### 7.6.6.3 Throat Plate

The Throat Plate is used to carry high concentrated loads into, for example, a beam flange or tension clip, where the flange, or base leg of the clip, is too weak according to the analysis in §7.6.5 & 6. This reinforcement, also known in the US as "a filler", is typically used where it is not possible to fit a bathtub fitting backup structure onto the web of a beam. Remember that the moment created during the load carry-through on a bathtub fitting, must be reacted by suitable structure. (Bathtub fittings are adequately dealt with in Ref. 5 and all company manuals)

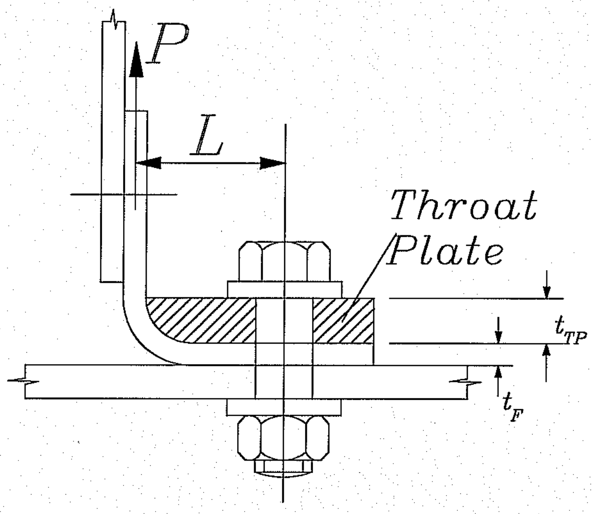



**Figure 7.6-31** **Throat Plate Critical Parameters**

Figure 7.6–31 represents the throat plate problem. The angle flange has thickness $\mathbf{t_F}$ and the TP has thickness $\mathbf{t_{TP}}$. The assembly has an effective width, say $\mathbf{w_e}$, so that inertia properties can be calculated.

The TP is considered as a cantilever of length **L** and section stiffness properties $\mathbf{(E\ I)_{TP}}$.

The tip deflection is $\delta_{TP}$, and is given by: -

$$\delta_{TP} = P_{TP} \cdot L^3 / [3 \cdot (E \cdot I)_{TP}]$$

The bent flange is a guided cantilever of length **L** and section properties $\mathbf{(E \cdot I)_F}$.

The tip deflection is $\delta_F$, (see §7.1.5.4, (x)) and is given by: -

$$\delta_F = P_F \cdot L^3 / [12 \cdot (E \cdot I)_F]$$

Also: -

$$P = P_{TP} + P_F$$

The deflections are set equal and the mathematical gymnastics performed to get the fraction of the load carried by the Throat Plate. This yields: -

$$P_{TP} = P / [1 + 4 \cdot (E \cdot I)_F / (E \cdot I)_{TP}]$$

Check stressing is done on the cantilevers using maximum moments of: -

$$M_{TP} = P_{TP} \cdot L$$

$$M_F = P_F \cdot L/2$$

### 7.6.6.3.1 Throat Plate Revisited

A query on the **Eng-Tips.com** Forum (05/2011) led the author to reassess the handy, but simple, method presented in §7.6.6.3 for analysing, and sizing up of a throat plate or filler. It was suggested, by a contributor to the Forum, that there may be some sort of pressure distribution between the flange and the throat plate. This will be true under initial low load conditions, but this will rapidly die away as the deflected shapes of the 2 beams develop.

For the configuration shown in Figure 7.6-31, with the corner of the flange of a bent-up clip using up a good proportion of the length **L**, the assumptions used in the derivation are probably pretty representative of that condition. However, the question was raised, as to how one would reinforce the endplate (end-pad) of a bath-tub fitting (BTF) in the event of an unexpected load increase or some such last-minute calamity that often befalls a project. The inner face of the BTF endplate is quite different from that of the tension clip in that it has a longer flat plate surface ending in a machined radius at the BTF sidewalls.

