

# RESEARCH REPORT

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## Tensile Capacity of Steel Connectors with Short Embedment Lengths in Concrete

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J.I. Restrepo and R. Park

August 1993

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**TENSILE CAPACITY OF STEEL CONNECTORS  
WITH SHORT EMBEDMENT LENGTHS  
IN CONCRETE**

by

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## ABSTRACT

The use of steel anchors with short embedment lengths in concrete has a wide range of application in the concrete industry. The load transfer by the anchors relies on the mechanical interlock between a bearing part of the device and the surrounding concrete. Among the diversity of these embedments, the precast concrete industry in New Zealand makes significant use of headed concrete inserts and hooked reinforcing bars. The current Concrete Design Code (NZS 3101, 1982) provides no guidelines for the design of these load transferring devices.

This report reviews several methods proposed for the evaluation of the load transferred by headed anchors and hooked bar anchorages with short embedment lengths. The results of experimental work on these two types of connectors, protruding from a prototype tilt-up concrete wall, are discussed.

A design equation for headed concrete inserts and hooked bars, that considers the possible variations in the material strengths as well as the construction tolerances, is also discussed. The design equation is based on the  $\psi$ -method developed at the University of Stuttgart for predicting the tensile capacity of headed anchors. The second moment probabilistic method is used to account for the possible deviations of the main variables affecting the tensile capacity of the connectors from their ideal or specified conditions.

## ACKNOWLEDGEMENTS

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## NOTATION

- $\beta$  = safety index
- $\lambda_s$  = material factor for steel
- $\xi_R$  = reduction factor that account for edge effects and overlapping of failure surfaces
- $\kappa$  = factor to account for the effect of bar diameter on the required embedment length
- $\phi$  = strength reduction factor
- $\phi_c$  = strength reduction factor associated with the concrete cone pull-out failure
- $\psi_c, \psi_{cx}, \psi_{cy}$  = reduction factors associated with the decrease of the concrete pull-out strength due to edge effects
- $\psi_{cr}$  = reduction factor to account for the detrimental effects of cracking on the concrete pull-out strength
- $\psi_s, \psi_{sx}, \psi_{sy}$  = reduction factors associated with the decrease of the concrete pull-out strength due to overlapping of failure surfaces
- 
- $A_e$  = effective projected area of failure surface ( $\text{mm}^2$ )
- $A_f$  = area of the surface failure of a truncated cone ( $\text{mm}^2$ )
- $A_p$  = projected area of conical failure surface of a single anchor, excluding its bearing area ( $\text{mm}^2$ )
- $A_s$  = area of steel anchored in a member ( $\text{mm}^2$ )
- $c$  = edge distance (mm)
- $c_x, c_y$  = edge distances in the x and y directions, respectively (mm)

$d$	=	effective depth of beam (mm)
$d_b$	=	bar diameter (mm)
$d_e$	=	diameter of the head of an anchor (mm)
$f_c^c$	=	compressive concrete cylinder strength (MPa)
$f_c^{cc}$	=	compressive concrete cube strength (MPa)
$f_s$	=	design steel stress (MPa)
$f_{sp}$	=	splitting tensile strength of concrete (MPa)
$f_{su}$	=	minimum specified tensile strength of steel (MPa)
$f_{sum}$	=	measured ultimate steel stress (MPa)
$f_t$	=	direct tensile strength of concrete (MPa)
$f_y$	=	lower characteristic yield strength of reinforcing steel (MPa)
$f_{ym}$	=	measured yield strength of reinforcing steel (MPa)
$\bar{f}_y$	=	mean yield stress of reinforcing steel (MPa)
$G$	=	dead service load (kPa)
$G_f$	=	fracture energy ( $MJ/m^2$ )
$h_e$	=	effective embedment length (mm)
$l_{db}$	=	full embedment length of a hooked bar anchorage (mm)
$M_l$	=	ideal moment strength (kN.m)
$M_u$	=	ultimate applied moment (kN.m)
$n$	=	number of anchors in a group
$n_x, n_y$	=	number of anchors in the same row in the x and y direction, respectively
$Q_p$	=	live service load (kPa)
$s$	=	centre-to-centre distance between anchors (mm)

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- $s_{cr}$  = critical distance when overlapping of the failure surfaces affects the individual pull-out strength of a group of anchors (mm)
- $s_x, s_y$  = centre-to-centre distance between anchors in the x and y direction, respectively (mm)
- $T$  = tensile capacity of an anchor or group of anchors (N)
- $T_e, T'_e$  = tension force in the concrete cone failure surface in the pre-cracked region (N)
- $T_C, T'_C$  = predicted concrete cone pull-out strength of an individual embedment (N)
- $\bar{T}_C$  = mean concrete pull-out strength (N)
- $T_s$  = tensile force in reinforcing steel (N)
- $T_w, T'_w$  = tension force in the concrete cone failure surface in the post-cracked region (N)
- $V_\lambda, V_\phi$  = global coefficient of variation of the inducing and resisting mechanisms
- $z$  = lever arm distance (mm)
- $z'$  = effective restraining distance in hooked bar anchorages (mm)

## 1. INTRODUCTION

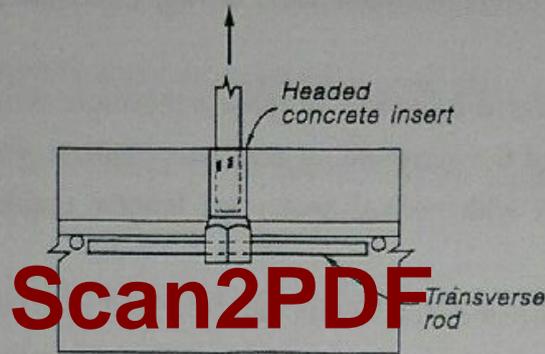
Concrete inserts and hooked bar anchorages have a wide range of application in the precast concrete industry. They are commonly utilized during lifting operations or to interconnect precast concrete elements together to enable the transfer of superimposed loads. In addition, they are also required for transferring forces arising from volumetric changes due to the rheological nature of concrete, from relative settlement of the supports, and from temperature variations or from inertial effects during earthquakes.

The force transfer mechanism of these connectors relies on the mechanical interlock between the connector and the surrounding concrete. This is why their embedment lengths are generally small compared with normal anchorage lengths required for deformed straight bars.

Extensive use of steel connectors can be found in tilt-up concrete construction. Steel anchors have been used to connect the proprietary precast concrete floor system to the external walls through a cast in place concrete topping of at least 50 mm thick. Figure 1.1 shows three connecting methods often used in practice in New Zealand. The first arrangement, illustrated in figure 1.1a, consists on a proprietary concrete insert with a headed base and a threaded sleeve. Additional reinforcement is usually passed through an eye-hole at the base of the insert to enhance the concrete pull-out strength. Threaded reinforcing bars are screwed into the insert. Concrete inserts are easily positioned and held during the casting of the concrete. However, one disadvantage is that some proprietary concrete inserts available in New Zealand possess rather short embedment lengths which might not prevent brittle concrete cone pull-out failure occurring before yielding of the reinforcing bar. Hence their use is not recommended when there are possibilities of spalling of the concrete in the region of the member where the insert is located. Another arrangement, depicted in figure 1.1b, makes use of hooked reinforcing bars anchored near the far side of the member. A transverse bar of the same diameter as the bent bar is commonly tied in contact with the inside of the hook in the belief that in shallow members it improves the anchorage conditions. This arrangement is very economical and efficient as it takes full advantage of the depth of the member for anchoring the bar. Its use is limited if the precast concrete members need to be stacked before construction. The practice of bending the reinforcing bars to enable stacking of the precast panels and straightening them once the member has been positioned should only be permitted if the reinforcing steel is not affected by strain age embrittlement (Erasmus, 1981). An alternative arrangement shown in figure 1.1c makes use of proprietary bar couplers and possesses the advantages of the two previous arrangements. One problem which the structural engineer faces when using hooked bar anchorages is that their anchorage length is limited by the thickness of the walls and often

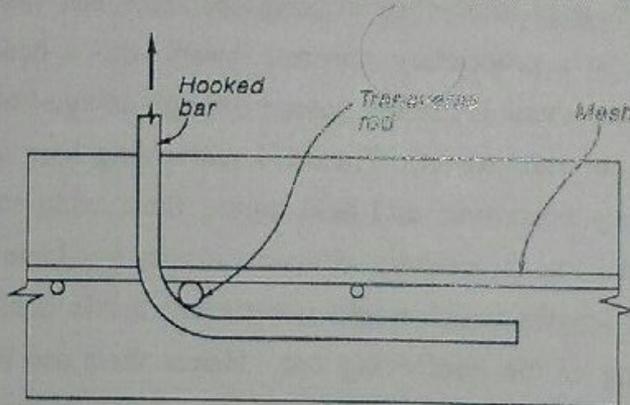
this length is smaller than the minimum anchorage length required by the current Concrete Design Code (NZS 3101, 1982).

One objective of this research is to report a literature review of different available methods for designing steel anchors loaded in tension. In addition, the results of two commonly utilized anchorage methods in tilt-up wall panels tested under short-term loading are discussed and design recommendations are made.

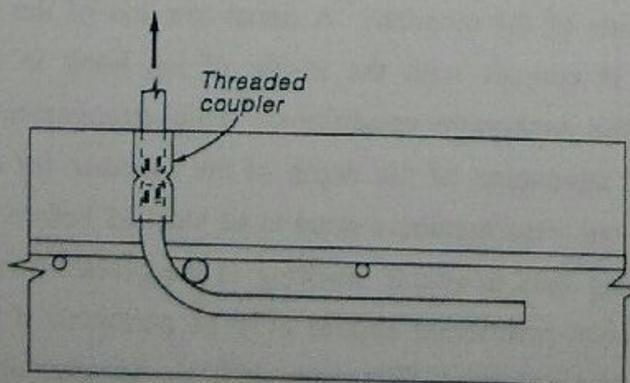


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(a) Threaded Concrete Insert with Transverse Rod



(b) Hooked Bar Anchorage



(c) Spliced Hooked Bar Anchorage

Figure 1.1 - Typical Connecting Methods Used in Tilt-up Construction.

## 2. ASSESSMENT OF THE TENSILE CAPACITY OF CONNECTIONS WITH SHORT EMBEDMENT LENGTHS IN CONCRETE

### 2.1 General

This section describes four of the most known methods available for the evaluation of the tensile capacity of headed steel connectors. Furthermore, two procedures for predicting the tensile capacity of hooked bars with short anchorage lengths are also outlined. A state-of-the-art report on this topic has been published by the Comité Euro-International du Béton (1991).

Figure 2.1 outlines the possible modes of failure displayed by steel anchors embedded in concrete. All concrete failures, perhaps except for the mode of failure caused by pull-out of the anchor, are characterized by a brittle behaviour with limited deformability. This is why most design recommendations aim to ensure a steel failure due to fracture of the threaded rod or bolt, or by extensive yielding of the reinforcing bar anchored in the member. Of the four possible concrete failures, the cone pull-out failure is the one that gives the higher load carrying capacity. The remaining modes of failure can be precluded by ensuring appropriate strength hierarchies.

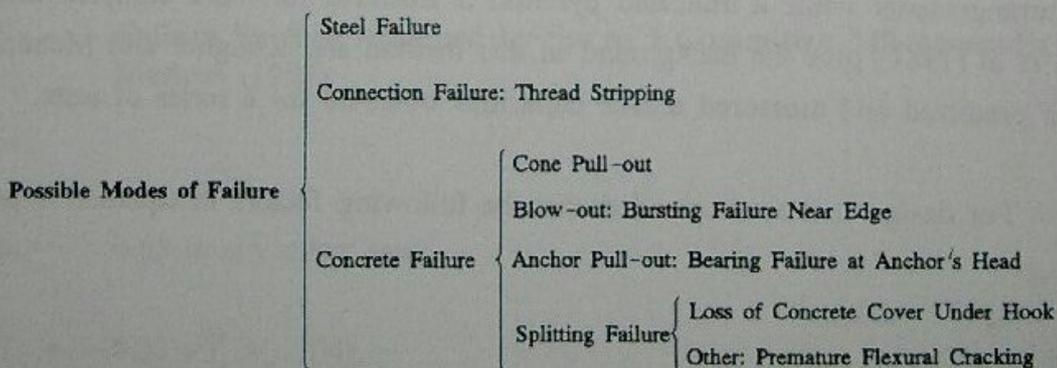


Figure 2.1 - Possible Modes of Failure Displayed by Steel Embedments Loaded in Tension.

A general expression for the design of steel anchors is of the form

$$\lambda_s A_s f_s \leq \phi_c \xi_R T_C \quad (1)$$

where  $\lambda_s$  and  $\phi_c$  are material factors for the applied load and strength reduction factors for the resisting mechanisms, respectively, to reduce the possibility of concrete failure,  $A_s$  is the section area of the bar being anchored,  $f_s$  is the design steel stress,  $\xi_R$  is a factor less than unity that accounts for the reduced loads caused by the edge effects and/or by the proximity between adjacent anchors, and  $T_C$  is the mean pull-out strength of a single embedment anchored in uncracked concrete and displaying a cone failure.

The way that four different methods use the above factors is discussed below.

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### 2.2 Tensile Capacity of Headed Steel Anchors

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#### 2.2.1 ACI Committee 349 - Appendix B Method (1985)

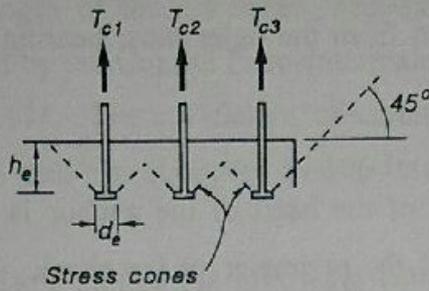
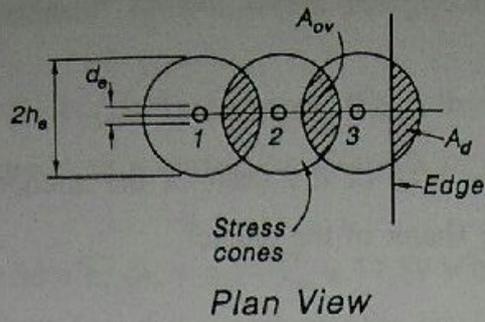
The ACI Committee 349 - Appendix B Method (1985) assumes that the pull-out strength of the concrete is determined as an uniform tensile stress in the concrete acting on an effective area projected from the surface failure. As depicted in figure 2.2, the surface failure is assumed to develop from the bearing edge of the anchor or from the far side of the member, depending on the embedment length to overall member thickness. The angle of inclination of the idealized surface failure is assumed to be 45°. A conical surface failure is assumed for simple arrangements while a truncated pyramid is assumed for more complex arrangements. Cannon et al (1981) give the background of this method and Klingner and Mendonca (1985) compare predicted and measured tensile capacities obtained for a series of tests.

