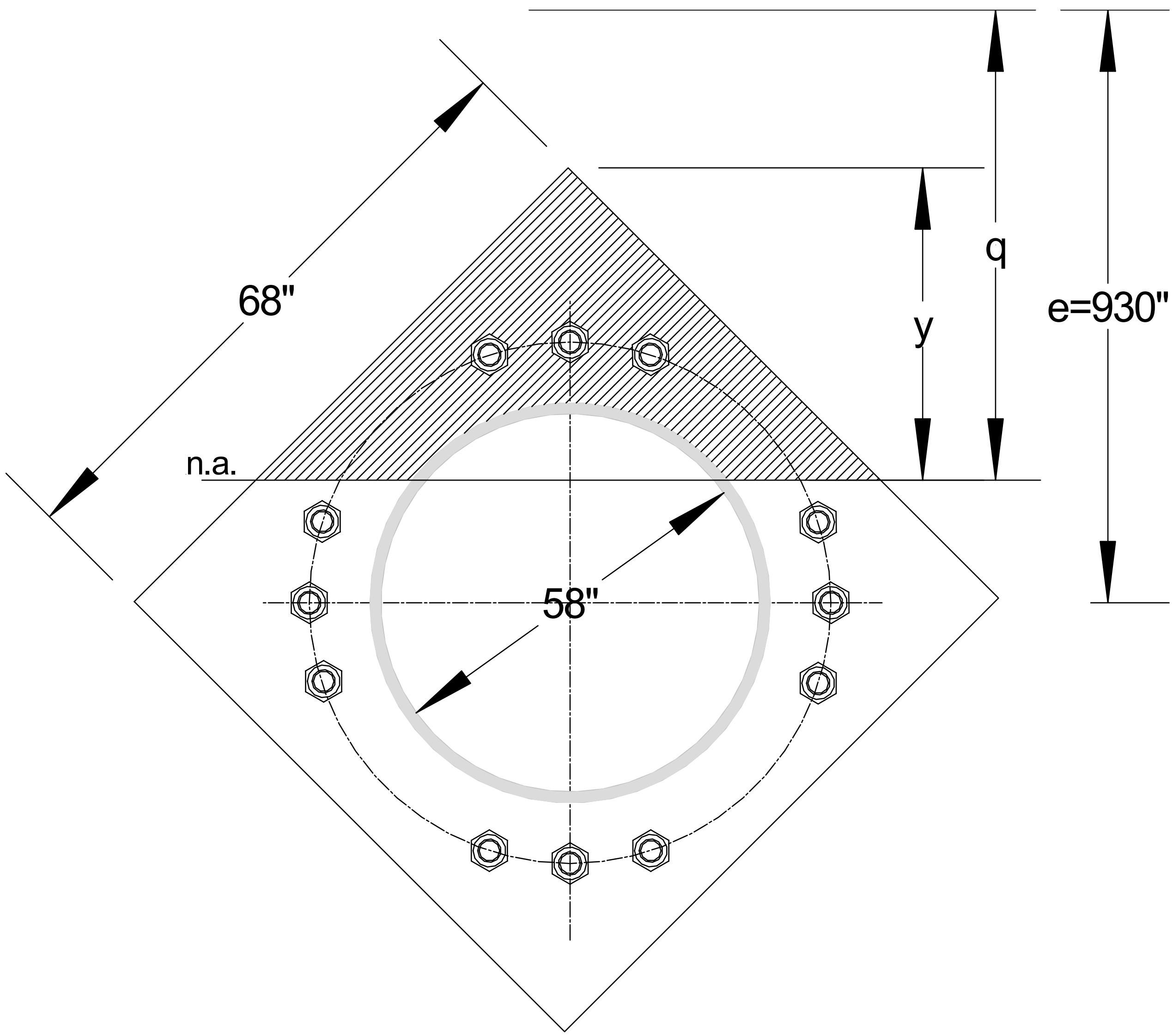


Fig 2- 6

The actual process requires iterating on q until a calculated value of q agrees to the fifth decimal place with the previous cycle. The final iteration of q yields q=911.33267.

Calculate the bolt values as follows

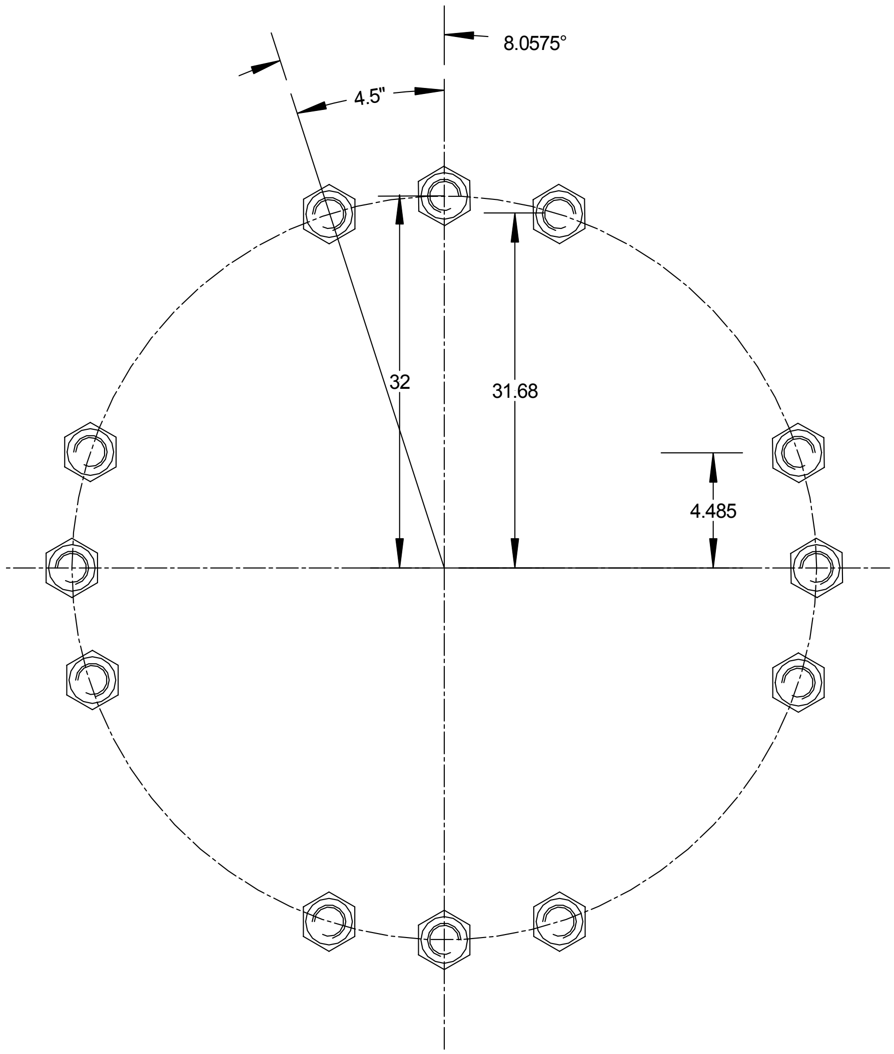


Fig. 2-7

$$\theta = 360\left(\frac{4.5}{64\pi}\right) = 8.0575°$$

$$n = \frac{29000}{57\sqrt{3000}} = 9$$

Bar #	ybolt	n	Abolt	nAbolt	Qbolt	Ibolt
1	0	9	3.25	29.25	27202.5	25298393
2	0	9	3.25	29.25	27202.5	25298393
3	32	8	3.25	26	23348	20966558
4	-32	9	3.25	29.25	28138.5	27069305
5	4.485	9	3.25	29.25	27071.31	25054975
6	4.485	9	3.25	29.25	27071.31	25054975
7	-4.485	9	3.25	29.25	27333.69	25542988
8	-4.485	9	3.25	29.25	27333.69	25542988
9	31.684	8	3.25	26	23356.22	20981316
10	31.684	8	3.25	26	23356.22	20981316
11	-31.684	9	3.25	29.25	28129.26	27051524
12	-31.684	9	3.25	29.25	28129.26	27051524
			Totals	341.25	317672	295894256

Calculating the plate areas

$$y = .707 \times 68 - 930 + 911.33267 = 29.41$$

$$A_1 = (29.41)^2 = 864.85$$

$$y_2 = \max(0, 29.41 - .707 \times 68) = 0$$

$$A_2 = 0$$

Calculating the deduction for the area of the hole

$$r_i = \frac{58}{2} = 29$$

$$l = 29 - 930 + 911.33267 = 10.33$$

$$y_0 = \max(r_i - l, -r_i) = 18.67$$

$$A_c = -\frac{29^2 \pi}{2} + 18.67\sqrt{29^2 - 18.67^2} + 29^2 \sin^{-1}\left(\frac{18.67}{29}\right) = -318.61$$

$$A_T = 864.86 + 0 + 341.25 - 318.61 = 887.49$$

Calculating the moment areas

$$Q_1 = 864.85\left(911.33267 - \frac{29.41}{3}\right) = 779690$$

$$Q_2 = 0$$

$$y_{1c} = \frac{-2\left(29^2 - 18.67^2\right)^{1.5}}{3(-318.61)} = 22.87$$

$$Q_c = -318.61(930 - 22.87) = -289020$$

$$Q_T = 779690 + 0 - 289020 + 317672 = 808342$$

Calculating the moments of inertia

$$I_1 = 864.85\left(\frac{29.41^2}{18} + \left[911.33267 - \frac{29.41}{3}\right]^2\right) = 702954883$$

$$I_2 = 0$$

$$I_c = -\left[\frac{\pi 29^4}{8} + \frac{18.67\sqrt{\left(29^2 - 18.67^2\right)^3}}{2} - \frac{29^2 18.67\sqrt{29^2 - 18.67^2}}{4} - \frac{29^4 \sin^{-1}\left(\frac{18.67}{29}\right)}{4}\right]\ldots$$

$$-(-318.61)22.87^2 + (-318.61)(930 - 22.87)^2$$

$$I_c = -262180695$$

$$I_T = 702954883 + 0 - 262180695 + 295894256 = 736668444$$

$$q_{calc} = \frac{I_T}{Q_T} = \frac{736668444}{808342} = 911.33267$$

Therefore, another iteration is not necessary and we can calculate final concrete and bolt stresses.

$$f_{cmax} = \frac{Py}{qA_T - Q_T} = \frac{46 \times 29.41}{911.33267 \times 887.49 - 808342} = 2.95\,\text{ksi}$$

$$y_{pole} = 911.33267 - 930 + 29 = 10.33$$

$$f_{cpole} = \frac{46 \times 10.33}{911.33267 \times 887.49 - 808342} = 1.04\,\text{ksi}$$

$$y_{bc} = 911.33267 - 930 + 32 = 13.33$$

$$f_{bc} = \frac{46 \times 13.33}{911.33267 \times 887.49 - 808342} = 1.34\,\text{ksi}$$

$$P_{bolt} = \frac{P\left(e - y_{bolt} - q\right)}{qA_T - Q_T}\left(nA_b\right)$$

$$P_{bolt} = \frac{46\left(930 - (-32) - 911.33267\right)}{911.33267 \times 887.49 - 808342}(9 \times 3.25)$$

$$P_{bolt} = 148.7\,\text{kips (tension)}$$

$$P_{compr} = \frac{46(930 - 32 - 911.33267)}{911.33267 \times 887.49 - 808342}(8 \times 3.25)$$

$$P_{compr} = -34.8\,\text{kips (compression)}$$

Likewise, for all of the bolts,

Bar #	ybolt	n	Pbolt
1	0	9	54.8084
2	0	9	54.8084
3	32	8	-34.79381
4	-32	9	148.76
5	4.485	9	41.6406
6	4.485	9	41.6406
7	-4.485	9	67.9763
8	-4.485	9	67.9763
9	31.684	8	-33.96912
10	31.684	8	-33.96912
11	-31.684	9	147.832
12	-31.684	9	147.832

The maximum plate stress using the maximum pressure over a 4.5" strip is then

$$M_{max(c)} = \frac{4.5\times1.04(.707\times68-29)^2}{2}+\frac{4.5\times(2.95-1.04)(.707\times68-29)^2}{3}+34.8\times3=1998 \text{ in}-\text{kips}$$

$$f_{bmax} = \frac{6\times1998}{4.5\times2.25^2}=526 \text{ ksi} \geq .75\times36\times1.33=36 \text{ ksi n.g.}$$

$$M_{max(t)} = 148.8\times3=446 \text{ in}-\text{kips}$$

$$f_{bmax} = \frac{6\times446}{4.5\times2.25^2}=117.5 \text{ ksi} \geq 36 \text{ ksi n.g.}$$

As can be seen, using the maximum stress over the plate results in an extremely high stress. This can only be achieved if the plate is extremely stiff. A more reasonable approach would be to modify the method to discount a portion of the plate similar to if the plate had a "clipped" corner.

Example 2.4

Modify Example 2.3 to discount a portion of the plate. Assuming that the bolt circle is at the mid-point between the pole and the clipped edge, the amount to be clipped would then be 13 inches from the top of the plate.

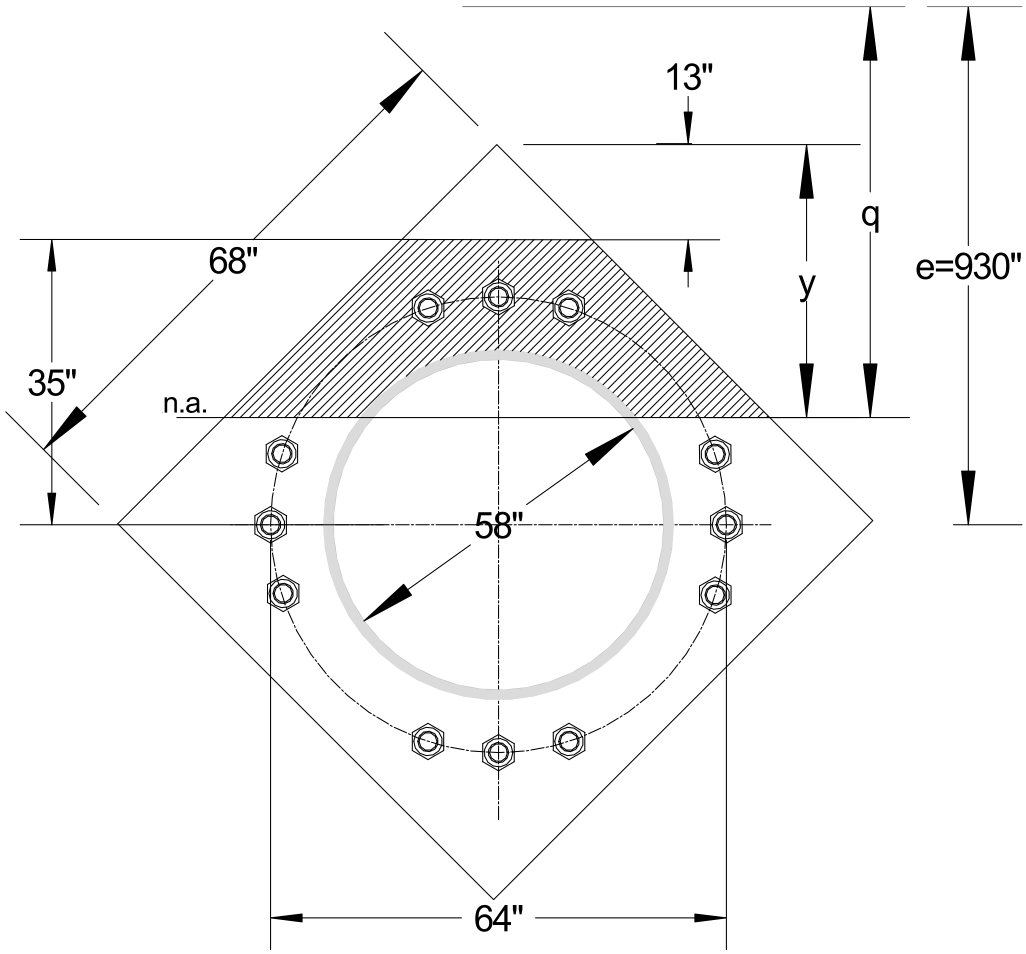


Fig. 2- 7

The previous equations may be modified using the clipped corner adjustments. After iterating, the new value for q is 915.860.

Calculate the bolt values as follows

Bar #	ybolt	n	Abolt	nAbolt	Qbolt	Ibolt
1	0	9	3.25	29.25	27202.5	25298393
2	0	9	3.25	29.25	27202.5	25298393
3	32	8	3.25	26	23348	20966558
4	-32	9	3.25	29.25	28138.5	27069305
5	4.485	9	3.25	29.25	27071.31	25054975
6	4.485	9	3.25	29.25	27071.31	25054975
7	-4.485	9	3.25	29.25	27333.69	25542988
8	-4.485	9	3.25	29.25	27333.69	25542988
9	31.684	8	3.25	26	23356.22	20981316
10	31.684	8	3.25	26	23356.22	20981316
11	-31.684	9	3.25	29.25	28129.26	27051524
12	-31.684	9	3.25	29.25	28129.26	27051524
			Totals	341.25	317672	295894256

Calculating the plate areas

$$y = .707 \times 68 - 930 + 915.86 = 33.94$$

$$A_1 = (33.94)^2 = 1151.64$$

$$y_2 = \max(0, 33.94 - .707 \times 68) = 0$$

$$A_2 = 0$$

$$A_3 = -13^2 = -169$$

$$l_3 = 915.86 - 33.94 + \frac{2 \times 13}{3} = 890.59$$

$$Q_3 = -169 \times 890.59 = -150509.82$$

$$I_3 = -160\left(\frac{13^2}{18} + 890.59^2\right) = -134044230$$

$$.707 \times B - y = .707 \times 68 - 33.94 = 14.13 \geq 13$$

$$\therefore y_4 = 0$$

Calculating the deduction for the area of the hole

$$r_i = \frac{58}{2} = 29$$

$$l = 29 - 930 + 915.86 = 14.86$$

$$y_0 = \max(r_i - l, -r_i) = 14.14$$

$$A_c = -\frac{29^2 \pi}{2} + 14.14\sqrt{29^2 - 14.14^2} + 29^2 \sin^{-1}\left(\frac{14.14}{29}\right) = -535.68$$

$$A_T = 1151.64 + 0 - 169 + 341.25 - 534.68 = 789.21$$

Calculating the moment areas

$$Q_1 = 1151.64\left(915.86 - \frac{33.94}{3}\right) = 1041713$$

$$Q_2 = 0$$

$$Q_3 = -169 \times 890.59 = -150509.82$$

$$y_{1c} = \frac{-2\left(29^2 - 14.14^2\right)^{1.5}}{3(-534.68)} = 20.24$$

$$Q_c = -534.68(930 - 20.25) = -486428$$

$$Q_T = 1041713 + 0 - 150510 - 486428 + 317672 = 722447$$

Calculating the moments of inertia

$$I_1 = 1151.64\left(\frac{33.94^2}{18} + \left[915.86 - \frac{33.94}{3}\right]^2\right) = 702954883$$

$$I_2 = 0$$

$$I_3 = -169\left(\frac{13^2}{18} + 890.59^2\right) = -134044230$$

$$I_c = -\left[\frac{\pi 29^4}{8} + \frac{14.14\sqrt{\left(29^2 - 14.14^2\right)^3}}{2} - \frac{29^2 14.14\sqrt{29^2 - 14.14^2}}{4} - \frac{29^4 \sin^{-1}\left(\frac{14.14}{29}\right)}{4}\right] \ldots$$

$$-(-534.68)20.24^2 + (-534.68)(930 - 20.24)^2$$

$$I_c = -442542378$$

$$I_T = 942352607 + 0 - 134044230 - 442542378 + 295894257 = 661660255$$

$$q_{calc} = \frac{I_T}{Q_T} = \frac{661660255}{722447} = 915.86$$

Therefore, another iteration is not necessary and we can calculate final concrete and bolt stresses.

$$y_{max} = y - c = 33.94 - 13 = 21.94$$

$$f_{cmax} = \frac{Py}{qA_T - Q_T} = \frac{46 \times 21.94}{915.86 \times 789.21 - 722447} = 2.67 \text{ ksi}$$

$$y_{pole} = 915.86 - 930 + 29 = 14.86$$

$$f_{cpole} = \frac{46 \times 14.86}{915.86 \times 789.21 - 722447} = 1.90 \text{ ksi}$$

$$P_{bolt} = \frac{P(e - y_{bolt} - q)}{qA_T - Q_T}(nA_b)$$

$$P_{bolt} = \frac{46(930 - (-32) - 915.86)}{915.86 \times 789.21 - 722447}(9 \times 3.25)$$

$$P_{bolt} = 171.8 \text{ kips (tension)}$$

$$P_{compr} = \frac{46(930 - 32 - 915.86)}{915.86 \times 789.21 - 722447}(8 \times 3.25)$$

$$P_{compr} = -35.96 \text{ kips (compression)}$$

Likewise, for all of the bolts,

Bar #	ybolt	n	Pbolt
1	0	9	**-52.6568**
2	0	9	**-52.6568**
3	32	8	**59.11848**
4	-32	9	**-171.822**
5	4.485	9	**-35.9551**
6	4.485	9	**-35.9551**
7	-4.485	9	**-69.3585**
8	-4.485	9	**-69.3585**
9	31.684	8	**58.07247**
10	31.684	8	**58.07247**
11	-31.684	9	**-170.645**
12	-31.684	9	**-170.645**

The maximum plate stress using the maximum pressure over a 4.5″ strip is then

$$M_{max(c)} = \frac{4.5 \times 1.90(35 - 29)^2}{2} + \frac{4.5 \times (2.67 - 1.90)(35 - 29)^2}{3} + 35.96 \times 3 = 303.4 \text{ in} - \text{kips}$$

$$f_{bmax} = \frac{6 \times 303.4}{4.5 \times 2.25^2} = 79.9 \text{ ksi} \geq .75 \times 36 \times 1.33 = 36 \text{ ksi n.g.}$$

$$M_{max(t)} = 171.8 \times 3 = 515 \text{ in} - \text{kips}$$

$$f_{bmax} = \frac{6 \times 515}{4.5 \times 2.25^2} = 135.6 \text{ ksi} \geq 36 \text{ ksi n.g.}$$

Complete Square Base Plate – Load Parallel

The Process Equipment Method assumes that the bolts can be idealized as a ring of steel and that the compression area can be idealized as a ring with the center of pressure at the center of the ring. Often base plates are encountered that have as few as four bolts. These cases can result in significant error when calculating pressures and bolt forces. Base plates that extend inside the pole will also have a triangular stress distribution that may differ from the assumed ring theory.