To investigate this further, this author decided to plot out the neutral axis deformed shapes of the guided cantilever flange and the cantilever throat plate with the compatibility condition of equal tip deflections as used in the load allocation derivation. The following result is presented in Figure 7.6-32: -

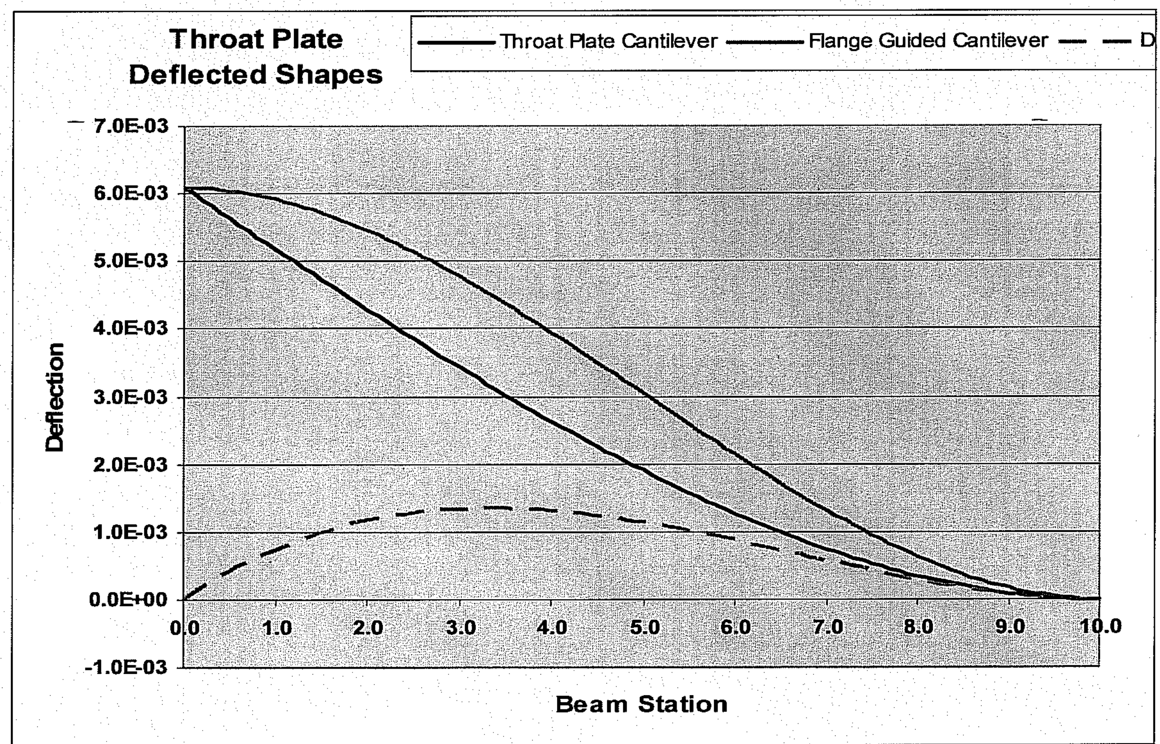



**Figure 7.6-32** <u>**Flange & Throat Plate Deflected Shapes**</u>

As can be seen, under the deflection compatibility load, the flange shape rises above the throat plate deflection curve. Thus ensuring that the throat plate was being loaded at its free end would only be possible if the lower surface of the throat plate was cut away to allow the flange to deflect according to its assumed boundary conditions. The **D** curve (Δ curve) represents the distance between the deflected shapes and hence the required theoretical shape modification to allow the 2 beams to deflect as per the boundary condition assumptions.

Machining such an elaborate undercut in the throat plate is not practical however and an engineering solution would be to reduce the thickness by a uniform amount dictated by the maximum of the D-curve in Figure 7.6-32. See Figure 7.6-33 for a schematic of an undercut throat plate setup. It is not uncommon for this kind of adaptation to be done on mating components such as bearings. Bearing surfaces that do not have a uniform load across the bearing length use this technique to "control" the magnitude and position of the bearing load. With the middle portion of the bearing interfaying surfaces removed, this ensures that the bearing stresses are directed towards the extremities of the bearing and appear as near uniform distributions on these outer bearing strips.