For design it is recommended that the following factors in equation 1 be used:

$$\lambda_s = 1.0$$

$$f_s = f_{su}, \text{ where } f_{su} \text{ is the minimum specified tensile strength of the steel.}$$

$$\phi_c = 0.85 \text{ for anchors embedded beyond the member face reinforcement; or embedded in a compression region a member; or anchored in the tension zone of a member providing that the tensile stress based on an uncracked section flexural analysis is less than } 0.41\sqrt{f'_c} \text{ (MPa).}$$



For anchor 3

$$A_p = \pi h_e (h_e + d_e)$$

$$A_e = A_p - \frac{A_{ov}}{2} - A_d$$

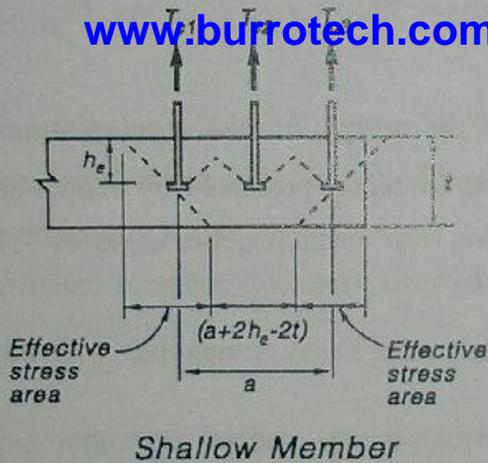


Figure 2.2 - Failure Surface Assumed by the ACI Committee 349-Appendix B Method (1985).

or  $\phi_c = 0.65$  in any other case.

$$T_C = 0.33 \sqrt{f'_c} A_p \quad (\text{N}), \tag{2}$$

where  $A_p$  is the projected area of the conical surface failure of a single anchor, excluding its bearing area (see figure 2.2).

$\xi_{R} = A_e/A_p$ , where  $A_e$  is the effective projected area of the surface failure reduced for overlapping regions due to the effect of multiple anchors and/or to the proximity of the anchor edge of the member (see figure 2.2).

The ACI Committee 349 also gives the following recommendations to avoid a premature anchor pull-out failure:

- (a) The effective bearing area of the head of the anchor is to be at least 1.5 times the area of the shank of the anchor.
- (b) The thickness of the head of the anchor is to be at least 1.0 times the greatest dimension from the outer most bearing edge of the head to the face of the shank.
- (c) The bearing area of the head of the anchor is to be approximately evenly distributed around the perimeter of the shank.

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#### 2.2.2 PCI Design Handbook (1985)

The recommendations for the design of steel embedments presented by the PCI Design Handbook (1985) are based on the report presented by Shaikh and Whayong Yi (1985). This method assumes a surface failure with an inclination angle of  $45^\circ$ .

The following factors, which were made compatible with the formulation presented in equation 1, apply:

$$\lambda_s = 1.0$$

$$f_s = 0.9f_{su}$$

$$\phi_c = 0.85$$

$$T_C = 0.23 \sqrt{f'_c} A_f \approx \sqrt{f'_c} h_e^2 \quad (\text{N}), \quad (3)$$

where  $A_f$  is area of the surface failure of the truncated cone and  $h_e$  is the effective embedment length of the anchor. For stud groups the pull-out strength is not based on the basic pull-out strength of a single anchor.

$$\xi_R = c/h_e, \text{ where } c \text{ is the edge distance measured from the centre of the anchor.}$$

### 2.2.3 Method Proposed by Bode and Roik (1987)

Bode and Roik (1987) obtained the following expression for the mean pull-out strength of anchors based on a regression analysis of existing test data:

$$T_c = 10.96 \sqrt{h_e} (h_e + d_e) \sqrt{f_{cc}} \approx 11.89 \sqrt{h_e} (h_e + d_e) \sqrt{f'_c} \quad (\text{N}), \quad (4)$$

where  $d_e$  is the diameter of the head of the anchor and  $f_{cc}$  is the compressive cube strength of the concrete. The second equation using  $f'_c$  instead of  $f_{cc}$  was given by the Comité Euro-International du Béton (1991), which assumed  $f_{cc}/f'_c = 1.176$ . The coefficient of variation found between the predicted and measured pull-out strengths in 106 tests with  $h_e < 175$  mm was 10.1%.

Bode and Roik also obtained the following equation for  $T_c$  ignoring the influence of the diameter of the head

$$T_c = 15.6 \sqrt{h_e} \sqrt{f_{cc}} \quad (\text{N}), \quad (5)$$

They point out that the value of  $T_c$  proposed should be reduced to 80% of the values obtained from equations 4 or 5 when  $h_e \leq 50$  mm. They also recommend that the thickness of the member anchoring the connector to be limited to  $t \geq 2h_e$ , since tests where flexural cracking developed due to insufficient member thickness showed a general trend to reduce the pull-out strength.

The following reduction factors were proposed to account for the edge effects and for the closeness between anchors:

$$\xi_R = \psi_c \psi_s$$

where

$$\psi_c = c / (1.5h_e) \leq 1.0, \text{ is the edge factor for an anchor with one free edge}$$

or 
$$\psi_c = c / (2.0h_e) \leq 1.0, \text{ for an anchor with two or more free edges}$$

and 
$$\psi_s = \{1 + (n - 1) (s/s_{cr})\} / n \leq 1.0, \text{ is the group factor in which } n \text{ is the number of anchors, } s \text{ is the centre-to-centre distance between anchors and } s_{cr} \text{ is the critical distance when overlapping of the surface failures will reduce the total concrete cone pull-out strength obtained as the sum of the individual capacities. The critical distance is defined as } s_{cr} = 4h_e.$$

## 2.2.4 $\psi$ -Method

The  $\psi$ -method for assessing the tensile capacity of anchors subjected to different load and geometric conditions has been developed during the past decade by Eligehausen and his associates at the University of Stuttgart (Comité Euro-International du Béton, 1991). A regression analysis, based on extensive laboratory test results and analytical work, forms the basis of this method. No material factors are provided.

The mean pull-out strength of a single headed anchor embedded in uncracked concrete displaying a full cone pull-out failure is given by

$$T_c = 17 h_e^{3/2} \sqrt{f_c} \quad (\text{N}), \quad (6)$$

The coefficient of variation of the measured and predicted loads using the above equation in some 100 tests was 14%. The statistical analysis indicated that for commonly used headed anchors the diameter of the anchor was not an important variable. In this method it is recommended that a splitting failure can be avoided if  $f_c \geq 20$  MPa;  $c \geq 1.5h_e$ ;  $s \geq 3.0h_e$  and  $t \geq 2.0h_e$ .

Laboratory tests conducted by these researchers have shown that the pull-out strength, as well as the stiffness of headed anchors, can be significantly reduced in the presence of cracks. For a crack width of 0.4 mm the pull-out strength in several tests decreased to between 25-45% of the pull-out strength in uncracked concrete.

Experimental work has also shown that anchor pull-out failure occurs when the bearing pressure at the head of the anchor is approximately 12 to 15 $f_c$ .

The reduction factor due to the overlapping of the failure surfaces or to edge effects is given as the product of several individual factors evaluated for two orthogonal directions,

$\xi_R = \psi_{sx} \psi_{sy} \psi_{cx} \psi_{cy}$ , where  $\psi_{sx}$  and  $\psi_{sy}$  account for the proximity between anchors in the x and y direction, respectively, and  $\psi_{cx}$  and  $\psi_{cy}$  account for the edge effects in the x and y directions, respectively. These factors are defined as:

$$\psi_{sx} = \{1 + (n_x - 1) (s_x/s_{cr})\} / n_x \leq 1, \text{ in which } n_x \text{ is the number of anchors in the x-direction, } s_x \text{ is the centre-to-centre distance in the x-direction and } s_{cr} \text{ is the critical distance taken as } s_{cr} = 3h_e.$$

$$\psi_{cx} = \{0.3 + (0.7c_y / 1.5h_e)\} \leq 1, \text{ where } c_y \text{ is the edge distance in the } y\text{-direction.}$$

Expressions for  $\psi_{sy}$  and  $\psi_{cy}$  are analogous to the previous two relations.

Figure 2.3 illustrates the meaning of parameters  $s_x$ ,  $s_y$ ,  $c_x$  and  $c_y$ .

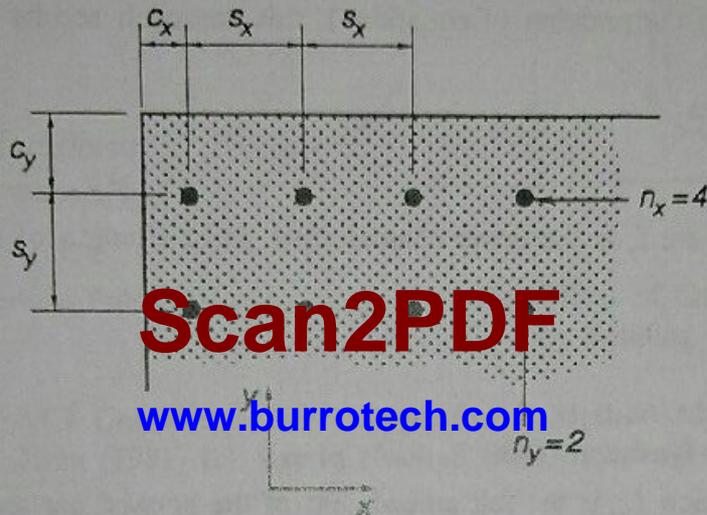


Figure 2.3 - Definition of Parameters Used by the  $\psi$ -Method.

### 2.3 Tensile Capacity of Hooked Bars with Short Anchorage Lengths in Concrete

The available research work conducted to assess the tensile capacity of hooked bars with short anchorages in wall members is rather limited. Johnson and Jirsa (1981), based on experimental work, concluded that two approaches can be used in design.

In the first approach they recommended the use of the proposal by ACI Committee 408 (1979) for the design of hooked bar anchorages with the following added restrictions:

- (a) Only standard  $90^\circ$  hooks are to be used.
- (b) The embedment length shall be measured from the start of the bend in the hook.
- (c) Bars with diameter larger than 44 mm shall not be used.

- (d) The spacing between hooked bars must be  $s \geq 12d_b$ , where  $d_b$  is the bar diameter.
- (e) No reduction in anchorage length shall be considered for confinement effects.

Conforming to the notation of equation 1, this approach results in:

$$\lambda_s = 1.0$$

$$f_s = f_y, \text{ where } f_y \text{ is the lower characteristic yield strength of the anchored bar.}$$

$$\phi_c = 0.80$$

$$T_C = 4 \sqrt{f'_c} \ell_{dh} d_b / \gamma \quad (\text{N}), \quad (7)$$

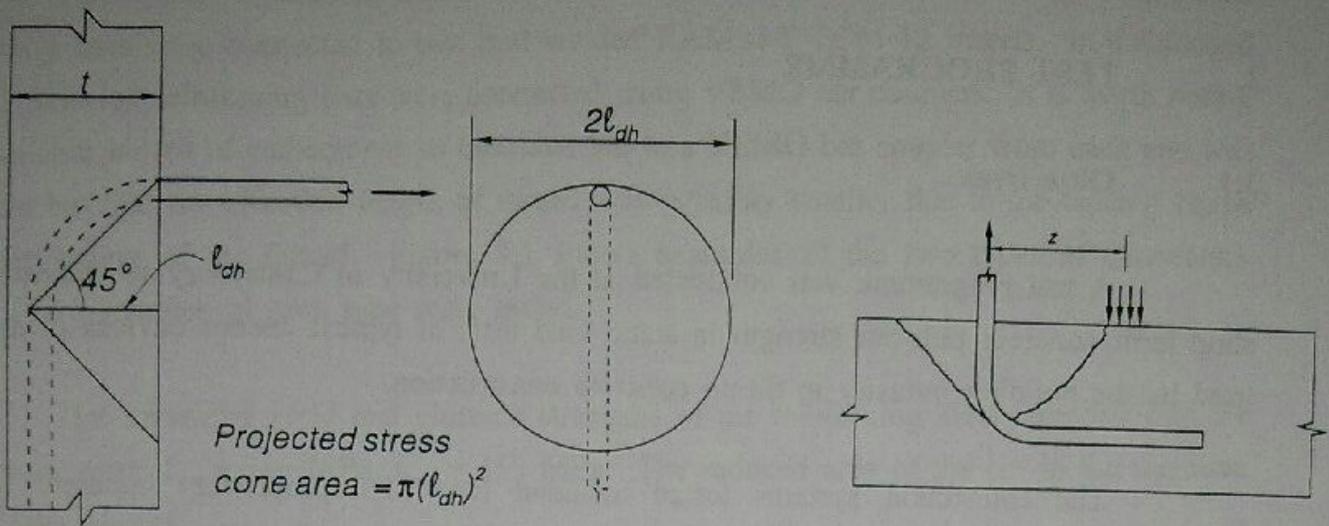
in which  $\ell_{dh}$  is the full embedment of the hooked bar and  $\gamma$  is either 1.0 or the product of the several applicable factors, among them:

0.7 for bars with diameter 28 mm or smaller with side cover normal to the plane of the hooked bar not less than 38 mm and cover on the tail extension of 90° hooks not less than 50 mm.

0.8 for additional confinement by closed stirrups or hoops at a spacing of  $3d_b$  or less.

The second approach is based on the recommendations made by the ACI Committee 349 (see Section 2.2.1) assuming an eccentric 45° pull-out concrete cone. The cone has its origin at the outside edge of the hook, as shown in figure 2.4a. Johnson and Jirsa observed in their experimental work that when the tensile force in the anchored bar is induced by bending of an adjacent member, the compressive force on the concrete caused by the bending may restrict the surface failure and increase its pull-out strength (see figure 2.4b). This effect was more noticeable in similar tests when the lever arm between the tensile and compressive forces was reduced. On this basis they modified the expression recommended by ACI Committee 349 and postulated that

$$T_C = 292 \ell_{dh}^2 \sqrt{f'_c} / z \quad (\text{N}), \quad (8)$$



(a) Eccentric Pull-out Cone

(b) Effect of Compressive Force Induced by Bending

Figure 2.4 - ACI Committee 309 - Appendix B Method Modified by Johnson and Jirsa (1981) for Use in Hooked Bar Anchorages.

### 3. TEST PROGRAMME

#### 3.1 Objectives

A test programme was conducted at the University of Canterbury to determine the short-term concrete pull-out strength in static load tests of typical anchor devices of systems used by the building industry in tilt-up concrete construction.

The connection systems tested included RAMSET proprietary concrete inserts, VEMO bar couplers, and hooked bars with embedment lengths shorter than those required by the Concrete Design Code (NZS 3101, 1982).

The main parameters studied were:

- (a) The capacity of the proprietary concrete inserts and steel couplers under axial tensile load.
- (b) The influence of the concrete compressive strength on the pull-out capacity of the embedments in relatively poorly cured walls.
- (c) The effect of the compression reactions normal to the wall on the capacity of hooked anchorages with short embedment lengths.
- (d) The possible detrimental effect of the small thickness of the prototype concrete wall.
- (e) The effect of a transverse rod placed through the eye-hole at the headed base in the RAMSET concrete inserts.

#### 3.2 Tensile Tests on Isolated Proprietary Connectors

Tensile tests were first performed on the proprietary connectors (not embedded in concrete) to verify that in practice failure of the connector due to thread stripping or to failure of the connector itself does not occur. The connectors chosen for the tests were 12 mm diameter bars since it is common practice in New Zealand to use threaded D12 reinforcing bars for connecting tilt-up walls with the slab floors.