Horn[21], developed a method for square base plates with a circular opening which takes into account individual bolt locations. Square plates with clipped corners or polygonal plates can be analyzed using the Complete Circular Base Plate Method discussed in the next section.

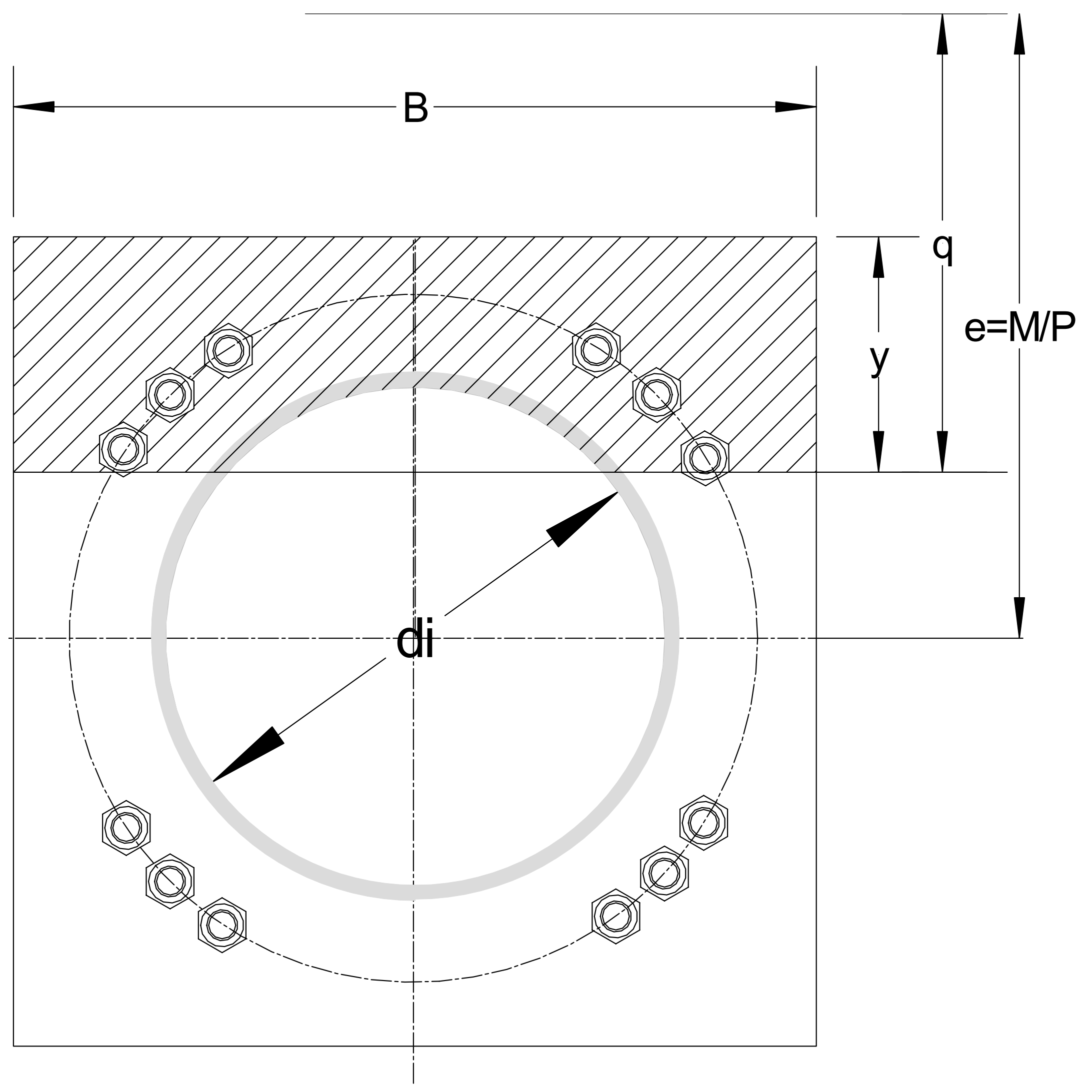


Fig. 2-8

The method is valid when there is net tension on the section, a condition that occurs when

$$e > \frac{\frac{B^4}{12} - \frac{\pi d_i^4}{64}}{\frac{B}{2}\left(B^2 - \frac{\pi d_i^2}{4}\right)}$$

The method will make use of transformed areas of the bolts. Bolts that are in the tension zone will have their areas transformed by

$$A_{bolt} = nA_b$$

Bolts within the compression zone will have their areas transformed by

$$A_{bolt} = (n-1)A_b$$

The method will calculate moment of inertia of the areas and the moment areas about the "e" line. The moment of inertia divided by the moment area will then be the location of the neutral axis. The process iterates on the value of q until it is determined to a sufficient degree of accuracy.

Defining

$$r_i = \frac{d_i}{2}$$
$$y = .5B - e + q$$
$$n = \frac{29000}{57\sqrt{f'_c}}$$
$$f_c = \frac{Py_{max}}{qA_T - Q_T}$$

$$A_T = A_1 + A_c + \sum\nolimits_{comp}(n-1)A_b + \sum\nolimits_{tens} nA_b$$

The area of the rectangle is defined as

$$A_1 = yB$$

Should the neutral axis fall below the top of the circular opening, the area of the semi-circle must be deducted

$$l = r_i - e + q$$
$$y_0 = \max(r_i - l, -r_i)$$
$$A_c = -\frac{r_i^2\pi}{2} + y_0\sqrt{r_i^2 - y_0^2} + r_i^2 \sin^{-1}\left(\frac{y_0}{r_i}\right), l > 0$$

The moment area is defined as

$$Q_T = Q_1 + Q_c + Q_{bolts}$$

$$Q_1 = A_1\left(q - \frac{y}{2}\right)$$

$$y_{1c} = 0, \text{if } l < 0$$

$$= \frac{-2\left(r_i^2 - y_0^2\right)^{1.5}}{3A_c}$$

$$Q_c = A_c\left(e - y_{1c}\right)$$

$$Q_{bolts} = \sum_{j=1}^{j=ntens} nA_b\left(e - y_j\right) + \sum_{j=1}^{j=ncomp} (n-1)A_b\left(e - y_j\right)$$

The moment of inertia of the areas is defined as

$$I_T = I_1 + I_c + I_{bolts}$$

$$I_1 = \frac{By^3}{12} + A_1\left(q - \frac{y}{3}\right)^2$$

$$I_c = -\left[\frac{\pi r_i^4}{8} + \frac{y_0\sqrt{\left(r_i^2 - y_0^2\right)^3}}{2} - \frac{r_i^2 y_0\sqrt{r_i^2 - y_0^2}}{4} - \frac{r_i^4 \sin^{-1}\left(\frac{y_0}{r_i}\right)}{4}\right] - A_c y_{1c}^2 + A_c\left(e - y_{1c}\right)^2$$

$$I_{bolts} = \sum\left[\frac{A_{bolt}^2}{4\pi} + A_{bolt}\left(e - y_j\right)^2\right]$$

$$q = \frac{I_T}{Q_T}$$

Example 2.4

Analyze the bolt forces on a grouted square base plate loaded parallel to the edge and a bolt circle of 64″ and (12) 2-1/4″ bolts spaced at 4.5″. The plate is 68″ square and the inside of the base plate has a diameter of 58″. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips). The concrete strength is f'_c=3000 psi.

$$e = \frac{M}{P} = \frac{42780}{46} = 930\,\text{in}$$

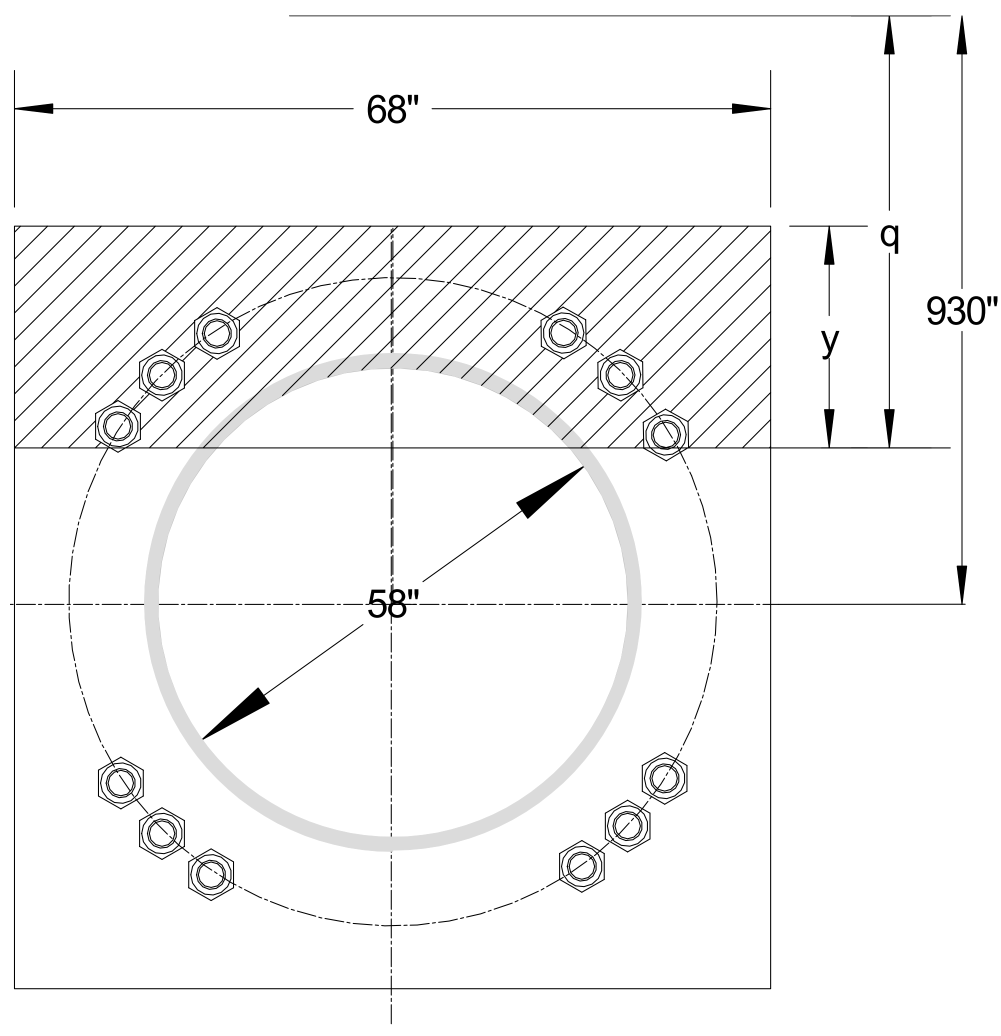


Fig.2-9

The actual process requires iterating on q until a calculated value of q agrees to the fifth decimal place with the value from the previous cycle. The final iteration of q yields q=911.6964.

Calculate the bolt values as follows

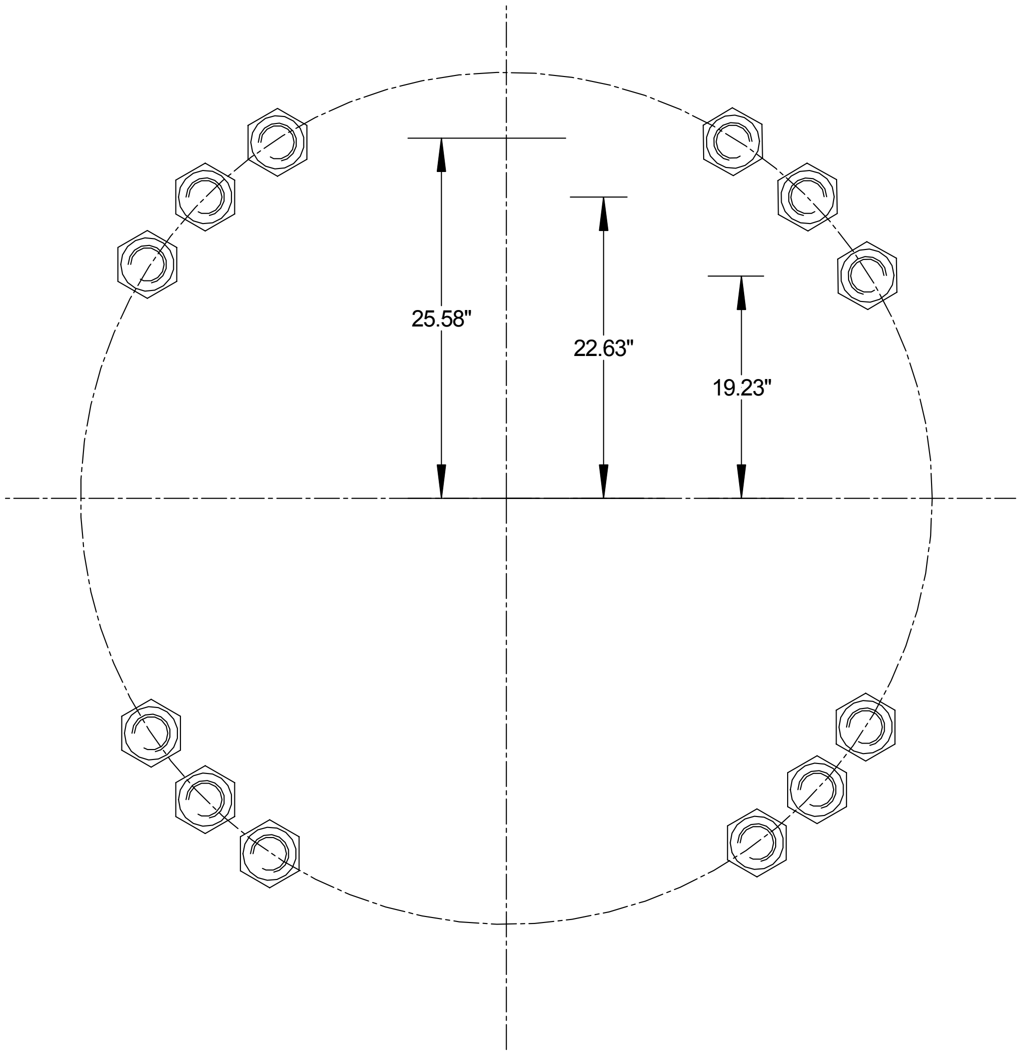


Fig. 2-10

$$\theta = 360\left(\frac{4.5}{64\pi}\right) = 8.0575^\circ$$

$$n = \frac{29000}{57\sqrt{3000}} = 9$$

Bar #	ybolt	n	Abolt	nAbolt	Qbolt	Ibolt
1	25.58	8	3.25	26	23514.92	21267418
2	25.58	8	3.25	26	23514.92	21267418
3	-25.58	9	3.25	29.25	27950.72	26709212
4	-25.58	9	3.25	29.25	27950.72	26709212
5	22.63	8	3.25	26	23591.62	21406382
6	22.63	8	3.25	26	23591.62	21406382
7	-22.63	9	3.25	29.25	27864.43	26544558
8	-22.63	9	3.25	29.25	27864.43	26544558
9	19.23	8	3.25	26	23680.02	21567106
10	19.23	8	3.25	26	23680.02	21567106
11	-19.23	9	3.25	29.25	27764.98	26355418
12	-19.23	9	3.25	29.25	27764.98	26355418
			Totals	331.5	308733	287,700,186

Calculating the plate areas

$$y = .5\times 68 - 930 + 911.6964 = 15.70$$

$$A_1 = 68\times 15.70 = 1067.36$$

Calculating the deduction for the area of the hole

$$r_i = \frac{58}{2} = 29$$

$$l = 29 - 930 + 911.6964 = 10.7$$

$$y_0 = \max\left(r_i - l, -r_i\right) = 18.3$$

$$A_c = -\frac{29^2\pi}{2} + 18.3\sqrt{29^2 - 18.3^2} + 29^2\sin^{-1}\left(\frac{18.3}{29}\right) = -334.88$$

$$A_T = 1067.34 + 331.5 - 334.88 = 1063.98$$

Calculating the moment areas

$$Q_1 = 1067.36\left(911.6964 - \frac{15.70}{2}\right) = 964727$$

$$y_{1c} = \frac{-2\left(29^2 - 18.3^2\right)^{1.5}}{3\left(-334.88\right)} = 22.66$$

$$Q_c = -334.88\left(930 - 22.66\right) = -303848$$

$$Q_T = 962727 - 303848 + 308733 = 969612$$

Calculating the moments of inertia

$$I_1 = \frac{68 \times 15.7^3}{12} + 1067.36\left(911.6964 - \frac{15.7}{2}\right)^2 = 871988746$$

$$I_c = -\left[\frac{\pi 29^4}{8} + \frac{18.3\sqrt{\left(29^2 - 18.3^2\right)^3}}{2} - \frac{29^2 18.3\sqrt{29^2 - 18.3^2}}{4} - \frac{29^4 \sin^{-1}\left(\frac{18.3}{29}\right)}{4}\right]\ldots$$

$$-(-334.88)22.66^2 + (-334.88)(930 - 22.66)^2$$

$$I_c = -275697158$$

$$I_T = 871988756 - 275697158 + 287700186 = 883991774$$

$$q_{calc} = \frac{I_T}{Q_T} = \frac{883991774}{969612} = 911.6964$$

Therefore, another iteration is not necessary and we can calculate final concrete and bolt stresses.

$$f_{cmax} = \frac{Py}{qA_T - Q_T} = \frac{46 \times 15.7}{911.6964 \times 1063.98 - 969612} = 1.75\,\text{ksi}$$

$$P_{bolt} = \frac{P\left(e - y_{bolt} - q\right)}{qA_T - Q_T}\left(nA_b\right)$$

$$P_{bolt} = \frac{46\left(930 - (-25.58) - 911.6964\right)}{911.6964 \times 1063.98 - 969612}(9 \times 3.25)$$

$$P_{bolt} = 143.1\,\text{kips}$$

Likewise, the other bolt forces are

Bar #	ybolt	n	Pbolt
1	25.58	8	-21.08987
2	25.58	8	-21.08987
3	-25.58	9	143.091
4	-25.58	9	143.091
5	22.63	8	-12.53961
6	22.63	8	-12.53961
7	-22.63	9	133.472
8	-22.63	9	133.472
9	19.23	8	-2.685071
10	19.23	8	-2.685071
11	-19.23	9	122.386
12	-19.23	9	122.386

Complete Circular Base Plate Method

The Process Equipment Method assumes that the bolts can be idealized as a ring of steel and that the compression area can be idealized as a ring with the center of pressure at the center of the ring. This assumption was valid for larger diameter pole with a relatively large number of bolts. Often base plates are encountered that have as few as four bolts. These cases can result in significant error when calculating pressures and bolt forces. Base plates that extend inside the pole will also have a triangular stress distribution that may differ from the assumed ring theory.

