

US Steel Sheet Piling Design Extracts

Lateral Pressures on Walls

Retaining walls are subjected to lateral pressures from the following load conditions:

1. Water.
2. Earth.
3. Earth submerged in water.
or a combination at different elevations of:
4. Water and earth submerged in water.
5. Earth and earth submerged in water.

Conditions 2 and 5 may be supporting a surcharge load which produces additional lateral pressure.

In the construction of such walls, steel sheet piling may act as a cantilever or, if the height of the wall is great, it may be braced or tied back to anchors, so as to bring it into the condition of a beam extending over two supports. In any event, the strength of the steel piling between the supports should be sufficient to sustain the lateral pressures, and its penetration into firm soil must be such as to prevent movement at the toe.

1. Water Pressures.

The direction of water pressure on an immersed plane is always normal to the plane. The fundamental laws of liquid pressures apply as follows:

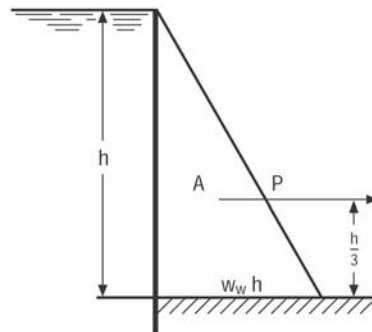


Fig. 1

$w_w h$ = pressure in pounds per square foot at depth h .

h = depth of water in feet.

w_w = weight of a cubic foot of water in pounds and also represents the hydrostatic pressure increment in pounds per square foot per foot of depth.

$w_w = 62.5$ for fresh water and 64.0 for salt water.

The total load on the wall, P , from the water level to the first support at h feet, is,

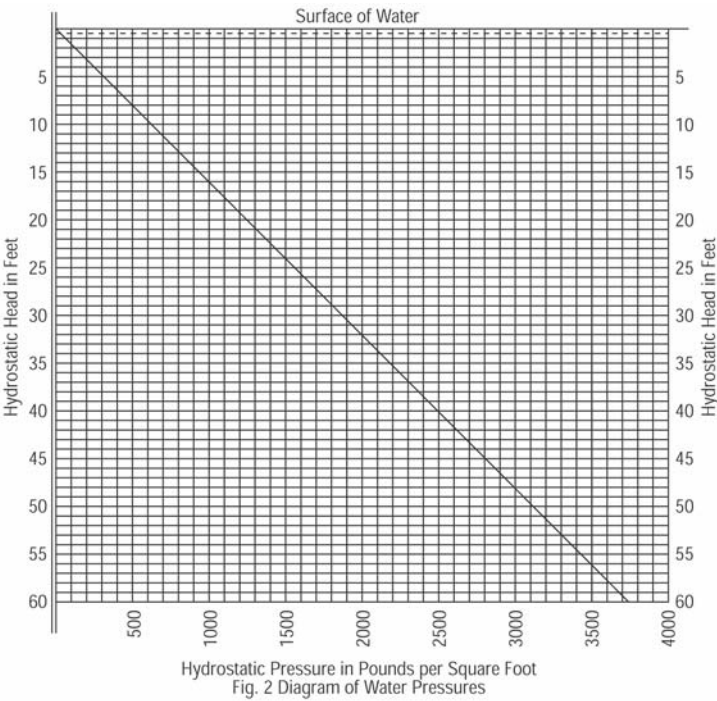
$$P = \frac{1}{2} w_w h^2, \text{ in pounds per foot of width}$$

The total load is distributed or applied as illustrated in Triangle A, Fig. 1 adjoining.

The total load, P , can be considered as concentrated at a distance of $\frac{1}{3}h$ from the bottom. The product $P \times \frac{1}{3}h$ is useful in the calculation of the overturning moment in a cellular cofferdam.

The following table will prove useful in giving the values of $w_w h$ and P for various depths of fresh water.

Table 5. Hydrostatic Pressures
Unit Bottom and Total Pressures on Surface One Foot in Width



Depth, Feet	Pressure, Pounds		Depth, Feet	Pressure, Pounds	
	Bottom, w _w h	Total, P		Bottom, w _w h	Total, P
1	62.5	31	31	1937.5	30031
2	125	125	32	2000	32000
3	187.5	281	33	2062.5	34031
4	250	500	34	2125	36125
5	312.5	781	35	2187.5	38281
6	375	1125	36	2250	40500
7	437.5	1531	37	2312.5	42781
8	500	2000	38	2375	45125
9	562.5	2531	39	2437.5	47531
10	625	3125	40	2500	50000
11	687.5	3781	41	2562.5	52531
12	750	4500	42	2625	55125
13	812.5	5281	43	2687.5	57781
14	875	6125	44	2750	60500
15	937.5	7031	45	2812.5	63281
16	1000	8000	46	2875	66125
17	1062.5	9031	47	2937.5	69031
18	1125	10125	48	3000	72000
19	1187.5	11281	49	3062.5	75031
20	1250	12500	50	3125	78125
21	1312.5	13781	51	3187.5	81281
22	1375	15125	52	3250	84400
23	1437.5	16531	53	3312.5	87581
24	1500	18000	54	3375	90812.5
25	1562.5	19531	55	3437.5	94031
26	1625	21125	56	3500	97250
27	1687.5	22781	57	3562.5	100500
28	1750	24500	58	3625	103750
29	1812.5	26281	59	3687.5	107000
30	1875	28125	60	3750	110250

2. Earth Pressures.

Much uncertainty exists as to methods of computation and also as to the exact weights of materials to be used in specific cases and to their angles of repose. The methods and formulas which follow are those devised by Rankine based on researches of Coulomb and represent generally accepted practice.

(a) **Level Bank.** The earth pressure is transferred to an equivalent horizontal liquid pressure increment,

p_e , in pounds per square foot per foot of depth, where p_e is the pressure increment corresponding to w_w for water. Then,

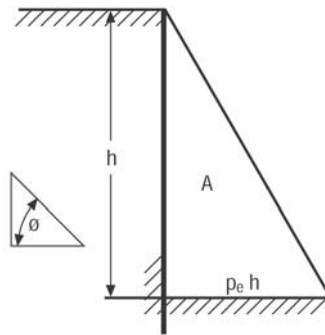


Fig. 3

$$p_e = w_e \tan^2 (45^\circ - \frac{1}{2}\theta).$$

w_e = weight per cubic foot of earth, in pounds.

θ = angle of repose of earth, in degrees.

The fundamental laws of liquid pressures then apply which makes,

$p_e h$ = pressure, in pounds per square foot, at depth, h , in feet, and the total load on the wall, P_e , in pounds per foot of width, is,

$$P_e = \frac{1}{2} p_e h^2$$

The total load is distributed or applied as illustrated in triangle A, Fig. 3 adjoining.

(b) Inclined Bank

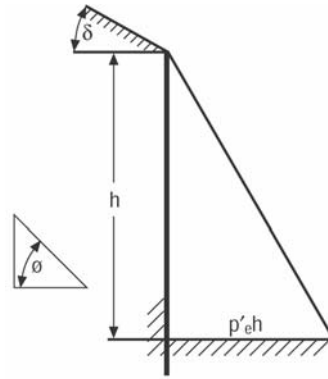


Fig. 4

$$p'_e = \frac{w_e \cos^2 \phi}{\left[1 + \sqrt{\frac{\sin \phi \sin(\phi - \delta)}{\cos \delta}} \right]^2}$$

When $\delta = \emptyset$, then,
 $p'_e = w_e \cos^2 \emptyset$, in pounds per square foot.

The same laws of liquid pressures then apply, as illustrated in Fig. 4 adjoining.

(c) **Surcharge Load.** Fig. 5 below shows a piling wall which retains earth pressure and is loaded with a surcharge of railroad cars or piled materials which increases the lateral pressure.

The lateral pressure, p_s , in pounds per square foot due to the surcharge load, W_s , in pounds per square foot is,

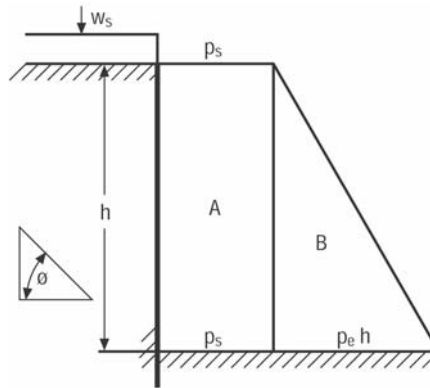


Fig. 5

$$p_s = w_s \tan^2 (45^\circ - \frac{1}{2} \theta), \text{ in pounds per square foot.}$$

The lateral pressure, p_s , is not subject to the laws of liquid pressures but is a uniform load throughout the height h , in feet, of the piling wall and the total load, in pounds, due to the surcharge is,

$$\text{Total uniform load} = p_s \times h, \text{ in pounds per foot of width.}$$

The total uniform load is distributed or applied as illustrated in rectangle A, Fig. 5 adjoining.

The lateral loads due to the earth loads alone are calculated as described in (a) just preceding and, since this pressure is liquid, the total load for these earth loads is illustrated in triangle B.

Thus the distribution or application of the total load, due to surcharge and earth pressures combined, is illustrated in the combination of rectangle A + triangle B, in pounds per foot of width.