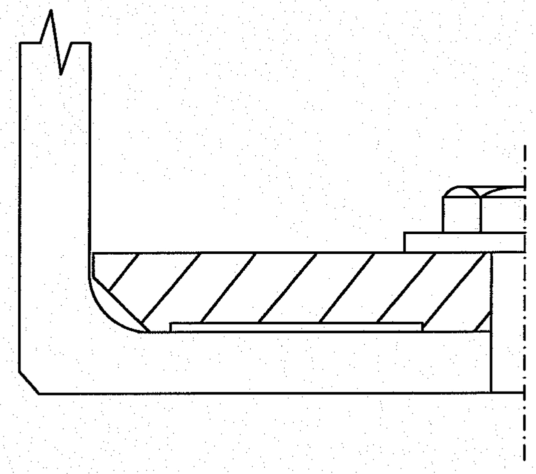


**Figure 7.6-33** **Undercut Throat Plate Reinforcement**

- Pretensioning the bolt improves the performance of the throat plate and the back-to-back BTF end-pads. This improves the actual encastré condition that is assumed at the bolt head.

  In the case of the BTF end-pad however, undercutting the throat plate would not be practical. A flat lower surface throat plate would tend to find a tangent position on the rising surface of the S-shaped end-pad "beam". For the S-shaped beam, with its change of slope roughly at the centre of the beam (**L/2**), this is a convenient position to choose such a tangent point. The problem setup is as shown in Figure 7.6-34: -

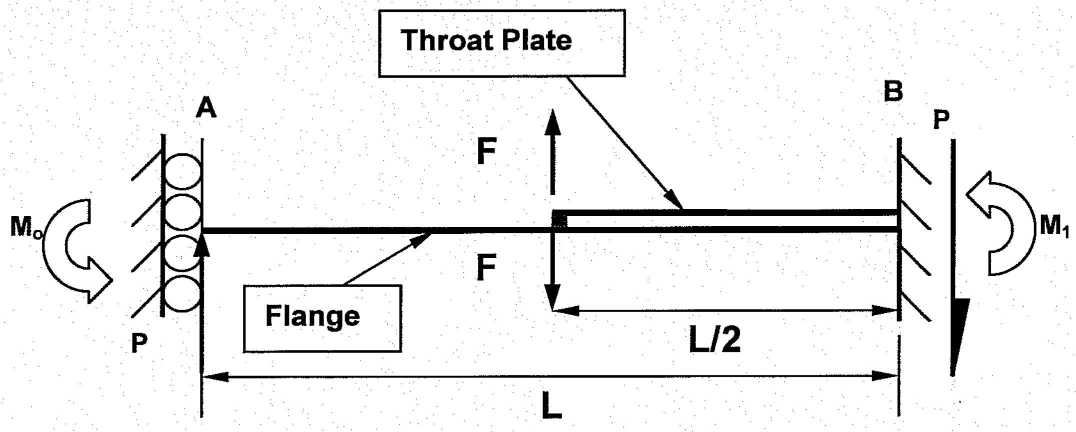



**Figure 7.6-34** **Midpoint Throat Plate System**

At the tip of the cantilever throat plate, the lower surface will touch the upper side of the end-pad at its mid-span as the latter bends upwards. This reduces the tip deflection of the end-pad, which naturally reduces the bending stresses therein. A deflected shape of the throat plate combining with the end-pad is shown in Figure 7.6.-35.