Horn(21), developed a method for circular or polygonal base plates with a circular opening which takes into account individual bolt locations. Square plates using the Complete Base Plate Method were discussed in the last two sections.

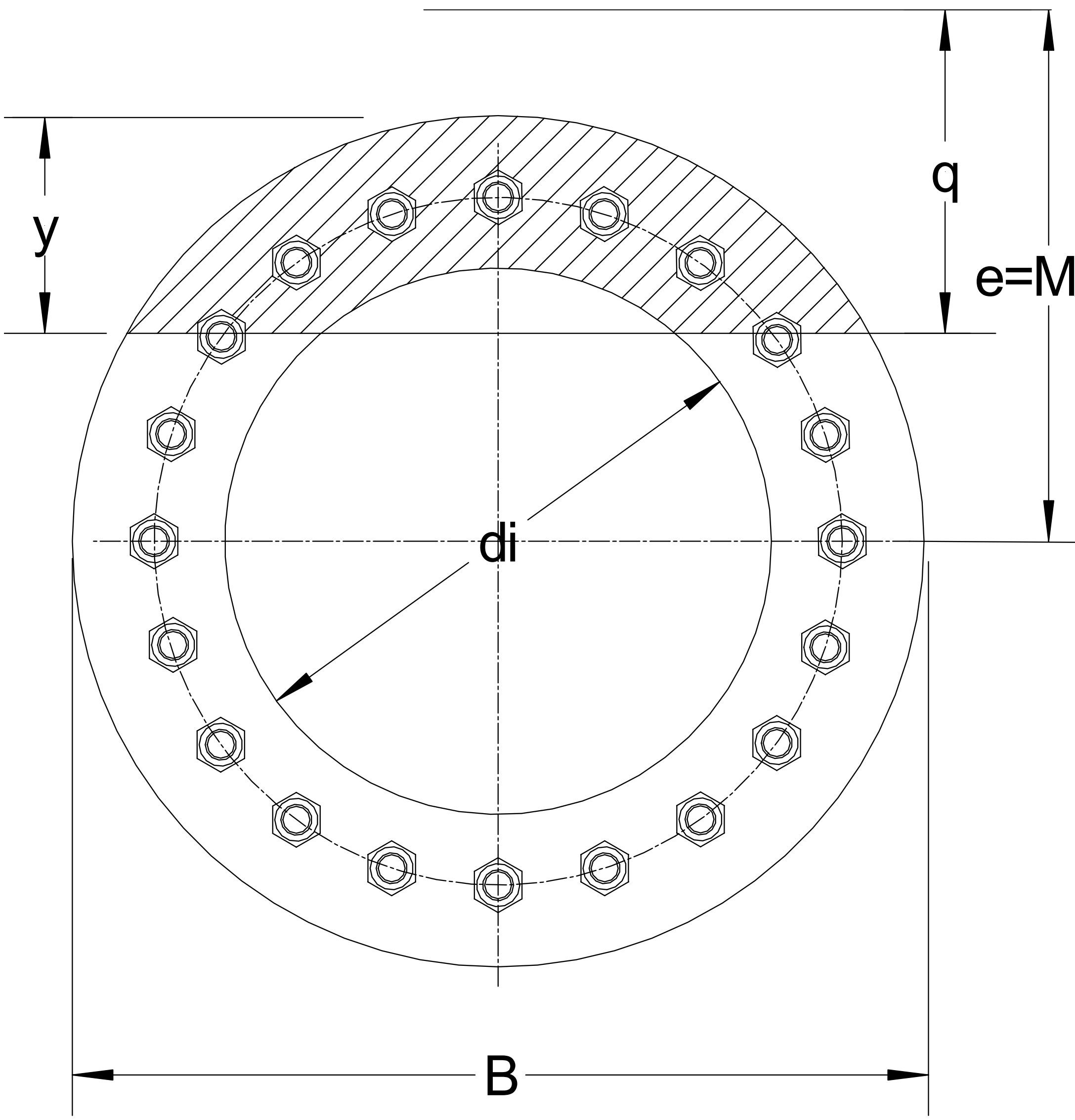


Fig.2-11

The method is valid when there is net tension on the section, a condition that occurs when

$$e > \frac{\left(B^4 - d_i^4\right)}{8B\left(B^2 - d_i^2\right)}$$

The method will make use of transformed areas of the bolts. Bolts that are in the tension zone will have their areas transformed by

$$A_{bolt} = nA_b$$

Bolts within the compression zone will have their areas transformed by

$$A_{bolt} = (n-1)A_b$$

The method will calculate moment of inertia of the areas and the moment areas about the "e" line. The moment of inertia divided by the moment area will then be the location of the neutral axis. The process iterates on the value of q until it is determined to a sufficient degree of accuracy.

Defining

$$r_i = \frac{d_i}{2}$$
$$y = .5B - e + q$$
$$n = \frac{29000}{57\sqrt{f'_c}}$$
$$f_c = \frac{Py_{max}}{qA_T - Q_T}$$

$$A_T = A_1 + A_c + \sum_{comp}(n-1)A_b + \sum_{tens} nA_b$$

The area of the outer circle is defined as

$$r = \frac{B}{2}$$
$$l_0 = r - e + q$$
$$y_0 = r - l_0$$
$$A_1 = \frac{r^2\pi}{2} - y_0\sqrt{r^2 - y_0^2} - r^2 \sin^{-1}\left(\frac{y_0}{r}\right)$$

Should the neutral axis fall below the top of the circular opening, the area of the semi-circle must be deducted

$$l = r_i - e + q$$
$$y_c = \max(r_i - l, -r_i)$$
$$A_c = -\frac{r_i^2\pi}{2} + y_c\sqrt{r_i^2 - y_c^2} + r_i^2 \sin^{-1}\left(\frac{y_c}{r_i}\right), l > 0$$

The moment area is defined as

$$Q_T = Q_1 + Q_c + Q_{bolts}$$
$$y_1 = \frac{2\left(r^2 - y_0^2\right)^{1.5}}{3A_c}$$
$$Q_1 = A_1(e - y_1)$$

$$y_{1c} = 0, \text{ if } l < 0$$

$$= \frac{-2\left(r_i^2 - y_c^2\right)^{1.5}}{3A_c}$$

$$Q_c = A_c\left(e - y_{1c}\right)$$

$$Q_{bolts} = \sum_{j=1}^{j=ntens} nA_b\left(e - y_j\right) + \sum_{j=1}^{j=ncomp}(n-1)A_b\left(e - y_j\right)$$

The moment of inertia of the areas is defined as

$$I_T = I_1 + I_c + I_{bolts}$$

$$I_1 = \left[\frac{\pi r^4}{8} + \frac{y_0\sqrt{\left(r^2 - y_0^2\right)^3}}{2} - \frac{r^2 y_0\sqrt{r^2 - y_0^2}}{4} - \frac{r^2 \sin^{-1}\left(\frac{y_0}{r}\right)}{4}\right] - A_1 y_1^2 + A_1\left(e - y_1\right)^2$$

$$I_c = -\left[\frac{\pi r_i^4}{8} + \frac{y_c\sqrt{\left(r_i^2 - y_c^2\right)^3}}{2} - \frac{r_i^2 y_c\sqrt{r_i^2 - y_c^2}}{4} - \frac{r_i^4 \sin^{-1}\left(\frac{y_c}{r_i}\right)}{4}\right] - A_c y_{1c}^2 + A_c\left(e - y_{1c}\right)^2$$

$$I_{bolts} = \sum\left[\frac{A_{bolt}^2}{4\pi} + A_{bolt}\left(e - y_j\right)^2\right]$$

$$q = \frac{I_T}{Q_T}$$

Example 2.5

Re-analyze Example 2.2 using the Complete Method.

$$e = \frac{M}{P} = \frac{42780}{46} = 930\,\text{in}$$

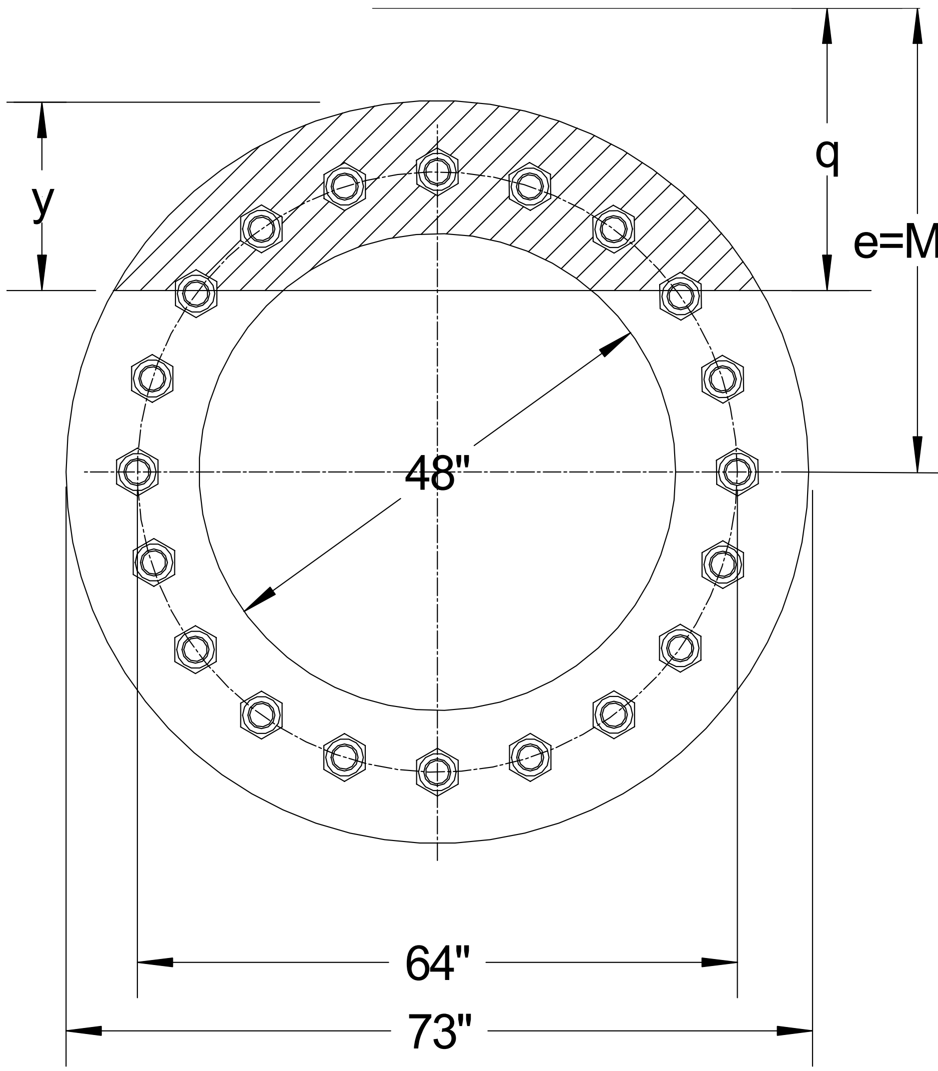


Fig. 2-12

The actual process requires iterating on q until a calculated value of q agrees to the fourth decimal place with the value from the previous cycle. The final iteration of q yields q=916.18446.

$$n = \frac{E_s}{E_c} = \frac{29000}{57\sqrt{3000}} = 9.29$$

Calculate the bolt values as follows

$$\theta = \frac{360}{20} = 18°$$

$$y_{bolt} = r \times \sin(j \times \theta)$$

Bar #	ybolt	n	Abolt	nAbolt	Qbolt	Ibolt
1	0.000	9.29	3.25	30.1925	28079.03	26113565.79
2	9.889	9.29	3.25	30.1925	27780.47	25561196.78
3	18.809	8.29	3.25	26.9425	24549.76	22369575.05
4	25.889	8.29	3.25	26.9425	24359.02	22023329.44
5	30.434	8.29	3.25	26.9425	24236.56	21802449.64
6	32.000	8.29	3.25	26.9425	24194.37	21726597.54
7	30.434	8.29	3.25	26.9425	24236.56	21802449.64
8	25.889	8.29	3.25	26.9425	24359.02	22023329.44
9	18.809	8.29	3.25	26.9425	24549.76	22369575.05
10	9.889	9.29	3.25	30.1925	27780.47	25561196.78
11	0.000	9.29	3.25	30.1925	28079.03	26113565.79
12	-9.889	9.29	3.25	30.1925	28377.58	26671839.45
13	-18.809	9.29	3.25	30.1925	28646.92	27180531.35
14	-25.889	9.29	3.25	30.1925	28860.66	27587651.45
15	-30.434	9.29	3.25	30.1925	28997.90	27850633.93
16	-32.000	9.29	3.25	30.1925	29045.19	27941540.51
17	-30.434	9.29	3.25	30.1925	28997.90	27850633.93
18	-25.889	9.29	3.25	30.1925	28860.66	27587651.45
19	-18.809	9.29	3.25	30.1925	28646.92	27180531.35
20	-9.889	9.29	3.25	30.1925	28377.58	26671839.45
			Totals	581.1	541015.35	503989684

The area of the outer circle is calculated as

$$r = \frac{73}{2} = 36.5$$

$$y = l_0 = 36.5 - 930 + 916.18446 = 22.68$$

$$y_0 = 36.5 - 22.68 = 13.82$$

$$A_1 = \frac{36.5^2 \pi}{2} - 13.82\sqrt{36.5^2 - 13.82^2} - 36.5^2 \sin^{-1}\left(\frac{13.82}{36.5}\right) = 1108.79$$

The inner circle is next calculated

$$l = 24 - 930 + 916.18446 = 10.18$$

$$y_c = \max(24 - 10.18, -24) = 13.82$$

$$A_c = -\frac{24^2 \pi}{2} + 13.82\sqrt{24^2 - 13.82^2} + 24^2 \sin^{-1}\left(\frac{13.82}{24}\right) = -280.34$$

$$A_T = 1108.79 + 581.1 - 280.34 = 1409.55$$

Calculating the moment areas

$$y_1 = \frac{2\left(36.5^2 - 13.82^2\right)^{1.5}}{3(1108.79)} = 23.18$$

$$Q_1 = 1108.79(930 - 23.18) = 1005464$$

$$y_c = \frac{-2\left(24^2 - 13.82^2\right)^{1.5}}{3(-280.34)} = 17.97$$

$$Q_c = -280.34(930 - 17.97) = -255674$$

$$Q_T = 1005464 + 541015 - 255674 = 1290806$$

Calculating the moments of inertia

$$I_1 = \left[\frac{\pi 36.5^4}{8} + \frac{13.82\sqrt{\left(36.5^2 - 13.82^2\right)^3}}{2} - \frac{36.5^2 13.82\sqrt{36.5^2 - 13.82^2}}{4} - \frac{36.5^4 \sin^{-1}\left(\frac{13.82}{36.5}\right)}{4}\right]\ldots$$

$$-(1108.79)23.18^2 + (1108.79)(930 - 23.18)^2$$

$$I_1 = 911810020$$

$$I_c = -\left[\frac{\pi 24^4}{8} + \frac{13.82\sqrt{\left(24^2 - 13.82^2\right)^3}}{2} - \frac{24^2 13.82\sqrt{24^2 - 13.82^2}}{4} - \frac{24^4 \sin^{-1}\left(\frac{13.82}{24}\right)}{4}\right]\ldots$$

$$-(-280.34)17.97^2 + (-280.34)(930 - 17.97)^2$$

$$I_c = -233183299$$

$$I_T = 911810020 + 503898968 - 233183299 = 1182616405$$

$$q_{calc} = \frac{I_T}{Q_T} = \frac{1182616405}{1290806} = 916.18446$$

Therefore, another iteration is not necessary and we can calculate final concrete and bolt stresses.

$$f_{cmax} = \frac{Py}{qA_T - Q_T} = \frac{46 \times 22.68}{916.18446 \times 1409.55 - 1290806} = 1.73\,\text{ksi}$$

$$y_{bc} = \frac{64}{2} - 930 + 916.18446 = 18.18$$

$$f_{bc} = \frac{Py}{qA_t - Q_t} = \frac{46 \times 18.18}{916.18446 \times 1409.55 - 1290806} = 1.39$$

$$P_{bolt} = \frac{P\left(e - y_{bolt} - q\right)}{qA_T - Q_T}\left(nA_b\right)$$

$$P_{bolt} = \frac{46\left(930 - \left(-32\right) - 916.18446\right)}{916.18446 \times 1409.55 - 1290806}\left(9 \times 3.25\right)$$

$$P_{bolt} = -105.6\,\text{kips}$$

The values from the Lutz Modification Example 2.2 compare as follows: 1.70 ksi vs. 1.73 ksi, 1.349 ksi vs. 1.39 ksi, and 110.4 kips vs. 105.6 kips.

Likewise, the other bolt forces may be calculated

Bar #	ybolt	n	Pbolt
1	0.000	9.29	**-31.85**
2	9.889	9.29	**-9.05**
3	18.809	8.29	**10.27**
4	25.889	8.29	**24.84**
5	30.434	8.29	**34.19**
6	32.000	8.29	**37.41**
7	30.434	8.29	**34.19**
8	25.889	8.29	**24.84**
9	18.809	8.29	**10.27**
10	9.889	9.29	**-9.05**
11	0.000	9.29	**-31.85**
12	-9.889	9.29	**-54.65**
13	-18.809	9.29	**-75.22**
14	-25.889	9.29	**-91.54**
15	-30.434	9.29	**-102.02**
16	-32.000	9.29	**-105.63**
17	-30.434	9.29	**-102.02**
18	-25.889	9.29	**-91.54**
19	-18.809	9.29	**-75.22**
20	-9.889	9.29	**-54.65**

The maximum plate stress with a pole diameter of 58″, using 50 ksi grade plate, is then

$$b_{eff} = \frac{\pi \times 58}{20} = 9.11$$

$$y = \frac{58}{2} - 930 + 916.18446 = 15.18$$

$$f_{pole} = \frac{46 \times 15.18}{916.18446 \times 1409.55 - 1290806} = 1.16$$

$$M_{max} = \frac{9.11 \times 1.16 \left(\frac{73-58}{2}\right)^2}{2} + \frac{9.11 \times (1.73 - 1.16)\left(\frac{73-58}{2}\right)^2}{3} + \ldots$$

$$37.41 \times \frac{64-58}{2} = 506.8 \text{ in} - \text{kips}$$

$$f_{max} = \frac{6 \times 506.8}{9.11 \times 2.25^2} = 65.93 \text{ ksi} \geq .75 \times 50 \times 1.33 = 50 \text{ ksi n.g.}$$

$$M_{max} = 105.6 \times \frac{64-58}{2} = 316.8 \text{ in} - \text{kips}$$

$$f_{max} = \frac{6 \times 316.8}{9.11 \times 2.25^2} = 41.2 \text{ ksi} \leq 50 \text{ ksi o.k.}$$

Ungrouted Base Plates

Determining Bolt Forces

Most monopoles built today do not have grouted base plates. Grouting of the plate can lead to corrosion problems if means are not provided to allow for drainage of condensation that can develop within the pole. Once grouted, the pole can no longer be adjusted for any out-of-plumb condition since the leveling nuts will be encased within the grout. Some manufacturers have specific warranty disclaimers if the pole is grouted.

The determination of bolt forces in ungrouted base plates is straightforward. The bolt group should be symmetrical about both axes and the bolts should be the same size. Square base plates usually have the bolt clustered in four groups located along the diagonals of the plate.

The maximum bolt force is determined by

$$F_{max} = \frac{P}{n} + \frac{My_{max}}{I}$$

Where n is the number of bolts and y_{max} and I are calculated as illustrated in the following examples.

Example 3.1

Analyze the bolt forces on a polygonal base plate with a bolt circle of 64″ and (20) 2-1/4″ bolts. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips).