Two types of tension tests were conducted. In the first set two D12 threaded reinforcing bars were connected to two butt welded RAMSET TCM-12 inserts. In the second set D12 threaded reinforcing bars were connected using VEMO bar couplers. It is worth noting that available length of embedment of threaded bar in a VEMO bar coupler from each end was 17.5 mm but that the effective length of thread was possibly smaller due to the tapered shape at the beginning of the thread. Figure 3.1 shows examples of the two types of connectors tested. Two samples of each type were tested.

The measured yield and ultimate strengths of the reinforcing steel threaded into the connectors were:  $f_{ym} = 346$  MPa,  $f_{sum} = 453$  MPa. The reduced area of the bar in the threaded region was  $84 \text{ mm}^2$ .

All tensile tests performed indicated that the ultimate strength of the D12 bars, based on the reduced bar area, could be attained by the connectors. At ultimate load stripping of the thread was observed. This resulted because of the reduction of the diameter of the threaded bar due to necking in the connection region.

From the results of these tension tests, it can be concluded that the use of RAMSET TCM-12 inserts and 12 mm VEMO bar couplers enable the development of the ultimate tensile strength of bars coupled using them. An aspect that deserves close attention during construction is to ensure that threaded reinforcing bars spliced by VEMO bar couplers similar to the ones tested will have an identical embedment at each side of the sleeve. An alternative

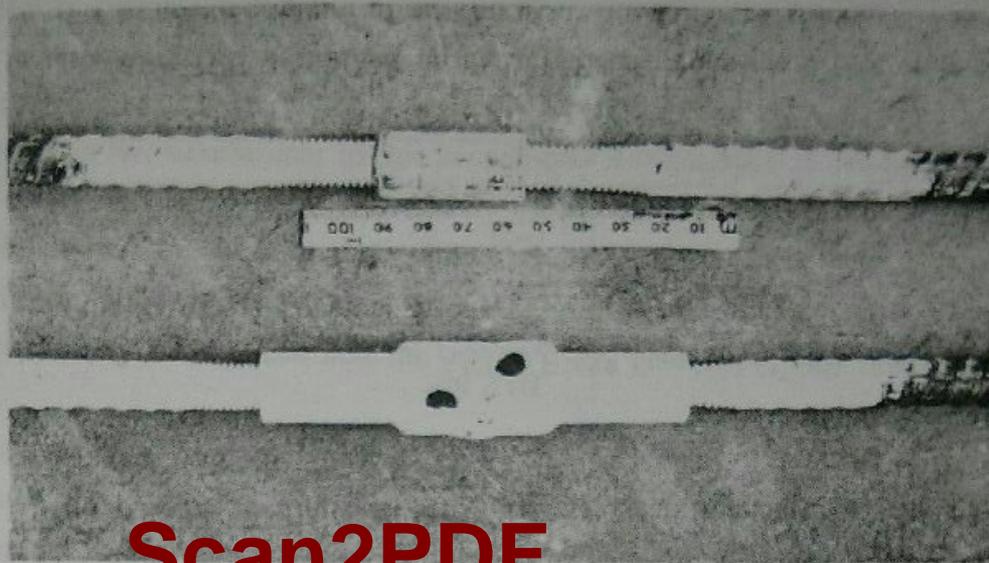


Figure 3.1 - Close-up of Proprietary Connectors after a Tensile Test (Top: two RAMSET proprietary inserts welded together, Bottom: VEMO bar coupler).

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solution, which is highly recommended, is to use commercially available bar couplers with stoppers at the middle of the sleeve (see figure 1.1c).

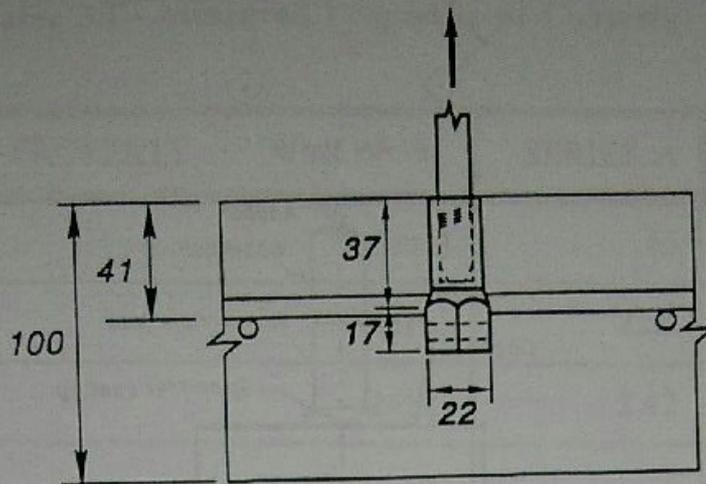
### 3.3 Pull-out Tests on Connectors Embedded in Concrete

Figure 3.2 shows the types of connections embedded in concrete selected for testing. The first and second connection types shown in figures 3.2a and 3.2b are RAMSET TCM-12 concrete inserts. The difference between these two arrangements is whether or not the 6.3 mm diameter transverse bar was present passing through the eye-hole at the headed base of the anchor. The transverse bars were from cuts of the 663 welded wire mesh that was used for reinforcing the concrete member. Figure 3.2c depicts the anchorage details of the D12 hooked reinforcing bar. A continuous transverse bar of the same diameter was tied in contact with the inside of the bend, as it is usually done in practice.

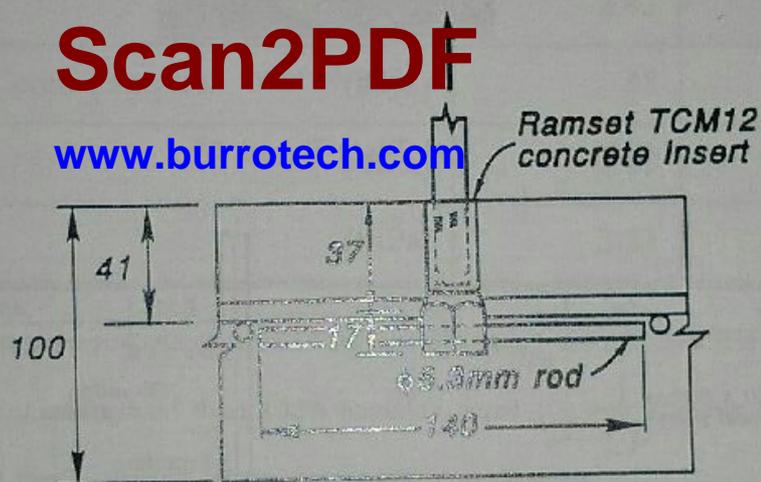
Three connections of the same type were cast in a 1 m wide by 2 m long slab with a centre-to-centre spacing between connectors of 250 mm and an edge distance of 250 mm. A member thickness of 100 mm was chosen to represent a 125 mm thick tilt-up wall panel with a rebate of 25 mm which could be used to seat a proprietary precast concrete floor system.

The slabs in which the connectors were embedded were constructed with different concrete strengths. Series A had a target concrete compressive cylinder strength of 20 MPa while Series B had a target of 40 MPa. Aggregate with a maximum size of 19 mm was specified. The concrete was supplied by Firth Concrete. The slabs were cast horizontally and cured for three days only. No curing compound was sprayed onto the surfaces of the slabs in order to simulate the worst scenario of relative poor curing practice. The measured properties of the concrete are given in table 3.1.

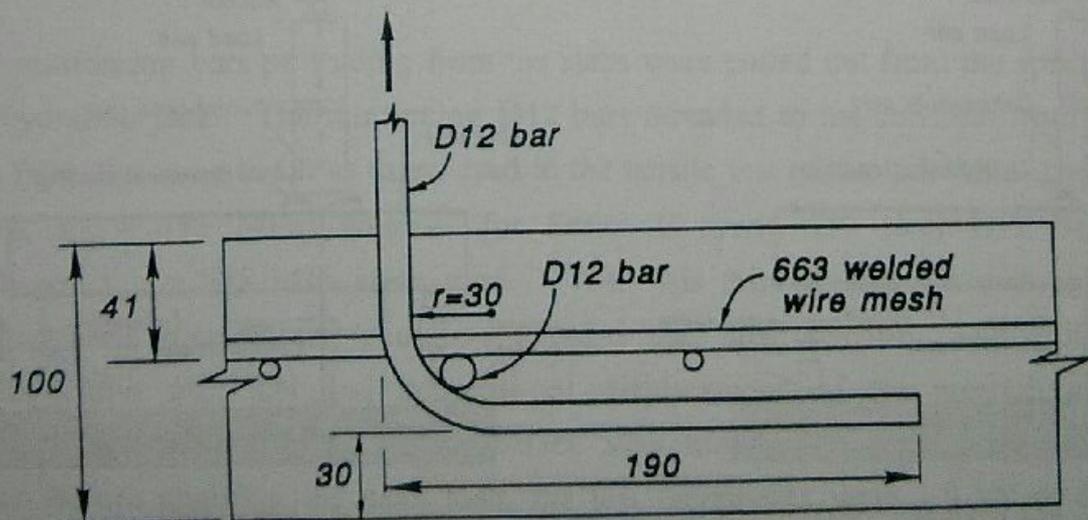
The test set-up conformed with the ASTM E 488 Standard (1990). Figure 3.3 depicts the test arrangement. For the tests on the RAMSET TCM-12 concrete inserts the positions of the four compressive reaction forces (see figure 3.3a) were chosen to be such that they would not enhance the capacity of the concrete inserts nor cause premature bending failure of the concrete slabs. The test arrangement for the hooked bars was designed to compare the possible restraining effect induced by the presence of a compression reaction force near the stem of the hook of the reinforcing bar (see figure 3.3b and c). During casting of the concrete it was difficult to exactly obtain the target member thickness of the concrete slabs and therefore the actual effective embedment length of the connectors was estimated from measurements taken at four different locations in the vicinity of the anchor.



(a) Test Series T - Concrete Inserts without Transverse Rods

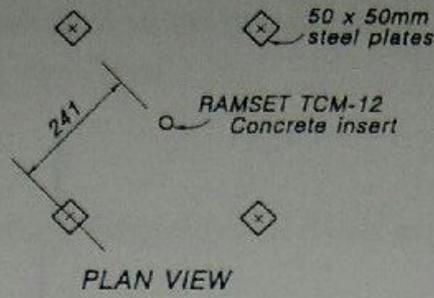


(b) Test Series TR - Concrete Inserts with Transverse Rods

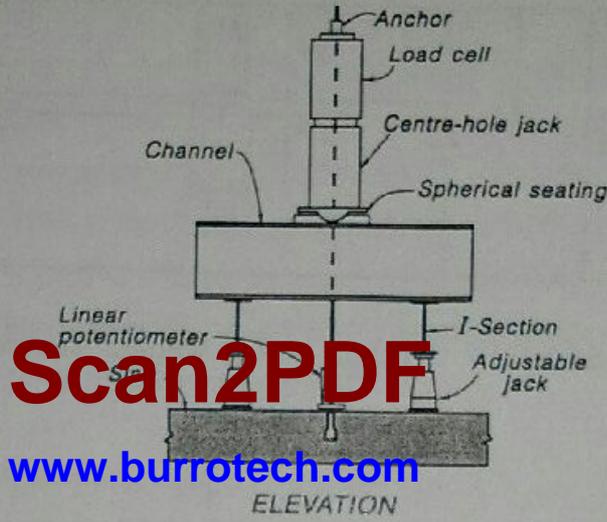


(c) Test Series HU and HR - Hooked Bar Anchorages

Figure 3.2 - General Details of Connectors Tested.

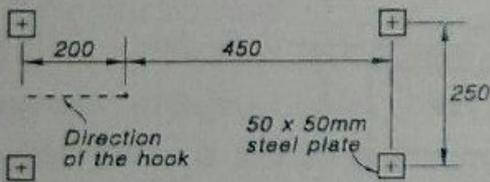


PLAN VIEW

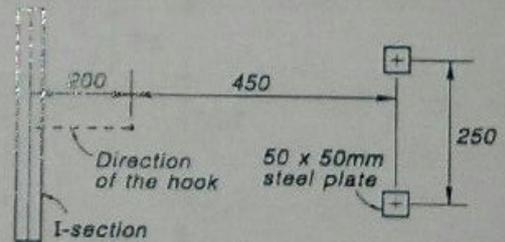


ELEVATION

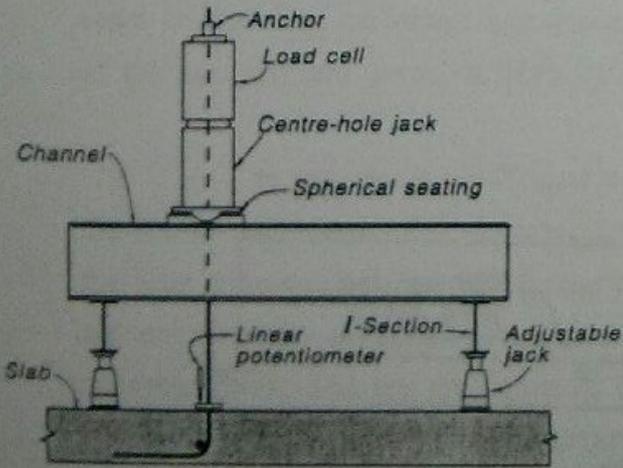
(a) Tests T and TR



PLAN VIEW

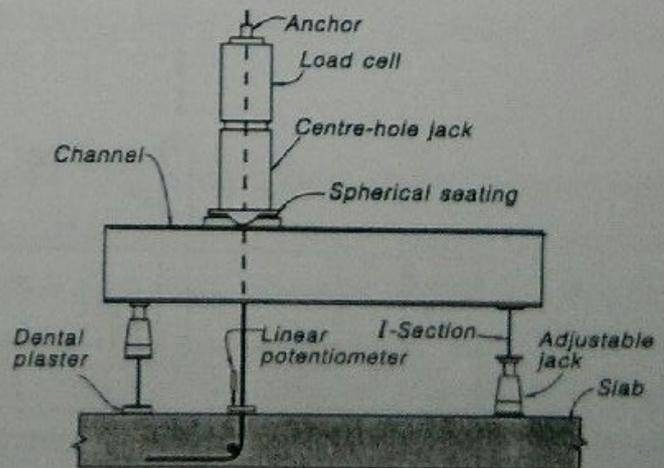


PLAN VIEW



ELEVATION

(b) Tests HU



ELEVATION

(c) Tests HR

Figure 3.3 - Test Arrangement

Table 3.1 - Measured Properties of Concrete

PROPERTY	SERIES A	SERIES B
Slump (mm)	45	35
$f'_c$ at 28 days <sup>(1)</sup> (MPa)	22.5	47.6
$f'_{sp}$ at 28 days <sup>(2)</sup> (MPa)	2.42	4.20
Age at Test (days)	28	32
$f'_c$ at Test <sup>(1)</sup> (MPa)	22.5	47.6
$f'_{sp}$ at Test <sup>(2)</sup> (MPa)	2.42	3.80
Age at Tests on Cored Samples (days)	48	39
$f'_t$ <sup>(3)</sup> (MPa)	2.80	3.05
$f'_{sp}$ <sup>(4)</sup> (MPa)	3.33	5.08

Notes:

- (1) Average compressive strength of three 100% humidity cured 100 mm diameter by 200 mm concrete cylinders.
- (2) Average splitting strength of three 100% humidity cured 100 mm diameter by 200 mm concrete cylinders.
- (3) Average direct tensile strength of two notched 75 mm diameter cored concrete cylinders.
- (4) Average splitting strength of three 50 mm diameter by 100 mm concrete cylinders.