Table 6. Typical Weights and Angles of Repose
Various Kinds of Loose and Dry Materials

Kind of Material	Size	Slope Ratio	Angle of Repose	Weight per Cubic Foot, Pounds	Equiv. Liquid Pressures, p_e , Lbs. per Sq. Ft.
Ashes, dry.....	1 on 1	45°	40	6.9
Cinders, bituminous, dry.....	1 on 1	45°	45	7.7
Clay, in lumps, dry.....	1 on 1½	36° 52'	63	15.8
Clay, damp, plastic.....	1 on 3	18° 26'	110	57.1
Clay and gravel, dry.....	1 on 1½	36° 52'	100	25.0
Clay, gravel and sand, dry.....	1 on 1½	36° 52'	100	25.0
Earth, perfectly dry, loose.....	1 on 1½	36° 52'	76	19.0
Earth, perfectly dry, packed.....	1 on 1½	36° 52'	95	23.8
Earth, slightly moist, loose.....	1 on 1½	36° 52'	78	19.5
Earth, more moist, packed.....	1 on 1	45°	96	16.5
Earth, soft flowing mud.....	1 on 3	18° 26'	108	56.1
Earth, soft mud, packed.....	1 on 3	18° 26'	115	59.7
Gravel, dry.....	1'' and under	1 on 1½	36° 52'	104	26.0
Gravel, dry.....	2 ½ '' and under	1 on 1½	36° 52'	96	24.0
Limestone fragments, dry.....	1'' and under	1 on 1	45°	85	14.6
Limestone fragments, dry.....	2 ½ '' and under	1 on 1	45°	80	13.7
Sand, clean and dry.....	1 on 1 ½	33° 41'	90	25.8
Sand, river, dry.....	1 on 1 ½	33° 41'	106	30.4
Sand, slag No.1, dry.....	1 on 1 ½	33° 41'	55	15.8
Sand, slag No.2, dry.....	1 on 1 ½	33° 41'	49	14.0
Sandstone fragments.....	1 on 1	45°	90	15.4
Shale fragments.....	1 on 1½	36° 52'	105	26.3
Slag, bank.....	½ '' to 1''	1 on 1½	36° 52'	67	16.8
Slag, bank.....	1'' to 2 ½ ''	1 on 1½	36° 52'	72	18.0
Slag, bank screenings.....	3/16'' and under	1 on 1½	36° 52'	117	29.3
Slag, bank screenings.....	½ '' and under	1 on 1½	36° 52'	98	24.5
Slag, machine.....	1'' to 2''	1 on 1½	36° 52'	96	24.0

Table 7. Coefficients of Active Earth Pressures
Trigonometric Functions for Various Angles of Repose

Slope Ratio	Angle ϕ	$\tan^2 (45^\circ - \frac{1}{2} \phi)$	$\tan^2 (45^\circ - \frac{1}{2} \phi)$
1 on 5	11° 19'	0.8198	0.6721
1 on 4	14° 2'	0.7808	0.6096
1 on 3	18° 26'	0.7208	0.5195
1 on 2	26° 34'	0.6180	0.3820
1 on 1 ½	33° 41'	0.5325	0.2864
1 on 1 ⅓	36° 52'	0.5000	0.2500
1 on 1	45° 0'	0.4142	0.1716
1 ½ on 1	56° 19'	0.3028	0.0917
2 on 1	63° 26'	0.2361	0.0557
3 on 1	71° 34'	0.1623	0.0263
4 on 1	75° 58'	0.1231	0.0152
5 on 1	78° 41'	0.0990	0.0098

Table 8. Total Loads due to Active Earth Pressures
Various Weights of Soils and Angles of Repose

$$\text{Total load, } P_e = \frac{1}{2} w_e h^2 \tan^2 (45^\circ - \frac{1}{2} \phi) = \frac{1}{2} p_e h^2$$

Depth, Feet	$w_e = 90$			$w_e = 100$			$w_e = 110$	
	$\phi = 26^\circ 34'$	$\phi = 33^\circ 41'$	$\phi = 36^\circ 52'$	$\phi = 26^\circ 34'$	$\phi = 33^\circ 41'$	$\phi = 36^\circ 52'$	$\phi = 36^\circ 52'$	$\phi = 45^\circ$
1	17	13	11	19	14	13	14	9
2	69	52	45	76	57	50	55	38
3	155	116	101	172	129	113	124	85
4	275	206	180	306	229	200	220	151
5	430	322	281	477	358	313	344	236
6	619	464	405	688	516	450	495	340
7	842	632	551	936	702	613	674	462
8	1100	825	720	1222	916	800	880	604
9	1392	1044	911	1547	1160	1013	1114	764
10	1719	1289	1125	1910	1432	1250	1375	944
11	2080	1559	1361	2311	1733	1513	1664	1142
12	2475	1856	1620	2750	2062	1800	1980	1359
13	2905	2178	1901	3228	2420	2113	2324	1595
14	3369	2526	2205	3744	2807	2450	2695	1850
15	3868	2900	2531	4298	3222	2813	3094	2124
16	4401	3299	2880	4890	3666	3200	3520	2416
17	4968	3725	3251	5520	4139	3613	3974	2728
18	5570	4176	3645	6188	4640	4050	4455	3058
19	6206	4653	4061	6896	5170	4513	4964	3407
20	6876	5155	4500	7640	5728	5000	5500	3775

3. Earth Submerged in Water.

Two assumptions are in general use:

1. The total pressure of earth and water is taken as the pressure of the earth only, where the weight of the earth per cubic foot is equal to the weight of a cubic foot of earth saturated with water.
2. The total pressure of the earth and water is taken as the full water pressure plus the earth pressure, where the weight of the earth is the weight minus the weight of water it displaces.

Assumption 2 gives the higher value for the total pressures but is the logical one because it is impossible to insure against water penetrating between the piling wall and the earth, no matter how impervious, with the corresponding development of hydrostatic pressure in addition to the earth pressures. Study of hydrostatic pressures and the means taken to prevent the same under the base of concrete dams will verify this conclusion.

The two possible loadings of submerged earth on a sheet piling wall are thus analyzed as follows:

(a) Submerged Earth on One Side Only.

Let $w_{e \text{ in } w}$ = weight, in pounds, of a cubic foot of earth submerged in water. Then the equivalent liquid pressure, $p_{e \text{ in } w}$, from the earth only, in pounds per square foot is,

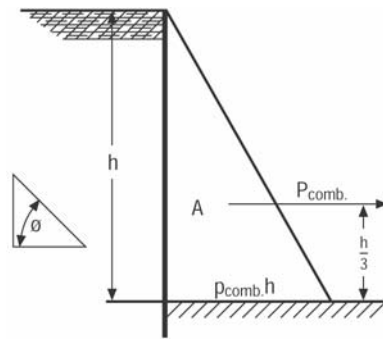


Fig. 6

$$p_{e \text{ in } w} = w_{e \text{ in } w} \tan^2 (45^\circ - \frac{1}{2} \theta), \text{ in pounds per square foot.}$$

The combined liquid pressure, $p_{comb.}$, in pounds per square foot of the earth and water is,

$$p_{comb.} = p_{e\ in\ w} + 62.5 \text{ pounds for fresh water or } 64.0 \text{ pounds for salt water.}$$

The total lateral load on the wall, $P_{comb.}$, in pounds per foot of width for a depth, h , in feet is,

$$P_{comb.} = \frac{1}{2} p_{comb.} h^2, \text{ in pounds per foot of width.}$$

The total load is distributed or applied as illustrated in triangle A, Fig. 6 adjoining.

(b) Submerged Earth on One Side, Water on the Other Side.

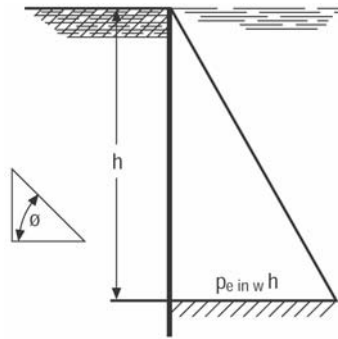


Fig. 7

The total liquid pressure will be that of the earth only.

$$p_{e\ in\ w} = w_{e\ in\ w} \tan^2 (45^\circ - \frac{1}{2} \theta), \text{ in pounds per square foot,}$$

and the total lateral load on the wall would be,

$$P_{e\ in\ w} = \frac{1}{2} p_{e\ in\ w} h^2, \text{ in pounds per foot of width.}$$

To obtain the weight per cubic foot of submerged earth, $W_{e\ in\ w}$, it is necessary to take samples of earth at the bottom as near as possible in its natural compacted state and weigh any convenient known volume when immersed in water. It may also be approximated theoretically by taking the weight of the original material and deducting the weight of a cubic foot of water multiplied by the percentage of solid material per cubic foot; that is, 100 per cent less the percentage of voids in the material.

The following table will prove useful in the determination of p_{comb} .

Table 9. Combined Pressure, p_{comb} , Sea Water and Earth
Pounds per Square Foot
(Equivalent Liquid Pressure)

(From Merriman's American Civil Engineers' Handbook)

Slope Ratio	Weight, $w_{e \text{ in } w}$, of Submerged Earth, Pounds Per Cubic Foot							
	40	44	48	52	56	60	64	68
1 on 6	104	108	112	116	120	124	128	132
1 on 5	90.9	93.6	96.3	99.0	102	104	107	109
1 on 4	88.4	90.8	93.3	95.7	98.1	101	103	105
1 on 3 ½	86.8	89.0	91.3	93.6	95.9	98.2	100	102
1 on 3	84.8	86.9	88.9	91.0	93.1	95.2	97.2	99.3
1 on 2 ½	82.3	84.2	86.0	87.8	89.7	91.5	93.3	95.2
1 on 2	79.3	80.8	82.3	83.9	85.4	86.9	88.4	90.0
1 on 1 ¾	77.5	78.8	80.2	81.5	82.9	84.2	85.6	86.9
1 on 1 ½	75.4	76.6	77.7	78.9	80.0	81.2	82.3	83.5
1 on 1 ¼	73.2	74.2	75.1	76.0	76.9	77.9	78.8	79.7
1 on 1	70.9	71.6	72.2	72.9	73.6	74.3	75.0	75.7
2 on 1 ½	68.4	68.9	69.3	69.8	70.2	70.7	71.1	71.6
2 on 1	66.2	66.4	66.7	66.9	67.1	67.3	67.6	67.8

NOTE: It will be unusual in practice to find conditions where a combined pressure, p_{comb} , in excess of 90 pounds per square foot should be used.

Table 10. Weights and Angles of Repose
Various Kinds of Materials Dumped into Water
(From Merriman's American Civil Engineers' Handbook)

Kind of Material	Slope Ratio	Angle of Repose	Weight per Cubic Foot, Pounds
Sand, clean.....	1 on 2	26° 34'	60
Sand and clay.....	1 on 3	18° 26'	65
Clay.....	1 on 3 ½	15° 57'	80
Gravel, clean.....	1 on 2	26° 34'	60
Gravel and clay.....	1 on 3	18° 26'	65
Gravel, sand and clay.....	1 on 3	18° 26'	65
Soil.....	1 on 3 ½	15° 57'	70
Soft rotten rock.....	1 on 1	45°	65
Hard rock, riprap.....	1 on 1	45°	65

The slope ratio of mud, silt, clay and sand deposited against retaining walls by suction dredge should be taken as not steeper than 1 on 3.

4. Combinations of Water and Earth Submerged in Water.

The problem of calculating the loads for the various combinations resolves itself into reducing all of the individual conditions which cause the lateral load on the wall to the individual lateral loads. For all except a surcharge load this means to their equivalent liquid pressures. These individual lateral loads are combined, after each one is studied as before described, and the total load laterally on the wall, for the most usual conditions, are best illustrated by the following sketches with the load diagrams.