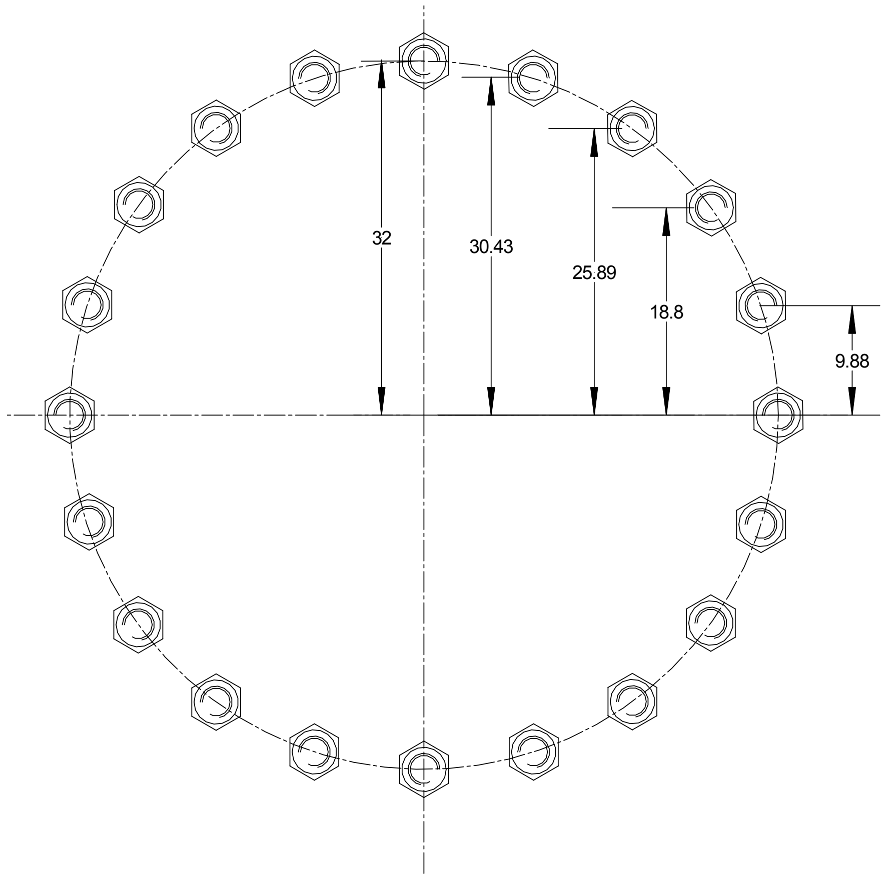


Fig 3- 1

$$n = 20$$

$$y_{max} = 32$$

$$I = \sum_{1}^{20} y^2 = \sum \left(r\sin(\theta)\right)^2$$

$$\theta = \frac{360}{20} = 18^\circ$$

$$I = 2\times 32^2 + 2\times 0^2 + 4\times\left(32\sin(18)\right)^2 + 4\times\left(32\sin(36)\right)^2 +$$

$$4\times\left(32\sin(54)\right)^2 + 4\times\left(32\sin(72)\right)^2$$

$$I = 10240$$

$$F_{max} = \frac{46}{20} \pm \frac{42780\times 32}{10240} = +135.99\,\text{kips}, -131.39\,\text{kips}$$

Example 3.2

Analyze the bolt forces on a square base plate loaded on the diagonal and a bolt circle of 64″ and (12) 2-1/4″ bolts spaced at 4.5″. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips).

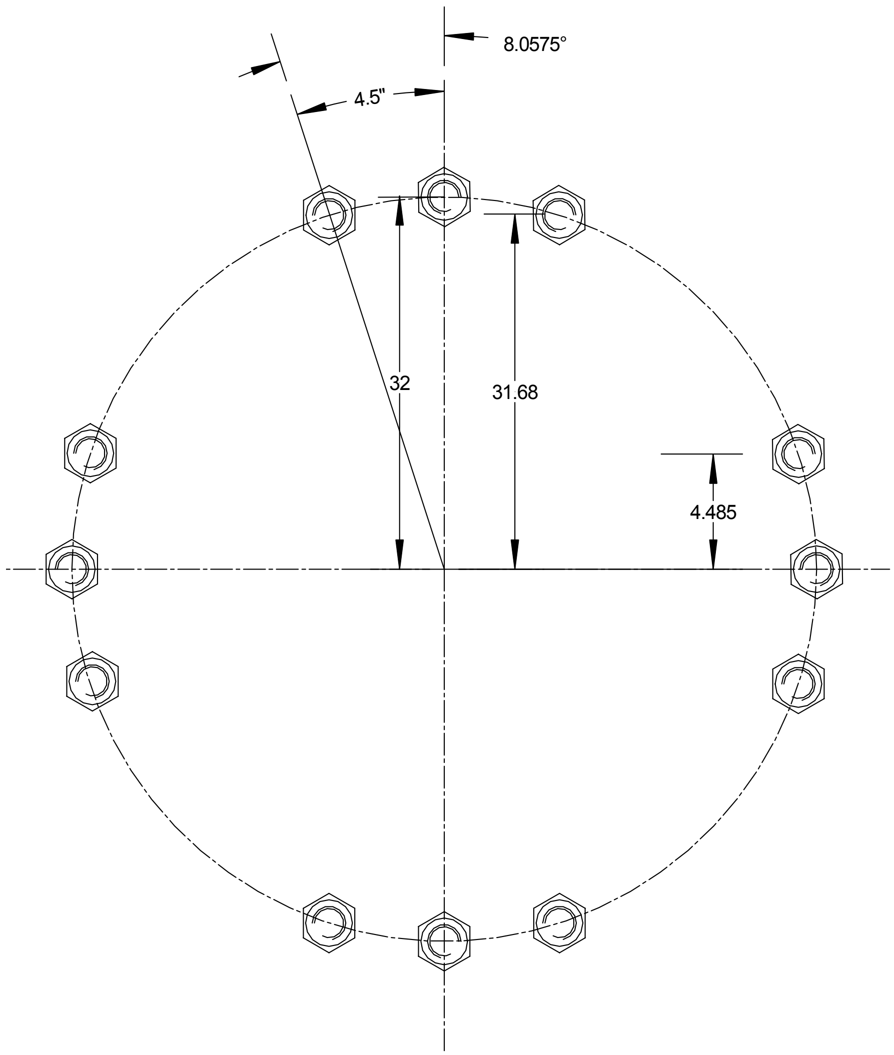


Fig. 3- 2

$$\theta = 360\left(\frac{4.5}{64\pi}\right) = 8.0575°$$

$$n = 12$$

$$y_{max} = 32$$

$$I = \sum_{1}^{12} y^2 = \sum \left(r \sin(\theta)\right)^2$$

$$I = 2 \times 32^2 + 2 \times 0^2 + 4 \times \left(32 \sin(8.0575)\right)^2 + 4 \times \left(32 \sin(81.9425)\right)^2$$

$$I = 6144$$

$$F_{max} = \frac{46}{12} \pm \frac{42780 \times 32}{6144} = +226.6\,\text{kips}, -219.0\,\text{kips}$$

Example 3.3

Analyze the bolt forces on a square base plate loaded parallel to the edge and a bolt circle of 64″ and (12) 2-1/4″ bolts spaced at 4.5″. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips).

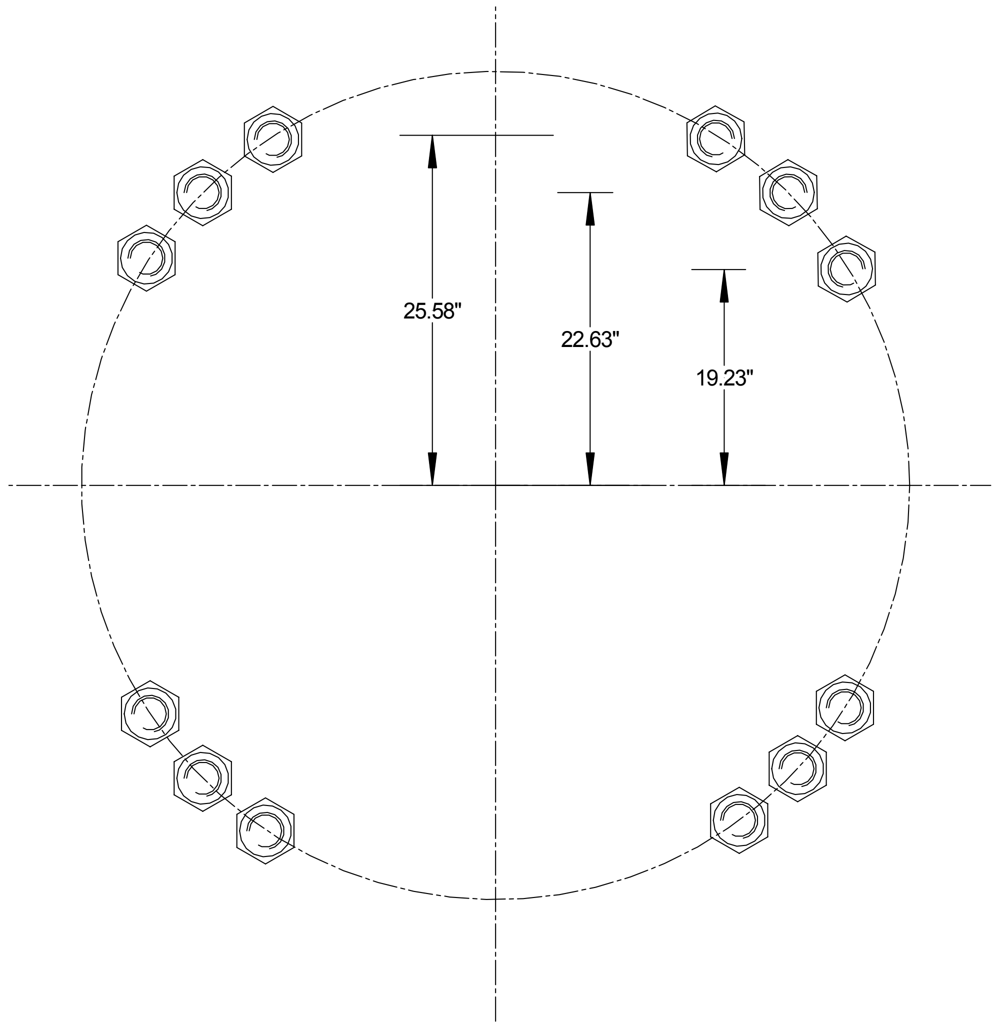


Fig. 3- 3

$$\theta = 360\left(\frac{4.5}{64\pi}\right) = 8.0575°$$

$$n = 12$$

$$y_{max} = 25.58$$

$$I = \sum_{1}^{12} y^2 = \sum \left(r \sin(\theta) \right)^2$$

$$I = 4 \times \left(32 \sin(45 - 8.0575) \right)^2 + 4 \times \left(32 \sin(45) \right)^2 + 4 \times \left(32 \sin(45 + 8.0575) \right)^2$$

$$I = 6144$$

$$F_{max} = \frac{46}{12} \pm \frac{42780 \times 25.58}{6144} = +182\,\text{kips}, -174.3\,\text{kips}$$

As can be seen from these two examples, the maximum bolt force is always determined from the loading along the diagonal for a square base plate.

Determining Bolt Shear

Shearing stresses in the pole are a maximum at the equator of the pole. A flexible base plate will therefore have shearing forces distributed primarily to the two side regions at the equator. The shear resisted by the bolts will then be

$$F_v = \frac{2V}{n}$$

When the base plate is solid (without a central hole), the plate would be sufficiently rigid to equally distribute the shear to all of the bolts.

$$F_v = \frac{V}{n}$$

Bending Stresses In Bolts

Bolts in ungrouted base plates may be subjected to bending stresses when the clear distance below the leveling nut is excessive. The TIA Standard has no requirement for this condition. However, AASHTO requires bending to be considered whenever the clear distance is greater than one bolt diameter. The ASCE Manual 72 recommends that bending be considered whenever the distance is greater than two bolt diameters. The bolt is considered to be bent in reverse curvature. The bending stress would then be

$$f_b = \frac{16cF_v}{\pi d_b^3}$$

where c is the clear distance (see Fig. 3-4). When threads extend well into the clear space, the root diameter of the threaded portion should be used for d_b.

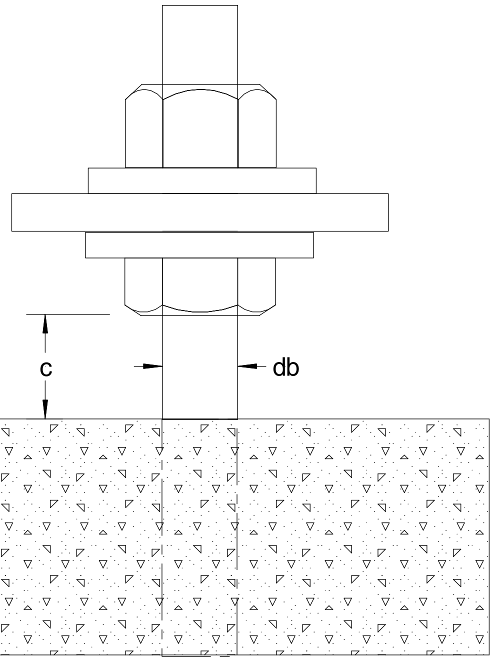


Fig 3- 4

Determining Plate Stresses

Introduction

The methods of determining bolt forces and concrete stresses presented in the previous chapters are commonly used by all designers. The methods for determining the base plate bending stresses, however, are not so uniformly accepted.

Effective Width Based Upon Plate Theory

Traditionally, most methods have used the peak bolt force or maximum concrete stress in determining a cantilever moment on an effective width of plate at the pole to determine the plate stress by simple beam theory. The moment would be equal to the

$$M = F \times d$$

where F is the force and d is the distance from the force to the face of the pole. The stress in the plate would then be

$$f = \frac{6M}{B_{eff} t^2}$$

To determine if this is realistic, consider a circular ring, guided at the pole but free to displace vertically, and simply supported continuously at the anchor bolt circle. Roarke[25] Table 24.1b states that the maximum moment in the ring will be

$$M = wa\frac{L_9}{C_8}$$

$$C_8 = \frac{1}{2}\left[1 + \upsilon + (1 - \upsilon)\left(\frac{b}{a}\right)^2\right]$$

$$L_9 = \frac{r_o}{a}\left\{\frac{1 + \upsilon}{2}\ln\frac{a}{r_o} + \frac{1 - \upsilon}{4}\left[1 - \left(\frac{r_o}{a}\right)^2\right]\right\}$$

where “a” is the radius of the outside of the ring and “b” is the radius of the inside of the ring (outside radius of the pole). Poisson’s ratio can be taken as 0.30 for steel. The unit load, “w” is taken as one and the position of the load, "r_o", is taken as "b" to simulate the load coming from the pole. Note that this condition is for an axial load without moment, but it will give us some indication for the correct value of effective width.

Monopole Examples			
a	24	32	36
b	15	27.5	30
b/a	.625	.86	.833
C_8	.787	.908	.893
L_9	.2576	.124	.143
M	7.85	11.94	5.77
d=a-b	9	4.367	6
B_{eff}=d/M	1.15	1.03	1.04

Although this may be a crude comparison, it leads us to conclude that the classical method of using a circumferential strip for determining plate stress should be reasonably accurate for plates that have a uniform spacing of anchor bolts in a circular pattern.

Process Equipment Methods

A base plate without stiffeners is assumed to behave as a cantilever beam. The allowable bending stress (AISC) would be limited to $.75F_y$, while industrial engineers designing stacks usually limit the stress to $.6F_y$.

The maximum bending moment in grouted base plates on the compression side is calculated per unit circumferential length (b=1 in) in the base plate as

$$d = \frac{OD - D}{2}$$

$$M_{press} = \frac{f_c d^2}{2}$$

where we use f_c as the average uniform pressure on the cantilevered plate, OC is the outside diameter of the plate and D is the pole diameter across the flats.

The maximum bending stress is therefore

$$f_b = \frac{6M_{press}}{t^2} = \frac{3f_c d^2}{t^2}$$

where t is the thickness of the plate.

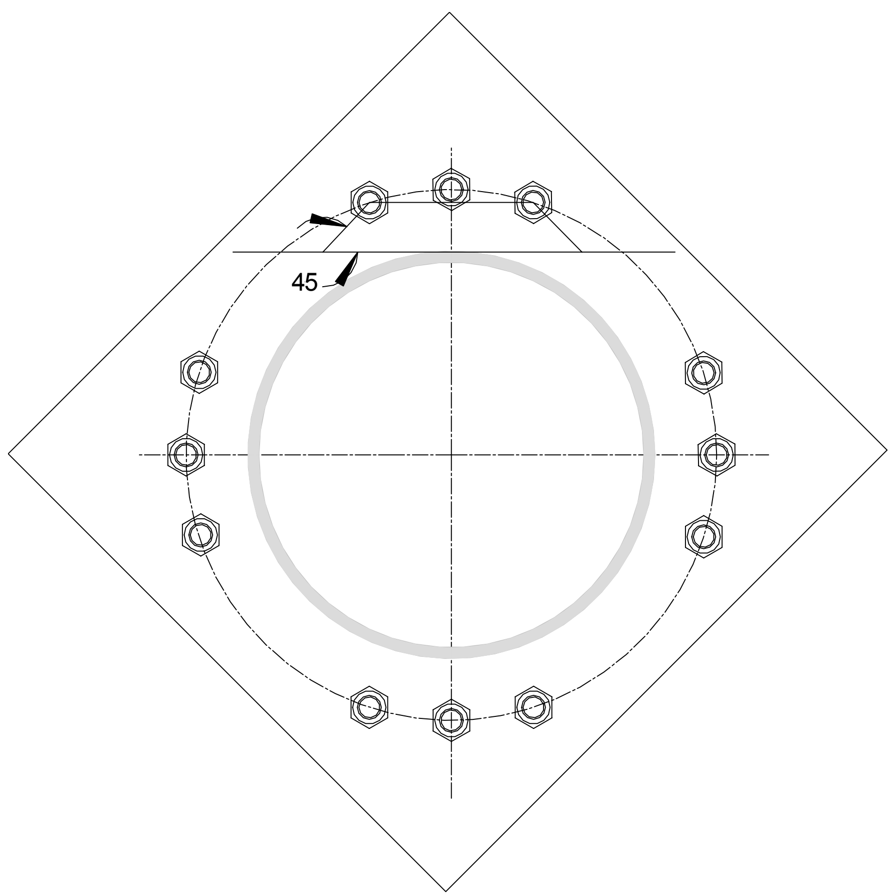
When using any of the Complete Methods, the moment would be a combination of the concrete pressure and the contribution of the bolts in compression. Both the compression and tension sides of the base plate should be checked.

The unit maximum bending moment in all base plates on the tension side and ungrouted base plates on the compression side is

$$d = \frac{BC - D}{2}$$

$$M_{bolt} = \frac{Tdn}{\pi D}$$

where we use T is the maximum bolt force on the cantilevered plate, BC is the bolt circle diameter, D is the pole diameter across the flats, and n is the number of bolts. Sometimes a 45 degree angle limitation is used[13] when there are a limited number of bolts (bolt spacing is greater than twice the distance to the pole).



