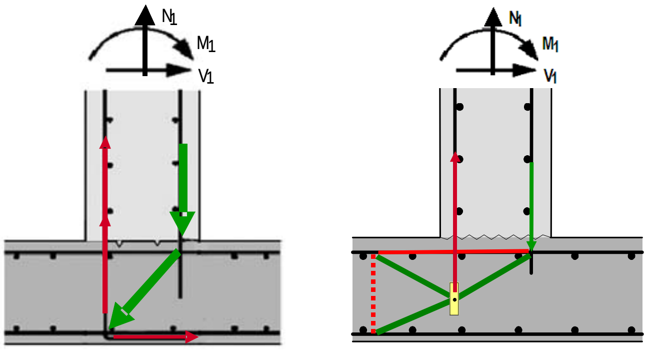
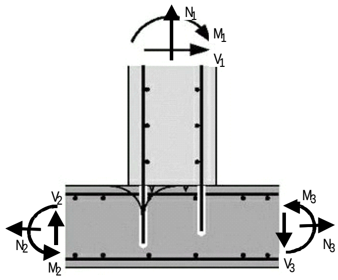
2.3.2 Strut and Tie Model for Frame Nodes

If frame nodes (or moment resisting connections in general) are designed with cast-in reinforcement, they usually require bent bars according to the standard reinforced concrete design rules. Anchoring the reinforcement of moment resisting connections with straight bars would, at least at first sight, result in concrete that is under tension, and therefore in a possible concrete cone failure. As this failure mode is brittle, such an anchorage is not allowed by the standard concrete design rules. In cooperation with the Technical University of Munich, Hilti performed a research programme in order to provide a strut-and-tie model for frame nodes with straight connection bars [6, 7]. The main differences to the standard cast-in solution are that the compression strut is anchored in the bonding area of the straight bar rather than in the bend of the bar and that, therefore, first the inner lever arm inside the node is reduced and second, splitting forces in the transition zone between D- and B-region must be considered.



Global Equilibrium of the Node

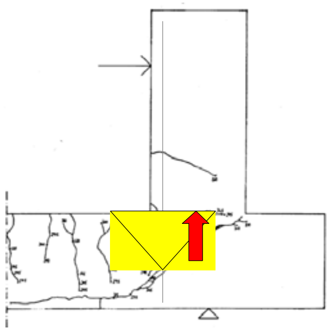
In order to check the struts and ties inside the node, the reactions N_2, V_2, M_2, N_3, V_3, M_3 at the other ends of the node need to be defined. Normally, they result from the structural analysis outside the node region and will be determined by the designer in charge.

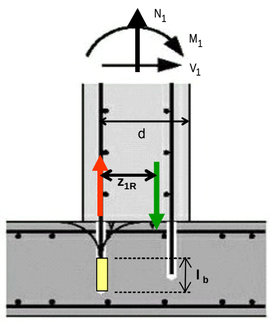


Global equilibrium of the node

Tension in connecting bars

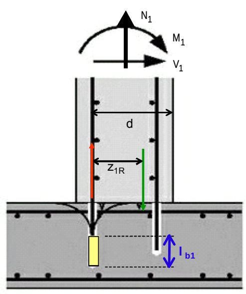
The loading of the wall in the figure above results in a tensile force in the reinforcement on the right hand side and in a compression force on the left hand side. Initial tests and computer simulations led to the consideration that the straight bar has a tendency to push a concrete cone against the interface with the wall. Thus the compressive stress is in the interface is not concentrated on the outside of the wall, but distributed over a large part of the interface, which leads to a reduced lever arm in the wall section. The recommended reduction factor is 0.85 for opening moments and 1.0 for closing moments.





