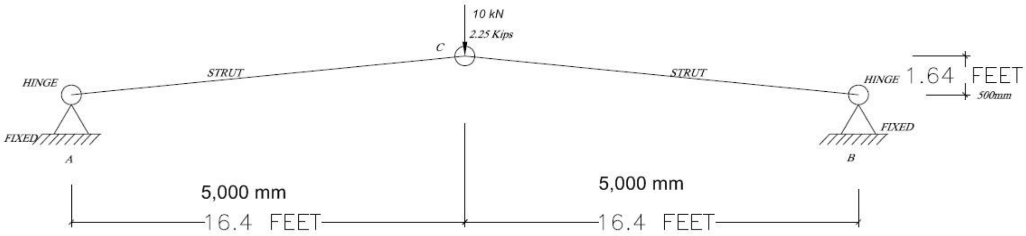


PROPERTIES OF STRUT
AREA = 0.155 in2 = 100 mm2
E = 14503 KSI = 10^5 MPa

FIND REACTIONS AT A & B,
LOADS IN STRUTS AND
DEFLECTION OF HINGE C

$$W_{midspan} := 10\text{kN}$$

$$A_s := 100\text{mm}^2$$

$$E_s := 10^5\text{MPa}$$

$$L_h := 5000\text{mm}$$

$$L_v := 500\text{mm}$$

$$L_{strut} := \sqrt{L_h^{\,2} + L_v^{\,2}} = 5024.938\cdot\text{mm}$$

Try a conventional "small displacements" static analysis:

$$V_{left.support} := \frac{W_{midspan}}{2} = 5\cdot\text{kN}$$

$$H_{left.support} := V_{left.support}\cdot\frac{L_h}{L_v} = 50\cdot\text{kN}$$

$$P_{left.strut} := \sqrt{V_{left.support}^{\,2} + H_{left.support}^{\,2}} = 50.249\cdot\text{kN}$$

$$\sigma_{left.strut} := \frac{P_{left.strut}}{A_s} = 502.494\cdot\text{MPa}$$

Note that while these results may seem rather high, the problem tells us nothing about material strength or section modulus, so we are not concerned with yielding, fracture or Euler buckling (although these would probably govern if this was a "real world" design problem).

$$\varepsilon_{left.strut} := \frac{\sigma_{left.strut}}{E_s} = 5.025 \times 10^{-3}$$

$$L_{strut.compressed} := L_{strut} \cdot \left(1 - \varepsilon_{left.strut}\right) = 4999.688 \cdot mm$$

Note that the compressed strut length is less than the half span, which means that this is a non-feasible solution, and the problem is a large displacements problem, so the frame will "snap through" the horizontal plane at something less than the nominated load.

For completeness, we could investigate the non-linear behaviour of the assembly, by setting strut force as functions of the central deflection δ, but this not a requirement to solve the problem as it has been set.