Example 4.1

Analyze the bolt forces on an ungrouted square base plate loaded on the diagonal and a bolt circle of 64″ and (12) 2-1/4″ bolts spaced at 4.5″. The plate is 68″ square and the inside of the base plate has a diameter of 58″. The thickness of the pole is .500″ and the plate is 2.25″ thick. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips).

$$e = \frac{M}{P} = \frac{42780}{46} = 930\,\text{in}$$

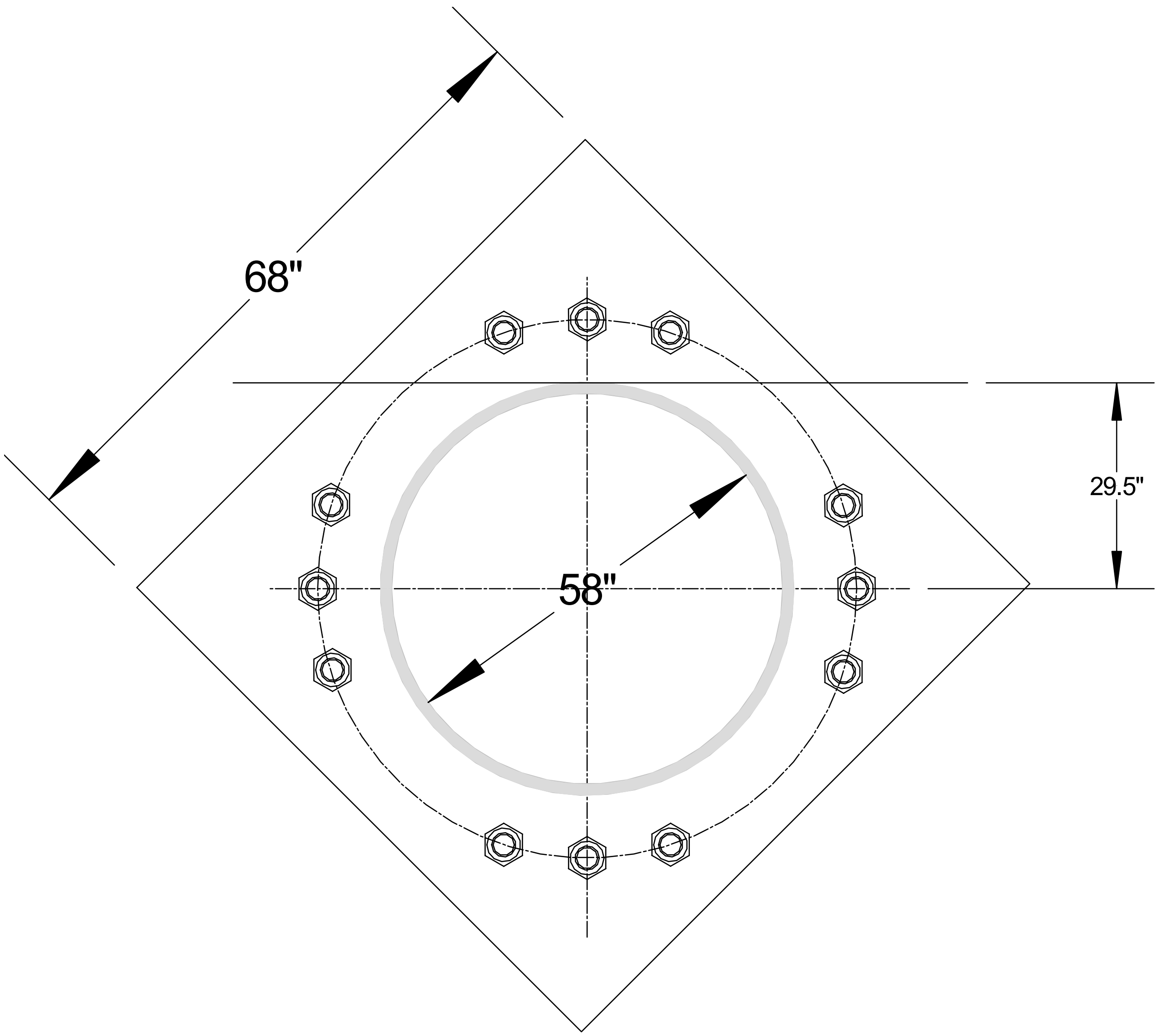


Fig 4- 1

Calculate the bolt values as follows

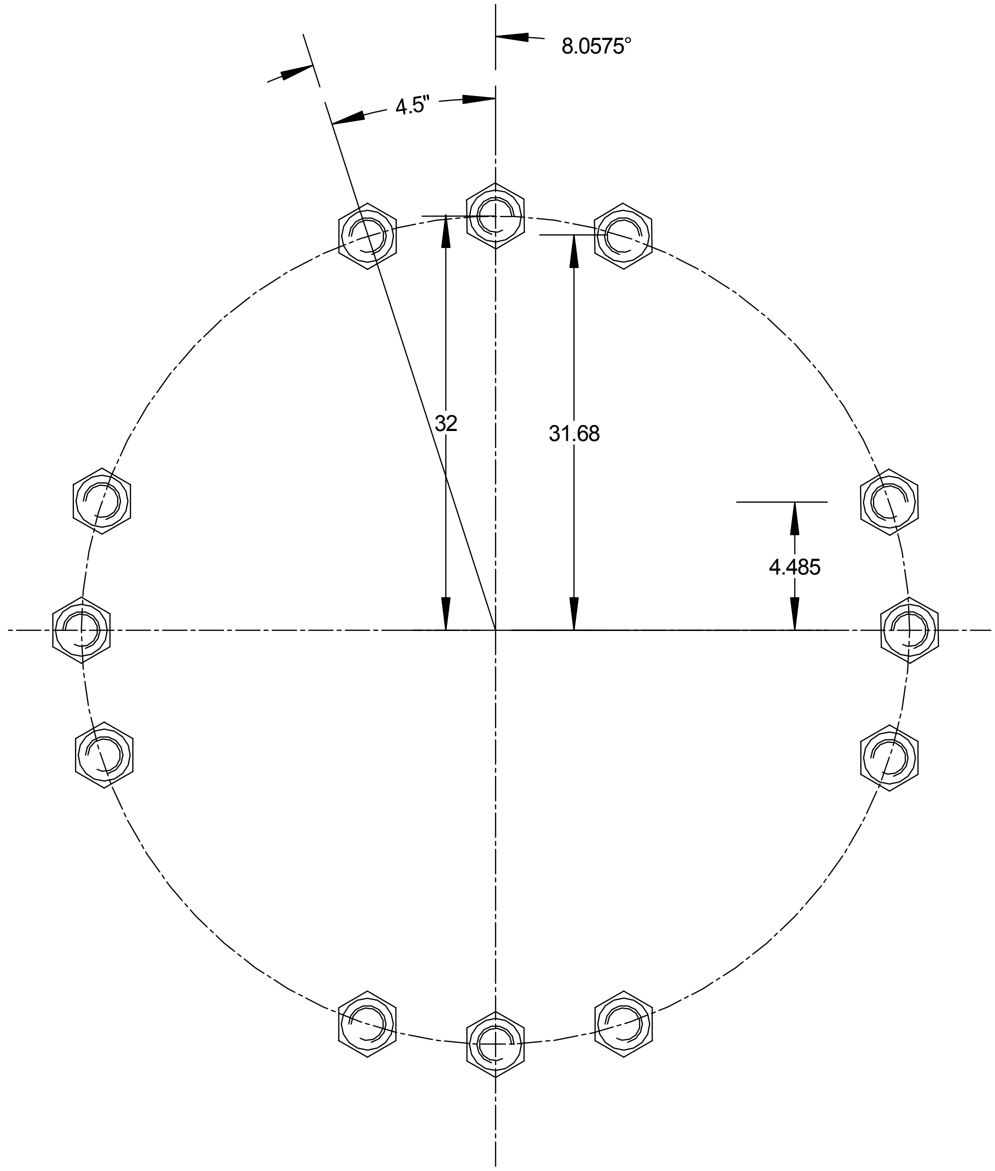


Fig. 4-2

$$\theta = 360\left(\frac{4.5}{64\pi}\right) = 8.0575^\circ$$

$$n = 12$$

$$y_{max} = 32$$

$$I = \sum_{1}^{12} y^2 = \sum \left(r \sin(\theta)\right)^2$$

$$I = 2 \times 32^2 + 2 \times 0^2 + 4 \times \left(32 \sin(8.0575)\right)^2 + 4 \times \left(32 \sin(81.9425)\right)^2$$

$$I = 6144$$

$$P_{max} = \frac{46}{12} \pm \frac{42780 \times 32}{6144} = +226.6\,\text{kips}, -219.0\,\text{kips}$$

$$y_1 = 32$$

$$P_1 = 226.6\,\text{kips}$$

$$y_2 = 31.68$$

$$P_2 = \left(\frac{46}{12} + \frac{42780 \times 31.68}{6144}\right) = 224.4\,\text{kips}$$

$$M = 226.6(32 - 29.5) + 2(224.4)(31.68 - 29.5) = 1544.88$$

If the effective width is limited to a 45 degree angle from the bolts (AISC approach),

$$b_{eff} = 2 \times 4.485 + 2(31.68 - 29.5) = 13.33$$

$$f_b = \frac{6 \times 1544.88}{13.33 \times 2.25^2} = 137.4\,\text{ksi}$$

Base Plates With Gussets

When gussets are used to stiffen the base plate, the loading condition on the section of the plate between the two gussets may be considered to act like a rectangular uniformly loaded plate with two opposite edges simply supported by the gussets, the third edge joined to the pole, and the fourth and outer edge free. The following table, taken from Timoshenko(26), tabulates the bending moments for this case.

ℓ/b	$M_x\begin{pmatrix} x = b/2 \\ y = \ell \end{pmatrix}$	$M_y\begin{pmatrix} x = b/2 \\ y = 0 \end{pmatrix}$
0	0	$-0.500 f_{cl} l^2$
.333	$0.0078 f_c b^2$	$-0.428 f_c l^2$
.500	$0.0293 f_c b^2$	$-0.319 f_c l^2$
.666	$0.0558 f_c b^2$	$-0.227 f_c l^2$
1	$0.0972 f_c b^2$	$-0.119 f_c l^2$
1.5	$0.123 f_c b^2$	$-0.124 f_c b^2$
2	$0.131 f_c b^2$	$-0.125 f_c b^2$
3	$0.133 f_c b^2$	$-0.125 f_c b^2$
∞	$0.133 f_c b^2$	$-0.125 f_c b^2$

In this table, b=gusset spacing (x direction) and l=base plate outside radius minus the pole radius (y-direction).

Note that when l/b=0 (no gussets or gusset spacing is very large), that the equations reduces back to the cantilever equation. Also, note that when l/b is equal to or less than 1.5, the maximum bending moment occurs at the junction with the pole because of cantilever action. If l/b is greater then 1.5, the maximum bending moment occurs at the middle of the free edge. The stress in the plate is

$$f_b = \frac{6M_{max}}{t_{bp}^2}$$

The gusset is usually designed to resist the entire shear force as a cantilever beam. Most process design texts design the gusset by ignoring any contribution of the base plate itself; the gusset acts as a vertical plate only. Some designers will assume a tee-beam with a portion of the base plate acting along with the vertical plate gusset.

The gusset will impart a bending stress on the wall of the pole, which can be estimated from the work of Bijilaard(22).

$$f_{b\,pole\,wall} = \frac{M_{gusset}}{t^2}\left[\frac{1.32Z}{\left(\frac{1.43ah^2}{Rt}\right)+\left(4ah^2\right)^{1/3}} + \frac{.031}{\sqrt{Rt}}\right]$$

where

$$Z = \frac{1.0}{\left(\frac{.177am}{\sqrt{Rt}}\right)\left(\frac{m}{t}\right)^2 + 1.0}$$

$$t = \text{thickness of pole wall}$$

$$a = 2\left(t_{gusset} + t\right)$$

$$R = \text{radius of pole wall}$$

$$h = \text{height of gusset}$$

$$m = \text{thickness of base plate}$$

Alternate Method 1

This method is identical to the Process Equipment Method with the following modifications.

1. D is taken to be the average diameter of the polygonal pole $D = \frac{D_{flat} + D_{tip}}{2}$.

2. d is taken from the edge of the bolt rather than from the bolt circle $d = \frac{BC - D - d_b}{2}$.

Alternate Method 2

This method is identical to the Process Equipment Method with following modifications.

1. D is taken to be the average diameter of the polygonal pole $D = \frac{D_{flat} + D_{tip}}{2}$.

2. d is taken from the edge of the nut to the fillet weld $d = \frac{BC - D - d_{nut}}{2} - t_w$.

Alternate Method 3

This method is identical to the Process Equipment Method with following modifications.

1. D is taken to be the average diameter of the polygonal pole $D = \frac{D_{flat} + D_{tip}}{2}$.

2. d is taken from the edge of the nut to the fillet weld $d = \frac{BC - D - d_{nut}}{2} - t_w$.

3. The plate is considered to be bent in double curvature such that the moment is one-half that of a cantilevered plate. This assumes that the bolt is considered capable of resisting such a moment. An equivalent procedure would be to adjust the bolt circle, BC, to a point midway between the bolt and the average diameter of the pole.

Method of Slices

The method evolved from the AISC method (see Introduction), which takes a slicing plane across the entire width of the base plate. The method ignores the tension side distribution shown in Fig 1-3 and assumes that the effective width resisting bending moment, b_{eff}, goes across the entire plate as shown in Fig. 4-3. Should a slice occur near a bolt hole, the width of the hole(s) is deducted from the effective plate width.

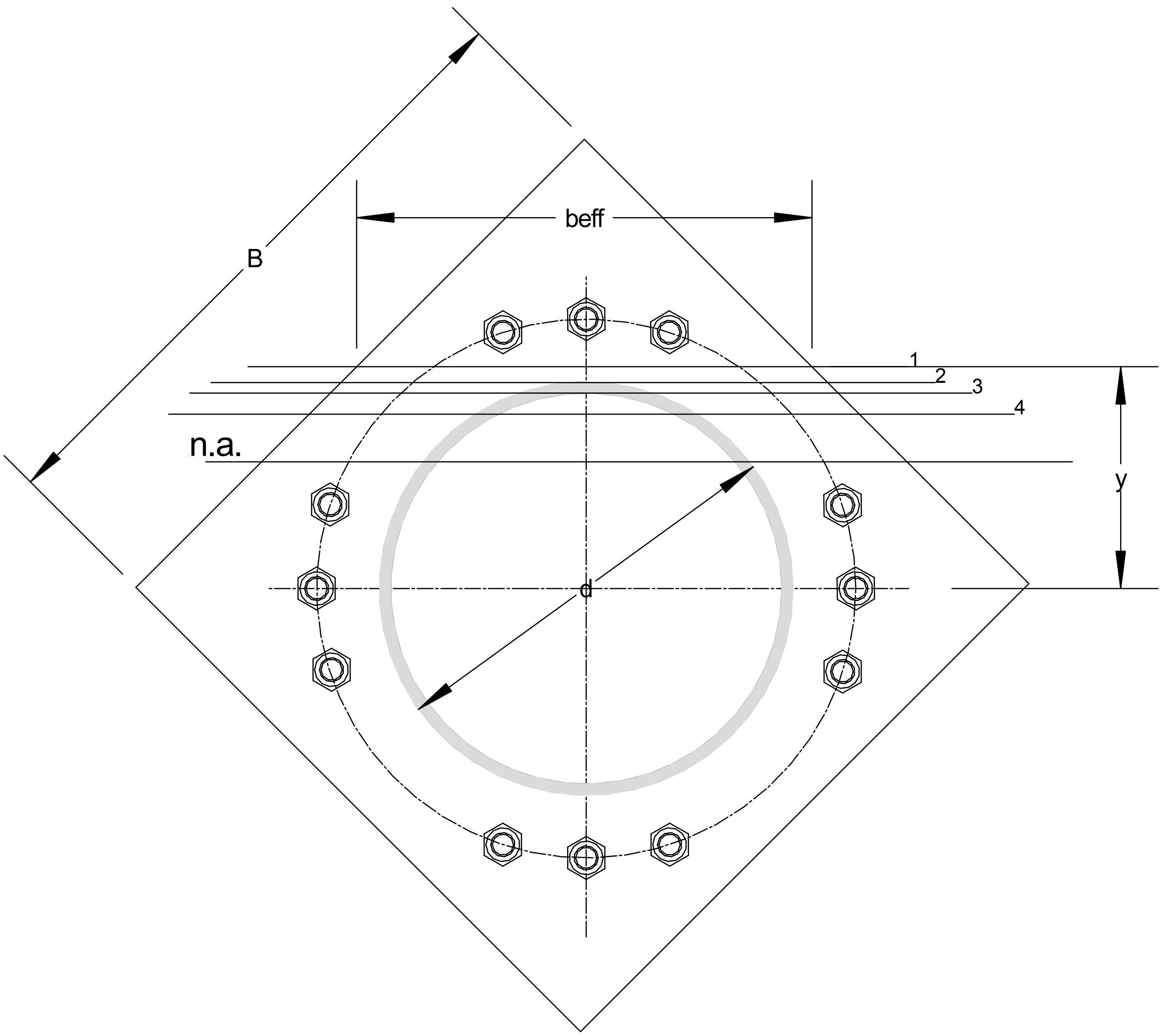


Fig 4- 3

The method requires that a number of slicing planes be taken between the bolt circle and the neutral axis. The net moment at any slice is then the sum of the moments due to the forces outside of the cutting plane minus the moment due to stresses in the pole when the slice occurs within the diameter of the pole. Most designers who use this method usually only take a slice at the face of the monopole, a point that may or may not be the most critical slice.

At any location from the center of a square plate for loading on the diameter, y

$$b_{eff}\Big|_{y=d/2}^{y=.707B} = 1.414B - 2y$$

$$b_{eff}\Big|_{y=n.a.}^{y=d/2} = 1.414B - 2y - 2\sqrt{\left(\frac{d}{2} - y\right)d - \left(\frac{d}{2} - y\right)^2}$$

$$M = \sum P_{bolt} \times (y_{bolt} - y) + \int f_i \times (y - y_i), y < y_i$$

$$f_b = \frac{6M}{b_{eff} t^2}$$

Parallel loading usually does not control in this method since the bolts are usually below the y location of the face of the pole.

At any location from the center of a square plate for loading parallel, y

$$b_{eff}\Big|_{y=d/2}^{y=B/2} = B$$

$$b_{eff}\Big|_{y=n.a.}^{y=d/2} = B - 2\sqrt{\left(\frac{d}{2} - y\right)d - \left(\frac{d}{2} - y\right)^2}$$

$$M = \sum P_{bolt} \times (y_{bolt} - y) + \int f_i \times (y - y_i), y < y_i$$

$$f_b = \frac{6M}{b_{eff} t^2}$$

At any location from the center of a polygonal or circular plate, y

$$b_{eff}\Big|_{y=d/2}^{y=B/2} = 2\sqrt{\left(\frac{B}{2} - y\right)B - \left(\frac{B}{2} - y\right)^2}$$

$$b_{eff}\Big|_{y=n.a.}^{y=d/2} = 2\sqrt{\left(\frac{B}{2} - y\right)B - \left(\frac{B}{2} - y\right)^2} - 2\sqrt{\left(\frac{d}{2} - y\right)d - \left(\frac{d}{2} - y\right)^2}$$

$$M = \sum P_{bolt} \times (y_{bolt} - y) + \int f_i \times (y - y_i), y < y_i$$

$$f_b = \frac{6M}{b_{eff} t^2}$$

The moment due to the pole shell stresses be calculated as follows

$$r = \frac{d}{2}$$

$$\alpha = \cos^{-1}\left(\frac{y}{r}\right)$$

$$M_{pole} = \frac{2Ptr^2}{A}\int_0^{\alpha}(\cos\theta - \cos\alpha)d\theta + \frac{2Mtr^2}{S}\int_0^{\alpha}\frac{(\cos\theta - \cos\alpha)^2}{1-\cos\alpha}d\theta$$

$$= \frac{2Ptr^2}{A}(\sin\alpha - \alpha\cos\alpha) + \frac{2Mtr^2}{S}\left[\frac{\alpha\cos^2\alpha - \frac{3}{2}(\sin\alpha\cos\alpha) + \frac{1}{2}\alpha}{\sin\alpha - \alpha\cos\alpha}\right]$$

Example 4.2

Analyze the bolt forces on an ungrouted square base plate loaded on the diagonal and a bolt circle of 64″ and (12) 2-1/4″ bolts spaced at 4.5″. The plate is 68″ square and the inside of the base plate has a diameter of 58″. The thickness of the pole is .500″ and the plate is 2.25″ thick. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips).

$$e = \frac{M}{P} = \frac{42780}{46} = 930\,\text{in}$$

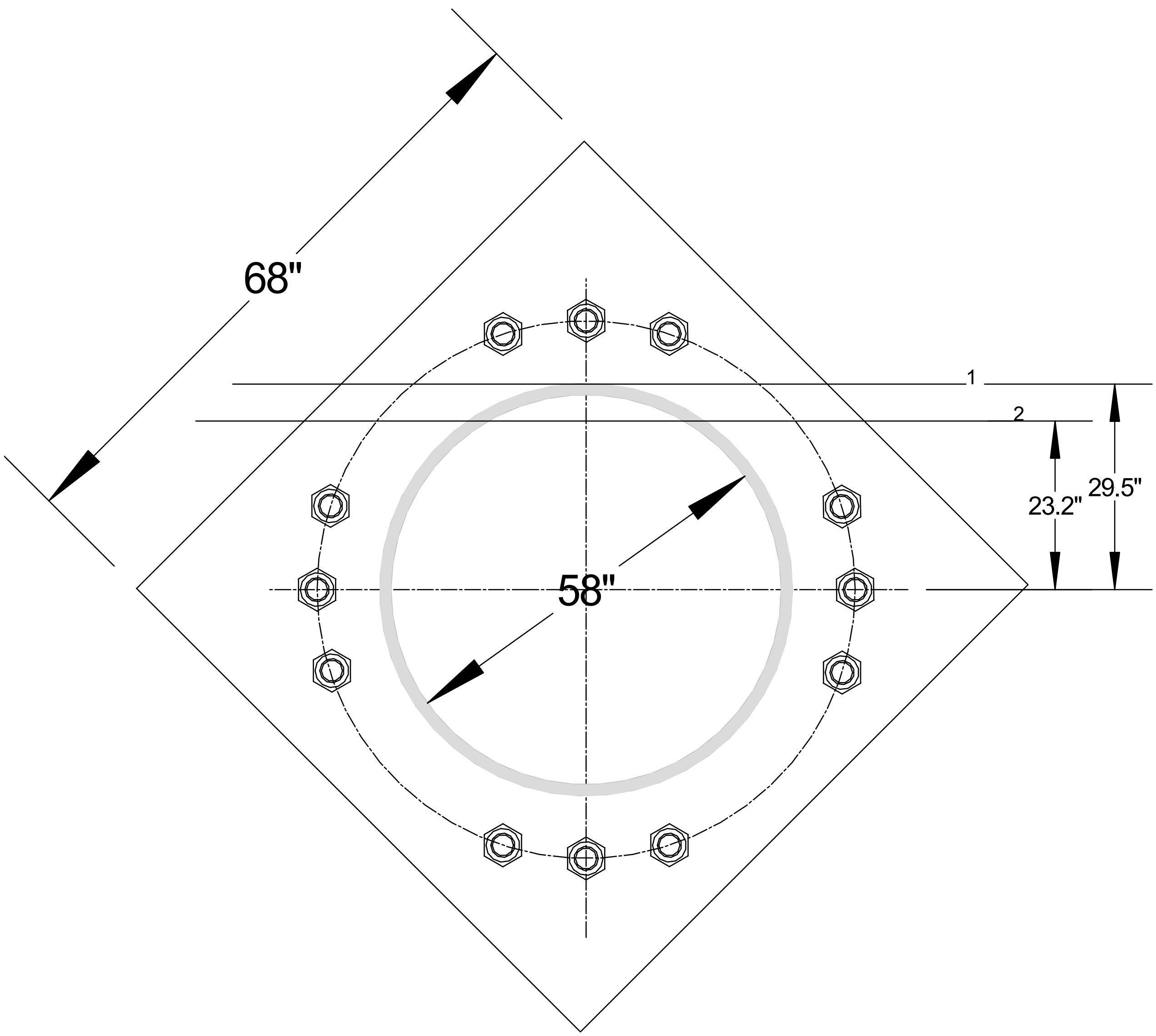


Fig 4- 4

Slice 1 occurs at the outside face of the pole. Slice 2 is at .8 times the diameter of the pole (AISC critical location).