The symbols shown in the sketches below have already been developed in the three paragraphs immediately preceding but for convenient reference they are again summarized as follows:

All Heights (h and h') are in feet; all pressures (p , etc.) in pounds per square foot.

Equivalent Liquid Pressure Increments

w_w = water.

p_e = earth only.

$p_{e \text{ in } w}$ = earth submerged in water.

$p_{comb.}$ = combined water and earth submerged in water.

Uniformly Distributed Lateral Pressure

p_s = Lateral pressure due to surcharge, W_8 , in pounds per square foot.

\emptyset = Angle of repose of earth, in degrees.

\emptyset' = Angle of repose of earth submerged in water, in degrees.

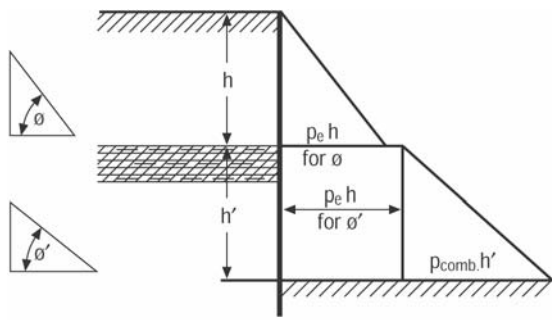


Fig. 8

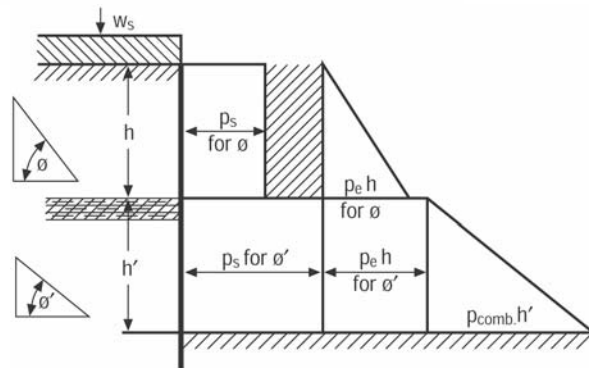


Fig. 9

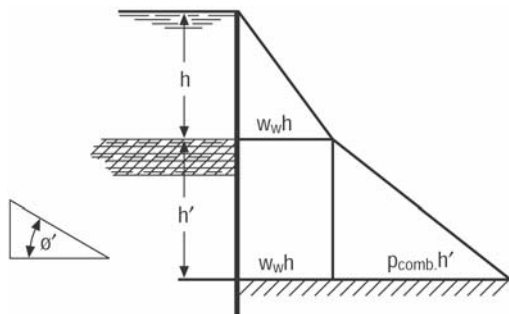


Fig. 10

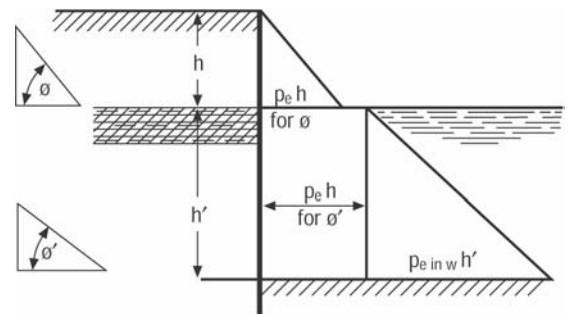


Fig. 11

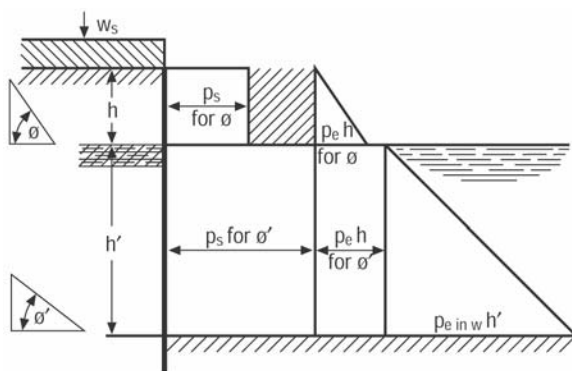


Fig. 12

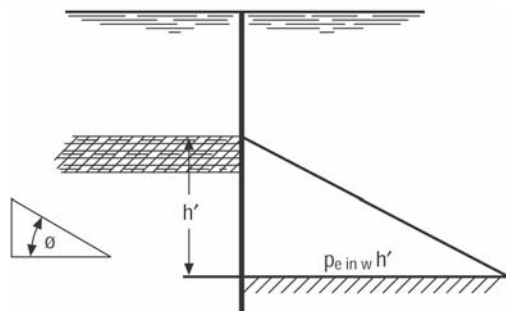


Fig. 13

Bending Moment Tie Rod and Wale Loads Working Stresses

1. Bending Moment.

When steel sheet piling is used for depths beyond its strength as a cantilever, it must be braced with a system of wales. For retaining walls these wales are usually tied back to an anchor system by means of steel tie rods, and for cofferdams the wales are generally braced internally across the cofferdam to the opposite wale.

The bending moment, M , in inch-pounds per foot of width of wall, developed from the top down to the first wale under liquid pressure, is that for a uniformly increasing load on a beam as follows:

(a) **Cantilever** with support h feet below the top,

$$M \text{ (inch-pounds per foot of width)} = \frac{P \times h \times 12}{3} = 4 Ph, \text{ when}$$
$$P = \frac{1}{2} p_{equiv.} h^2, \text{ in pounds per foot of width,}$$

in which $p_{equiv.}$ is the equivalent liquid pressure increment in pounds per square foot per foot of depth.

The calculation of $p_{equiv.}$ and application or distribution of the total load, P , has been developed in paragraphs 1 to 4 inc. immediately preceding, the $p_{equiv.}$ corresponding to either w_w , p_e , $p_{e \text{ in } w}$, or $p_{comb.}$ in these paragraphs.

(b) **Simple Beam** with first support at the top and the second h feet below.

$$M \text{ (inch-pounds per foot of width)} = \frac{2 \times P \times h \times 12}{9\sqrt{3}} = 1.54 Ph,$$

in which P is determined as above.

The necessary section modulus of the steel sheet piling per foot of width is,

$$\frac{M}{f},$$

where f is the safe working stress, in pounds per square inch, for the steel.

Given a steel piling section with a section modulus, S , per foot of wall, the necessary distance from the top, h , in feet, for the wale can be calculated by reversing the formula for a beam with a uniformly increasing load in which f is equal to the safe working stress per square inch of steel as follows:

(c) Top Section a Cantilever.

$$h^3 = \frac{6fS}{p_{equiv.} \times 12} \text{ or } h^3 = \frac{fS}{2p_{equiv.}}$$

in which $p_{equiv.}$ Is the equivalent liquid pressure in pounds per square foot.

For fresh water $p_{equiv.}$ is 62.5 and for salt water, 64.0. Using a working stress of 24,000 pounds per square inch for the steel, then,

$$h^3 = 192 \text{ S for fresh water} \quad h^3 = 187.5 \text{ S for salt water.}$$

(d) Top Section a Beam supported at Ends.

$$h^3 = \frac{9\sqrt{3}fS}{p_{equiv.} \times 12} \quad \text{or} \quad h^3 = \frac{1.30fS}{p_{equiv.}}$$

Using a working stress of 24,000 pounds per square inch,

$$h^3 = 500 \text{ S for fresh water} \quad h^3 = 487 \text{ S for salt water.}$$

For the sections or section below h feet, the calculation of the bending moment M becomes a combination of a simple beam uniformly loaded (rectangle B) and a uniformly increasing load (triangle C) in Fig. 14 below.

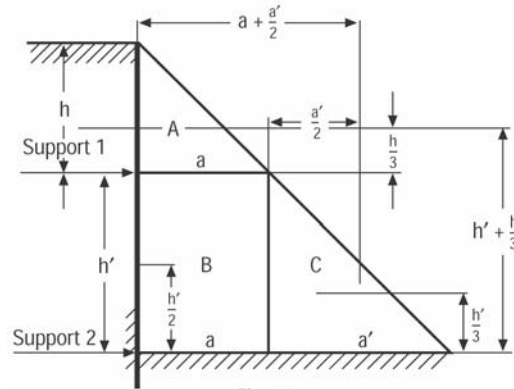


Fig. 14

Uniformly distributed load,

$$M = \frac{P \times h' \times 12}{8} = 1.50 Ph' \text{ when } P = ah'.$$

Uniformly increasing load,

$$M = \frac{2 \times P' \times h' \times 12}{9\sqrt{3}} = 1.54 P'h' \text{ when } P' = \frac{a'h'}{2}$$

The actual combined moment cannot be obtained by adding M+M' but it is a figure between 1.50 (P+P') h' and 1.54 (P+P') h'.

Since the error is inconsequential, a very close figure to the actual bending moment can be obtained by treating the combined load of:

$\left(a + \frac{a'}{2}\right)h'$ as a uniformly distributed load which makes the formula,

$$M \text{ (combined) in inch pounds per foot of width} = \frac{(P + P') \times h' \times 12}{8} = 1.50(P + P')h'$$

2. Tie Rod and Wale Loads.

The load in pounds per foot of width of wall for support 1 is,

$$\frac{\text{triangle}A}{h'} \left(h' + \frac{h}{3} \right) + \frac{h'(\text{rectangle}B)}{2h'} + \frac{h'(\text{triangle}C)}{3h'}$$

or

$$\frac{\text{triangle}A}{h'} \left(h' + \frac{h}{3} \right) + \frac{\text{rectangle}B}{2} + \frac{\text{triangle}C}{3}$$

On support 2 the load is,

$$(\text{triangle}A + \text{triangle}B + \text{triangle}C) - \text{load on support 1.}$$

The size of the wale can be selected by multiplying the load per foot on the wale by the selected distance between the struts or tie rods and considering this total load as a uniformly distributed load. The size of the tie rod can be determined by dividing this total load by the safe working stress of the steel and selecting one having the necessary area at the root of the threads.