The reinforcing bars protruding from the slabs were pulled out from the specimen by a centre-hole hydraulic jack. The reinforcing D12 bars threaded to the concrete inserts in the Series A were from the same batch as those used in the tensile test of the individual connectors ( $f_{ym} = 346$  MPa,  $f_{sum} = 453$  MPa) whereas for Series B grade 430 steel D12 bars with  $f_{ym} = 425$  MPa and  $f_{sum} = 588$  MPa were used. With this reinforcement it was practically assured that a concrete cone failure would occur rather than steel fracture. The applied load was monitored within  $\pm 0.1$  kN and the pull-out displacement of the protruding rod at approximately 5 mm above the concrete surface was measured using a 30 mm linear potentiometer. Before attaining the peak load, the test increments were 1.0 kN or 0.1 mm, whichever was the smallest. The peak load was attained after 6 to 10 minutes from the initiation of the test. The post-peak response was displacement controlled in increments which varied from 0.5 to 1.0 mm. Cracking around the anchorage was also monitored during the test.

### 3.4 Test Results

#### 3.4.1 Tests on the RAMSET Concrete Inserts

Table 3.2 summarizes the test results obtained in the experimental programme. Also shown in this table are the effective embedment lengths,  $h_e$ , estimated from the local thickness of the constructed slab and the assumed origin of the surface failure, that will be discussed later in this report. Except for the tests on RAMSET TCM-12 concrete inserts without a transverse rod, the projected failure surfaces, visible on the surface of the slab, were unexpectedly large, rendering most of the remaining central specimens ineffective for analysis.

Figure 3.4 illustrates the measured load versus displacement pull-out behaviour of the RAMSET TCM-12 test specimens. The key to the labels on the curves is given at the bottom of table 3.2. In all tests the ascending branch before the peak load was stable and stiff. These characteristics indicate that crushing of the concrete bearing against the headed base of the anchors did not occur. The vertical component of the bearing stresses on the concrete ranged between  $3.3$  and  $7.6f'_c$ . Upon peak load a flat truncated cone popped off in a very brittle way, leaving a peripheral crack pattern. The post-peak response displayed in figure 3.4 has a very steep descending branch. The base of the truncated cone was located at the beginning of the headed base in all tests on RAMSET TCM-12 concrete inserts without transverse rods (see figure 3.5). In the tests including transverse rods the truncated cone was pulled-out from the level at which the rods were placed. This observation can be used to explain the higher load carrying capacity measured in the tests having a transverse rod. The eventual kinking of the transverse rods also enhanced the post-peak performance. Figure 3.6 shows a close-up of one of the specimens tested with a transverse rod. However the overall performance of this arrangement with a transverse rod cannot be considered to provide a large deformability at near full strength.

#### 3.4.2 Tests on Hooked Bars

The measured tensile load versus bar pull-out performance of the hooked reinforcing bars is illustrated in figure 3.7. No major differences in strength or load-displacement behaviour were noticed between the specimens in which the failure surfaces were restrained or not restrained by the external compression reaction (see figures 3.3b and c). Hence it can be concluded that the restraining of the surface failure at 200 mm from the protruding reinforcement had no effects. Figures 3.8 and 3.9 show two of these tests. The test arrangement devised for restraining or not the surface failures can clearly be seen in these

figures. This recorded behaviour contradicts Johnson and Jirsa's observations. The main reason appears to be that any enhancement of the load carrying capacity should be given as a proportion of the ratio between the effective embedment length and the distance between the origin of the pull-out cone and the restraining plane, instead as a proportion of lever arm distance alone as they proposed.

One major difference observed in these tests was that the post-peak behaviour of the specimens in Series A, with lower concrete strength, was reasonable stable and displayed good

Table 3.2 - Test Results

TEST	Series A - $f'_c = 22.3 \text{ MPa}$		Series B - $f'_c = 47.6 \text{ MPa}$	
	Effective Embedment Depth $h_e$ (mm)	Pull-out Strength $T_c$ (kN)	Effective Embedment Depth $h_e$ (mm)	Pull-out Strength $T_c$ (kN)
T-L	32.5	19.1	41.3	31.1
T-C	31.5	22.8	43.3	29.1
T-R	31.9	17.9	42.5	28.2
TR-L	44.4	31.1	46.3	39.0
TR-C	44.5	28.0	-	-
TR-R	43.8	28.5	46.3	33.7
HU-L	50.1	31.4	51.0	40.6
HU-R	50.2	32.5	51.3	36.0
HR-L	46.5	33.1	51.1	35.4
HR-R	48.1	30.1	51.8	42.8

Key:

- T- RAMSET TCM-12 concrete insert without transverse rod.
- TR- RAMSET TCM-12 concrete insert with transverse rod.
- HU- Hooked D12 bar with unrestricted surface failure.
- HR- Hooked D12 bar with restricted surface failure.
- L embedment located at the left side of the prototype wall.
- C embedment located at the centre of the prototype wall.
- R embedment located at the right side of the prototype wall.

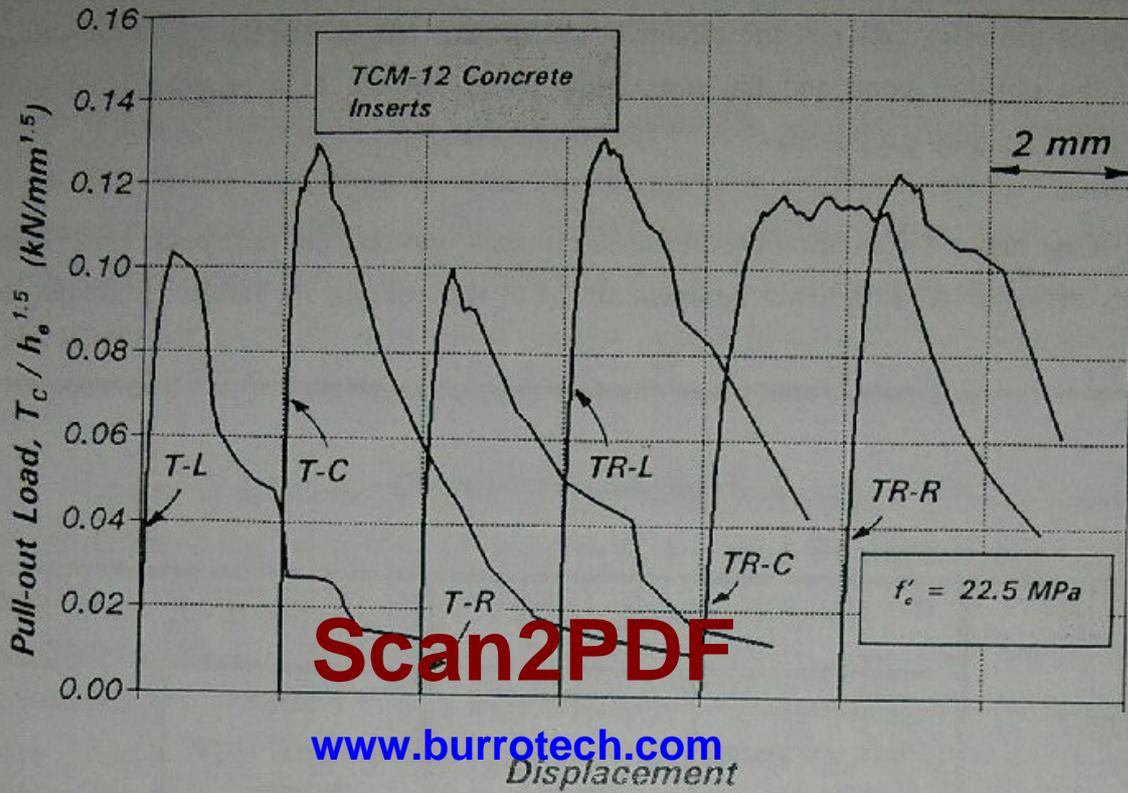
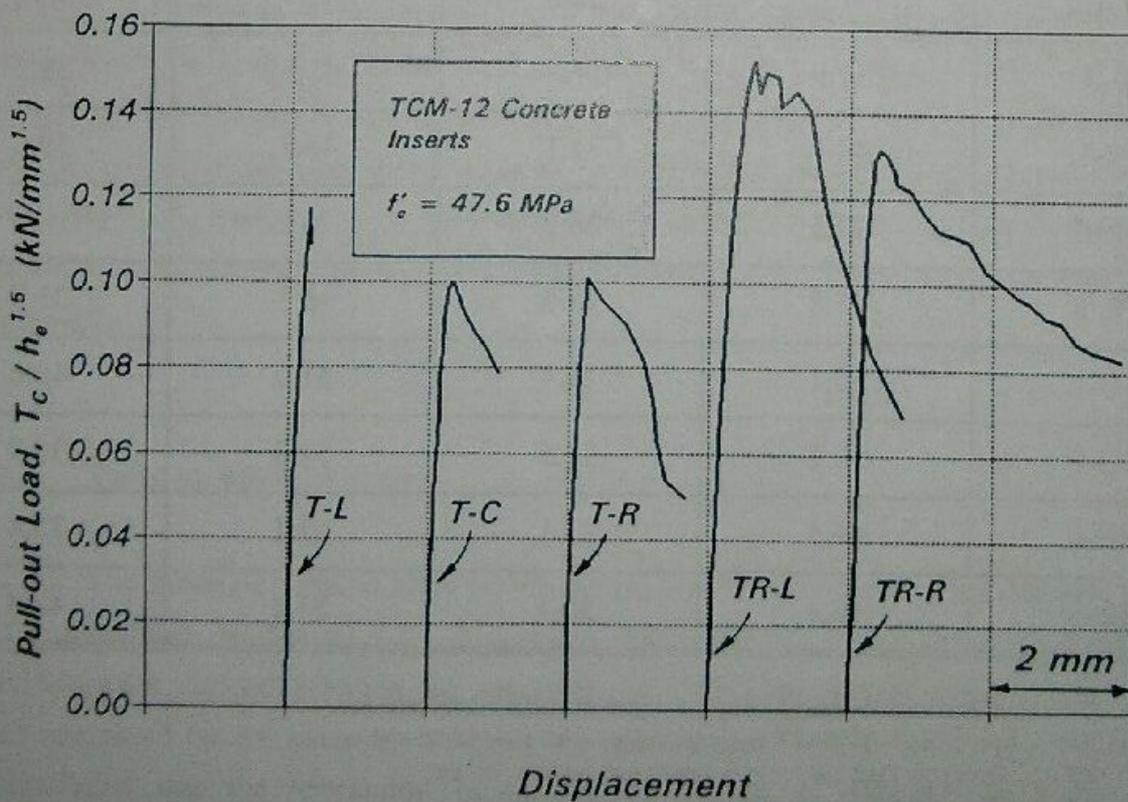
(a) Series A -  $f'_c = 22.5 \text{ MPa}$ (b) Series B -  $f'_c = 47.6 \text{ MPa}$ 

Figure 3.4 - Results of Tests on RAMSET TCM-12 Concrete Inserts.

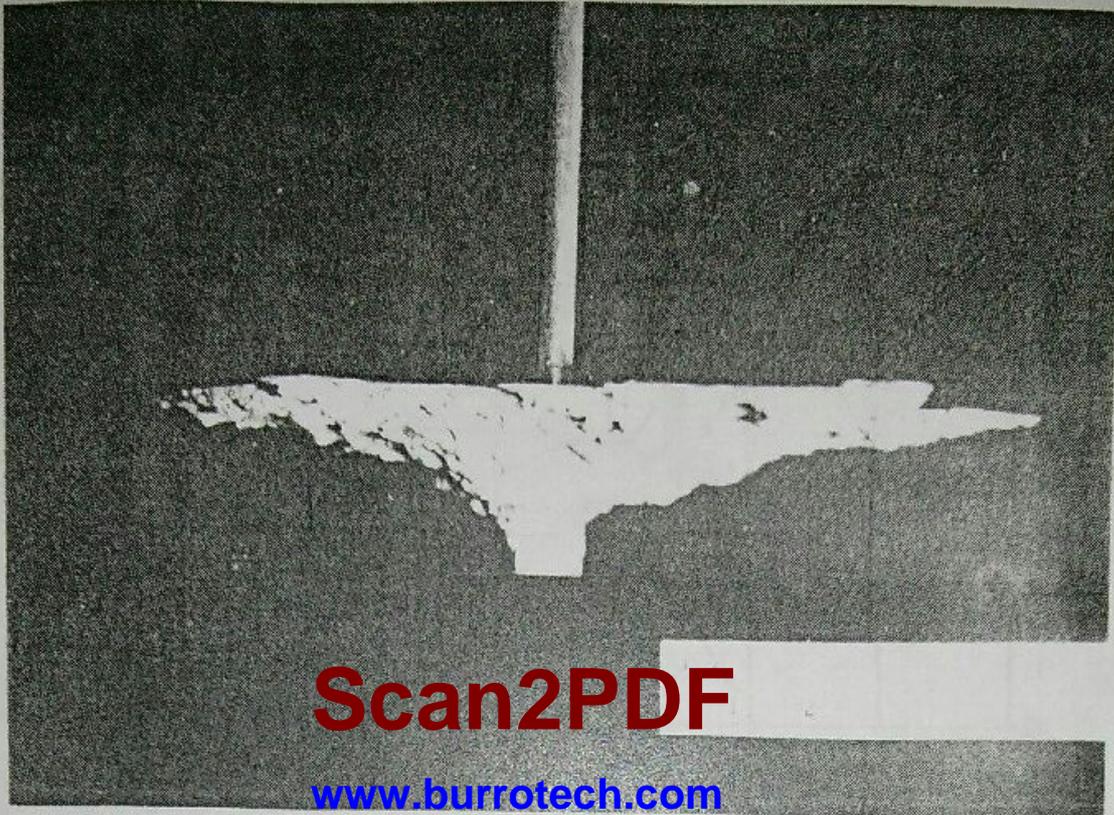


Figure 3.5 - Truncated Failure Cone of Concrete around a RAMSET TCM-12 Concrete Insert.