Calculate the bolt values as follows

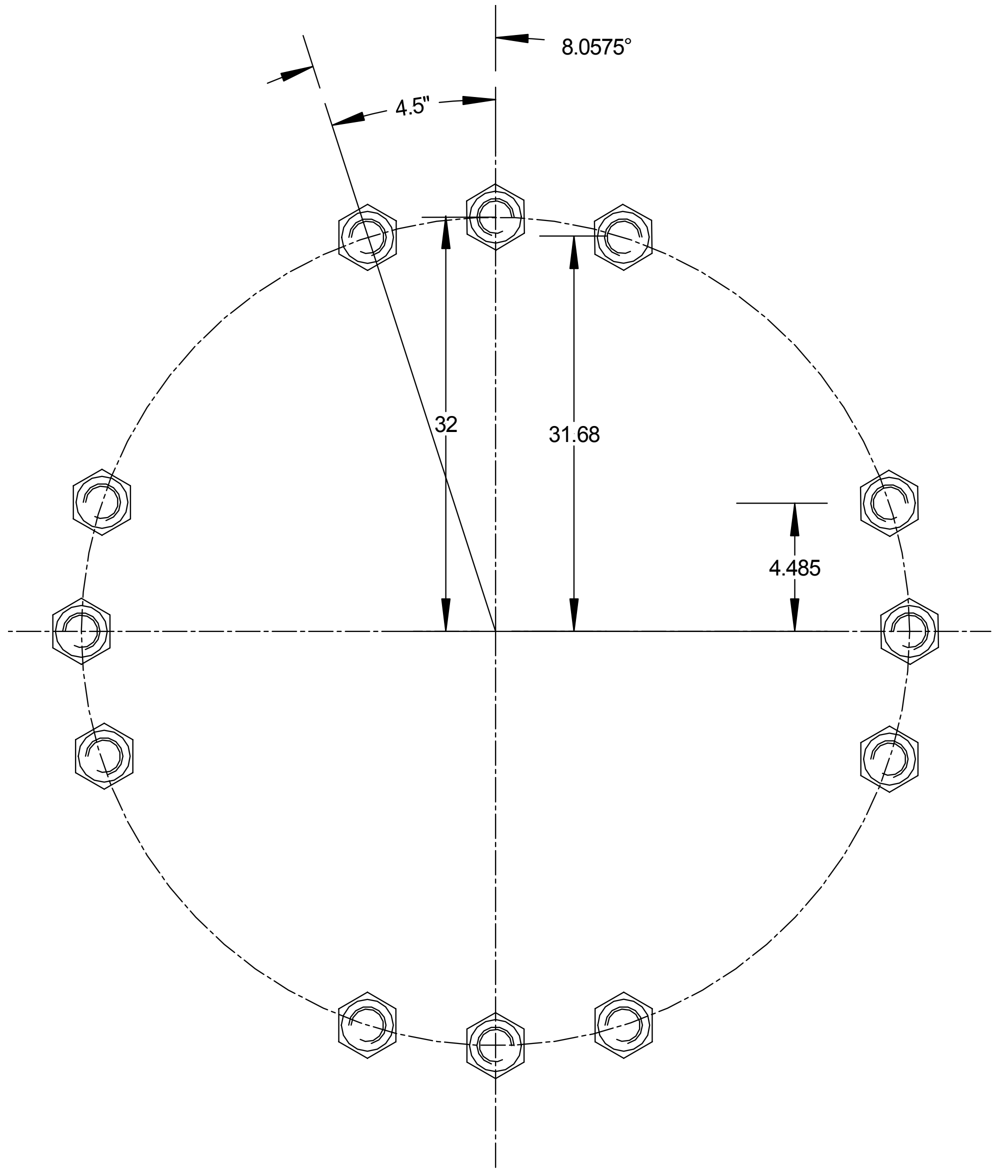


Fig.4-5

$$\theta = 360\left(\frac{4.5}{64\pi}\right) = 8.0575°$$

$$n = 12$$
$$y_{max} = 32$$
$$I = \sum_{1}^{12} y^2 = \sum \left(r\sin(\theta)\right)^2$$
$$I = 2\times 32^2 + 2\times 0^2 + 4\times\left(32\sin(8.0575)\right)^2 + 4\times\left(32\sin(81.9425)\right)^2$$
$$I = 6144$$
$$P_{max} = \frac{46}{12} \pm \frac{42780\times 32}{6144} = +226.6\,\text{kips}, -219.0\,\text{kips}$$
$$y_{n.a.} = \frac{46\times 6144}{12\times 42780} = .55"$$
$$y_{1a} = 32$$
$$P_{1a} = 226.6\ \text{kips}$$
$$y_{1b} = 31.68$$
$$P_{1b} = \left(\frac{46}{12} + \frac{42780\times 31.68}{6144}\right) = 224.4\ \text{kips}$$
$$b_{eff1} = 1.414\times 68 - 2\times 29.5 = 37.152$$
$$M_1 = 226.6(32-29.5) + 2(224.4)(31.68-29.5) = 1544.88$$
$$f_1 = \frac{6\times 1544.88}{37.152\times 2.25^2} = 49.28\ \text{ksi}$$

This compares to 130.8 ksi using the Process Equipment Method.

Next, calculating the stress on slice 2

$$y_2 = .80d = 23.2$$
$$\alpha = \cos^{-1}\left(\frac{23.2}{29.5}\right) = .665$$
$$A = \frac{\pi\left(59^2 - 58^2\right)}{4} = 91.88\ \text{in}^2$$
$$I = \frac{\pi\left(59^4 - 58^4\right)}{64} = 39311\ \text{in}^4$$
$$S = \frac{39311}{29.5} = 1333$$
$$b_{eff} = 1.414\times 68 - 2\times 23.2 - 2\sqrt{(29.5-23.2)59 - (29.5-23.2)^2} = 13.3$$

$$M_{pole} = \frac{2 \times 46 \times .5 \times 29.5^2}{91.88}\left(\sin(.665) - .665\cos(.665)\right) + \ldots$$

$$\frac{2 \times 42780 \times .5 \times 29.5^2}{1333}\left[\frac{.665\cos^2(.665) - \frac{3}{2}\left(\sin(.665)\cos(.665)\right) + \frac{1}{2}.665}{\sin(.665) - .665\cos(.665)}\right]$$

$$M_{pole} = 41 + 4495 = 4535$$

$$M_2 = 226.6(32 - 23.2) + 2(224.4)(31.68 - 23.2) - 4535 = 1265$$

$$f_2 = \frac{6 \times 1265}{13.3 \times 2.25^2} = 112.73\ \text{ksi}$$

Alternate Method 1 – Square Base Plates

This alternate uses one slice at the face of the pole and limits the value of b_{eff} to no greater than the diameter of the pole.

Alternate Method 2 – Circular Base Plates

This alternate uses one slice at the face of the pole and uses 90% of b_{eff}.

Boulos Method (New York Dept. of Transportation)

As can be seen from Examples 4.1 and 4.2, there is a wide discrepancy between stresses calculated by the Process Equipment Method (used by industrial engineers for stacks) and those calculated by the Method of Slices (used by a number of pole manufacturers), especially if the slice location is limited to the face of the pole.

A research project was undertaken in 1993 at the New York State Department of Transportation Engineering Research and Development Bureau and sponsored by the Federal Highway Administration[(5)]. The project performed full-scale testing and finite element analysis of traffic signal poles (square poles with four anchor bolts) to evaluate their structural adequacy. Results of testing and analysis of four poles indicated that they were structurally inadequate. The support base plate and anchor bolts were found to be deficient components, but the pole's post was adequately designed. FEA models of the signal poles that verified the test results were then used to evaluate a representative sample of poles from three major suppliers. Results showed that plates designed by the manufacturers' current (1993) methods of design were not adequate to carry the design loads. The report proposed a new design method for the base plate. This method is only applicable to square base plates. Since the testing was done on smaller ungrouted base plates commonly used for traffic poles having only four bolts, the method may not be totally applicable to larger diameter pole used for telecommunication purposes; however, it will provide educational background into the problem of determining stresses in base plates.

Boulos et al determined that both the parallel load and diagonal load cases resulted in three critical zones of stress. The diagonal case had a single zone at the narrowest part of the plate (45 degrees to the diagonal). The parallel case had two zones, each one at the narrowest part of the plate (0 degrees and 90 degrees).

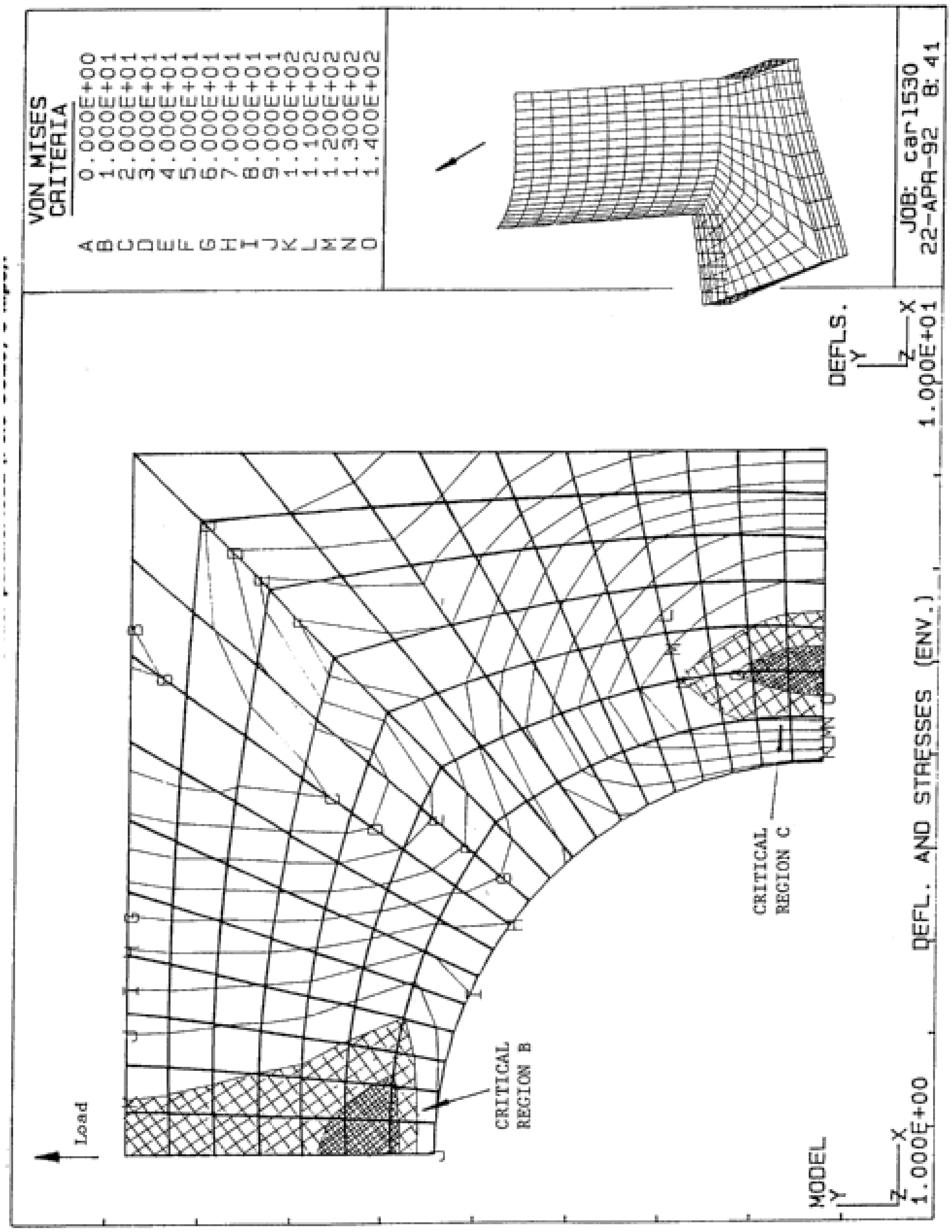

VON MISES
CRITERIA
A 0.000E+00
B 1.000E+01
C 2.000E+01
D 3.000E+01
E 4.000E+01
F 5.000E+01
G 6.000E+01
H 7.000E+01
I 8.000E+01
J 9.000E+01
K 1.000E+02
L 1.100E+02
M 1.200E+02
N 1.300E+02
O 1.400E+02
Load
CRITICAL REGION B
CRITICAL REGION C
MODEL
1.000E+00
DEFL. AND STRESSES (ENV.)
DEFLS.
1.000E+01
JOB: car1530
22-APR-92 B: 41

Fig. 4- 6

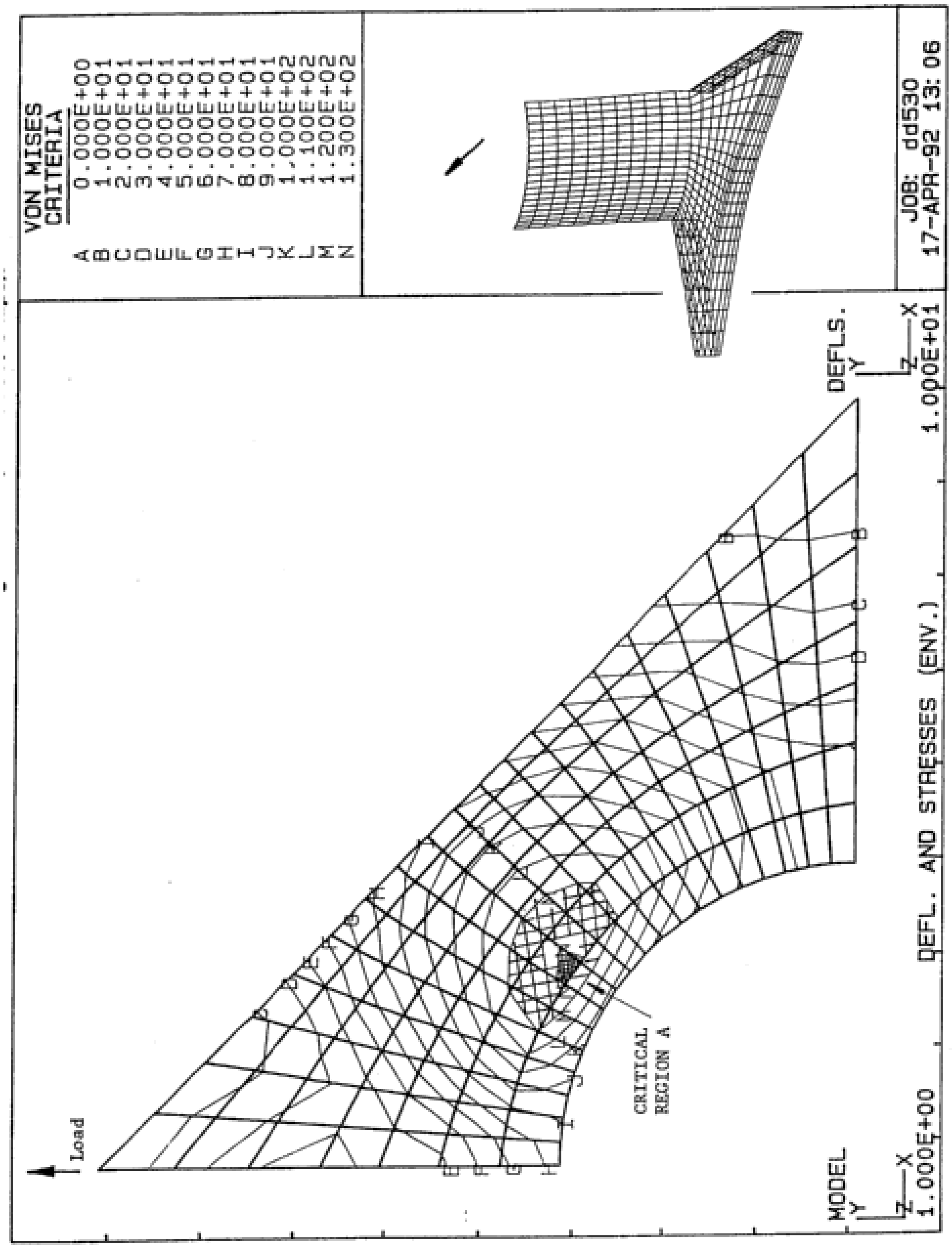

VON MISES
CRITERIA
A 0.000E+00
B 1.000E+01
C 2.000E+01
D 3.000E+01
E 4.000E+01
F 5.000E+01
G 6.000E+01
H 7.000E+01
I 8.000E+01
J 9.000E+01
K 1.100E+02
L 1.200E+02
M 1.300E+02
JOB: dd530
17-APR-92 13:06
DEFLS.
1.000E+01
DEFL. AND STRESSES (ENV.)
CRITICAL
REGION A
Load
MODEL
1.000E+00

Fig. 4- 7

Diagonal Loading Case

The method proposed by Boulos et al calculates the maximum bending stress as

$$d = \frac{BC - DB}{2}$$

$$f_b = \frac{6M \times d}{\alpha(1.414L - DB) \times BC \times T^2}$$

Where M/BC is the bolt force, BC is the bolt circle diameter, L is the width of the plate and DB is the diameter of the pole. Upon careful examination of this equation, it can be seen that this is identical to the method of slices with a correction term for the effective width, α.

$$\alpha = \left(\begin{array}{l} 4.304 - 0.02021\frac{BC}{T} - 4.304\frac{DB}{L} + 4.503\left(\frac{DB}{L}\right)^2 - \ldots \\ 0.9750\frac{L - .707BC}{L - DB} - 1.686\frac{BC}{L} \end{array} \right) / C_\alpha$$

$$C_\alpha = 1.097$$

Typically, the value of α is $.45 \le \alpha \le .75$.

