

Figure 15-3 Pressure thrust- Q_p at a bend (elbow) in a pipe due to internal pressure, P , showing the free-vector-diagram for calculating Q_p .

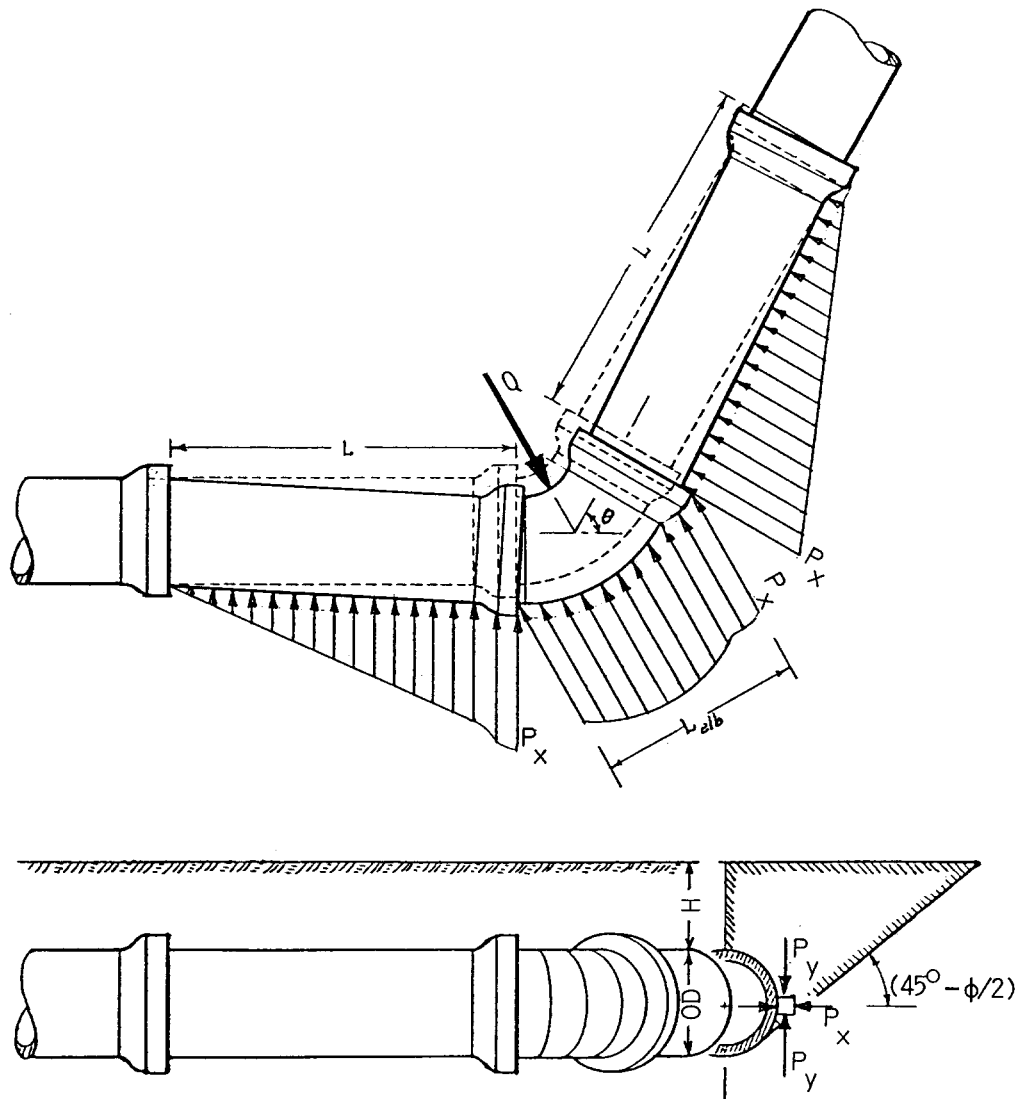


Figure 15-4 Passive soil resistance on an elbow and on contiguous gasketed pipe sections showing how the soil envelope can provide thrust restraint.

Pressure Thrust Q_p

See Figure 15-3; where

- D = inside diameter = $2r$,
- P = internal fluid pressure,
- Q_p = thrust due to internal pressure,
- θ = offset angle of the bend (elbow).

A free-body-diagram of the elbow with pressurized fluid contents is shown cross-hatched. Neglecting the small friction loss of flow around the bend, from the free-vector-diagram,

$$2Q_p = \pi D^2 P \sin(\theta/2) \quad \dots \dots (15.2)$$

Q_p is at an angle of $\theta/2$ with the y-axis. Consequently, thrust, Q , is the sum, $Q_i + Q_p$; i.e.,

$$2Q = \pi D^2 (P + v^2 \rho) \sin(\theta/2) \quad \dots \dots (15.3)$$

where

- v = average velocity of fluid flow,
- ρ = mass density of the fluid,
- θ = offset angle of the bend.