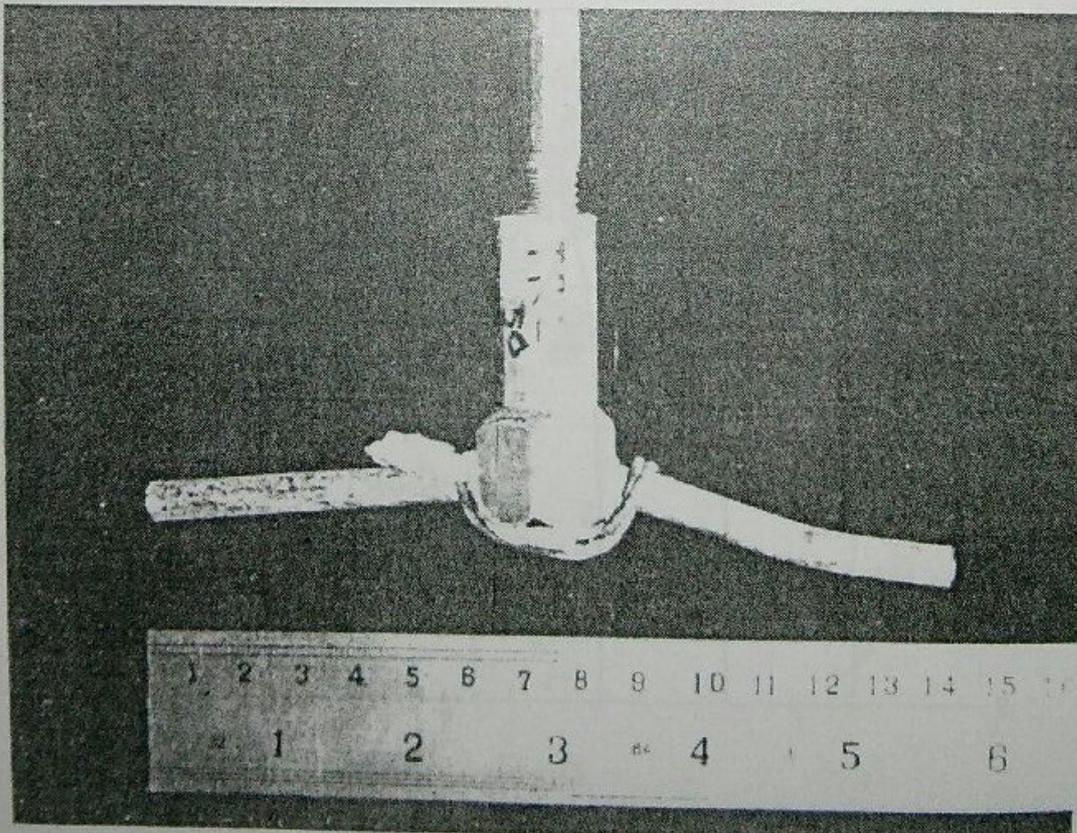


Figure 3.6 - Close-up of a RAMSET TCM-12 Insert with a Transverse Rod after Testing.

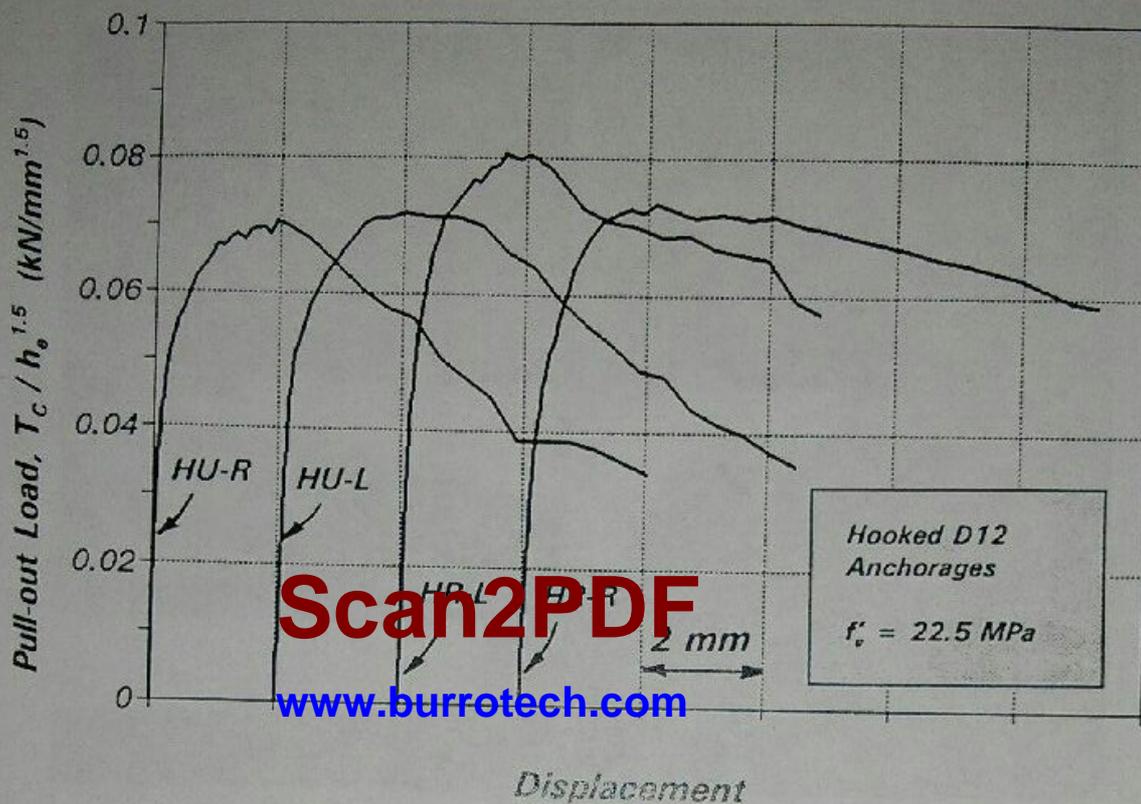
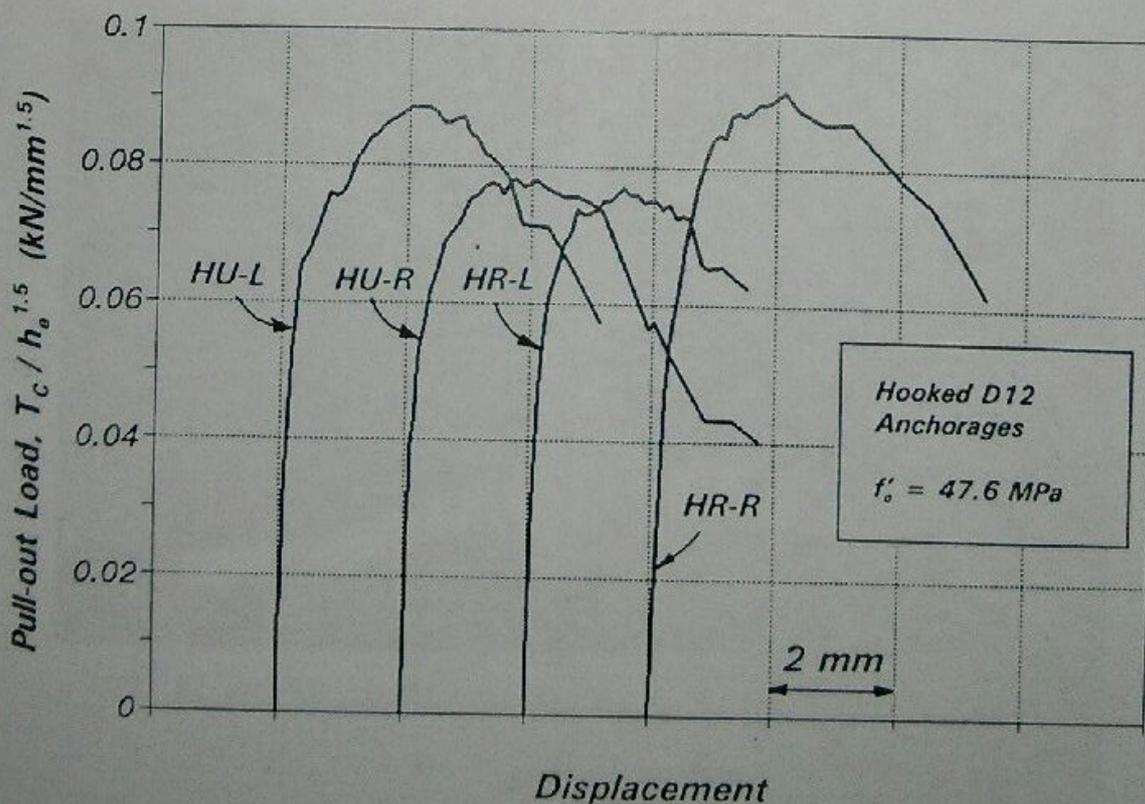
(a) Series A -  $f'_c = 22.5$  MPa(b) Series B -  $f'_c = 47.6$  MPa

Figure 3.7 - Results of Tests on Hooked Bar Anchorages.



Figure 3.8 - Crack Pattern of a Hooked Bar Pull-out Test with the Failure Surface Unrestrained by External Compression Force.



Figure 3.9 - View of a Hooked Bar Pull-out Test with a Failure Surface Restrained by Nearby External Compression.

deformation characteristics. In Series B a reasonable post-peak behaviour was also observed but not as stable as for the Series A, presumably due to the slightly more brittle nature of the higher strength concrete. The good post-elastic performance of these tests is due to the effect of the transverse bar in contact with the inside of the hook. As can be seen in figure 3.9 this bar kinked and prevented the hooked bar from totalling pulling out. Similar behaviour was also reported by Johnson and Jirsa (1981). They also pointed out that the transverse rod had no enhancing effects on the cone pull-out strength.

Another observation made during the test is that the relatively small thickness of the slabs did not appear to have a detrimental effect on the pull-out strength of the hooked anchorages. The effective embedment length  $h_e$  to wall thickness for these tests was about 0.50, a similar than that recommended by Peden and Roik (1987) and the  $\psi$ -method (Comité Euro-International du Béton, 1991).

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### 3.4.3 Pull-Out Strength Compared with the Tensile Yield Strength of the Reinforcing Bars

It was of interest to note that in the tests on RAMSET inserts without transverse rods the pull-out strengths attained by the concrete truncated cone failures were in the range of 61-87% of the yield forces calculated on the basis of the reduced area of the bar in the threaded region. In tests on RAMSET inserts with transverse rods, pull-out strengths close to or slightly above the yield force of the reinforcing bars, calculated using the reduced area of the bars, was observed in both series of tests (96-107% and 94-109% in Series A and B, respectively). The pull-out strengths measured in the tests HU and HR were in the range 77-85% and 91-109% in Series A and B, respectively, of the yield force calculated using the nominal area of the bars.

## 4. DISCUSSION

### 4.1 Prediction of the Tensile Capacity

Table 4.1 contains the pull-out capacity of the RAMSET TCM-12 concrete inserts tested as predicted by equations 2, 3, 4 and 6 of the methods described in Section 2.2. These specimens were chosen for comparison since all methods have been previously calibrated for anchor bolts with similar shapes.

It is evident that the  $\psi$ -method (Comité Euro-International du Béton, 1991) and the method proposed by Bode and Roik (1987) give the closest prediction of the test results obtained. Both the ACI Committee 308 Appendix B (1985) and PCI Design Handbook (1985) methods give a significant underestimation of the concrete pull-out strength.

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Table 4.1 - Measured and Predicted Pull-out Strengths of RAMSET Concrete Inserts - Series T

Test	$f'_c$ (MPa)	Pull-out Strength $T_c$ (kN)	Predicted Pull-out Strength <sup>(1)</sup>			
			$T_c$ (kN)			
			ACI-349 Eq. 2	PCI method Eq. 3	Bode- Roik <sup>(2)</sup> Eq. 4	$\psi$ - method Eq. 6
T-L	22.5	19.1	8.7	5.0	14.0	14.9
T-C		22.8	8.3	4.7	13.5	14.3
T-R		17.9	8.5	4.8	13.7	14.5
T-L	47.6	31.1	18.7	11.8	26.7	31.1
T-C		29.1	20.2	12.9	28.2	33.4
T-R		28.2	19.6	12.5	27.6	32.5

Notes:

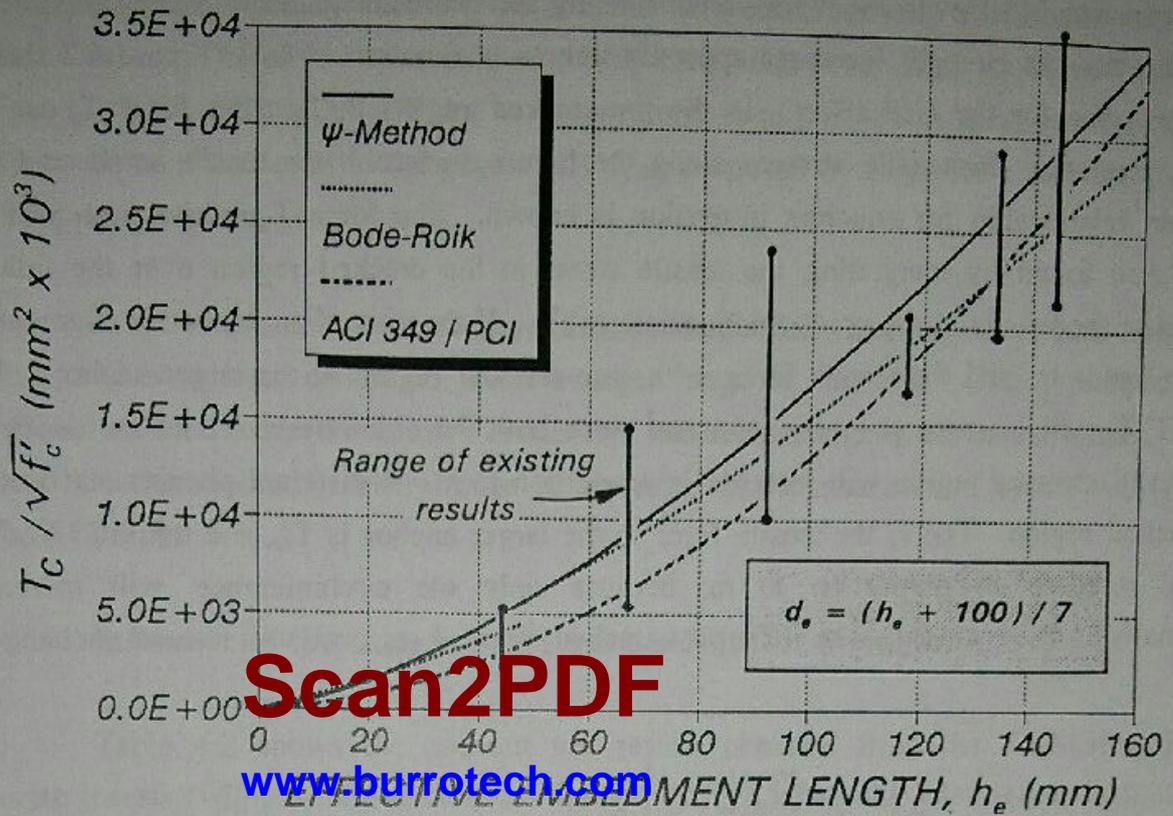
(1) Calculated using  $\phi_c = 1$

(2)  $T_c$  reduced by 20% since  $h_c < 50$  mm

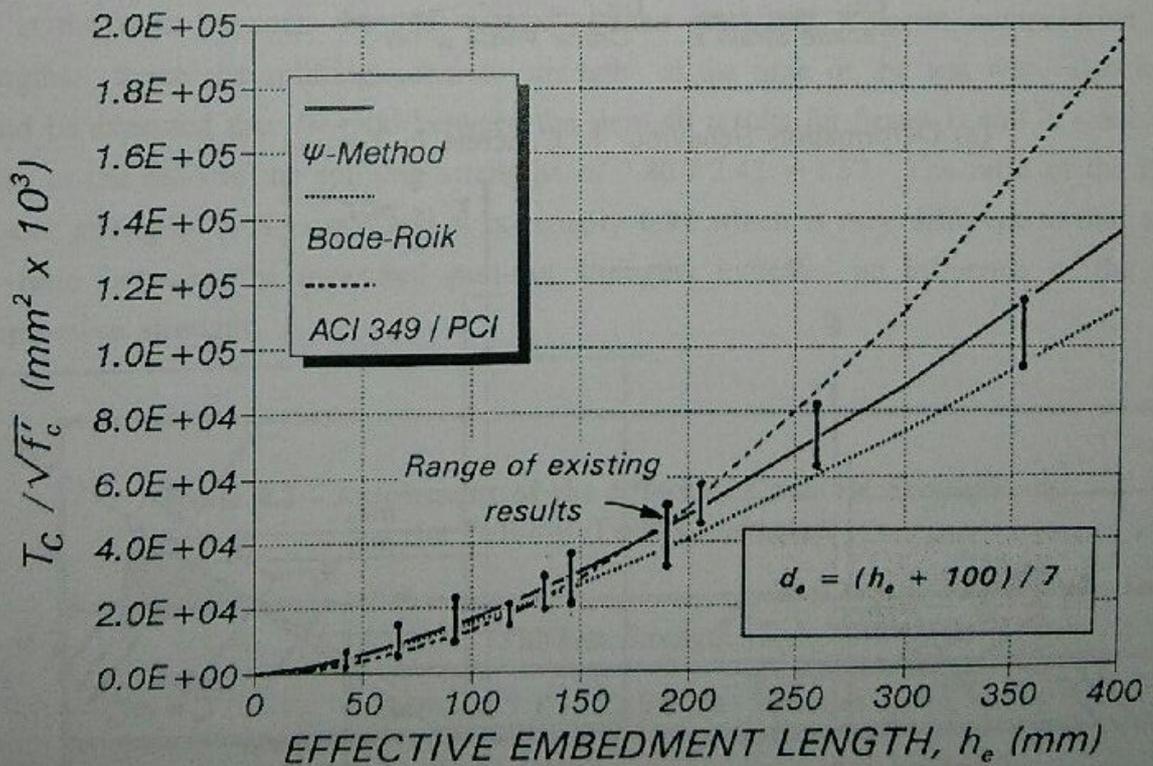
A plot of the predicted concrete pull-out strengths against the effective embedment length of the methods described in Section 2 is illustrated in figure 4.1. Evidently the ACI Committee 349 or PCI Design Handbook approaches are very conservative for small embedment lengths. However, for deep embedment lengths the ACI Committee 349 and PCI Design Handbook methods predict a concrete pull-out strength larger than the other two methods. There are two main reasons which explain these differences. First, the ACI Committee 349 and PCI Design Handbook methods were never calibrated against results obtained from tests with shallow or deep embedment lengths. Second, recent research (Comité Euro-International du Béton, 1991) has shown that the actual assumption of a uniform "shear stress" around the idealized 45° truncated cone concrete failure is conservative for widely spaced anchors with small or medium embedment lengths but can become unconservative if used under other conditions. For instance, the degree of conservatism given by the ACI Committee 349 and PCI Design Handbook methods when used for grouped shallow anchors can be significantly reduced since overlapping of the small individual failure regions is only considered when their centre to centre spacing is less than  $2h_e$ . It is apparent that to properly account for many variables of embedment length, spacing of anchors and edge distance, the actual shape of the failure cone (not necessarily a 45° surface) and the stress along the failure surface (not uniform tension) need to be considered more accurately.

Extensively instrumented pull-out tests in headed anchors have been conducted by Eligehausen and Sawade (Comité Euro-International du Béton, 1991). Their tests have shown that the actual failure region gradually develops from the edge of the bearing end. The crack growth is stable up to peak load. Based on these measurements they were able to correlate the pull-out strength of headed steel connectors with that predicted using the known stress versus displacement behaviour of concrete in tension. From this study the authors concluded that the pull-out strength of a headed anchor should be a function of the whole stress-displacement relation of concrete in tension rather than on the tensile strength of the concrete alone. That is, a main parameter which should be included in the strength equations is the concrete tension fracture energy,  $G_f$ , which is the area under the concrete stress-displacement curve in tension. However, since the only parameter that the designer has control over is the concrete compressive strength, they concluded that for simplicity the fracture energy could be assumed proportional to  $\sqrt{f_c}$ , as the tensile strength of the concrete is often expressed in that manner.

From the measured development of the failure region and the typical stress-displacement behaviour of concrete in tension is possible to explain a size effect not recognized by the ACI Committee 349 and PCI Design Handbook approaches. In these two methods, it would be expected that the concrete pull-out strength,  $T_c$ , will increase by approximately 400% if the embedment length is doubled. Eligehausen and Sawade have tested headed anchors in



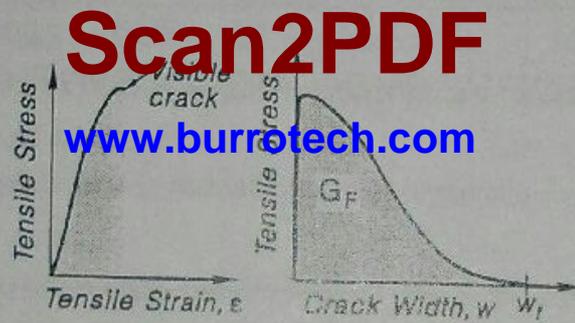
(a) Effective Embedment Length between 0 and 160 mm



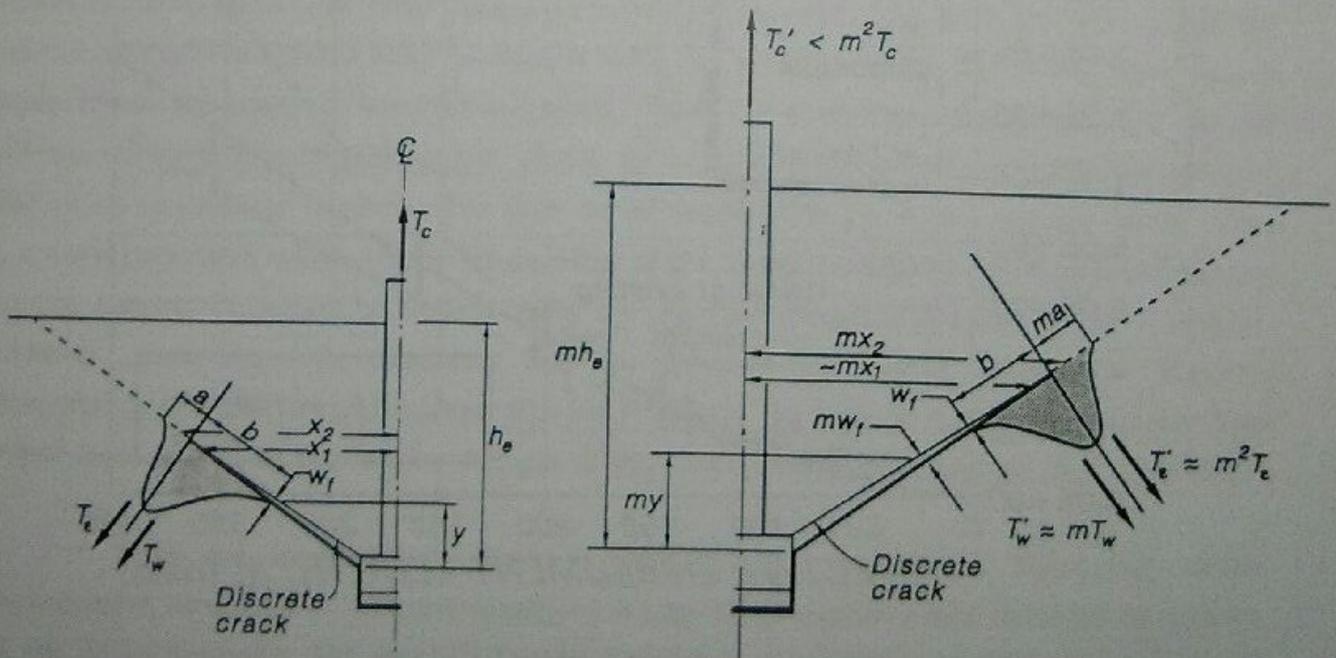
(b) Effective Embedment Length between 0 and 400 mm

Figure 4.1 - Comparison of Methods to Predict the Pull-out Strength of Headed Steel Anchors.

the same concrete block with the main variable being the embedment length. They found that the concrete pull-out strength increases approximately as a function of  $h_e^{1.5}$ . Figure 4.2 shows the main reasons for the size effect. In the pre-cracked region the tension force  $T_e$  can be found by integrating the tensile stresses along the failure surface if the tensile strain and the stress-strain relationship for concrete in tension is known. The force  $T_w$  in the post-cracked region is also found by integrating the tensile stress in the cracked region over the failure surface using the tensile stress crack width relationship. If the size of an anchor is augmented by a scale factor  $m$ , and the tensile force in the pre-cracked region of the larger anchor is  $T'_e$ , the ratio  $T'_e/T_e$  will increase proportional to  $m^2$  since both the circumference and the depth of the uncracked stressed region will increase in proportion to  $m$ . A different phenomena occurs in the cracked region. There, the tensile force in the larger anchor is  $T'_w$ , and the ratio  $T'_w/T_w$  will only increase in proportion to  $m$ , because only the circumference will increase proportionally to the scale factor  $m$ . The post-cracked stressed region  $b$  will remain unchanged



(a) Deformation Behaviour of Concrete in Tension



(b) Stress Distribution in Pull-out Tests

Figure 4.2 - Idealized Stress Distribution in the Failure Surface of Headed Anchors and the Effect of Scale.

in both cases since at crack width  $w_f$  the concrete will cease carrying any tensile stresses (see figure 4.2a). Therefore it would be expected that the total pull-out force ratio  $T'_C/T_C = (T'_e + T'_w)/(T_e + T_w)$  will not be proportional to  $m$  nor  $m^2$ . A good approximation for the proportional increase is to the power of 1.5, as proposed by Bode-Roik and the  $\psi$ -method, which appear to give good results.

In summary, the method proposed by Bode-Roik and the  $\psi$ -method seem more accurate and have been calibrated under a larger range of variables. The  $\psi$ -method will be adopted as the basis for the further discussion and recommendations in this study. It is of interest that this method is likely to be incorporated in the next edition of the ACI Building Code, ACI 318.

#### 4.2 Influence of the Concrete Strength on Pull-Out Strength

Table 4.2 shows the pull-out test results obtained from the RAMSET TCM-12 concrete inserts (without transverse rods) divided by  $h_e^{1.5}$ , where  $h_e$  is defined as in figure 4.3a. These results are presented that way to assess the influence of the concrete compressive strength,  $f'_c$ , on the pull-out strength. It would be expected that the ratio between the strength results shown in table 4.2 for Series B and A would be close to  $\sqrt{47.6} / \sqrt{22.5} = 1.45$ , since that is the ratio between the square root of their measured concrete compressive cylinder strengths. Since the splitting concrete strengths at the time of the test were also known, it would be expected that the ratio between the strength results for Series B and A would be even closer to the ratio of the splitting strengths of  $3.80 / 2.42 = 1.57$ . The ratio of the measured pull-out strengths of Series B and A is actually 0.97 which is very different to that expected. This ratio between the measured pull-out strengths indicates no influence of the concrete compressive strength.

**Table 4.2 - Assessment of the Effect of Concrete Strength on the Pull-out Tests Conducted**

Test	Pull-out Strength $T_C/h_e^{1.5}$ (kN/mm <sup>1.5</sup> )	
	Series A - $f'_c = 22.5$ MPa	Series B - $f'_c = 47.6$ MPa
T-L	0.103	0.117
T-C	0.129	0.102
T-R	0.099	0.102
Average	0.110	0.107

Splitting and direct tension tests on concrete cylinders cored from the slabs showed different trends and therefore no conclusive remarks can be made regarding the influence of the poor curing conditions on the tensile strength of the concrete. No explanation for the observation that the pull-out strength was independent of the concrete compressive strength was found.

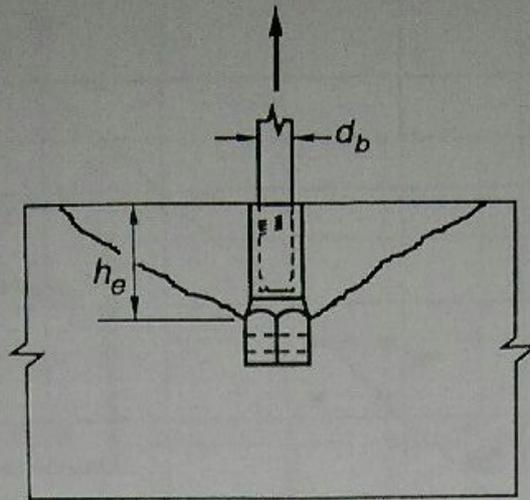
#### 4.3 Evaluation of the Effective Embedment Length of Steel Anchors

So far the discussion of the test results has concentrated on the T series of tests on RAMSET TCM-12 concrete inserts without a transverse rod passing through the eye-hole at the headed base. When a transverse rod is not present the effective embedment length  $h_e$  is defined as shown in figure 4.3a.

It was mentioned earlier that during the tests when the RAMSET TCM-12 concrete inserts had a transverse rod through their eye-hole, the observed pull-out cone failure originated at the level of the rod. In lieu of more test data it is suggested that the effective embedment length  $h_e$  of this type of arrangement be measured from the top part of the transverse rod as shown in figure 4.3b, providing that the transverse rod is tied so as to be in contact with the bottom end of the eye-hole. The extension of the transverse rod, measured from each side of the insert shall not be less than  $5d_b$  and its diameter shall not be less than  $d_b/2$ .

The test results obtained by Johnson and Jirsa (1981) provide a good data base to verify whether the expression developed by the  $\psi$ -method for headed anchors can be modified or adapted for the evaluation of the concrete pull-out strength of shallow hooked anchorages. The assumption for the effective embedment length (equal to the full embedment of the hooked bar  $\ell_{dh}$ ) postulated by these researchers does not appear realistic and does not conform with the experimental observations.