3. Working Stresses

Steel Sheet Piling produced in the United States is now available in at least four strength levels from 38,500 to 55,000 psi minimum yield points. For permanent structures general practice is to base maximum allowable working stresses on 60% of the yield point of the material. (S.F. = 1.67)

Steel Sheet Piling Grade Y.P. - Psi	Working Stress Psi
38,500	24,000
45,000	27,000
50,000	30,000
55,000	33,000

These working stresses may be increased or reduced of course depending on the factor of safety desired in the steel design. Higher stresses than shown are generally acceptable where such stresses are temporary or short term.

For cellular cofferdams subjected to ring tension, maximum working interlock stresses of 8,000 pounds per lineal inch for flat web sections similar to the U.S. Steel MP-101, and 3,000 pounds per lineal inch for shallow arch sections similar to U. S. Steel MP-112 Sections, will provide a safety factor of at least 2 against rupture.

Other structural steel items such as wales and tie rods are available in a number of grades providing a selection of strength levels. For the more commonly used grades A-7 and A-36, working stresses of 20,000 and 22,000 psi are generally acceptable and provide factors of safety equivalent to those selected for the sheet piling.

Wharves, Slips and Retaining Walls

Modern wharves and retaining walls are constructed with a continuous wall of steel sheet piling in front of which the water is either already deep enough for the channel desired or is dredged to the required depth after the steel sheet piling is driven. A system of wales, tie backs and anchors is constructed and a fill is made back of this wall.

The problem resolves itself into a study of the following:

1. Depth to which the steel sheet piling should be driven.
2. Line of support of the piling in the bottom.
3. Bending strength of the steel sheet piling to resist the lateral loads due to the fill with a surcharge load if the latter is contemplated.
4. Size of wales and tie rods.
5. Location and design of the anchor system.

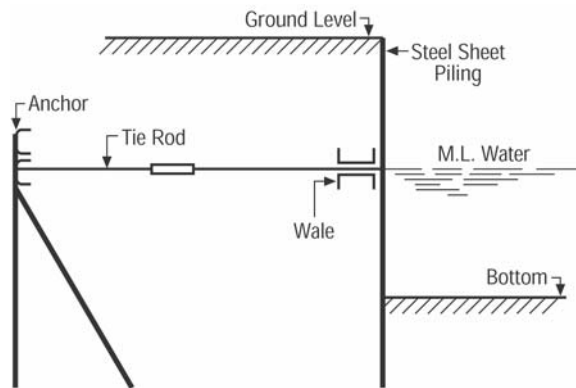


Fig. 15

The simplest form of the problem is illustrated by the above Fig. 15.

The tie rod and top of the anchor system is usually placed at or slightly below the water line in order to keep them in the area of saturation to preserve them against rust and rot. Lowering the wale reduces the unsupported span between the wale and the bottom, thus reducing the bending stress in the steel sheet piling wall. Two feet below water mark is the limit of depth at which men in wading boots can work for the drilling or flame cutting of holes, attachment of waling and bolts; and even working at this depth is slow and expensive during the short intervals at low tide. Work below a depth of 2 feet must be done by divers.

Except in the cases of wharves carrying very heavy surcharge loads, or when the cantilever span from the wale to the top of the wall is excessive, the critical stresses on the steel sheet piling wall occur when, and immediately after, the fill is made and in the span between the wale and the bottom. To obtain the most economical design, the angle of repose of the soil and weight, both submerged in water and above the water line, should be carefully determined. Hydraulic fills in which the earth is almost liquid, having a very small angle of repose, may exert liquid pressures up to 90 pounds per square foot. Compact soil, having a steeper angle of repose for the fill, greatly reduces the liquid pressure.

1. Depth to Which Steel Sheet Piling Should Be Driven.

The steel sheet piling must be driven to sufficient penetration below the bottom to insure stability of the base of the wall against lateral movement. There are two methods of determining this depth:

- (a) Mathematical determination.
- (b) Experience.

(a) Mathematical Determination. Lateral movement at the base of the steel sheet piling wall is prevented by the resistance of the soil to sliding laterally, being pushed up against the force of gravity and breakage of its cohesion. This resistance of the soil is calculated as a counter pressure against the base of the sheet piling wall and designated as the passive pressure of the bottom. This passive pressure is liquid because it increases in direct proportion to the depth.

Coulomb's law for passive pressures is:

Let p_p = equivalent liquid passive pressure in pounds per square foot,

\emptyset = angle of repose of the soil at the bottom in degrees,

w_e = weight per cubic foot of the soil at the bottom in pounds,

then,

$$p_p = w_e \tan^2 (45^\circ + \frac{1}{2} \emptyset).$$

When the soil at the bottom is submerged, the weight of submerged soil, w_e in w , set up in previous calculations, should be used in place of w_e .

The total passive pressure (P_p) in pounds per foot of width is,

$$P_p = \frac{1}{2} p_p X^2, \text{ when } x \text{ is the depth of penetration below the bottom in feet.}$$

The total passive pressure can be considered as concentrated at an elevation of $x/3$ feet above the bottom of the steel sheet piling.

It is obvious from the above formula that the angle of repose and the weight of the soil into which the steel sheet piling is to penetrate must be carefully determined. This is accomplished either by dredging samples of the soil and obtaining the data with the soil kept as near as possible in its native condition or

by means of drillings in which a comparison can be made by experience with other soils of a similar character.

Table 11, following, gives the values of $\tan^2 (45^\circ + \frac{1}{2} \phi)$ for various angles of repose.

Table 11. Coefficients of Passive Earth Pressures
Trigonometric Functions for Various Angles of Repose

Slope Ratio	Angle ϕ	$\tan^2 (45^\circ - \frac{1}{2} \phi)$	$\tan^2 (45^\circ + \frac{1}{2} \phi)$
1 on 6	9° 28'	1.1806	1.3938
1 on 5	11° 19'	1.2200	1.4884
1 on 4	14° 2'	1.2807	1.6402
1 on 3	18° 26'	1.3874	1.9249
1 on 2	26° 34'	1.6181	2.6182
1 on 1 $\frac{1}{2}$	33° 41'	1.8683	3.4905
1 on 1 $\frac{1}{3}$	36° 52'	1.9999	3.9996
1 on 1	45° 0'	2.4142	5.8284
1 $\frac{1}{2}$ on 1	56° 19'	3.3035	10.9131
2 on 1	63° 26'	4.2358	17.9420
3 on 1	71° 34'	6.1628	37.9801
4 on 1	75° 58'	8.1248	66.0124
5 on 1	78° 41'	10.0930	101.8686

Professor Franzius of Hanover has conducted very thorough and careful experiments to determine the passive pressures of moist sand. His report of 55 tests are published in Der Bauingenieur-1924, Vol. 10. The results were consistent and indicated that the passive pressure of sand is approximately double that indicated by the Coulomb formula, or,

$$p_p = (\text{approximately}) 2 w_e \tan^2 (45^\circ + \frac{1}{2} \phi).$$

The dimensions of the box of sand used for the experiment were about 16'-5" by 4'-11" and the depths of sand investigated ranged from about 16 to 40 inches. Although these experiments were conducted on a very small scale and were few in number they are described because they indicate that, when the Coulomb formula is used for the determination of the passive pressure, the equivalent liquid pressure derived therefrom is a conservative figure with a large factor of safety.

With the passive pressure (p_p) in pounds per square foot determined, the depth necessary to drive the steel sheet piling below the bottom to insure stability can be calculated by the method of moments outlined in Merriman's American Civil Engineers' Handbook. Fig. 16 below illustrates the problem.

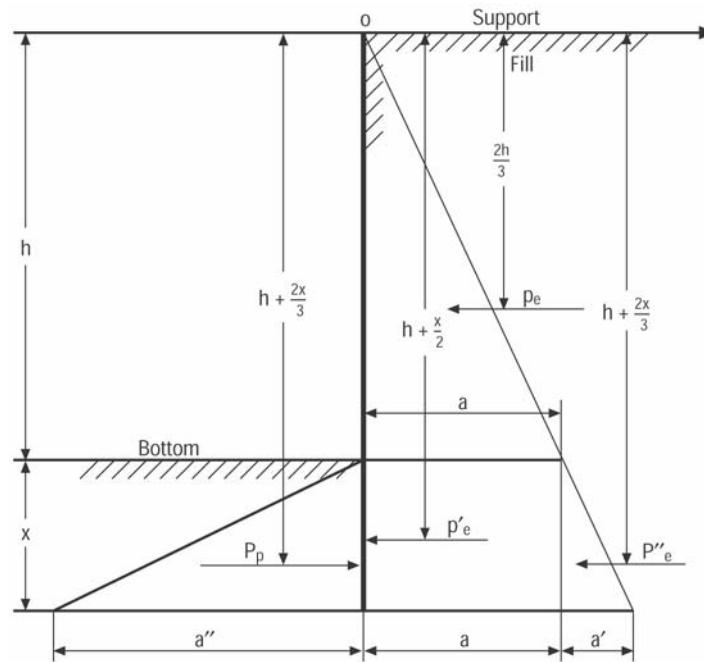


Fig. 16

$p_{equiv.}$ = equivalent liquid pressure of the fill in pounds per square foot.

p_p = equivalent liquid passive pressure in pounds per square foot.

Then,

$$a = p_{equiv.} h$$

$$a' = p_{equiv.} x$$

$$a'' = p_p x$$

all in pounds per foot of width.

Then the total pressures, as represented by the respective triangles and rectangles, are,

$$P_e = \frac{p_{equiv.} h^2}{2}$$

$$P'_e = p_{equiv.} h x$$

$$P''_e = \frac{p_{equiv.} x^2}{2}$$

$$P_p = \frac{p_p x^2}{2}$$

Equating moments about o,

$$P_p \left(h + \frac{2x}{3} \right) = P_e \left(\frac{2h}{3} \right) + P'_e \left(h + \frac{x}{2} \right) + P''_e \left(h + \frac{2x}{3} \right). \text{ Solve for } x.$$

This formula is involved and is most conveniently solved by assuming a value for x of about 35 per cent of h and solving, thus finding how closely the equation balances.

(b) **Experience.** Engineering experience, precedent and judgment can also be exercised in this determination. Consideration should be given to the unsupported height of the sheet piling between the tie rod and the bottom, as well as the intensity of the pressure at the bottom and the stability of the soil into which the piling is driven. For unstable soil and deep channels (25 to 35 feet), the maximum depth of penetration may be as high as 75 per cent of the unsupported height. For shallower water, less intensity of pressure and hard, stable bottom, the safe penetration may reduce to as low as 20 per cent of the unsupported height.

For walls entirely cantilever, the penetration below bottom should equal approximately the unsupported height above.