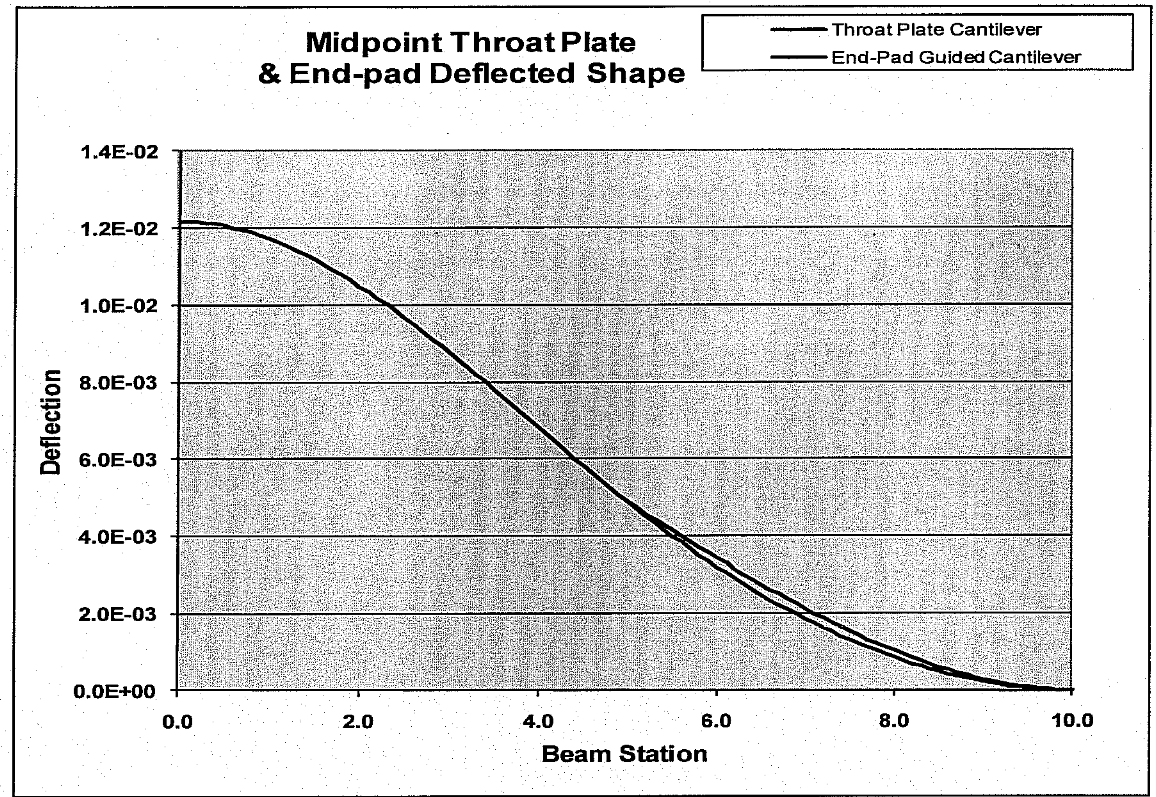



**Figure 7.6-35** **Midpoint Throat Plate Deflected Shape**

Analysis of the 2 deflected shape equations, by setting their mid-span deflections equal, yield the following results.

The ratio of the loads **P** and **F** can be written as follows: -

$$P/F = (E\cdot I)_F/(E\cdot I)_{TP} + 0.625 \quad \text{----------Eqn. 7.6.6.3.1 – 1}$$

The maximum bending moment in the end-pad "beam" alone is as follows: -

$$M_{Fmax} = P\cdot L/2 - F\cdot L/8$$

The maximum bending moment in the throat plate "beam" alone is as follows: -

$$M_{Fmax} = F\cdot L/2$$

## Discussion

The reader should bear in mind that this solution is likely to be a "repair" or recovery from an unexpected load increase. The long-term and most mass efficient solution would be to redesign the fitting with a thicker end-pad.

Applying this fairly simple tension clip result to the end-pad of a BTF can only be an approximate design tool that precedes the use of a more rigorous FE analysis, and possibly laboratory proving tests, of the joint. The 3-dimensional nature of the low aspect ratio end-pad plate ensures mutual interaction between the supported sides of the end-pad. When the length to width ratio of the end-pad plate reaches 3 or more, the width dimension dominates the stress level along the long side of the plate, and the problem becomes more like a tension clip with an "effective" width.

Using the plate reaction result from a known solution to a point load located at the equivalent position to a bolt on a plate supported and clamped on three sides would provide a starting point for a solution. Armed with the distribution of the reaction loads on three sides of the BT end-pad, or on two sides in the case of an angle tension fitting, the impact of inserting a half length throat plate under the bolt washer could be estimated in the 2 or 3 directions.

This method should not be classed as an "accurate" analysis of the throat plate's effectiveness relative to the end-pad, but should provide a first-pass solution to the problem.

Another problem (or challenge perhaps) is the limiting nature of closed form analytical solutions, such as those proposed in this section. The problem lies in the mathematical assumptions possible to obtain "convenient" boundary conditions (zero deflection, or zero slope). A zero slope (fully encastré) boundary condition is difficult to achieve in reality. The physical reality is usually somewhere in-between a simply supported and a fully clamped condition. Most difficult to predict is the degree of clamping at the load application end of the guided cantilever flange. Under the bolt head, with preload, the assumption of a fully clamped restraint is feasible. A certain amount of clamping at the load application end of the flange is certain and will modify the deflected shape of the flange relative to the free-ended cantilever throat plate. This means that a distributed pressure load between the 2 should not occur. A pressure distribution between the 2 plates would make a simple closed-form solution more difficult to achieve.