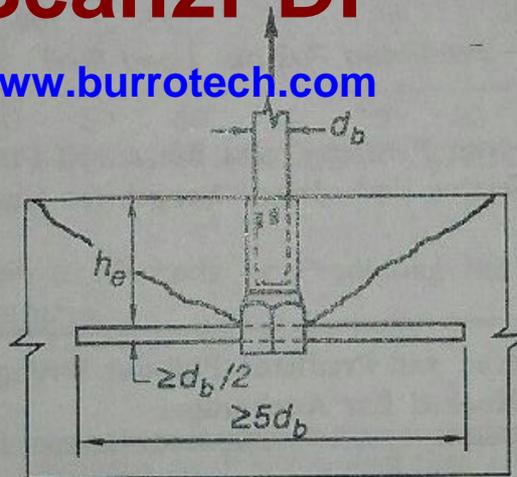
The failure cone for hooked anchorages is usually pulled out at some distance along the inside side of the bend as depicted in figure 4.3c. A statistical regression analysis was carried out in this study to find the effective embedment length and with it the predicted pull-out strength using the  $\psi$ -method. All tests with  $z'/h_e > 2.4$  (see figure 4.3c) were considered in the analysis. Tests with  $z'/h_e < 2.4$  were found to be gradually affected by the confining effect of the restraining compressive reaction applied at a distance  $z$  from the pull-out force. No attempt was made to account for this variable. The analysis indicated that the  $\psi$ -method predicts accurately the pull-out strength if the effective embedment length is taken as  $h_e = \ell_{dh} - 1.5d_b$ , as shown in figure 4.3c. Figure 4.4 shows the predicted against measured



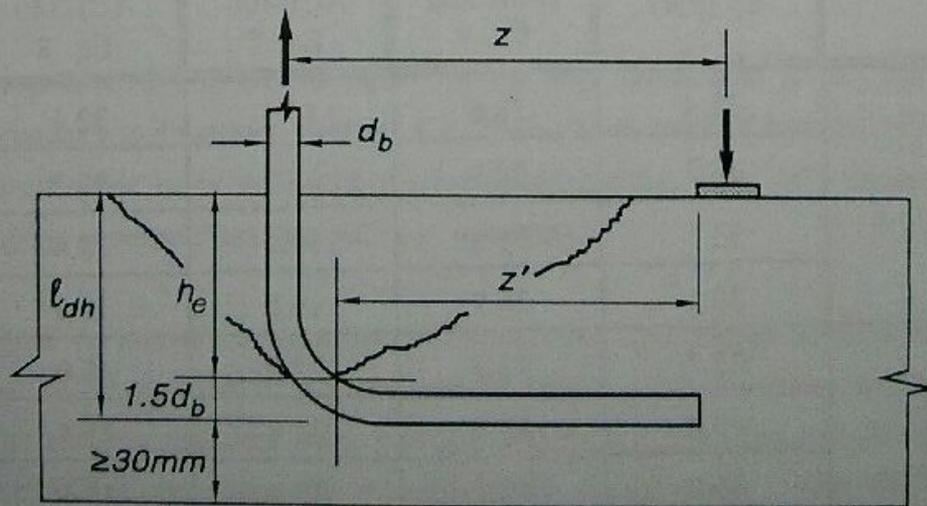
(a) Headed Insert without Transverse Rod

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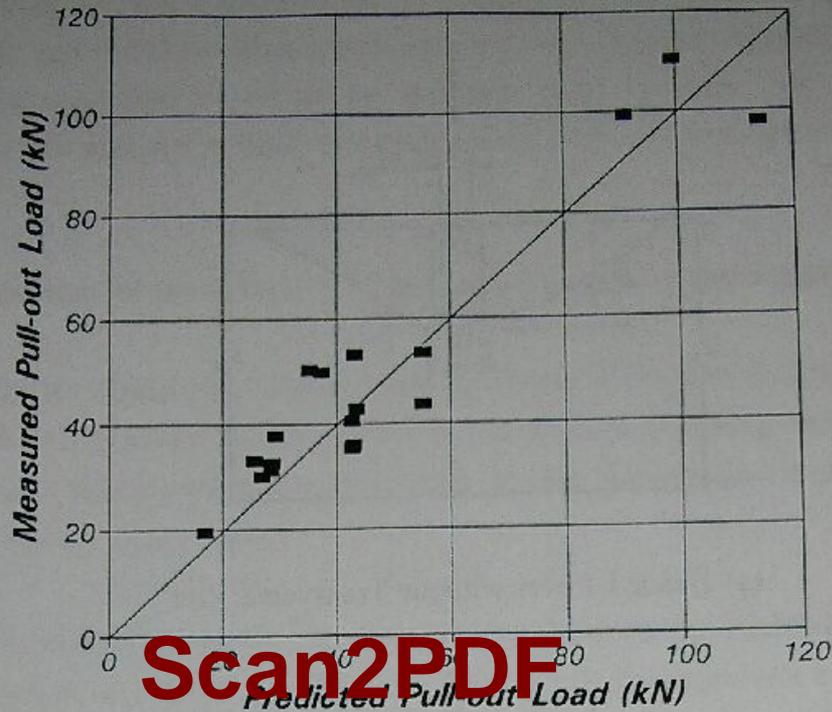


(b) Headed Insert with Transverse Rod



(c) Hooked Bar Anchorage

Figure 4.3 - Definition of Effective Embedment Length.



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Figure 4.4 - Comparison between Predicted and Measured Pull-out Strengths Using the Proposed Effective Embedment Length for Hooked Bar Anchorages.

Table 4.3 - Measured and Predicted Pull-out Strengths of Hooked Bar Anchorages					
Test	$f'_c$ (MPa)	Pull-out Strength $T_c$ (kN)	Predicted Pull-out Strength <sup>(1)</sup> $T_c$ (kN)		
			$\psi$ -method Eq. 6	ACI-408 Eq. 7	ACI-349 Eq. 8
HU-L	22.5	31.4	28.6	15.5	32.1
HU-R		32.5	28.7	15.5	32.2
HR-L		33.1	25.6	14.7	28.8
HU-L		30.1	26.9	15.0	30.3
HU-L	47.6	33.7	42.7	22.9	48.0
HU-R		40.6	43.1	22.9	48.4
HR-L		36.0	42.8	22.9	48.1
HR-R		42.8	43.7	23.1	49.1

(1) Calculated using  $\phi_c = 1.00$

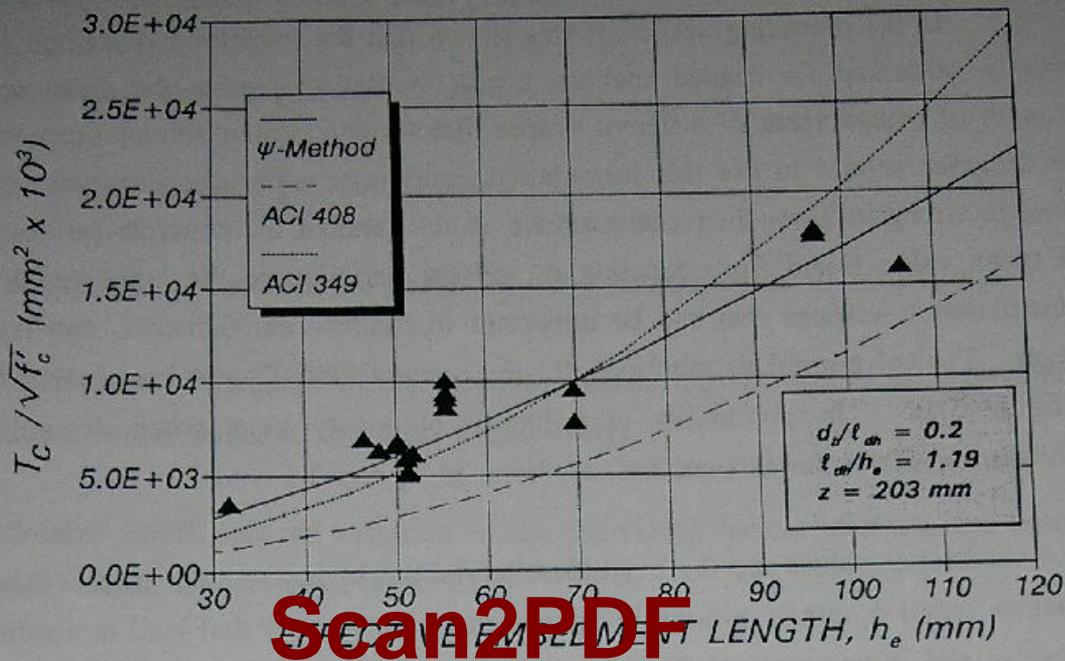


Figure 4.5 - Comparison of Methods for Predicting the Pull-out Strength of Hooked Bar Anchorages.

pull-out loads for 18 tests conducted by Johnson and Jirsa (1981) and in this current study, calculated using  $h_e$  so defined. The mean ratio of the measured to predicted strengths was 1.08 and the coefficient of variation was 18.9%.

The predicted pull-out strengths for the tests conducted in this current study using different approaches is illustrated in table 4.3. Both, the  $\psi$ -method and the ACI Committee 349 (1985) approach agree quite well with the measured results. The ACI Committee 408 (1979) approach largely underpredicts the pull-out strengths.

Figure 4.5 shows a comparison of the pull-out strengths obtained using the  $\psi$ -method, incorporating the proposed effective embedment length, with the other two approaches. Also plotted are the existing test data from Johnson and Jirsa (1981) and the current series of tests. Evidently the ACI Committee 408 method is too conservative. The ACI Committee 349 approach fits well the data with shallow embedments but overestimates it for deeper embedments. The  $\psi$ -method approximates the test data well over the whole range of embedment lengths.

## 5. DESIGN RECOMMENDATIONS

In the preceding section it was shown that the  $\psi$ -method described in Section 2.2.4 initially calibrated for headed anchors, could be used to predict the mean concrete pull-out strength of embedments of different shapes that show a similar failure mechanism. However the designer should use this formulation cautiously, since the variations of the steel and concrete strengths from their characteristic values, and of the concrete pull-out strength from the mean value found from equation 6, and the variation in the embedment lengths due to construction tolerances that can be important in shallow embedments, can lead to an unsafe design. Typical normal distributions for the applied load  $T_s$  and load resisted  $T_c$  are plotted in figure 5.1a. The probability of failure is obviously large when designing without any modification factors to account for variations in  $T_s$  and  $T_c$ .

Proper values for the magnification factor  $\lambda_s$  due to steel overstrength and strength reduction factor  $\phi_c$  due to pull-out concrete strength in equation 1 that lead to a reliable design will be determined in this section. The objective is to prevent a premature brittle failure by aiming for yield of the steel before pull-out of the concrete. Figure 5.1b depicts the physical meaning of a reliable design using these factors. The second moment probabilistic method is ideal for this procedure because of its simplicity. This method only requires as input data the mean and coefficient of variation of the different variables affecting the tensile capacity of the anchors. The second moment probabilistic method has been used by MacGregor (1976, 1988) for checking the load factors of the ACI Building Code (ACI Committee 318, 1971), which formed the basis of load factors recommended by NZS 3101 Concrete Design Code (1982) and previous NZS 4203 Loadings Code (1984). If the loads of the applied and resisting mechanisms can be assumed to be normally-distributed, the factors  $\lambda_s$  and  $\phi_c$  are given by

$$\lambda_s = \frac{\bar{f}_y}{f_y} e^{0.75\beta V_{\lambda}} \quad (9)$$

and

$$\phi_c = \frac{\bar{T}_c}{T_c} e^{-0.75\beta V_{\phi}} \quad (10)$$

where

## 5. DESIGN RECOMMENDATIONS

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and

$$\phi_c = \frac{\bar{T}_c}{T_c} e^{-0.75\beta V_{\phi}} \quad (10)$$

where

- $\bar{f}_y$  = mean yield strength of reinforcing steel,
- $f_y$  = specified yield strength of reinforcing steel, or lower characteristic yield strength according to NZS 3402 (1989),
- $\bar{T}_c$  = measured concrete pull-out strength,
- $T_c$  = predicted concrete pull-out strength,
- $\beta$  = safety index related to the probability and consequences of failure, and,
- $V_\lambda$  and  $V_\phi$  = coefficient of variation of the variables influencing the applied steel force and the concrete pull-out strengths, respectively.

The estimated coefficient of variation of the individual factors affecting the tensile capacity of the steel anchor are given in table 5.1.

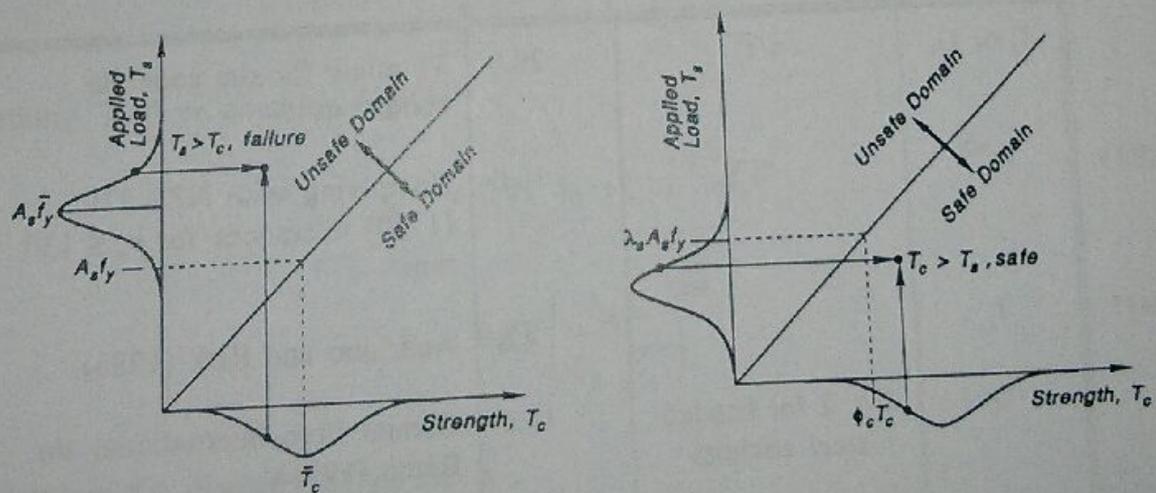
The global coefficient of variation  $V_\lambda$  is equal to the coefficient of variation of the yield strength of the steel

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$$V_\lambda = 0.048$$

(11)

and the global coefficient of variation  $V_\phi$  is found from the different factors involved in the local variations of the  $\sqrt{f'_c}$  and  $h_e$  and the coefficient of variation of the equation itself, namely



(a) Design using  $T_s = A_s f_y$  and  $T_c$

(b) Design using  $T_s = \lambda_s A_s f_y$  and  $\phi_c T_c$

Figure 5.1 - Probability of Failure of Steel Anchors Designed with or without Load Factors.

$$\phi_{1.1} = \phi_{1.2} \phi_{1.3} \phi_{1.4} \phi_{1.5} = 1.0 \text{ for welded steel sections} \quad (12.9)$$

$$\phi_{1.1} = \phi_{1.2} \phi_{1.3} \phi_{1.4} \phi_{1.5} = 1.0 \text{ for welded steel cast-in-place} \quad (12.10)$$

Bracing (12.9) recommends a safety index value of  $\beta = 1.5$ , which corresponds to a probability of failure of  $10^{-7}$  per year for ductile structures with normal consequences of failure. A value of  $\beta = 4.0$  is recommended for brittle failures or failures with serious consequences. A value of  $\beta = 1.5$  will be obtained here because a pull-out failure, although brittle in nature, does not lead to a service failure, providing that alternative load paths have been allowed during design and construction.

Analysis and Test (12.10) has involved a statistical analysis of New Zealand manufacturing steel and has resulted in a yield strength of Grade 275 steel (now Grade 350) of  $f_y = 275 \text{ MPa}$ . This  $f_y = 1.0 \phi_{1.1} f_y$ .