2. Line of Support of Steel Sheet Piling at the Bottom.

Obviously the line of support of the steel sheet piling wall is not at the surface of the soil into which the piling penetrates but is a line somewhere below the surface. In order to calculate the bending moment on the wall due to the lateral pressure of the earth, this line of support is determined by two methods.

- (a) Mathematical determination.
- (b) Experience.

(a) **Mathematical Determination.** The depth x , in feet, as the proper depth to drive the sheet piling below the bottom, has been determined as explained in paragraph (a) preceding.

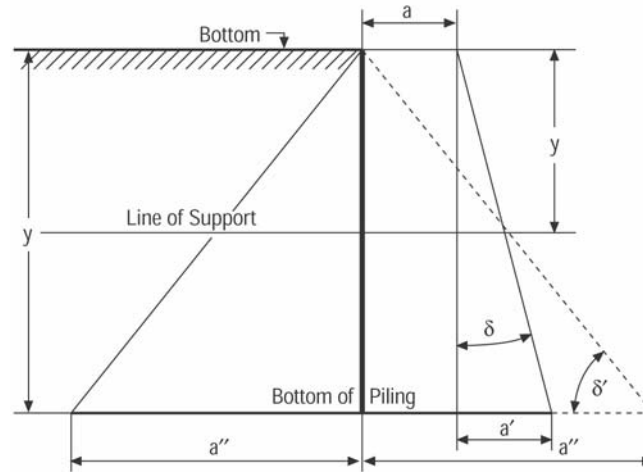


Fig. 17

Let y , in feet, be the unknown distance below the bottom which can be assumed as the line of support of the sheet piling wall carrying the load of the fill. Use the same symbols as in paragraph 1 (a) preceding and reconstruct the load diagram below the bottom only. The problem is illustrated in Fig. 17 adjoining. Superimpose the load triangle representing the passive pressure of the soil, as shown by the dotted line.

The point of intersection of the passive pressure line (dotted) with the active pressure line (full) locates the line of support of the sheet piling wall.

$$\tan \delta' = \frac{x}{a''}, \quad \tan \delta = \frac{a'}{x}$$

then,

$$x - y = (a'' - a) \tan \delta' - y \tan \delta \tan \delta'.$$

Substituting and solving,

$$y = \frac{ax}{a'' - a'} = \frac{a}{p_p - p_{equiv.}}$$

The accuracy of this entire computation is of course dependent on the successful determination of p_p

(b) **Experience.** Engineering judgment and experience can also be exercised here in this determination by assuming a line of support, taking into consideration the same conditions for the determination of the depth to which the steel sheet piling should be driven. For unstable bottoms, deep channels and great intensity of pressure at the bottom, the line of support may be assumed as low as 8 feet below the bottom and, for more favorable conditions, up to a few inches below the bottom.

3. Bending Strength.

The approximate bending moment on the sheet piling wall for the span between the tie rod and line of support results from a total load, in pounds per foot of width, as represented, in Fig. 18 adjoining, by,

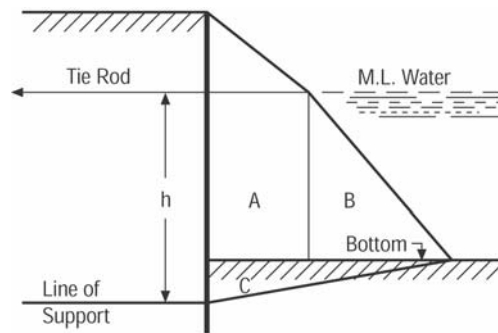


Fig. 18

$$\text{Total Load} = A + B + C$$

and the unsupported height as h , in feet.

The bending moment M , in inch pounds per foot of width is approximately,

$$M = \frac{\text{total load} \times h \times 12}{8}.$$

Details of the necessary calculations are shown in paragraph 1, Bending Moment, pages 7 and 8.

4. Size of Wales and Tie Rods.

The calculation for the required size of wales and tie rods is outlined in paragraph 2, page 8.

5. Location and Design of Anchor System.

(a) **Location.** The location of the anchor system should be far enough back of the sheet piling wall to be in stable ground.

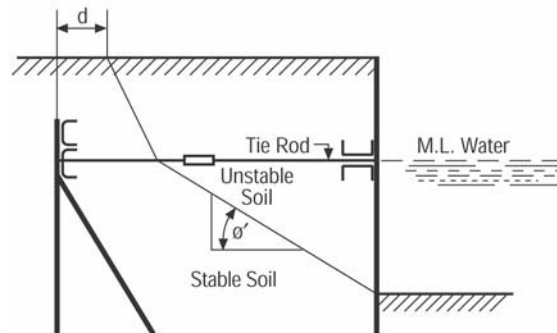


Fig. 19

The angle of repose of the fill locates the plane of stable soil as illustrated in Fig. 19 above, and the line of the anchors should be placed at distance, d , equal to 10 to 15 feet back of this plane if practicable, for absolute safety.

(b) **Design.** The proper design is generally based on judgment, precedent and experience. Driving of test piles to determine their bearing capacity is of great assistance. Authorities generally assume that it requires the same force to extract the pile as to drive it. Determination of the passive resistance of the soil is also of assistance in deciding upon the necessary area of the anchor system bearing upon the soil.

Where the anchor system extends above the level of the original soil or extends into the contemplated fill, extreme care should be exercised in making certain that the vertical piles of the anchorage are driven deep enough to develop enough skin friction to resist extraction and that the batter piles are safe in bearing to resist the horizontal pull of the tie rods.

Placing the fill around the anchor system and allowing it to settle before the fill is placed against the piling wall is advisable wherever possible.

Relieving Platforms

When the lateral loads become so heavy, due to deep channels and heavy surcharge loads, that the bending strength required in the steel sheet piling wall exceeds the strength of any rolled section, relieving platforms can be constructed, supported by steel or wood bearing piles as illustrated in Fig. 20 adjoining.

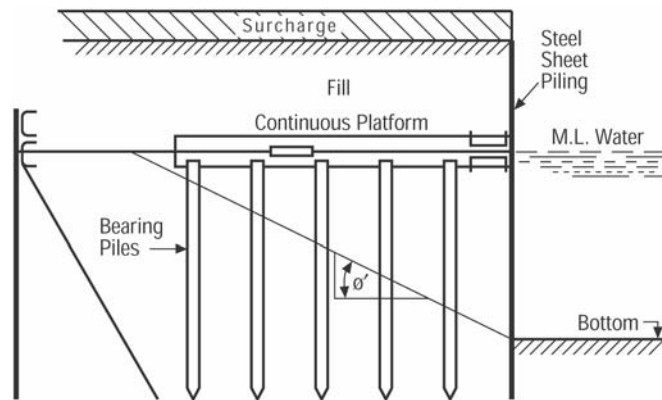


Fig. 20

The platform is best located at the water line because it relieves the lateral load on the sheet piling wall below the platform, due to the weight of the earth and surcharge above the platform, and also allows the tops of the bearing piles to remain in the zone of saturation, which preserves them from rot.

1. Bearing Piles.

The bearing piles should be spaced and driven to the necessary penetration to support the load upon the platform due to the earth and surcharge above. The following tables, published by permission of the McKiernan-Terry Corporation and Vulcan Iron Works, are useful.

Cofferdam Design

There are two main types of steel sheet piling cofferdams:

1. Single wall internally braced.
2. Cellular.

1. Single Wall Braced Cofferdams.

There are no standard types of construction in framing and bracing cofferdams. However, the usual type of wooden braced cofferdam for small piers is made with wale timbers, struts, posts and tie rods. To facilitate computation of sizes required, reference may be had to tables 18 and 19, pages 19 and 20, giving safe loads for square timbers used as beams and also as struts, in accordance with specifications of United States Forest Service and Department of Commerce.

In construction of piers requiring very large cofferdams, steel may be used in place of wood, the wales, struts and posts being made of structural shapes and diagonal bracing either of wooden timbers or rods. At interior junctions of this bracing, framing should conform to that in general use in structural work. At ends of struts, provision should be made for adjustment by use of wooden wedges driven, on one hand, to compel the struts to bear tightly against the posts or waling pieces and, on the other hand, to permit quick removal of timbers when cofferdam is completed. The plan of construction is often based on use of a single-size section for wales throughout depth of cofferdam. In the deeper panels, bending stress on these sections may be too great to allow their use, safely, on clear spans center to center of posts, in which case short foot blocks may be used; in upper panels these will not as a rule be required.

Steel bracing for cofferdams is often economical and advantageous, especially for large or deep cofferdams subjected to hydrostatic or heavier pressures, and is practical to greater depths than wood.

Steel may be economical because rolled steel beams are offered which will carry far greater loads, and on longer spans, than the largest available timber. The number of wales required is thus reduced because they can be spaced to develop the full strength of the steel sheet piling rather than to the limit of the strength of the timber. The longer unsupported spans possible with steel materially reduce the amount of cross bracing required, thereby greatly enlarging the free space in the cofferdam for excavation and concreting as well as requiring less cutting of the form work and reducing the loss of time required to remove bracing while pouring the concrete. Steel can be cast permanently into concrete and the small loss of bracing therefrom may prove to be a great saving of time and labor. Steel bracing in some structures will reduce the size of the cofferdam which is an economy in piling, bracing and excavation.

CB Sections with their wide flanges are ideal for this type of work as they have great strength combined with light weight, offer great resistance against overturning, and connections are very simple and easily assembled. The steel, if properly handled, has a large salvage value.

Flame cutting and fitting in the field is often advantageous and economical.

A combination of steel wales and wooden cross bracing at times will be advisable, as well as timber for the upper wales and lighter loads, and steel for the lower wales carrying the heavier loads.

Bracing:

Where steel sheet piling is used for cofferdams or other structures of depths beyond its cantilever strength (see table 15), it becomes necessary to provide an adequate system of bracing. Spacing of

rangers and struts will depend upon character of soil, hydrostatic pressure, etc. Economy in total cost of construction is better attained by use of heavy piling and a small amount of bracing than by use of light piling with a large number of braces. It is better to err on the side of excessive bracing than to take chances of failure. Rules and formulas for bracing design should, therefore, be interpreted in the light of actual experience.

(1). **Wale Spacing.** Location and spacing of wales or rangers are determined by the general formulas for flexure with appropriate values for loading, as computed from formulas already given for earth and water pressures. Top wale may be located at the surface, in which case the piling may be considered as beams supported at both ends and partly continuous over supports, or may be located at such a distance below the surface as to develop full cantilever strength of piling above. These conditions give rise to two methods of computation.