Anchorage length

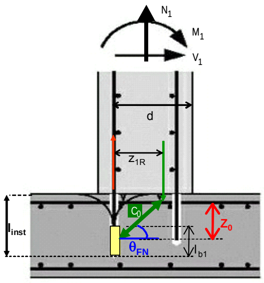
While the equilibrium inside of frame nodes with cast-in hooked bars can be modeled with the compression strut continuing from the vertical compression force and anchored in the bend at the level of the lower reinforcement, straight bars are anchored by bond stresses at a level above the lower reinforcement.
As bending cracks are expected to occur along the bar from the top of the base concrete, the anchorage zone is developing from the lower end of the bar and its length ℓ_b is that required to develop the steel stress calculated form the section forces M_1, N_1 and V_1.



$$\ell_b = \frac{\sigma_{sd} \cdot \phi}{4 \cdot f_{bd}}$$

with σ_{sd} design steel stress in the connection bars [MPa]
ϕ diameter of the vertical bar [mm]
f_{bd} design bond strength of cast-in bar to concrete or of the adhesive mortar [MPa]

<u>Installation length</u>



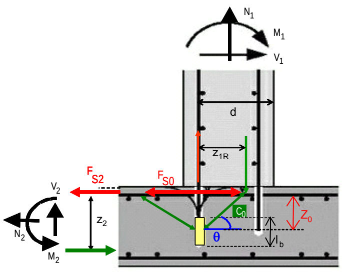
The strut-and-tie model requires that the angle θ between the inclined compression strut C_0 and the horizontal direction is 30º to 60º. For low drill hole lengths the resulting strut angle will be less than 30º. In such situations the design will not work as tests have shown. Also in order to remain as close as possible to the original solution with the bent bar, it is recommended to drill the holes as deep as possible in order to achieve a large strut angle θ_{FN}.

Note that PROFIS Rebar will preferrably propose the installation length such that the strut angle θ_{FN} is 60º. In cases where the existing section is too thin for this, it will propose the maximum possible embedment depth which is defined for bonded anchors in ETAG 001, part 5, section 2.2.2 as

$\ell_{inst,max} = h_{member} - \max(2 \cdot d_0;\ 30mm)$

with $\ell_{inst,max}$ maximum possible installation length [mm]
h_{member} thickness of the existing concrete member [mm]
d_0 diameter of the drilled hole [mm]

<u>Tension in Existing Reinforcement</u>



For a drilled hole depth t_b and a concrete cover of the upper reinforcement to the center of the bars of c_s, the lever arm inside z_0 the node is:

$$z_0 = l_{inst} - \frac{\ell_b}{2} - c_s$$

The lever arm inside the node z_0 is smaller than the lever arm of the slab z_2. The tension in the upper slab reinforcement in the node region, F_{S0}, is higher than the tension calculated for the slab with z_2; the tensile resistance of the existing upper reinforcement $A_{s0,prov}$ must therefore be checked separately as follows:

$F_{s2} = M_2/z_2 + N_2/2$ (tension in existing reinforcement outside node area)

$H_{s2} = \left(M_1 + (V_2 + V_3) \cdot \frac{z_1}{2} \right) \cdot \left(\frac{1}{z_0} - \frac{1}{z_2} \right) + V_1 \cdot \left(\frac{z_1}{z_0} - 1 \right)$ (additional tension in node due to reduced lever arm)

$F_{s0} = F_{s2} + H_{s2}$ (steel tension in node area)

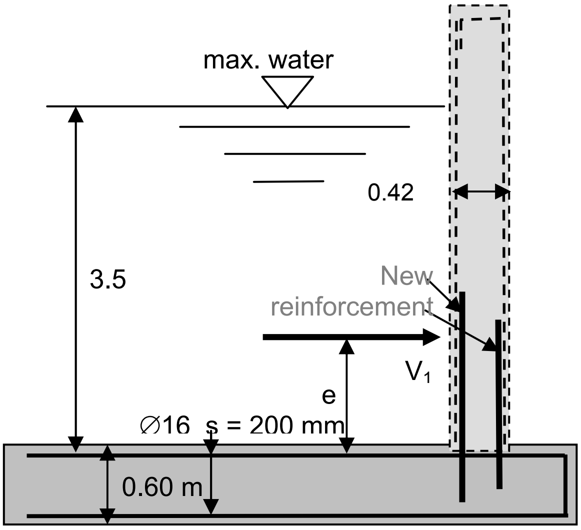
$A_{s0,rqd} = F_{s0} + H_{s2}$ (steel area required in existing part for forces from new part)

If $A_{s0,prov} \geq A_{s0,rqd}$ the reinforcement of the existing part is sufficient, provided that the forces from the new part are the only load on the section. This is the analysis obtainable from PROFIS Rebar.