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**Table 2.1** Coefficients of Variation (C.V.) of Factors affecting the Yield Capacity of Steel sections Encased in Concrete

Property	Expression used	C.V. (%)	Source
$f_{yk}$	$\phi_{1.1}$	0.0	To allow for site concrete conditions
$\phi_{1.2}$	$\phi_{1.2}$	0.0	Comprising with BS 5400 (1989) increases for $\phi_{1.2}$ to 1.0 etc.
$\phi_{1.3}$		0.0	Anderson and Park (1989)
$\phi_{1.4}$	Eq. 2 for welded steel sections	0.0	Current Euro-International de Beton (1991)
	Eq. 2 for welded steel cast-in-place	0.0	This report

$$h_e \geq 0.23 \left( \frac{f_s}{\xi_R \sqrt{f'_c}} \right)^{2/3} d_b^{4/3} \quad (17)$$

For simplicity, good approximations for  $d_b^{4/3}$  for use in equation 17 are

$$d_b^{4/3} \approx \begin{cases} 2.29d_b, & \text{for } d_b \leq 16 \\ 2.88d_b, & \text{for } 16 < d_b \leq 28 \end{cases} \quad (18)$$

Then from equations 17 and 18 the required effective embedment length  $h_e$ , defined as illustrated in figure 4.3, is given by

$$h_e \geq \left( \frac{f_s}{\xi_R \sqrt{f'_c}} \right)^{2/3} \frac{d_b}{\kappa}, \quad \text{where } \kappa = \begin{cases} 1.9, & \text{for } d_b \leq 16 \text{ mm} \\ 1.5, & \text{for } 16 < d_b \leq 28 \text{ mm} \end{cases} \quad (19)$$

where the following values are recommended:

$f_s$  = the lower characteristic yield strength  $f_y$  for non-threaded reinforcing bars anchored in the concrete member,

or  $1.2f_y$  for threaded reinforcing bars attached to an embedded connector,

or the ultimate strength,  $f_{su}$ , for bolts.

The larger stress  $f_s$  recommended for bolts is due to the smaller plastic elongation expected from them at fracture. In reinforcing bars embedded in a concrete member, the degree of plastic elongation is expected to be larger due to yield penetration in both directions from the critical section.

$d_b$  = the nominal bar diameter in a non-threaded reinforcing bar,

or diameter of the reduced section in a threaded reinforcing bar or bolt.

Substituting equations 11, 12a and 12b and

$$\beta = 3.5,$$

$$\bar{f}_y/f_y = 1.07,$$

$$\bar{T}_c/T_c = 1.00, \text{ for headed steel anchors, and,}$$

$$\bar{T}_c/T_c = 1.08 \text{ for hooked bar anchorages}$$

into equations 9 and 10 results in

$$\lambda_s = 1.21 \quad (13)$$

and

$$\phi_c = 0.51, \text{ for both anchorage types} \quad (14)$$

Note that the value of  $\lambda_s$  found by the second moment probabilistic method is quite similar to the ratio of upper to lower characteristic yield strengths of about 1.17 specified by NZS 3402 (1989).

Substituting equations 13 and 14 into equation 1 gives

$$1.21 A_s f_y \leq 0.51 \xi_R T_c$$

and substituting  $T_c$  from equation 6 gives

$$1.21 A_s f_y \leq 0.51 \xi_R 17 h_e^{3/2} \sqrt{f'_c} \quad (15)$$

$$\therefore h_e \geq 0.27 \left( \frac{A_s f_y}{\xi_R \sqrt{f'_c}} \right)^{2/3} \quad (16)$$

which in terms of the diameter of the bar is

$$h_e \geq 0.23 \left( \frac{f_s}{\xi_R \sqrt{f_c'}} \right)^{2/3} d_b^{4/3} \quad (17)$$

For simplicity, good approximations for  $d_b^{4/3}$  for use in equation 17 are

$$d_b^{4/3} \approx \begin{cases} 2.29d_b, & \text{for } d_b \leq 16 \\ 2.88d_b, & \text{for } 16 < d_b \leq 28 \end{cases} \quad (18)$$

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where the following values are recommended:

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$d_b$  = the nominal bar diameter in a non-threaded reinforcing bar,

or diameter of the reduced section in a threaded reinforcing bar or bolt.

The value of  $\xi_R$  is as given by the  $\psi$ -method except that it is modified by a factor  $\psi_{cr}$  to take into account for the detrimental effects of cracking. Thus

$$\xi_R = \psi_{cr} \psi_{sx} \psi_{sy} \psi_{cx} \psi_{cy} \quad (20)$$

where  $\psi_{cr} = 0.75$  if the combined tensile stresses due to flexure and shrinkage in the concrete member anchoring the connector exceed  $0.6\sqrt{f'_c}$  at the side where the connector protrudes,

or 1.0 in any other case.

$\psi_{sx}$  and  $\psi_{sy}$  are to take into account the proximity between anchors,

$\psi_{sx} = \{1 + (n_x - 1)(s_x / s_{cr})\} / n_x$ , where  $n_x$  is the number of anchors in the x-direction,  $s_x$  is the centre-to-centre distance in the x-direction and  $s_{cr}$  is the critical distance taken as  $s_{cr} = 3h_s$ .

$\psi_{sy} =$  as above for  $\psi_{sx}$  with subscript y substituted for x.

$\psi_{cx}$  and  $\psi_{cy}$  are to take into account edge effects,

$\psi_{cx} = \{0.3 + (0.7c_y / 1.5h_s)\} \leq 1$ , where  $c_y$  is the edge distance in the y-direction.

$\psi_{cy} =$  as above for  $\psi_{cx}$  with subscript x substituted for y.

The minimum permitted thickness of the member with embedded connectors is governed by the required effective embedment length and the minimum cover of 30 mm at the back of the connector.

The attached Appendix give examples of the use of equation 19 in design.

## 6. CONCLUSIONS

The literature review presented shows that there are several methods available which can be used to predict the tensile capacity of anchors embedded in relatively thin concrete members. One method in particular, the  $\psi$ -method, has had very comprehensive theoretical and experimental verification.

Tensile tests were conducted on RAMSET TCM-12 concrete inserts and 12 mm VEMO bar couplers. The tests showed that the ultimate load of D12 threaded bars can be sustained by these connecting systems. At failure necking of the reinforcing bars occurred in the threaded region. The use of these proprietary systems can therefore assure that failure will occur in the connected bar. Caution should be taken during construction when using short bar couplers to ensure that the two bar ends have equal screwed lengths in the coupler. The use of bar couplers with mid-stoppers is strongly recommended since they are less prone to errors during construction.

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An experimental programme on connectors commonly used in the tilt-up industry was carried out. The aim of this programme was to assess the pull-out strength of the connectors embedded in concrete slabs and to monitor the load-displacement characteristics. RAMSET TCM-12 with and without transverse rods through the head as well as hooked bar anchorages were tested in two series having different concrete strengths.

It was found that the  $\psi$ -method predicted with good accuracy the concrete pull-out strength of the headed anchors tested. A statistical analysis indicated that this method can also be used to predict the pull-out strength of hooked bar anchorages providing the effective embedment length is appropriately defined.

A design equation for determining the tensile capacity of headed anchors and hooked bars is proposed based on the  $\psi$ -method and accounting for the possible variations of the material properties and the effect of tolerances on the effective embedment length. The second moment probabilistic method was used to determine the design equation from a basic equation proposed by Eligehausen et al. The design equation is based on the principle that at ultimate load extensive yielding of the steel should occur rather than a brittle failure pull-out of the concrete.

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## APPENDIX: DESIGN EXAMPLES

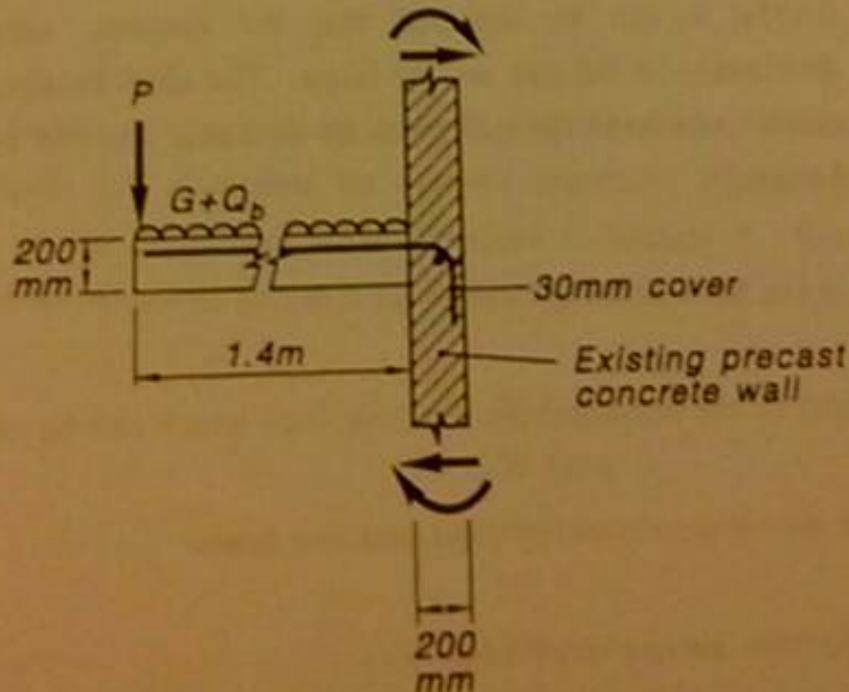
### Example 1.

A 200 mm thick cast in place concrete slab is to cantilever 1.4 m from an existing precast concrete wall as shown in figure A.1.

- Calculate the maximum service live load which can be carried by the slab if the flexural top reinforcement in the slab is D20 @ 400 mm centres (785 mm<sup>2</sup>/m) anchored as shown.
- Calculate the flexural top reinforcement in the slab if the service live load is 3 kPa.

Consider that the concrete wall may crack under the existing boundary conditions. The following parameters are known:

Superimposed service dead load	1.5 kPa
Edge service dead load, P	1.3 kN/m
$f_y$	300 MPa
$f'_c$	25 MPa



Note: Other reinforcement not shown

Figure A.1 - Section of Wall with Existing Connection Detail.

Solution (a)  $d_b = 20 \text{ mm}$ ,  $s = 400 \text{ mm}$

- (1) Evaluation of the available ideal moment of resistance at the critical section in the slab at the wall face.

$$\psi_{cr} = 0.75$$

$$\text{Now } h_e = 200 - 30 - 1.5 d_b = 140 \text{ mm}$$

$$s_{cr} = 3h_e = 420 \text{ mm} > s$$

$$\therefore \psi_{sx} = (1 + (n_x - 1) s_x/s_{cr})/n_x \approx s_x/s_{cr} = 0.95$$

$$\text{Also } \psi_{sy} = \psi_{cx} = \psi_{cy} = 1.00$$

$$\text{Hence } \xi_R = 0.95 \times 0.75 = 0.71$$

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Find  $f_s$  from equation 19, substituting  $\kappa = 1.5$  and the above values for  $f_c$ ,  $d_b$ ,  $h_e$  and  $\xi_R$ .

$$f_s = 121 \text{ MPa} < f_y$$

Yielding of the reinforcement cannot be achieved before pull-out failure. When  $f_s = 121 \text{ MPa}$ , and noting the steel ratio  $\rho$  is low being about 0.49%, it can be expected that the concrete compressive stress distribution in the slab will be linear. The ideal bending moment at the critical section of the slab given by an elastic analysis for when  $f_s = 121 \text{ MPa}$  is

$$M_i = 13.87 \text{ kN.m/m}$$

- (2) Evaluation of the maximum service live load which can be carried.

Ultimate moment induced by dead and live loads

Uniform service dead load

$$G = 23.5 \times 0.2 + 1.5 = 6.2 \text{ kPa}$$

$$s_{cr} = 456 \text{ mm} > s$$

$$\psi_{sx} \approx 200/456 = 0.66$$

$$\text{Also } \psi_{sy} = \psi_{cx} = \psi_{cy} = 1.00$$

$$\text{Hence } \xi_R = 0.49$$

Required embedment length from equation 19

$$h_e \geq 156 \text{ mm} \approx 152 \text{ mm ok}$$

**Notes:** The two designs using D20 @ 400 (785 mm<sup>2</sup>/m width) and D12 @ 300 mm (357 mm<sup>2</sup>/m width) in the slab result in maximum permitted service live loads of 1.47 kPa and 3.00 kPa, respectively. This difference is due to the D20 bars being able to reach  $f_y = 411 \text{ MPa}$ , whereas the D12 bars could reach  $f_y = 300 \text{ MPa}$ , because the available anchorage in the 200 mm thick wall.

If the bar diameter and/or spacing of the bars is such that yield of the reinforcement cannot be reached, then the bar diameter or spacing should be changed to ensure yielding of the reinforcement, and thus avoid a brittle concrete failure.

### Example 2.

A group of eight 12 mm anchors are to be embedded in a reinforced concrete wall as shown in figure 2.3. Calculate the required minimum effective embedment length,  $h_e$ , according to the design procedure proposed in Section 5. Consider in the analysis that the wall is uncracked and that the following parameters are known:

$f'_c$ .....	20 MPa
$f_y$ .....	300 MPa
$h_e$ .....	110 mm
$c_x$ .....	90 mm
$c_y$ .....	175 mm
$s_x$ .....	250 mm
$s_y$ .....	275 mm

Using load factors of 1.2 for dead load and 1.6 for live load  $Q_b$ .

$$\begin{aligned} M_u &= (1.2 \times 6.2 + 1.6 Q_b) \times 1.4^2 / 2 + 1.2 \times 1.3 \times 1.4 \\ &= 9.48 + 1.57 Q_b \end{aligned}$$

Using a strength reduction factor  $\phi = 0.85$  for flexure

$$M_u = \phi M_i = 0.85 \times 13.87 = 11.79 \text{ kN.m/m}$$

$\therefore$  Maximum uniform service live load is

$$Q_b = 1.47 \text{ kPa}$$

Solution (b)

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(1) Determination of the required steel area.

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$$\text{Now } M_u = (1.2 \times 6.2 + 1.6 \times 3) \times 1.4^2 / 2 + 1.2 \times 1.3 \times 1.4 = 14.2 \text{ kN.m/m}$$

Therefore required  $M_i = M_u / \phi$

$$M_i = 16.7 \text{ kN.m/m}$$

$$A_s \approx \frac{M_i}{0.95 d f_y} = 357 \text{ mm}^2/\text{m}$$

(2) Determine the tensile capacity of the hooked embedment.

Use D12 @ 300 ( $A_s = 377 \text{ mm}^2/\text{m}$ )

Maximum available embedment length

$$\psi_{cr} = 0.75$$

$$\text{Now } h_e = 200 - 1.5 \times 12 - 30 = 152 \text{ mm}$$

**Solution:**

Calculate  $\xi_R$  from equation 20:

$$\psi_{cr} = 1.00$$

$$s_{cr} = 3 \times 110 = 330 \text{ mm}$$

$$\psi_{sx} = \{1 + (4-1) \times 250/330\} / 4 = 0.82$$

$$\psi_{sy} = \{1 + (2-1) \times 275/330\} / 2 = 0.92$$

$$\psi_{cx} = 0.3 + \{0.7 \times 170 / (1.5 \times 110)\} = 1.02 \geq 1.00 \Rightarrow \psi_{cx} = 1.00$$

$$\psi_{cy} = 0.3 + \{0.7 \times 90 / (1.5 \times 110)\} = 0.68$$

$$\therefore \xi_R = 1.00 \times 0.82 \times 0.92 \times 1.00 \times 0.68 = 0.51$$

Now find  $h_e$  from equation 19 substituting  $\alpha = 1.9$ :

$$h_e \geq \left( \frac{300}{0.51 \sqrt{20}} \right)^{2/3} \frac{12}{1.9} = 163 \text{ mm}$$