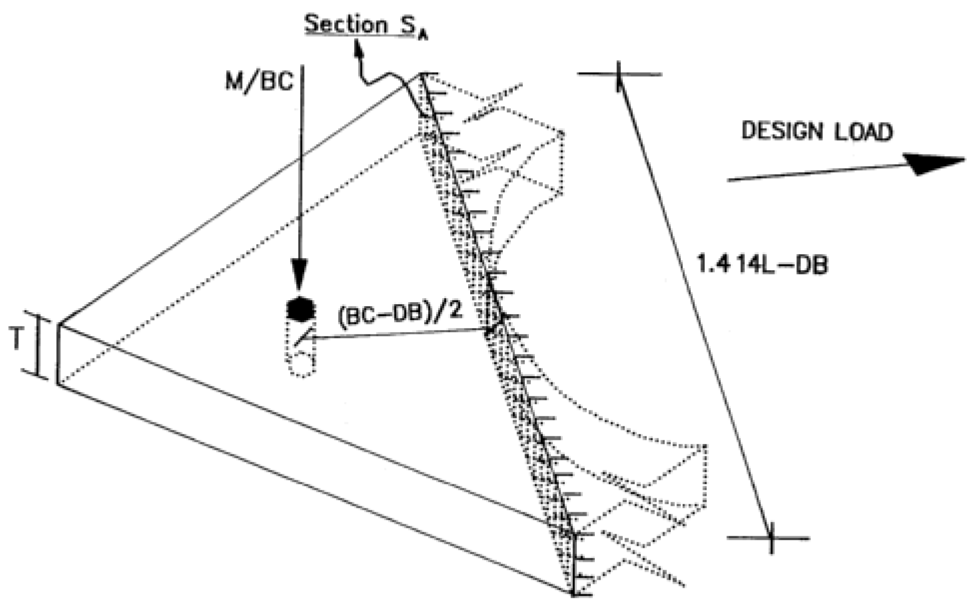


Fig. 4- 8

Parallel Loading Case

Bending Stress

The maximum bending stress was found to occur midway between the bolts as the point of maximum pole stress pushes down analogous to a triangular load at midspan spanning between the two bolts. In fact, Boulos constructed a mathematical model which assumed that the plate spanned between the two bolts with fixed ends. The loading was then assumed to be a triangular load acting between the bolts.

$$f_b = \frac{3M}{\beta t^2 (L - DB)\left(\frac{DB}{L'}\right)^2} \left[\frac{1}{4} - \frac{1 - \frac{DB}{L'}}{3} + \frac{\left(1 - \frac{DB}{L'}\right)^4}{12} \right]$$

$$L' = \max(.707BC, D)$$

$$\beta = \begin{pmatrix} 157.6 - 21.85\frac{L}{DB} - .33\frac{BC}{T} - 259.3\frac{DB}{L} - 48.13\frac{LT}{DB\sqrt{L-DB}} + 194.6\left(\frac{DB}{L}\right)^2 + \ldots \\ 127.4\frac{T}{BC} - 21.65\frac{DB}{BC} \end{pmatrix} / C_\beta$$

$$C_\beta = 1.080$$

Typically, the value of β is $0.5 \le \beta \le 1.5$.

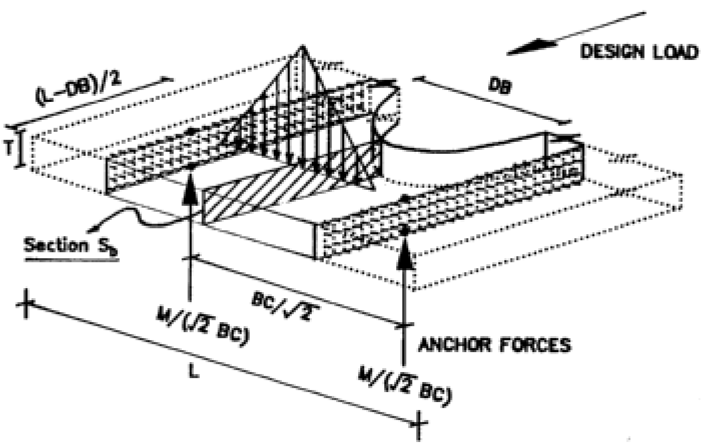


Fig. 4- 9

Torsional Shear Stress at Midpoint of Side

The side of the plate has one bolt in compression and the other in tension. This results in a torsional stress that is calculated as

$$f_v = \frac{M}{2\left(\gamma C' b T^2\right)}$$

$$b = \min\left(.707BC, DB\right)$$

b/t	C’
1.0	0.208
1.2	0.219
1.5	0.231
2.0	0.246
2.5	0.258
3.0	0.267
4.0	0.282
5.0	0.291
10.0	0.312
∞	0.333

$$\gamma = \begin{pmatrix} 210 - 66.9\dfrac{BC}{DB} - .1719\dfrac{BC-DB}{T} - 714.8\dfrac{DB}{L} + 358.3\left(\dfrac{DB}{L}\right)^2 - \ldots \\ 48.16\dfrac{L-.707BC}{L-DB} - 288.2\dfrac{BC-DB}{1.414B-DB} + 381\dfrac{BC}{L} \end{pmatrix} / C_\gamma$$

$$C_\gamma = 1.094$$

Typically, the value for γ is $1.3 \le \gamma \le 2.3$.

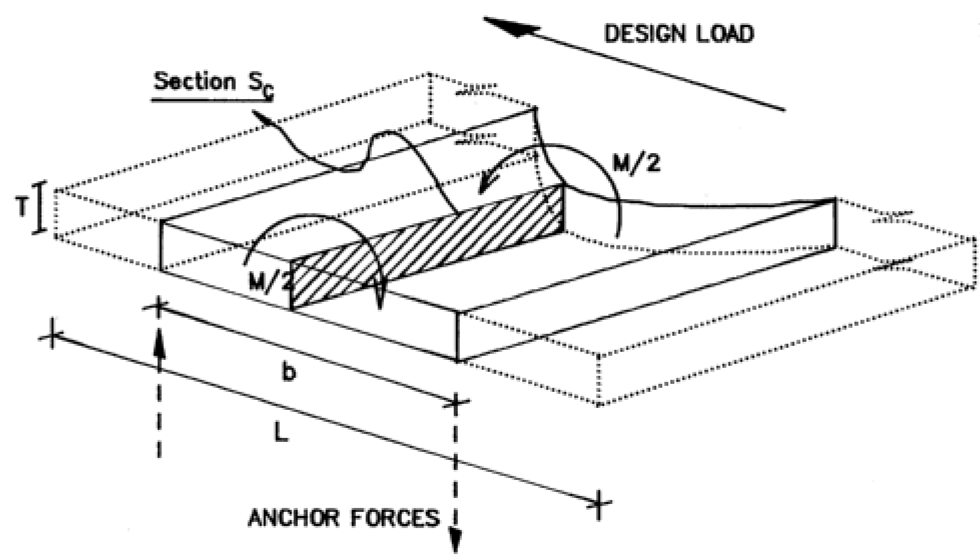


Fig. 4- 10

The method is valid for square base plates within the following ranges

$$8.0 \leq \frac{BC}{T} \leq 16.5$$

$$0.5 \leq \frac{DB}{L} \leq 0.75$$

$$0.95 \leq \frac{BC}{L} \leq 1.05$$

These ranges are usually exceeded for telecommunication poles and the corresponding values will be < 0, thus suggesting that the lower bounds should be used. Therefore, it may be prudent to impose an upper limit on the effective width equal to .40 of the full slice width at the face of the pole for poles with four bolts. However it is reasonable to assume that, as the number of bolts increases, the effective width would increase above this value.

Owens Method (New York Dept. of Transportation)

Owens[6,7] et. al., extended the Boulos method to accommodate grouted base plates. He incorporated yet another check using yield line theory.

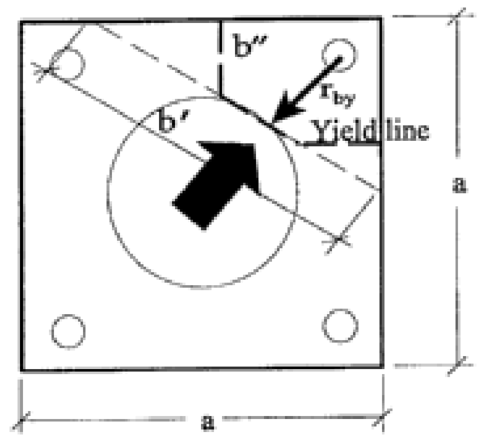


Fig. 4- 11

The research confirmed the Boulos findings that base plates previously designed by manufacturers were deficient.

The stress at the yield line is calculated as follows

$$M_{by} = T \times r_{by}$$

$$S_x = b'' \times \frac{t^2}{6} = \frac{.707t^2}{6}\left(b' - \frac{a}{4}\right)$$

$$f_{by} = \frac{M_{by}}{S_x}$$

Comparing to the Method of Slices, the effective width is

$$b_{eff} = b'' = .823B - .707d$$

Which is about 40-45% of the full diagonal width at the face of the pole for common telecommunication poles.

ASCE Manual 72 (Proposed 2003 Revision)

ASCE Manual 72[27] contains a base plate design section which uses the Method of Slices with the 45 degree limitation discussed in the Process Equipment section. The ASCE method checks two bend lines on each facet of the polygon, a line parallel to the pole face and another line tangential to the corner of the polygon. The bend line for a circular pole is tangent to the pole. See Fig. 4-11.

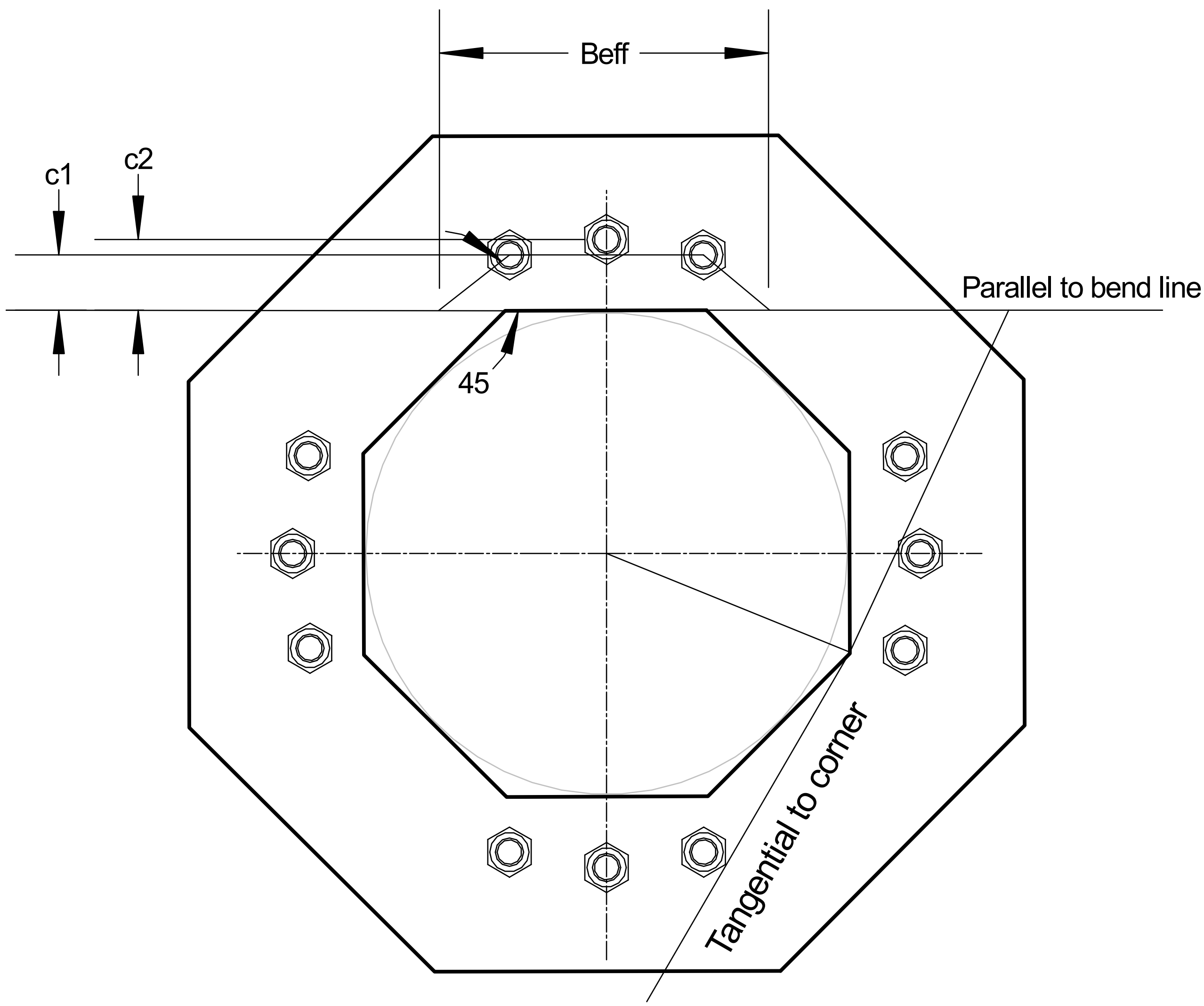


Fig. 4- 11

The bending moment on the bend line is then the sum of the bolt forces outside of the bend line times the distance of the bolt from the bend line (c1 and c2 in Fig. 4-11). The tangential to corner situation is only applicable to power line transmission poles where the forces are primarily parallel and perpendicular to the transmission lines. Telecommunication poles can be oriented at any possible angle to the wind and therefore the pole would be assumed to be oriented such that the bolts and the polygonal faces are in the worst possible condition with the resultant moment in only one direction. The the parallel case would be the worst case scenario for this situation.

Example 4.3

Analyze the bolt forces on an ungrouted square base plate loaded on the diagonal and a bolt circle of 64″ and (12) 2-1/4″ bolts spaced at 4.5″. The plate is 68″ square and the inside of the base plate has a diameter of 58″. The thickness of the pole is .500″ and the plate is 2.25″ thick. The vertical load is 46 kips and the moment is 3565 kip-ft (42780 in-kips).

$$e = \frac{M}{P} = \frac{42780}{46} = 930\,\text{in}$$

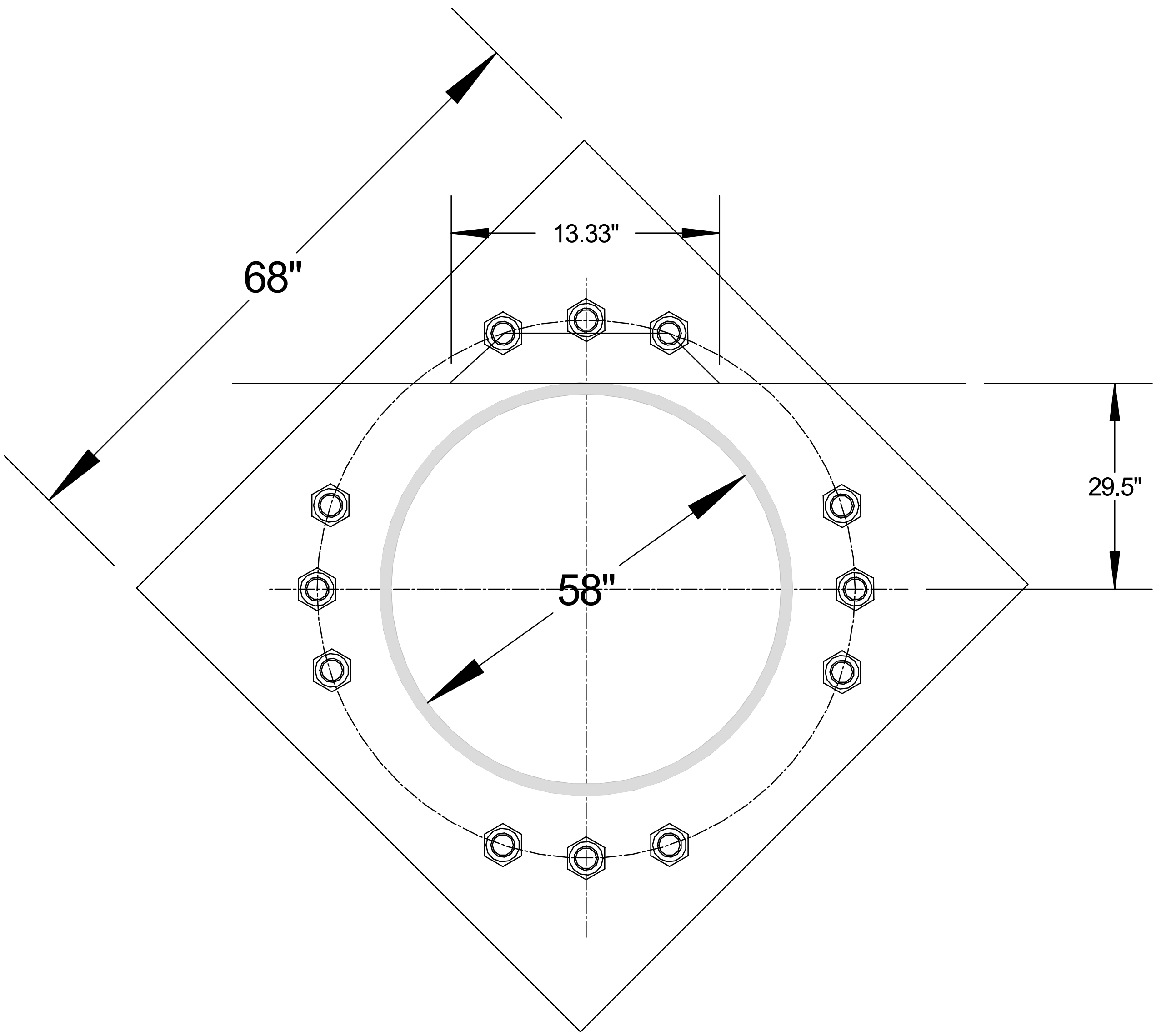


Fig 4- 12

Calculate the bolt values as follows

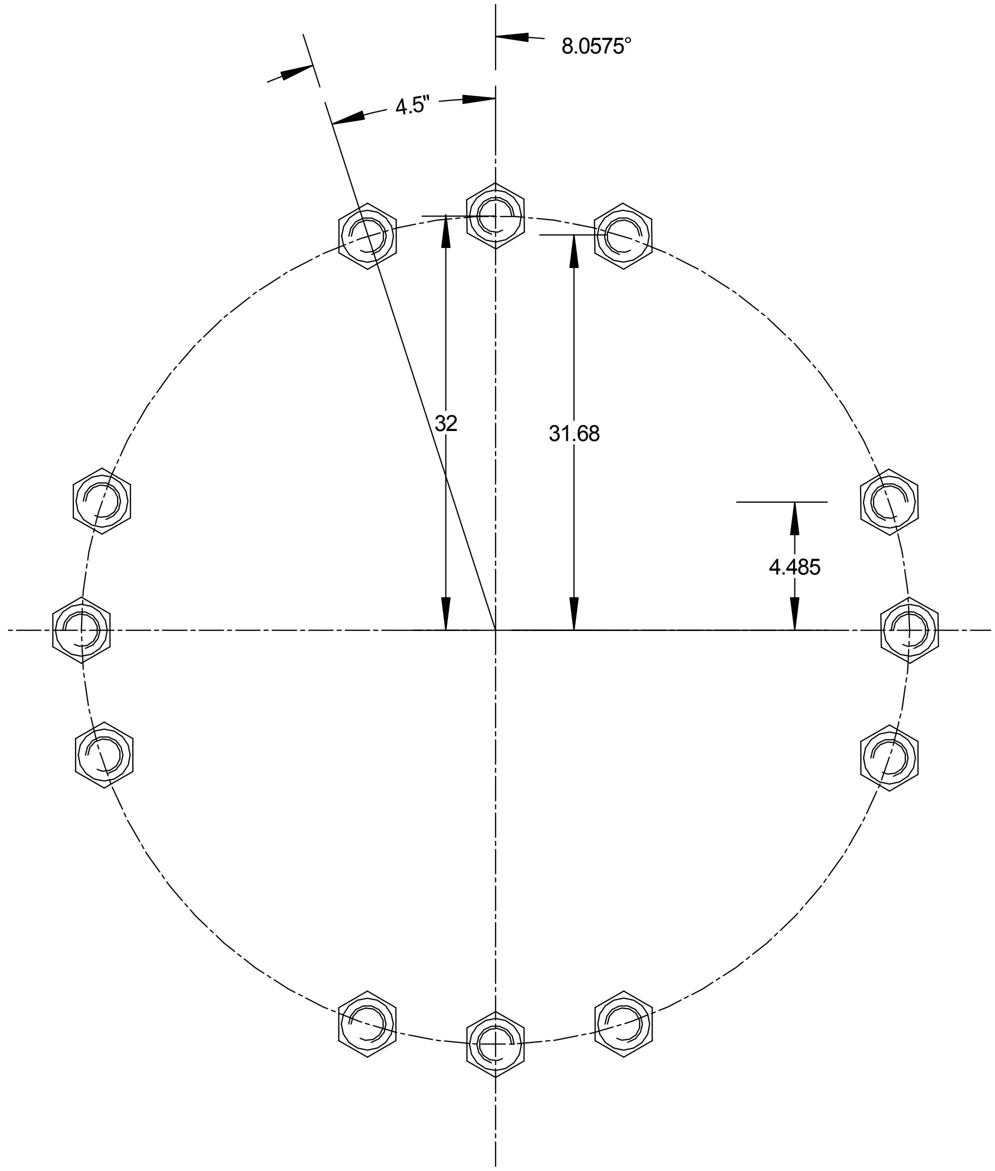


Fig.4-5

$$\theta = 360\left(\frac{4.5}{64\pi}\right) = 8.0575^\circ$$

$$n = 12$$

$$y_{max} = 32$$

$$I = \sum_{1}^{12} y^2 = \sum \left(r \sin(\theta) \right)^2$$

$$I = 2 \times 32^2 + 2 \times 0^2 + 4 \times \left(32 \sin(8.0575) \right)^2 + 4 \times \left(32 \sin(81.9425) \right)^2$$

$$I = 6144$$

$$P_{max} = \frac{46}{12} \pm \frac{42780 \times 32}{6144} = +226.6\,\text{kips}, -219.0\,\text{kips}$$

$$y_{1a} = 32$$

$$P_{1a} = 226.6\text{ kips}$$

$$y_{1b} = 31.68$$

$$P_{1b} = \left(\frac{46}{12} + \frac{42780 \times 31.68}{6144} \right) = 224.4\text{ kips}$$

$$b_{eff1} = 2 \times 4.485 + 2 \times (31.68 - 29.5) = 13.33\,\text{in}$$

$$M_1 = 226.6(32 - 29.5) + 2(224.4)(31.68 - 29.5) = 1544.88$$

$$f_1 = \frac{6 \times 1544.88}{13.33 \times 2.25^2} = 137.4\text{ ksi}$$

This compares to 49.28 ksi using the Method of Slices (Example 4.2).