(a) **Top Section, a Cantilever.**

ϕ = angle of repose of retained material, in degrees.

h = unsupported height, in feet.

w = weight of retained material, in pounds per cubic foot.

S = section modulus, in inches', per foot of wall.

f = working stress of the steel sheet piling, in pounds per square inch (assumed as 24,000 pounds).

$$h^3 = \frac{12,000S}{w \tan^2 \left(45^\circ - \frac{1}{2} \phi \right)}, \text{ for earth.}$$

For water pressures,

$\phi = 0$.

$w = 62.5$ for fresh water or
64.0 for salt water.

Substituting in above formula,

$h^3 = 192 S$ for fresh water and

$h^3 = 187.5 S$ for salt water.

(b) Top Section, a Beam Supported at Ends.

$$h^3 = \frac{31,000S}{w \tan^2 \left(45^\circ - \frac{1}{2} \phi \right)}, \text{ for earth,}$$

and, substituting as above,

$$h^3 = 500 S \text{ for fresh water and } h^3 = 487 S \text{ for salt water.}$$

If it is desirable to use any other unit stress, multiply the results obtained by the desired unit stress and divide by 24,000.

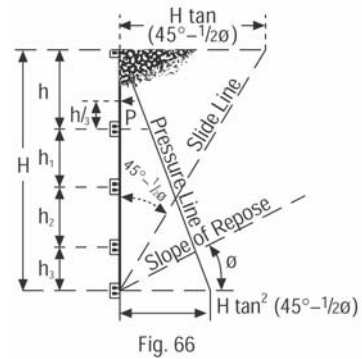
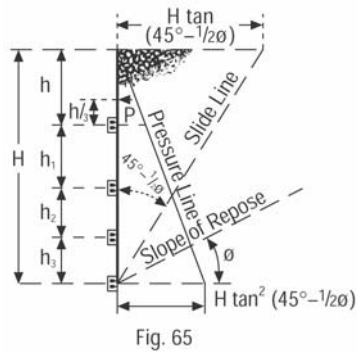
(2). Wale Computation. When value of h has been determined by these formulas, and first or second wale located in such a way as to develop full strength of piling in first panel of cofferdam or other structure, location of next and subsequent wales may be determined in accordance with two methods of construction: wales may be placed at such a distance apart as to make stresses in piling equal, in which case size of wale timbers will increase from top downward; or wales may be so located that stresses in all wales are equal, in the case of Fig. 65 and in all except top wale Fig. 66. In this case, stresses in piling will decrease from top downward and there will be an increase in margin of strength as depth increases while size of wales remains constant. In either method, the calculated space between wales decreases as depth of excavation increases.

Formulas which underlie computation of successive distances between wales are somewhat intricate, but the distances themselves are so related to distance h from surface to first or second wale that they may be expressed in terms of the latter. Table below gives successive wale spaces for these two conditions and is based on assumption that effect on wales or piling is produced by a load increasing uniformly from one end and that, with exception of top section, piling is in condition of a beam partly continuous over two supports so that bending moments may be computed from flexure formula: $M_{\max} = Wl/10$ where W is the total pressure load between supports and not the unit weight of materials.

To design cofferdam or other bracing systems, decide whether or not to place a wale at the surface. Determine distance h by formula. Compute successive distances to second or third and subsequent wales by multiplying h by constants given in table below for desired construction.

Table 15. Wale Spacing Coefficients

Equal Stresses, Cantilever Span at Top		Equal Stresses, Wale at Top	
Wales	Piling	Wales	Piling
$h_1 = 0.828 h$	$h_1 = 1.046 h$	$h_1 = 0.565 h$	$h_1 = 0.691 h$
$h_2 = 0.539 h$	$h_2 = 0.823 h$	$h_2 = 0.404 h$	$h_2 = 0.570 h$
$h_3 = 0.436 h$	$h_3 = 0.718 h$	$h_3 = 0.334 h$	$h_3 = 0.505 h$
$h_4 = 0.372 h$	$h_4 = 0.653 h$	$h_4 = 0.291 h$	$h_4 = 0.463 h$
$h_5 = 0.336 h$	$h_5 = 0.606 h$	$h_5 = 0.261 h$	$h_5 = 0.432 h$
$h_6 = 0.307 h$	$h_6 = 0.570 h$	$h_6 = 0.239 h$	$h_6 = 0.408 h$
$h_7 = 0.284 h$	$h_7 = 0.541 h$	$h_7 = 0.222 h$	$h_7 = 0.388 h$
$h_8 = 0.265 h$	$h_8 = 0.518 h$	$h_8 = 0.208 h$	$h_8 = 0.372 h$
$h_9 = 0.250 h$	$h_9 = 0.498 h$	$h_9 = 0.196 h$	$h_9 = 0.358 h$
$h_{10} = 0.237 h$	$h_{10} = 0.481 h$	$h_{10} = 0.186 h$	$h_{10} = 0.346 h$



Equal stresses apply only to wales below top wale. Stresses in top wale are appreciably less than in other wales.

(3). Pressure Loads on Wales and Struts. When the location and spacing of wales have been determined, the next step is to compute the pressure loads on wales and struts. Compute total thrust in upper panel (for cases A and B in second panel also) by formula $P = \frac{1}{2} wh^2 \tan^2 (45^\circ - \frac{1}{2}\phi)$, (for water $P = \frac{1}{2}wh^2$), and then proceed by the following practical approximate methods:

(a) Equal Stresses in Piling, Cantilever Top Section. Add to total thrust in upper panel one-half the thrust in second panel and multiply the sum by length of wale between struts.

(b) Equal Stresses in Piling, Wale at Top. For top wale, multiply one-third the thrust in upper panel by length of wale between struts. For second wale, add two-thirds the thrust in upper panel to one-half the thrust in second panel and multiply the sum by length of wale between struts.

For all other wales in both these cases, compute pressure in pounds per square foot at wale. Multiply result by one-half the distance between adjacent wales above and below and that product by length of wale between struts.

(c) Equal Stresses in Wales, Cantilever Top Section. Multiply the thrust per foot of wall in upper panel by 2.25, and that product by length of wale between struts.

(d) Equal Stresses in Wales, Wale at Top. Multiply the thrust per foot of wall in upper panel by length of wale between struts and that product by 0.3333 to obtain load pressure on top wale, and by 1.4216 to obtain load pressure on second and subsequent wales.

Note that equal stresses apply only to wales below top wale. Stresses in top wale are appreciably less than in other wales.

(e) Compression in Struts. Compression in any strut will be half the load pressure on each of the two adjacent wales.

Example:

Assume a cofferdam with a hydrostatic head of 29 feet.

For piling Section MP 116 with cantilever span at the top

$$h \text{ max.} = \sqrt[3]{192 \times 10.7} = 12.7 \text{ feet}$$

and for equal stresses in wales the distance from wale No. 1 to No. 2 = .828 x 12.7 = 10.5 feet, No. 2 to No. 3 = .539 x 12.7 = 6.9 feet.

Round these to 12 feet, 10 feet and 6 feet respectively.

From table 5 on page 1, pressures in pounds per square foot are as follows:

12 feet down = 750

22 feet down = 1,375

28 feet down = 1,750

Loads in pounds per foot of wall are:

On top span = 4,500

On second span = 15,125 - 4,500 = 10,625

On third span = 24,500 - 15,125 = 9,375

On wale 12 feet down,

$$4500 + \frac{10625}{2} = 9,812 \text{ pounds}$$

Assuming steel bracing and 9 foot spacing between supports, the section modulus required will be

$$S = \frac{M}{f} = \frac{wl^2}{8f} = \frac{9812 \times 9 \times 9 \times 12}{8 \times 20,000} = 60 \text{ in}^3$$

M = maximum bending moment, inch pounds

f = allowable unit working stress, pounds per square inch

S = section modulus, inches cubed

w = load on wale, pounds per foot

l = unsupported span, in inches

Select a 16" wide flange beam at 40 pounds per foot.

$$S = 64.4 \text{ in}^3$$

Load on strut = 9 x 9812 = 88,308 pounds

Struts are designed according to column formulas.

Table 18. Bracing—Square Wooden Beams
Safe Loads in Thousands of Pounds for Select Grade Occasionally Wet

Douglas Fir
Maximum Bending 1385; Maximum Shear 90 Lbs. Sq. In.

Span in Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
	4.3							
8	4.2							
9	3.7							
10	3.3	7.7						
11	3.0	7.2						
12	2.8	6.6	12.0					
13	2.6	6.1	11.8					
14	2.4	5.6	11.0					
15	2.2	5.3	10.3	17.3				
16	2.1	4.9	9.6	16.6				
17	2.0	4.6	9.1	15.6	23.5			
18		4.4	8.5	14.8	23.5			
19		4.1	8.1	14.0	22.2			
20		3.9	7.7	13.3	21.1	30.7		
21		3.8	7.3	12.7	20.1	30.0		
22		3.6	7.0	12.1	19.2	28.6		
23		3.4	6.7	11.6	18.4	27.4	38.9	
24		3.3	6.4	11.1	17.6	26.3	37.4	
25		3.2	6.2	10.6	16.9	25.2	35.9	48.0

Red and White Oak
Maximum Bending 1200; Maximum Shear 125 Lbs. Sq. In.

Span in Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
	6.0							
5	5.8							
6	4.8							
7	4.1	10.7						
8	3.6	9.8	16.7					
9	3.2	8.5	14.8	24.0				
10	2.9	7.6	13.3	23.0				
11	2.6	6.8	12.1	20.9	32.7			
12	2.4	6.2	11.1	19.2	30.5	42.7		
13	2.2	5.7	10.3	17.7	28.1	42.0		
14	2.1	5.3	9.5	16.5	26.1	39.0		
15	1.9	4.9	8.9	15.4	24.4	36.4	54.0	
16		4.6	8.3	14.4	22.9	34.1	51.8	66.7
17		4.3	7.8	13.6	21.5	32.1	48.6	66.7
18		4.0	7.4	12.8	20.3	30.3	45.7	62.7
19		3.8	7.0	12.1	19.3	28.7	43.2	59.3
20		3.6	6.7	11.5	18.3	27.3	40.9	56.1
21		3.4	6.3	11.0	17.4	26.0	38.9	53.3
22		3.3	6.1	10.5	16.6	24.8	37.0	50.8
23		3.1	5.8	10.0	15.9	23.8	35.3	48.5
24		3.0	5.6	9.6	15.2	22.8	33.8	46.4
25		2.8	5.3	9.2	14.6	21.8	32.4	44.4
		2.7					31.1	42.7

Hemlock

Maximum Bending 1100; Maximum Shear 75 Lbs. Sq. In.