All of the above assumes elastic behaviour of the interacting beams, and thus for airframe work should function accordingly up to Limit Load. Assessing Ultimate Load interaction of the throat plate and end-pad, or flange, would require further plasticity effects analysis. Of course the end-pad/throat plate failure could be limited to elastic yielding at ultimate load. It is common practice in critical aircraft joints to provide them with a liberal MS at ultimate load to ensure more than adequate fatigue and damage tolerance strength, so elastic design for the throat plate/end-pad would not be that unusual.

The author has developed an extension to the mid-point throat-plate sizing formula provided in Eqn. 7.6.6.3.1 – 1, above. If one assumes that the clamping at the guided end of the flange is not fully encastré, but some fraction "**k**" thereof, then the ratio of the loads becomes: -

$$P/F = [\tfrac{2}{3}\cdot(E\cdot I)_F/(E\cdot I)_{TP} - \tfrac{1}{4}\cdot k + \tfrac{2}{3}]/(5/3 - k) \text{----- Eqn. 7.6.6.3.1 – 2}$$

For k = 1, the A and B ends (Figure 7.6-34) are fully encastré.

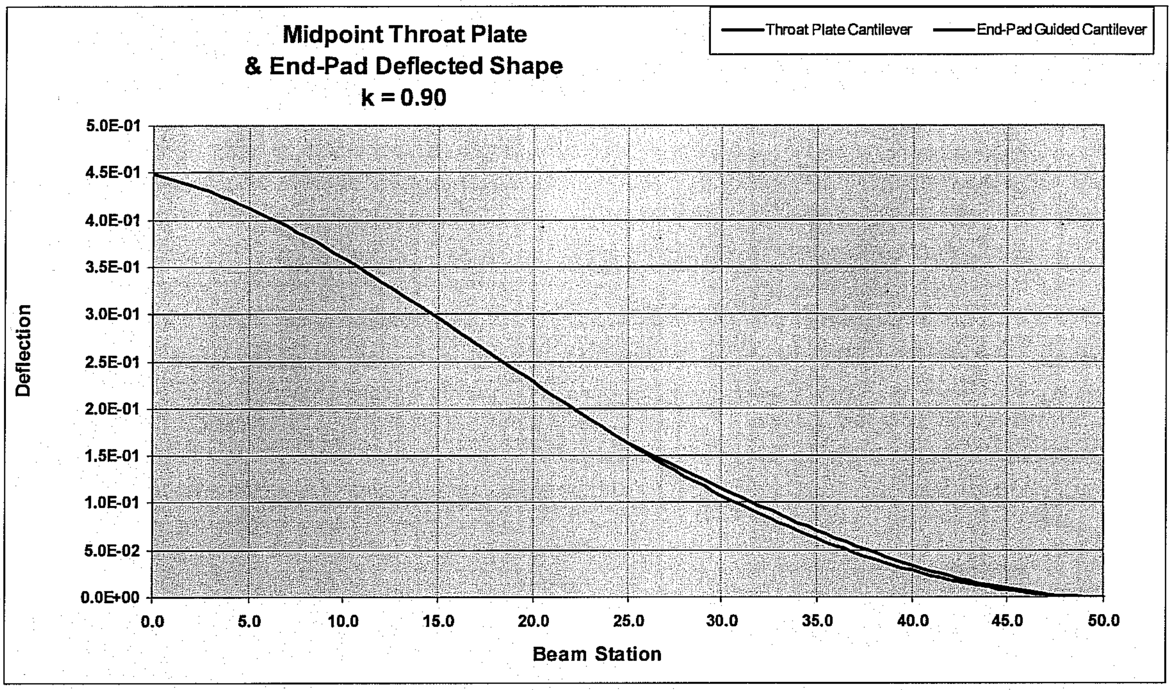
For k = 0, the A end has no clamping at all and the B end is fully encastré.

Looking at some of the results achieved with this formula, a clamping coefficient of k = 0.90 is probably a reasonable one to use when the A-end is not fully encastré.

The following two figures show plots of a partially clamped end-pad (flange) with a mid-point cantilever throat plate and the flange slope across the beam.

The formulae upon which these plots are based are derived in §10.22, (Appendix V), Volume 2. of these notes.

The bending moments at the critical points of the flange and throat plate can be determined from Appendix V, and thus the approximate stress levels in both of the members.