Example

Find thrust- Q at a 90° elbow in a water pipe for which,

- $\theta = 90^\circ$,
- $D = 30$ inches,
- $P = 200$ psi = internal pressure,
- $v = 15$ ft/second = flow velocity,
- $\rho = \gamma_w / g$ = mass density of water,
- $\gamma = 62.4$ lb/ft³ = unit wt. of water,
- $g = 32.2$ ft/second² = gravity.

Substituting into Equation 15.1, $Q_i = 3$ kips.

Substituting into Equation 15.2, $Q_p = 200$ kips.

Combined, $Q = 203$ kips. Impulse thrust, Q_i is usually neglected.

If a large diameter pipe with high internal pressure has an elbow with a large offset angle, θ , thrust- Q is enormous.

SPECIAL SECTIONS

Special sections redirect or alter flow. Examples include elbows, wyes, tees, valves, reducers, caps, plugs, etc. The following analyses for elbows can be applied to any special section. In every case, thrust, Q is the sum of impulse thrust, Q_i , and pressure thrust, Q_p .

COMMON THRUST RESTRAINTS

1. Welded or Bolted Joints at Special Sections

In a pressurized pipe, at a *gasketed* elbow, Q must be resisted by the soil or by a thrust restraint (thrust block). For a *welded* elbow, Q is resisted by the pipe. Two analyses of a *welded* elbow follow.

a) If the contiguous pipes are *unrestrained and uncapped* (like a garden hose), normal force, F , and shearing force, S , act on the elbow. Analysis is conservative because soil resistance reduces F and S .

$\sigma = F/2\pi r t$ = average normal stress,

$\tau = S/2\pi r t$ = average shearing stress.

From the equations of static equilibrium,

$$\sigma/P(r/t) = (1 - \cos \theta) \quad \dots \dots (15.4)$$

NORMAL STRESS TERM

$$\tau/P(r/t) = \sin \theta \quad \dots \dots (15.5)$$

SHEARING STRESS TERM

These stress terms are upper limits — twice the force-per-unit-area — to account for eccentricity of the F -force and redistribution of stresses. The outside of a bend can stretch more than the inside. Therefore, stresses are greater on the inside. See Problem 15-12. Wall thickness is sometimes increased for elbows. In general, greater wall thickness is not justified.

b) If the contiguous pipes are restrained and capped, from the equations of equilibrium, longitudinal stress is,

$$\sigma = Pr/2t \quad \dots \dots (15.6)$$

This is only half as great as circumferential stress, and is independent of offset angle, θ . A more precise analysis would show that stress, σ , on the inside of the bend is increased slightly as the offset angle, θ , is increased. Most pipes are ductile enough that the material "plastic-flows" at yield, and does not fail. Moreover, soil friction resists thrust. In practice, contiguous pipes are seldom capped. Longitudinal stress is not critical for isotropic plain steel and plastic pipes. Of course, joints must be adequate.

For non-isotropic pipes (corrugated, ribbed, or wrapped with fiberglass or wire), longitudinal strength must be assured. Neglecting impulse force and soil resistance, for uncapped, unrestrained contiguous pipes:

$$\text{At elbows, for longitudinal design,} \\ P\pi r^2(1-\cos\theta) = A\sigma_f/sf \quad \dots \dots (15.7)$$

where

A = area of longitudinal fibers,
 σ_f = strength of the fibers.

$$\text{At valves or caps (not at bends) for design,} \\ P\pi r^2 = A\sigma_f/sf \quad \dots \dots (15.8)$$

2. Embedment As Thrust Restraint

If thrust-Q is not large, the embedment is able to develop adequate passive resistance. It may not be necessary to provide additional thrust restraint. Consider in [Figure 15-4](#) the free-body-diagram of an elbow and one section of pipe on each side. The joints are gasketed so the pipe can take no longitudinal force. Thrust-Q can be restrained only by the soil bearing against the pipe. The maximum soil pressure bearing horizontally against the elbow is passive resistance P_x at the average depth of soil, $H + OD/2$,

$$P_x = (2H + OD)\gamma/2K$$

where

K = $P/P_x = (1-\sin\phi)/(1+\sin\phi)$,
 ϕ = soil friction angle,
 γ = unit weight of soil,
 OD = outside diameter,
 H = height of soil cover,
 L = length of pipe section.

The restraint capacity of soil against elbow is,
 $Q_{elb} = (\text{area}) \text{ times } P_x$

where

(area) = (OD) L_{elb} ,
 L_{elb} = cord length (approximate) of elbow from coupling to coupling as shown.