[illegible]

Spruce

Maximum Bending 900; Maximum Shear 85 Lbs. Sq. In.

Span in Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	4.1							
	3.6							
	3.1							
	2.7	7.3						
	2.4	6.4	11.3					
	2.2	5.7	11.1					
	2.0	5.1	10.0	16.3				
		4.7	9.1	15.7				
		4.3	8.3	14.4	22.2			
		3.9	7.7	13.3	21.1			
		3.7	7.1	12.3	19.6	29.0		
		3.4	6.7	11.5	18.3	27.3		
		3.2	6.3	10.8	17.2	25.6	36.7	
		3.0	5.9	10.2	16.1	24.1	34.3	45.3
		2.8	5.6	9.6	15.2	22.8	32.4	44.4
		2.7	5.3	9.1	14.4	21.6	30.7	42.1
		2.6	5.0	8.6	13.7	20.5	29.2	40.0
			4.8	8.2	13.1	19.5	27.8	38.1
			4.5	7.9	12.5	18.6	26.5	36.4
			4.3	7.5	11.9	17.8	25.4	34.8
			4.2	7.2	11.4	17.1	24.3	33.3
			4.0	6.9	11.0	16.4	23.3	32.0

Southern Yellow Pine
Maximum Bending 1385; Maximum Shear 110 Lbs. Sq. In.

Span in Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
7	5.3	9.4	14.7	21.1	28.7	37.5	47.5	58.7
8	4.7							
9	4.2	8.8	12.8	20.5	26.4	33.2	42.7	52.7
10	3.7							
11	3.3	7.9	11.0	14.0	17.7	22.2	27.4	33.5
12	3.0	7.2	10.3	13.3	16.6	21.1	26.3	32.5
13	2.8	6.6	9.6	12.1	15.6	20.1	25.2	31.5
14	2.6	6.1	9.1	11.6	14.8	19.2	24.4	30.5
15	2.4	5.6	8.5	11.1	13.8	18.4	23.6	29.5
16	2.2	5.3	8.1	10.6	13.3	17.6	22.8	28.5
17	2.1	4.9	7.7	10.1	12.8	17.1	22.3	28.0
18	2.0	4.6	7.3	9.6	12.4	16.7	21.9	27.6
19		4.4	7.0	9.3	12.1	16.4	21.6	27.3
20		4.1	6.7	8.9	11.8	16.1	21.3	27.0
21		3.9	6.4	8.6	11.5	15.8	21.0	26.7
22		3.8	6.2	8.4	11.3	15.6	20.8	26.5
23		3.6	6.0	8.2	11.1	15.4	20.6	26.3
24		3.4	5.8	8.0	10.9	15.2	20.4	26.1
25		3.2	5.6	7.8	10.7	15.0	20.2	25.9

NOTE: Unit Stresses as recommended by Forest Products Laboratory, U.S. Forest Service.
Maximum safe load = $\frac{4}{3}$ Area of Section \times Maximum Unit Shearing Stress.

Table 19. Bracing—Square Wooden Columns
Safe Loads in Thousands of Pounds for Select Grade Occasionally Wet
 U. S. Forest Service and Department of Commerce Formula

Douglas Fir $f_c = 1065$

Length Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
5	38.3	68.2	106.5	153.4	208.7	272.6	345.1	426.0
6	37.6							
7	37.0							
8	36.1							
9	34.8	66.0	105.7	153.4	208.2	271.4	342.8	422.8
10	32.9	65.0	104.5					
11	30.6	63.6	103.5	151.9	208.2	272.6	345.1	426.0
12	27.3	62.0	102.3	150.5				
14	20.2	56.3	98.9	148.0	204.8	271.4	342.8	422.8
16	15.4	48.6	93.6	144.4	202.1	267.5	338.6	418.0
18	12.2	38.7	86.2	139.4	198.2	264.2	335.0	414.0
20	9.9	31.2	75.9	131.8	192.9	259.8	330.2	409.2
22		25.9	63.1	122.3	185.0	254.5	325.0	
24		21.6	53.1	109.3	175.4	247.8		

Red and White Oak $f_c = 900$

Length Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
5	32.4	57.6	89.4	129.6	176.4	230.4	291.6	360.0
6	31.9							
7	31.5							
8	30.9							
9	30.0	56.2	89.4	129.6	176.0	229.4	290.0	357.6
10	28.7	55.5	88.6					
11	27.0	54.5	88.0	128.6	176.4	230.4	291.6	360.0
12	24.7	53.2	87.2	127.6				
14	18.9	49.5	84.8	126.0	173.7	229.4	290.0	357.6
16	14.5	43.9	81.1	123.6	171.9	226.8	287.1	354.4
18	11.4	36.3	75.9	119.8	169.3	224.8	284.8	352.0
20	9.3	29.2	68.6	114.8	165.4	222.0	281.6	348.8
22		24.3	59.1	107.9	159.9	217.9		
24		20.3	49.8	98.8	153.5	213.0		

Hemlock $f_c = 900$

Length Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
5	^{32.4} 32.4							
6	31.9							
7	31.4	^{57.6} 57.3						
8	30.6	56.6						
9	29.7	56.0	^{90.0} 89.4	^{129.6} 129.6				
10	28.2	55.1	88.5					
11	26.2	54.0	87.7	128.6	^{176.4} 176.0			
12	23.7	52.7	86.7	127.4		^{230.4} 229.4		
14	17.6	48.4	84.0	125.4	173.5		^{291.6} 290.0	^{360.0} 357.6
16	13.5	42.0	79.9	122.5	171.1	226.6	290.0	
18	10.7	33.9	74.0	118.7	168.0	224.0	286.7	357.6
20	8.6	27.3	65.7	112.8	163.9	220.4	283.8	354.0
22	7.1	22.7	55.2	105.0	158.0	216.1	279.9	350.8
24	6.0	18.9	46.4	94.6	150.3	210.9	275.7	346.8

Spruce $f_c = 750$

Length Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
5	^{27.0} 27.0							
6	26.6							
7	26.2	^{48.0} 47.8						
8	25.6	47.2						
9	24.8	46.8	^{75.0} 74.5	^{108.0} 108.0				
10	23.7	46.0	73.8					
11	22.1	45.2	73.2	107.1	^{147.0} 146.6			
12	20.1	44.2	72.5	106.3		^{192.0} 191.2		
14	15.1	40.7	70.3	104.8	144.6		^{243.0} 241.7	^{300.0} 298.0
16	11.6	35.6	67.0	102.5	143.1	188.9	241.7	
18	9.1	29.1	62.3	99.4	140.5	187.1	239.1	298.0
20	7.4	23.4	55.7	94.6	137.2	184.1	237.2	295.2
22		19.4	47.4	88.4	132.3	180.7	233.9	292.8
24		16.3	39.8	80.2	126.2	176.6	230.7	290.0

Southern Yellow Pine $f_c = 1065$

Length Feet	Side of Square—Inches							
	6	8	10	12	14	16	18	20
	38.3							
5	38.3							
6	37.6	68.2						
7	37.0	67.8						
8	36.1	66.9	106.5					
9	34.8	66.0	105.7	153.4				
10	32.9	65.0	104.5	153.4				
11	30.6	63.6	103.5	151.9	208.7			
12	27.3	62.0	102.3	150.5	208.2	272.6		
14	20.2	56.3	98.9	148.0	204.8	271.4	345.1	
16	15.4	48.6	93.6	144.4	202.1	267.5	342.8	426.0
18	12.2	38.7	86.2	139.4	198.2	264.2	338.6	422.8
20	9.9	31.2	75.9	131.8	192.9	259.8	335.0	418.0
22		25.9	63.1	122.3	185.0	254.5	330.2	414.0
24		21.6	53.1	109.3	175.4	247.8	325.0	409.2

NOTE: f_c = Maximum allowable Compression in Pounds per Square Inch. Stresses parallel to Grain.

2. Cellular Cofferdams. For large and deep cofferdams, in which bracing would either be impractical or very heavy and expensive, the steel sheet piling may be driven to form a series of connected cells to be filled with whatever soil, gravel or rock is native to the project. Such structures are either one of two types or combination of the two, but in either case the structure is dependent on the mass of the fill for stability.

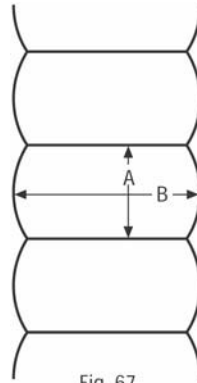


Fig. 67

One type, illustrated in Fig. 67, consists of a series of arcs connected to straight diaphragm walls by means of fabricated Y pieces (see details in Piling Catalog). The legs of the Y pieces form three angles of 120 degrees each, thus making the chord equal to the radius of the arc, and the tensions in the arcs and cross walls are equal.

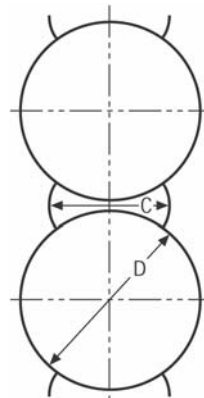


Fig. 68

The other type, illustrated in Fig. 68, consists of a series of complete circles connected by short arcs C. These connecting arcs driven to a comparatively small radius (seven or eight feet) are joined to the circles by means of fabricated T pieces (see details in Piling Catalog).

The first type, Fig. 67, is more economical of steel sheet piling and fabricated connections but must be filled by stages, the height of the fill in each cell being kept reasonably close to that in the adjacent cells in order to avoid distortion of the cross walls. The circular type, Fig. 68, requires more steel sheet piling and more fabricated connections but has the advantage that each cell may be filled immediately to the top without distortion of the cell.

The complete elimination of bracing for these cellular cofferdams requires that each cell must be stable against:

- a. Overturning.
- b. Sliding.
- c. Tension in the interlock.

(a) Overturning. Stability against overturning is secured when the resultant of the force tending to overturn the cell and the weight of the cell acting through its center of gravity falls within the middle third of the width. It is not advisable to narrow the cells to the extent of permitting the resultant to fall outside the middle third as this condition will produce an uplift at the back, thus tending to overturn the cells by withdrawing the piling, and no definite figure can be assumed for the resistance to withdrawal.