As mentioned further above, a more sophisticated check needs to be made if there are also other loads in the system. Basically it would mean replacing F_{s2} as evaluated by under “global equilibrium” above by that evaluated in the complete static design.

The shallower the embedment of the post-installed vertical bar is, the more the moment resistance of the slab in the node region is reduced compared to a node with hooked bar. For this reason, it is also recommended to provide deep embedment of the connecting bars rather than trying to optimize mortar consumption by trying to recommend the shortest possible embedment depth.

b) Wall bending connection



Note: transverse reinforcement not

Geometry:

h_1 = 420 mm; $h_2 = h_3$ = 600 mm;

d_1 = 380 mm; $d_2 = d_3$ = 560 mm;

z_1 = 360 mm; $z_2 = z_3$ = 520 mm

$A_{s0} = A_{s2} = A_{s3}$ = 1005 mm²/m (∅16 s = 200 mm)

$c_S = h_2 - d_2$ = 40 mm

Material:

Concrete: C20/25 (new and existing parts), γ_s = 1.5

Steel grade: 500 N/ mm², γ_s = 1.15

Safety factor for variable load: γ_Q = 1.5

HIT-RE 500-SD (temperature range I)

Acting loads:

$V_{1d} = \gamma_Q \cdot p \cdot h^2 / 2 = 1.4 \cdot 10 \cdot 3.5^2 / 2$ = 92 kN/m

$e = h / 3 = 3.5 / 3$ = 1.17 m

$M_{1d} = V_{1d} \cdot e = 92 \cdot 1.17$ = 107 kNm/m

Force in post-installed reinforcement

$z_{1r} = 0.85 \cdot z_1 = 0.85 \cdot 360$ = 306 mm (opening moment → reduced inner lever arm)

$F_{s1d} = M_{1d} / z_{1r} = 107 / 0.306$ = 350 kN/m

$A_{s1,rqd} = F_{s1d} / (f_{yk}/\gamma_{Ms}) = 350'000 / (500 / 1.15)$ = 805 mm²/m

Select ϕ12mm, spacing s_1 = 125mm → $A_{s1,prov}$ = 905 mm²

→ drilled hole diameter: d_0 = 16 mm

Stress in bar: $\sigma_{sd} = F_{s1d} / A_{s1,prov}$ = 386 N/mm²

anchorage length

$f_{bd,EC2}$ = 2.3 N/mm2 (EC 2 for minimum length)

$\ell_{b,rqd,EC2} = (\phi/4) \cdot (\sigma_{sd}/f_{bd,EC2})$ = 504 mm

$\ell_{b,min} = \max\{0{,}3\ell_{b,rqd,EC2};\ 10\phi;\ 100\text{ mm}\}$ = 151 mm

$f_{bd,b}$ = 8.3 N/mm² (see tech. data, sect. 6)

$c_d = s_1/2 - \phi/2$ = 56.5 mm > 3ϕ

$$\alpha_2' = \frac{1}{\max\left[\frac{1}{0.7} + \delta \cdot \frac{c_d - 3\phi}{\phi};0.25\right]}$$ = 0.512

$$f_{bd,spl2} = \frac{f_{bd}}{\max[\alpha_2'\ ;\ 0.25]}$$ = 4.5 N/mm²

$f_{bd} = \min\{f_{bd,b};\ f_{bd,spl}\}$ = 4.5 N/mm²

$\ell_{b1} = \max\{(\phi/4)\cdot(\sigma_{sd} / f_{bd});\ \ell_{b,min})$ = 258 mm

Drilled hole length

$\ell_{inst,max}$ = $h_2 - \max\{2d_0; 30mm\}$ = 568 mm (maximum possible hole length)