Anchor Bolts

Material Grades

Over the years, a wide variety of bolt grades have been used for pole base plates. The most commonly used grades are listed below.

ASTM Designation	Tensile Strength, ksi
A307	60
A325 (SAE Gr 5 similar)	105
A354 Gr BD	150, 140 (over 2 ½”)
A354 Gr BC	125, 115 (over 2 ½”)
A449	90
A490 (SAE Gr 8 similar)	150
A618 Gr 75 (Anchor bolt grade, #18J bars)	100
A687	150
A36	58
A572 Gr 50	65
A572 Gr 42	60
A588	70
F1554 Gr 36	58
F1554 Gr 55	75
F1554 Gr 105	125

The current standard has anchor bolts unified under ASTM F1554 . Only grades 36 and 55 permit welding.

Hooked anchor bolts have fallen into disfavor as tests have shown that local crushing occurs at the bend point leading to a reduction in pullout capacity. The standard method detail is to provide a threaded portion at the bottom of the anchor bolt and attach a nut or nut with washer. When the grade permits welding, the nut is usually tack-welded to the bolt to insure that the nut does not loosen or fall off. High strength grades usually do not permit welding and a second “jam” nut is often provided.

The number of anchor bolts required is a function of the maximum net uplift on the column and the allowable tensile load for the grade chosen. Prying forces in anchor bolts are typically neglected.

Fatigue is often ignored in the selection and sizing of anchor bolts. However, in cases of high seismic/wind situations fatigue may be a consideration. The following table shows some recommended values for bolt fatigue stresses.

Number of Cycles[a]	Allowable Tensile Stress, ksi
20,000 to 100,000	40
100,000 to 500,000	25
500,000 to 2,000,000	15
Over 2,000,000	10
[a]Categories correspond to AISC Appendix K	

Anchorage

Prior to publication of ACI 318-02, anchorage of bolts was based upon approximate methods using pull-out "cones" as developed in AISC[12] and PCI[24]. ACI 318 presented a new method for determining anchor bolt capacities. Appendix D of ACI 318-02 (ACI 2002) and Appendix B of ACI 349-01 both address the anchoring to concrete of cast-in-place anchors. Adhesive anchors and grouted anchors are not covered by these appendices. The provisions in both of these appendices are based upon the Concrete Capacity Design (CCD) Method.

In the CCD method the concrete cone is considered to be formed at a slope of 1 in 1.5 rather than the older 45 degree assumption. The cone is considered to have a square shape in plan rather than the classical round shape to simplify calculations.

Section 15.8.3.3 of ACI 318-02 requires that anchor bolts reach their design strength before anchorage failure or failure of the surrounding concrete (ductile failure) yet no code section is provided to state how this is to be accomplished.

Lutz-Fisher[23], outline a procedure to insure this ductility requirement. When an anchor bolt is designed to lap with reinforcement, the anchor capacity can be taken as $\phi A_{se} F_y$ since the lap splice will insure that ductile behavior will occur. A_{se} is the tensile stress area of the anchor bolt and $\phi = .9$ as given in ACI 318-02 Chapter 9. To insure ducitlity for breakout, there must be sufficient lap length (with a standard hook on the reinforcing bars if required) to adequately develop the capacity. The distance "g" as shown in Fig. 5-1 is the distance from the centerline of the anchor bolt to the centerline of the nearest reinforcing bar. Using a Class B splice, the required embedment (top of concrete to top of nut) would be

$$h_{ef} = 1.3 l_d + \frac{g}{1.5} + \text{top cover}$$

When the anchor is solely resisted by concrete, as it would be for pullout and side-face blowout, one needs to have the concrete designed with additional capacity in order to insure ductility of the connection. To achieve this, Lutz proposed that the concrete must reach a capacity of $1.25\left(\phi A_{se} F_y\right)$ thereby insuring that the steel would yield prior to concrete rupture.

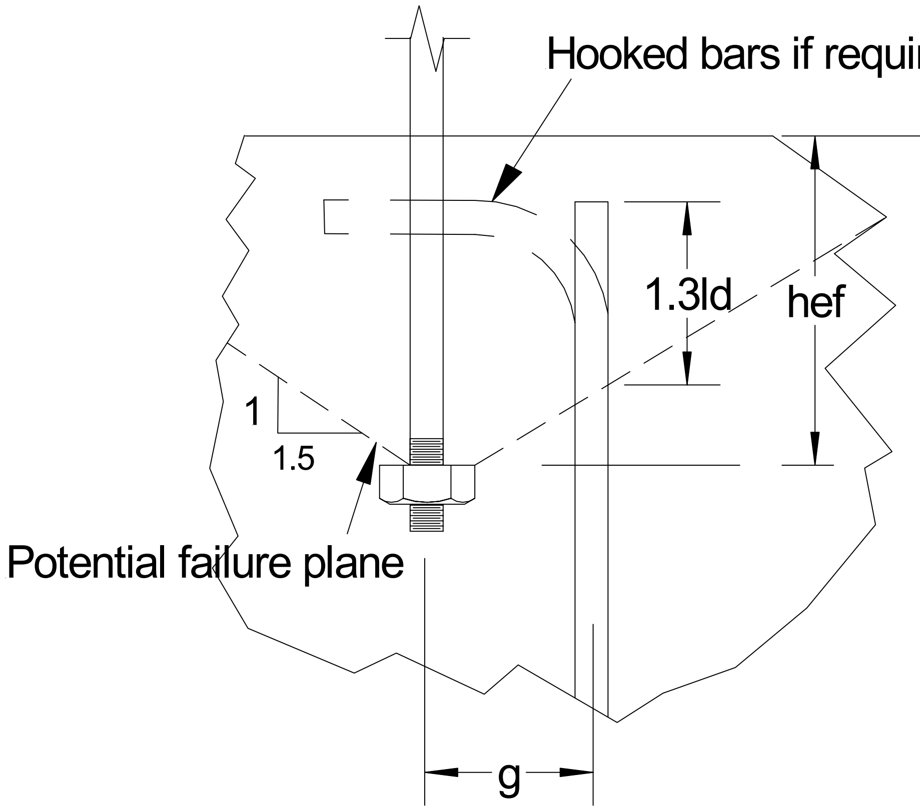


Fig. 5- 2 Development of bar for concrete breakout

Example 4.1

An anchor bolt has a factored (USD) bolt tension of 50 kips. There are 8 anchor bolts and the anchor bolts are 10 inches from the edge of a 60 inch diameter concrete pier and the bolts are 1-1/4" F1554 Gr 55. There are 24-#9 Gr 60 reinforcing bars bundled in groups of 2, 3 inches clear from the edge of the pier and the concrete strength is 3500 psi. The basic development length of a #9 bar with 3 inches clear is 37 inches (straight bar).

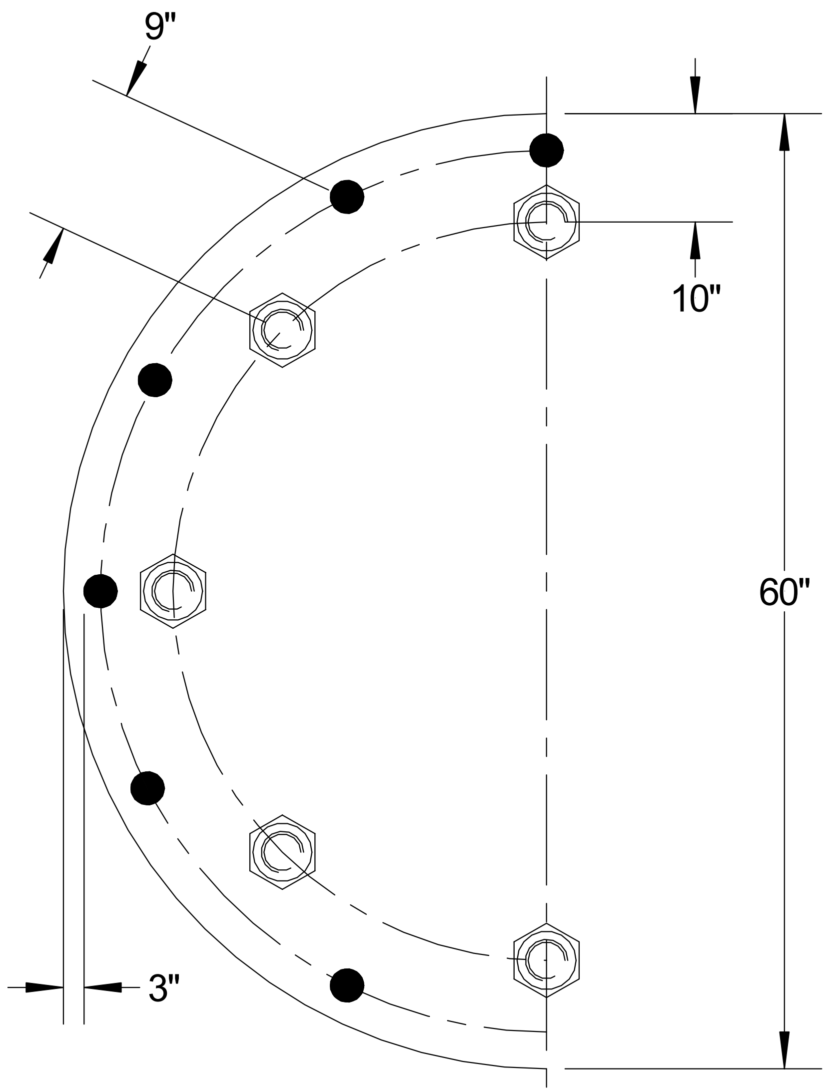


Fig 5-2

Check tension capacity of 2-#9 bars

$$\phi P_{cap} = .90 \times nA_b \times F_y = .90 \times 2 \times 1.0 \times 60 = 108 \text{ o.k.}$$

The anchor bolt capacity is

$$\phi N_s = .75 \times A_{se} \times F_{ut} = .75 \times .969 \times 75 = 54.5 \text{ kips}$$

Breakout

Ductility for breakout will be insured by lapping the bolt with the reinforcing bars.

$$l_d = 37 \text{ in}$$
$$1.3 l_d = 48 \text{ in}$$
$$h_{ef} \geq 1.3 l_d + \frac{g}{1.5} + \text{cover} = 48 + \frac{9}{1.5} + 3 = 57 \text{ in, say } 5'$$

Pullout

To insure ductility for pull-out and side-face blowout, the concrete strength must be greater than

$$1.25\left(\phi A_{se} F_y\right) = 1.25 \times 0.9 \times .969 \times 55 = 60 \text{ kips/bolt}$$

Using a nut diameter of 2.1", the pullout strength is

$$\phi N_{pn} = \phi \psi_4 N_p = .70(1.4) A_{brg}\left(8 f_c'\right)$$
$$A_{brg} = \pi\left(\frac{2.1^2 - 1.25^2}{4}\right) = 2.236$$
$$\phi N_{pn} = .70 \times 1.4 \times 2.236 \times 8 \times 3.5 = 61 \text{ kips} \geq 60 \text{ o.k.}$$

Often the anchor bolts are held together with a ½" setting plate ring. This would increase the bearing area but might not be fully effective since the plate would bend during pullout. An assumption could be made that the bearing would be equivalent to a washer which would have an outside diameter of two times the bolt diameter.

Side-Face Blowout

The side clearance, c, is 10". Since $c \leq .4 h_{ef}$, side-face blowout must be checked. The side-face blowout strength is

$$\phi N_{sbg} = .75\left(1 + \frac{s_o}{6c}\right) 160 c \sqrt{A_{brg} f_c'}$$
$$= .75\left(1 + \frac{18.85}{6 \times 10}\right) 160 \times 10 \sqrt{2.236 \times 3500} = 139.5 \text{ kips} \geq 60 \text{ o.k.}$$

Normally, side-face blowout would not need to be checked if ties are provided in the region at the anchor bolt nut location such that the ties would cross the failure plane.

Recommendations

Determining Base Plate Forces

Grouted Base Plates

All of the methods presented yield approximately the same results. However, the Complete Methods are more accurate when there are a limited number of bolts. The Complete Methods also have specific methods for square base plates, which can only be approximated as circular using the Process Equipment or Lutz Methods.

Therefore, the Complete Method should be modified to obtain a more reasonable distribution of plate stresses considering that the plate is not infinitely rigid. The following two modifications should be made:

1. **The distance from the bolt circle to the outside edge of the plate should be limited to the distance from the pole face to the bolt circle.**
2. **The plate hole opening diameter should be taken as no less than the inner diameter of the pole minus the distance from the pole face to the adjusted outside edge of the plate.**

Ungrouted Base Plates

The method of calculating bolt forces using the section modulus of the bolt group is the only method currently being used.

Determining Base Plate Stresses

The Method of Slices without any adjustment to the effective width was proven inaccurate by the work of Boulos and Owens. The method should only be used for square base plates whenever the effective width of plate at the face of the pole is multiplied by an adjustment factor . The factor would be approximately 0.40 for four bolt base plates and would increase for poles with more than four bolts.

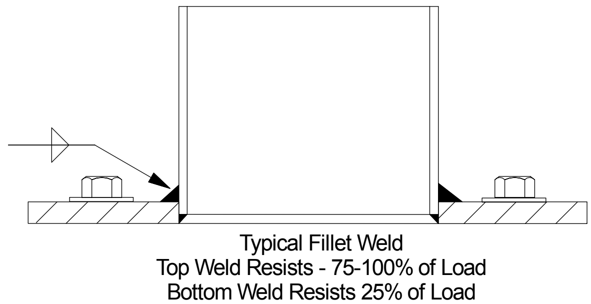
The Process Equipment Method of considering the plate as a simple cantilever from the center of the bolt circle to the face of the pole is still the simplest method for circular or polygonal plates and yields results for square base plates, when using the 45 degree rule, that are comparable to those obtained by using the Boulos method when using a lower bound of .40 on effective width. There was no evidence in the research that would support the assumptions in any of the Alternate Methods although they may have some validity.

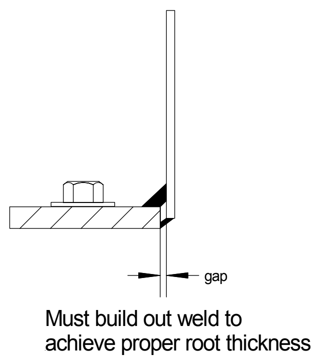
The ASCE Manual 72 (Proposed) Method is identical to the aforementioned Process Equipment Method using the 45 degree rule.

The Process Equipment Method (45 degree rule)/ASCE Manual 72 Method is therefore recommended using the parallel bend line and the worst case positioning of pole facets and bolts.

Construction Details

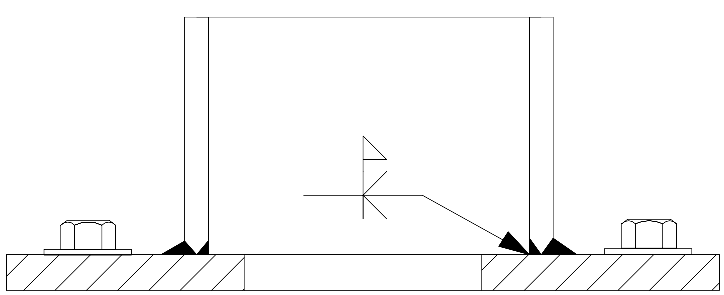
The majority of poles manufactured today are small diameter poles used for traffic control purposes. The poles are circular and extend through the plate as shown below.





This detail subjects the weld to a constant state of stress. Larger diameter polygonal poles may have manufacturing tolerances such that a "gap" may occur during fit-up. This requires that the fillet welds be built-up in order to maintain the proper root thickness.

A better detail for larger diameter poles is to extend the plate inside the pole and to use a full penetration weld as shown below. A covering fillet(s) may be added to prevent water collection at the base plate.



Bibliography

References

1. *Design of Steel Transmission Pole Structures, Manual* 72, American Society of Civil Engineers (ASCE).

2. *Standard Specifications for Structural Supports for Highway Signs, Luminaires and Traffic Signals*, American Association of State Highway Transportation Offices (AASHTO).

3. *Guide Specifications for Design of Metal Flagpoles*, The National Association of Architectural Metal Manufacturers (NAAMM).

4. *Metal Flagpole Manual*, The National Association of Architectural Metal Manufacturers (NAAMM).

5. *Research Report 159, Load Testing, Finite Element Analysis, And Design of Steel Traffic-Signal Pole,* Boulos, Fu, and Alampalli, New York State Department of Transportation.

6. *Research Report 131, A Simplified Design Procedure For End-Plates and Base-Plates Of Cantilevered Traffic Structures,* Frank Owens, Osman Hag-Elsafi, Sreenivas Alampalli, New York State Department of Transportation.

7. *New Procedure For Design of End Plates and Base Plates of Cantilevered Traffic Support Structures,* Osman Hag-Elsafi, Sreenivas, and Frank Owens, ASCE Journal of Structural Engineering, October 2001.

8. *Design of Welded Structures*, Blodgett, Lincoln Arc Welding.

9. *Design of Steel Structures*, Gaylord and Gaylord.

10. S*teel Structures*, McGuire.

11. *Steel Structures, Design and Behavior*, Salmon and Johnson.

12. *Steel Design Guide Series 1, Column Base Plates*, American Institute of Steel Construction (AISC).

13. *Column Base Plates with Axial Loads and Moments,* DeWolf, ASCE Journal of Structural Engineering, November 1980.

14. *Base Plates Under Axial Loads and Moments,* Thambiratnam, ASCE Journal of Structural Engineering, May 1986.

15. *Concrete Plain and Reinforced,* Taylor, Thompson, and Smulski, 1925.

16. *Process Equipment Design*, Brownell and Young , 1957.

17. T*ubular Steel Structures, Theory and Design*, Troitsky , Lincoln Arc Welding 1990.

18. *Pressure Vessel Handbook*, Megyesy.

19. *Pressure Vessel Handbook*, Bednard , 1991.

20. *Internal Memorandum, Computerized Structural Design*, Dr. Leroy Lutz, 2004.

21. *Internal Memorandum, Computerized Structural Design,* Daniel Horn, 2004.

22. *Design of Plate Structures – Volume 2*, Bijilaard, American Iron and Steel Institute (AISI).

23. S*teel Design Guide Series #7, Industrial Buildings, Roofs to Column Anchorage,* Lutz-Fisher, American Institute of Steel Construction (AISC), 2004.

24. *Prestressed Concrete Design Handbook*, Presressed Concrete Institute (PCI).

25. *Formulas for Stress and Strain*, Roarke.

26. *Theory of Plates and Shells*, Timoshenko.

27. *ASCE Manual 72, Guide for Design of Steel Transmission Pole Structures*, American Society of Civil Engineers (ASCE)

Glossary of Terms

AASHTO

American Association of State Highway Transportation Officials.

AISC

American Institute of Steel Construction.

AISI

American Iron and Steel Institute.

ASCE

American Society of Civil Engineers.

Base Plate

A steel plate that transmit forces from a column to the foundation.

Monopole

A stepped or tapered steel pole. Poles can be round or polygonal (8,12,16, or 18 sides).

NAAMM

National Association of Architectural Metal Manufacturers.

PCI

Prestressed Concrete Institute.

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