$$L_{strut.comp}(\delta) := \sqrt{L_h^{\,2} + \left(L_v - \delta\right)^2}$$

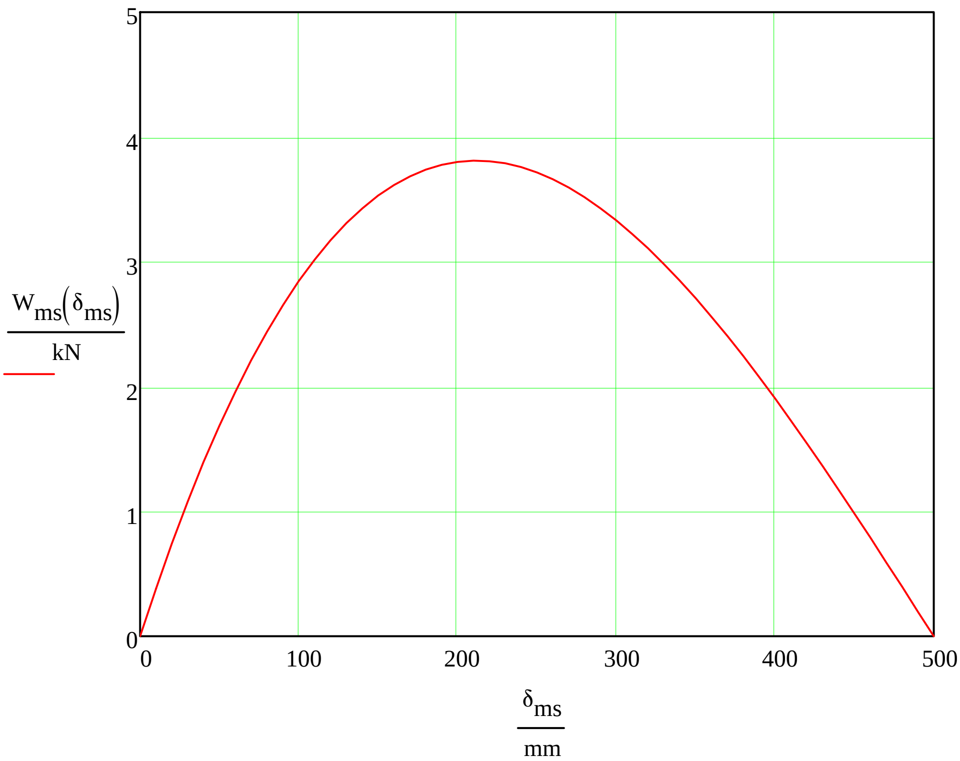
$$P_{strut.comp}(\delta) := \frac{L_{strut} - L_{strut.comp}(\delta)}{L_{strut}} \cdot E_s \cdot A_s$$

$$V_{left}(\delta) := \frac{L_v - \delta}{L_{strut.comp}(\delta)} \cdot P_{strut.comp}(\delta)$$

$$H_{left}(\delta) := \frac{L_h}{L_{strut.comp}(\delta)} \cdot P_{strut.comp}(\delta)$$

$$W_{ms}(\delta) := 2 \cdot V_{left}(\delta)$$

$$\delta_{ms} := 0mm, 10mm .. 500mm$$



We see that the maximum load the assembly can support with the struts in compression is approximately 3.8 kN with a mid-span deflection of approximately 210 mm, after which its load-carrying capacity actually reduces as the load increases.

Try a large displacements analysis of the assembly after it has "snapped through" such that the struts are now acting in tension:

Initial defection of 1,000 mm will result in the assembly taking up a symmetric position below the horizontal plane, in an unstressed condition; further displacement will be required to carry load, with the "struts" now acting in tension. The assembly is expected to show increasing stiffness as the deflections increase, due to the increasing angle of the members.

$$L_{strut.ext}(\delta) := \sqrt{L_h^{\,2} + \left(\delta - L_v\right)^2}$$

$$T_{strut.ext}(\delta) := \frac{L_{strut.ext}(\delta) - L_{strut}}{L_{strut}} \cdot E_s \cdot A_s$$

$$V_l(\delta) := \frac{\delta - L_v}{L_{strut.ext}(\delta)} \cdot T_{strut.ext}(\delta)$$

$$H_l(\delta) := \frac{L_h}{L_{strut.ext}(\delta)} \cdot T_{strut.ext}(\delta)$$

$$W_{m.s}(\delta) := 2 \cdot V_l(\delta)$$

We can now calculate the deflection which results in a mid-span load of 10 kN:

$$\delta_{10.kN} := 1164.79\,mm \qquad \delta_{10.kN} = 3.821\,ft$$

$$W_{m.s}\left(\delta_{10.kN}\right) = 10.000 \cdot kN \qquad W_{m.s}\left(\delta_{10.kN}\right) = 2.248\,kip$$

$$T_{strut.ext}\left(\delta_{10.kN}\right) = 37.937 \cdot kN \qquad T_{strut.ext}\left(\delta_{10.kN}\right) = 8.529\,kip$$

$$V_l\left(\delta_{10.kN}\right) = 5.000 \cdot kN \qquad V_l\left(\delta_{10.kN}\right) = 1.124\,kip$$

$$H_l\left(\delta_{10.kN}\right) = 37.606 \cdot kN \qquad H_l\left(\delta_{10.kN}\right) = 8.454\,kip$$

$\delta_{ms} := 1000mm, 1010mm .. 1500mm$