$\ell_{inst,60}$ = $c_s + z_{1R} \cdot \tan 60° + \ell_{b1} / 2$ = 672 mm (hole length corresponding to θ=60°)

$\ell_{inst,60}$ > $\ell_{inst,max}$ → select hole length $\ell_{inst} = \ell_{inst,max}$ = 568 mm

Strut angle with $\ell_{inst,max}$: $\tan\theta = (\ell_{inst,max} - c_s - \ell_{b1}/2)/z_{1R}$ → θ_{FN} = 53°

check: $\theta > 30°$ → ok

Reaction in Foundation:

$-M_{2d}$ = $M_{1d} + V_{1d} \cdot z_2 / 2 = 107 + 0.25 \cdot 92$ = 131 kNm/m

N_{2d} = $-V_{1d}$ = -92 kN/m

$M_{s3} = 0$; $V_{2d} = V_{3d} = 0$; $N_1 = N_3 = 0$

Check of foundation reinforcement

F_{s2d} = $M_{2d} / z_2 + N_{2d} / 2$ = 298 kNm/m (tension outside node area)

z_0 = $\ell_{inst} - c_s - \ell_{b1} / 2 = 568 - 40 - 258/2$ = 399 mm (lever arm in node area)

H_{s2d} = $M_{1d} \cdot (1/z_0 - 1/z_2) + V_{1d} \cdot (z_1/z_0 - 1)$ = 53 kNm/m (additional force in node area)

$F_{s2d,node}$ = $F_{s2d} + H_{s2d}$ = 351 kNm/m (tension in node area)

$A_{s2,rqd}$ = $F_{s2d,node} / (f_{yk}/\gamma_{Ms})$ = 351'000 / (500 / 1.15) = 808 mm^2/m

A_{s2} > $A_{s2,rqd}$ → ok (A_{s2} is given)

Check concrete compressive strut

F_{c0d} = M_{1d} / z_0 = 268 kN/m

D_{0d} = $F_{c0d} / \cos\theta_{FN}$ = 441 kN/m

α_{ct} = 1.0 (EC2: EN 1992-1-1:2004, 3.1.6(1))

ν' = $1 - f_{ck}/250$ = 0.92 (EC2: EN 1992-1-1:2004, 6.5.2(2))

k_2 = 0.85 (EC2: EN 1992-1-1:2004, 6.5.4(4b))

D_{0Rd} = $\alpha_{ct} \cdot \nu' \cdot k_2 \cdot f_{ck} / \gamma_c \cdot \ell_{b1} \cdot \cos\theta_{FN}$ = 1639 kN/m

D_{0Rd} > D_{0d} → ok

Check concrete splitting in plane of foundation

α_{ct} = 1.0 (EC2: EN 1992-1-1:2004, 3.1.6(2))

$f_{ctk,0.05}$ = $\alpha_{ct} \cdot 0.7 \cdot 0.3 \cdot f_{ck}^{2/3} / \gamma_c$ = 1.03 N/mm^2 (table 3.1, EC2: EN 1992-1-1:2004)

$M_{sp,d}$ = $F_{c0d} \cdot z_0 \cdot (1 - z_0/z_2) \cdot (1 - \ell_{b1}/(2z_2))$ = $1.87 \cdot 10^7$ Nmm/m

W_{sp} = $1000mm \cdot z_2^2 / 2.41$ = $1.12 \cdot 10^8$ mm^3/m

$\max\sigma_{sp}$ = $M_{sp,d} / W_{sp}$ = 0.17 N/mm^2

$f_{ctk,0.05}$ > $\max\sigma_{sp}$ → ok