Multiplying (area) times P_x ,

$$Q_{elb} = (2H + OD)\gamma L_{elb} OD/2K$$

Added to this is the restraint capacity of the first section of pipe on each side of the elbow. Full passive resistance of the soil would be developed at the elbow end of each section. At the opposite end, each pipe section could rotate, because of the gasket. But there would be no lateral movement. Passive soil resistance would not be developed. A crude, but reasonable and conservative assumption, is that passive resistance varies linearly from P_x at the elbow end to zero at the opposite end. Due to soil supporting the two pipe sections, the component of restraint in the direction of Q is,

$$Q_{secs} = (OD)LP_x \cos(\theta/2)$$

or, substituting for P_x ,

$$Q_{secs} = (OD)L(2H + OD)\gamma \cos(\theta/2)/2K$$

Combining the thrust restraints provided by the elbow and the two pipe sections,

$$\text{Restraint-Q} = \\ OD(2H + OD)\gamma [L_{elb} + L\cos(\theta/2)]/2K \quad \dots \dots (15.9)$$

Rewriting Equation 15.3,

$$\text{Thrust-Q} = \pi(\text{ID})^2(P + v^2\rho)\sin(\theta/2)/2 \quad \dots\dots (15.10)$$

Equation 15.10 for thrust-Q was derived for a horizontal bend. For a vertical bend (in a vertical plane), thrust-Q has a vertical component. If soil cover alone is to resist the upward component of thrust-Q, then soil cover H must be great enough that soil weight can hold the pipe down. A conservative restraint-Q for this vertical bend is,

$$\text{Restraint-Q} = \text{OD}(2H + \text{OD})\gamma[L_{\text{elb}} + L\cos(\theta/2)]/2 \quad \dots\dots (15.11)$$

This is the same as Equation 15.9 except that K is eliminated. For design, restraint-Q must be greater than thrust-Q. A safety factor should be included.

3. Thrust Block as Thrust Restraint

Thrust blocks are the most common restraints in use for pressurized gasketed pipes. See [Figure 15-5](#). Thrust blocks are usually concrete. A reasonable analysis for design starts with the free-body-diagram. Assuming a cubical block,

$$\begin{aligned} B &= \text{lengths of sides of the cube,} \\ \gamma &= \text{unit weight of soil,} \\ \gamma_c &= \text{unit weight of the thrust block,} \\ jB &= \text{distance down to thrust-Q from the top of the block,} \\ K &= (1-\sin\phi)/(1+\sin\phi), \\ \phi &= \text{soil friction angle.} \end{aligned}$$

Other data are shown on the sketch. Friction on the sides of the block is undependable and is conservatively neglected.

Two modes of failure are considered: overturn about point O, and slip. The conditions under which each mode controls are described by an example of a cubical thrust block.

Example — Assumptions

$$\begin{aligned} h &= H/B = \text{ratio of soil cover H to side B,} \\ j &= \text{ratio of distance between top of block and thrust-Q, to side B,} \\ \phi &= 30^\circ = \text{soil friction angle,} \\ K &= 1/3 = (1-\sin\phi)/(1+\sin\phi), \\ \gamma &= 120 \text{ pcf} = \text{unit weight of soil,} \\ \gamma_c &= 144 \text{ pcf} = \text{unit weight of concrete.} \end{aligned}$$

Taking the sum of the moments of force about overturn fulcrum O,

$$Q/\gamma B^3 = (2h + 1.10)/(1-j) \quad \text{OVERTURN} \quad \dots\dots (15.12)$$

Taking the sum of the horizontal forces,

$$Q/\gamma B^3 = (3.577h + 2.193) \quad \text{SLIP} \quad \dots\dots (15.13)$$

The dimensionless quantity $Q/\gamma B^3$ is the thrust block restraint number. A table of values is shown as [Table 15-1](#) for typical design based on the assumptions indicated.

Overturn

In order to design a cubical thrust block with the typical soil properties assumed in the analysis above, it is only necessary to guess a trial value for B from which values of h and j can be calculated. Entering [Table 15-1](#) with h and j, the restraint number, $Q/\gamma B^3$ can be found in the overturn columns.

For a soil unit weight of $\gamma = 120$ pcf, $Q/B^3 = (120 \text{ pcf})(\text{restraint number})/\text{sf}$. Solve for B. If not the same as the assumed B, using the new B recalculate values for h and j. Enter [Table 15-1](#) for a second trial solution of the restraint number from which a new value of B is calculated. If this new B is unchanged, then the answer has been found. If not, recycle the analysis with the new B.

Slip

The left of the two SLIP columns of [Table 15-1](#)