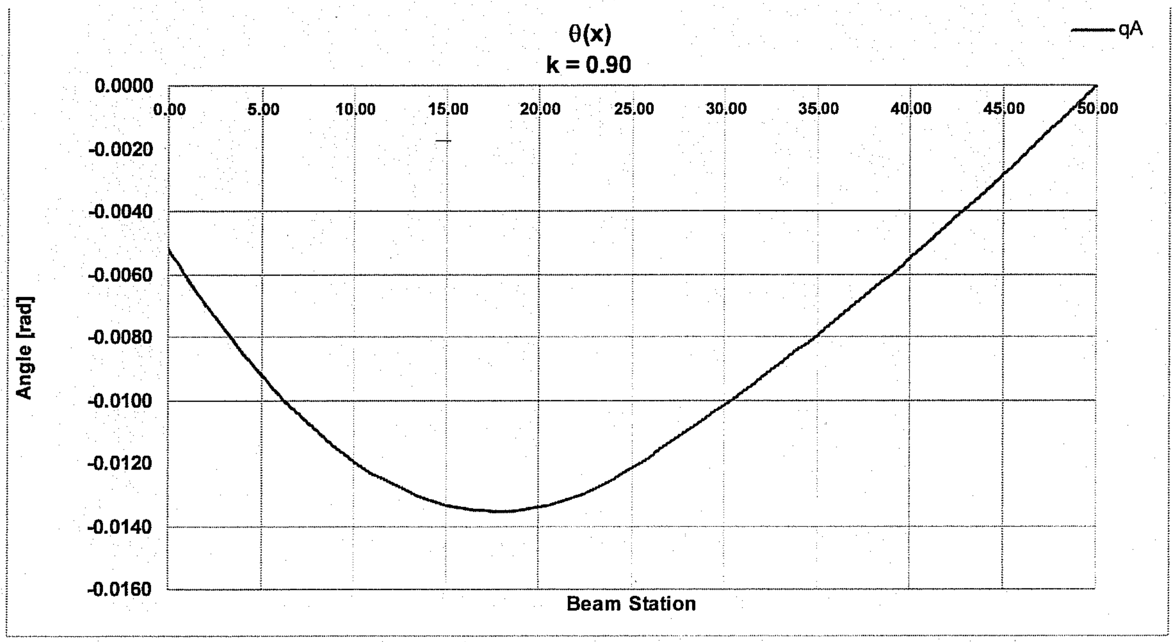



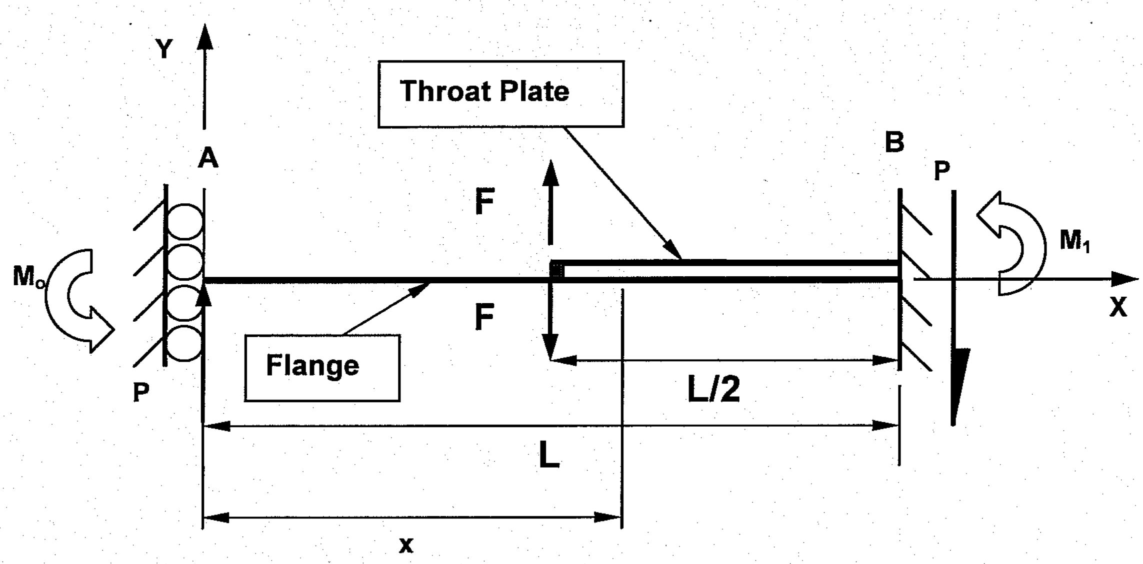

**Figure 7.6-36** **k = 0.90 Deflected Shapes & Beam Slope Plot**

On a practical physical level, inserting a steel throat plate in direct contact with say an aluminium flange, or end-pad, may cause rubbing or galling to occur between the surfaces. So besides the possible effects of dissimilar metal corrosion, the long term impact of this rubbing, or galling, on the joint should also be assessed, or provision should be made minimise this effect at the interfaying surfaces.

## 10.22 APPENDIX V (MIDPOINT THROAT PLATE FORMULAE)

The following section provides the derivation of the mathematical formulae used in producing the throat plate to flange interaction reinforcement presented in §7.6.6.3.1.

The following diagram is the basis for the axis system and sign convention used.




**Throat Plate**

The bending moment for the throat plate cantilever is straight forward.

$$M(x) = F\cdot(x - b)$$

Where: - $b = L - L/2 = L/2$

With: - $(EI)_{TP} d^2y/dx^2 = M(x) = F\cdot(x - b)$

Integrating this function twice w.r.t. **x** and, inserting the boundary conditions and beam stiffness, the following deflection is obtained for the cantilever beam over half the length of the flange or effective end-pad.

$$\delta_{TP} = F\cdot L^3/[24\,(EI)_{TP}]$$

**Flange (End-pad)**

$$M(x) = P\cdot x - M_o - F\cdot\langle x - b\rangle$$

**Note: -** The < > brackets indicate that this term only becomes "active" when **x ≥ b**.

With: - $(EI)_F d^2y/dx^2 = M(x) = P\cdot x - M_o - F\cdot\langle x - b\rangle$

Integrating this function once w.r.t. **x**: -

$$(EI)_F dy/dx = P/2\cdot x^2 - M_o\cdot x - F/2\cdot\langle x - b\rangle^2 + C_1$$

When deriving the guided cantilever deflected shape per the boundary condition of fully encastré at end A, for x = 0, the value of $C_1$ = 0. However at x = L, dy/dx = 0, thus: -

$$M_o = [P\cdot L/2 - F/(2\cdot L)\cdot(L - b)^2]$$

If $\mathbf{M_o}$ is reduced at end A by the factor **k**, then $\mathbf{M_o}$ becomes $\mathbf{k\cdot M_o}$, and there will now be a remainder at end A, and we can now write the slope equation as follows: -

$$\mathbf{(EI)_F dy/dx = P/2 \cdot x^2 - k \cdot [P \cdot L/2 - F/(2 \cdot L) \cdot (L - b)^2] \cdot x - F/2 \cdot \langle x - b \rangle^2 + C_2}$$

And in order to still get $dy/dx = 0$ at $x = L$, we need calculate $C_2$, which yields: -

$$\mathbf{C_2 = -(1 - k) \cdot [P \cdot L^2/2 - F/2 \cdot (L - b)^2] = -(1 - k) \cdot [M_o \cdot L]}$$

Notice that $\mathbf{C_2 = 0}$ **when** $\mathbf{k = 1}$, which is correct.

Integrating this function once w.r.t. $\mathbf{x}$**: -**

$$\mathbf{(EI)_F y_F = P/6 \cdot x^3 - (k/2) \cdot [Mo] \cdot x^2 - F/6 \cdot \langle x - b \rangle^3 - (1 - k) \cdot [M_o \cdot L] \cdot x + C_3}$$

At $x = L$, $y_F = 0$, thus $\mathbf{C_3}$ can be calculated: -

$$\mathbf{C_3 = -P/6 \cdot L^3 - (k \cdot L^2/2) \cdot [Mo] - F/6 \cdot \langle L - b \rangle^3 - (1 - k) \cdot [M_o \cdot L] \cdot L}$$

Substituting $\mathbf{x = L/2}$, gathering terms and simplifying, yields the following result: -

$$\mathbf{\delta_F = L^3/[16\ (EI)_F]\ [P \cdot (5/3 - k) + F \cdot (k/4 - 2/3)]}$$

Now setting the deflections equal to one another: -

$$\delta_F = \delta_{TP}$$

Yields the relationship between **P** and **F** as follows: -

$$\mathbf{P/F = [2/3 \cdot (EI)_F/(EI)_{TP} - k/4 + 2/3]/(5/3 - k)}$$