(b) Sliding. Stability against sliding is assured when the ratio of the force tending to cause sliding and the weight of the fill within the cell does not exceed what may be considered a safe coefficient of friction for the soil at the base of the cell.

Lack of frictional resistance to sliding is rarely a consideration in the design of cellular cofferdams. Smooth rock only, on top of which is a fill of wet clay, should be avoided as the figures in Table 22 indicate. In actual practice, rock is rarely absolutely smooth and, in addition, the soft layer of rotten rock generally encountered on top of hard bed rock allows enough penetration of the steel sheet piling to secure a toe hold which will greatly resist sliding.

The following tables will assist in determining the safety of cellular cofferdams against sliding.

Table 20.
Cohesive Strength of Clays
(Merriman, 4th Edition)

Description of Material	Cohesive Strength Tons per Square Foot
Puddle clay, wet.....	0.20 to 0.45
Stiff clay puddle.....	0.62
Sandy clay, wet.....	0.6
Stiff sandy clay.....	0.5
Moderately firm boulder clay.....	0.7
Very stiff boulder clay, fairly dry.....	1.6

Table 21.
Coefficients of Internal Friction of Various Earths
(Merriman, 5th Edition)

Description of Material	Coefficient of Friction
Earth.....	0.25 to 1.00
Dry sand, clay mixed earth.....	0.38 to 0.75
Wet clay.....	0.31
Damp clay.....	1.0
Shale and gravel.....	0.81 to 1.11

Table 22.

Coefficients of Friction—Soils on Various Rocks
Brennecks-Lohmeyer

Description of Material	Rough Granite	Smooth Sandstone	Rough Sandstone	Smooth Masonry	Rough Masonry
Gravel and sand					
Dry.....	.54
Wet.....	.48
Fine sand					
Dry.....	.70
Wet.....	.53
Fluid slime.....05	.10
Firmer slime.....10	.20
Wet clay and loam.....20	.30
Dry sand.....60	.70
Wet sand.....30	..
Dry gravel.....	..	.57	.61	.40	.50
Wet gravel.....	..	.60	.62	.40	.50

(c) Tension in Interlock. The maximum tension in the interlock of the type of cofferdam having straight cross walls, Fig. 67, occurs at the Y junction pieces at the base of the cell and is the component of the forces originating in all the intermediate piles in one-half the arc. In the case of the type of cofferdam consisting of a series of circles, Fig. 68, the maximum tension, equal at all interlocks at the plane of the base, occurs at the base of the cell and is equal to one-half the total horizontal pressure on the diameter.

Formulas and example following are to be used in investigating and designing cellular structures for overturning, sliding and tension in the interlock.

Symbols Used in Formulae See Fig. 69

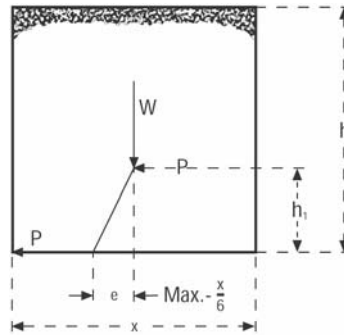


Fig. 69

h = height of cell in feet.

h_1 = distance in feet from bottom of cell to where P is applied.

P = overturning force in pounds per foot of width.

p = pressure in pounds per square foot at bottom.

w = weight of contents per cubic foot in pounds.

W = weight of structure per foot of length in pounds = whx .

x = dimension of cell from front to back in feet.

t = tension in interlock in pounds per lineal inch.

r = radius of curved portion of cell in feet.

c = coefficient of friction.

A = width of cell in feet. See Fig. 67.

e = distance in feet from center of cell to point where resultant falls (should not exceed $x/6$)

(a) Overturning.
$$e = \frac{Ph_1}{whx}; \text{ then } (whx) \frac{x}{6} = Ph_1, whx^2 = 6Ph_1 \text{ and}$$

$$x^2 = \frac{6Ph_1}{wh}; \text{ this is for rectangular structures.}$$

To compensate for curved front of cell: Dimension B, Fig. 67 = $\frac{x}{.9}$

Diameter D, Fig. 68 = $\frac{x}{.85}$

(b) Sliding. Coefficient of friction, $c = \frac{P}{W}$. This usually will prove to be less than .5 when the other conditions are satisfied.

(c) Tension. Having determined x and h ,

$$p = wh \tan^2 (45^\circ - \frac{1}{2}\phi)$$

$$t = \frac{pr}{12} \text{ or } \frac{pA}{12} \text{ and if } t \text{ max} = 6,000 \text{ pounds, then,}$$

$$r \text{ max} = \frac{72,000}{p}$$

Example: Assume $h = 60$ feet from top of cells to bottom of excavation on rough granite bed rock. Cells are to be filled with gravel, etc., from which water is discharged by means of weep holes at the bottom of the inside face of cells, to eliminate hydrostatic head within the cells.

Assume weight of fill 100 pounds per cubic foot, slope 1 on 2 in which the $\tan^2 (45^\circ - \frac{1}{2}\phi) = .38$. The lateral pressure from gravel fill will then be $100 \times .38 = 38$ pounds per square foot, equivalent liquid pressure. Fig. 70, below, illustrates the problem.

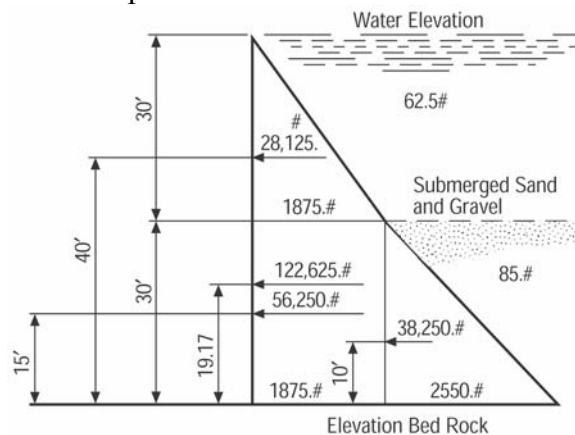


Fig. 70

Overturning. The force tending to cause overturning results from a 30 foot head of water at $62 \frac{1}{2}$ pounds per cubic foot, and the remaining 30 feet submerged gravel, assumed slope 1 on 2, weight 60 pounds per cubic foot. Lateral pressure = $60 \times .38 = 22 \frac{1}{2}$ pounds per square foot, equivalent liquid pressure. Add to this figure the full hydrostatic pressure of $62 \frac{1}{2}$ pounds per square foot and use the total or 85 pounds per square foot, equivalent liquid pressure.

$p, 30 \text{ feet down} = 62 \frac{1}{2} \times 30 = 1,875 \text{ pounds.}$

$p, 60 \text{ feet down, due to submerged sand and gravel alone,} = 85 \times 30 = 2,550 \text{ pounds.}$

Total forces:—Upper 30 feet,

$$\frac{1,875 \times 30}{2} = 28,125 \text{ pounds acting 40 feet from bottom.}$$

Lower 30 feet,

$$1,875 \times 30 = 56,250 \text{ pounds acting 15 feet from bottom.}$$

$$\frac{2,550 \times 30}{2} = 38,250 \text{ pounds acting 10 feet from bottom.}$$

Total P = 28,125 + 56,250 + 38,250 = 122,625 pounds per foot of width acting

$$\frac{28,125 \times 40 + 56,250 + 38,250 \times 10}{122,625} =$$

19.17 feet from bottom.

Using formula given in (a) preceding:

$$x^2 = \frac{6 \times 122,625 \times 19.17}{100 \times 60} = 2,350 \text{ and } x = 48.47 \text{ feet.}$$

$$\frac{48.47}{.9} = 54 \text{ feet, minimum advisable width, dimension B, for cellular structure with straight walls,}$$

Fig. 67, and

$$\frac{48.47}{.85} = 57 \text{ feet, minimum advisable diameter, D, for complete circles, Fig. 68.}$$

Sliding

Force tending to cause sliding is same as that for overturning, 122,625 pounds.

Weight of cell = $54 \times .9 \times .60 \times 100 = 291,600$ pounds.

The coefficient of friction necessary for safety is $\frac{122,625}{291,600}$, or .42. Coefficient of friction for wet gravel on rough granite according to Table 22 is .48. Therefore structure is safe from sliding.

Tension

p, at bottom due to fill in the cells = $38 \times 60 = 2,280$ pounds per square foot.

$$r \text{ max.} = \frac{72,000}{2,280} = 31.58 \text{ feet.}$$

Assume $r = 25$ feet which is also the distance between cross walls, dimension A, Fig. 67, or $\frac{1}{2}$ diameter

$$D, \text{ Fig. 68, then actual } t \text{ at base of cell} = \frac{2,280 \times 25}{12} = 4,750 \text{ pounds per lineal inch of interlock.}$$

Dr. Hermann Blum in Germany was making very exhaustive studies of the problem of calculating the bending moments of steel sheet piling walls. The results of his studies, published in 1931, coincide in essential features with the methods published in our catalog, and, in addition, offer considerable supplemental information.

On pages 25 to 37 is given a condensed and rearranged translation of Dr. Hermann Blum's work "Einspannungsverhaeltnisse bei Bohlwerken" in order that the Engineering profession may have more complete information for the design of steel sheet piling structures. It is interesting to note how the two studies, made independently, coincide in their essential features. Dr. Blum's principles are based upon more precise methods with the addition of graphical solutions. The approximations, made for simplicity, on pages 9 to 13, will err only on the side of safety.

Bending Moment Acting on Anchored Steel Sheet Piling Walls

The problem of calculating the bending moment on anchored sheet piling walls has been very carefully studied by Dr. Hermann Blum, with particular attention to the variations in the bending moment which could be caused by the wide range of soil conditions met in practice and to the restraining effect of earth pressures acting at the lower end of the steel sheet piling.

Penetration and Restraint.

Deviation from the ideal, of the restraining effect of the pressure exerted by the earth into which the lower end of the sheet piling is driven, must first be considered.

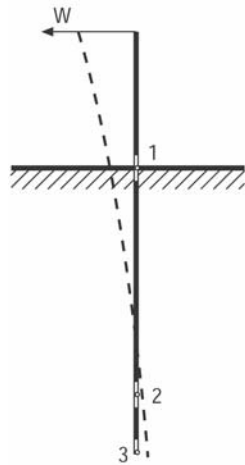


Fig. 88

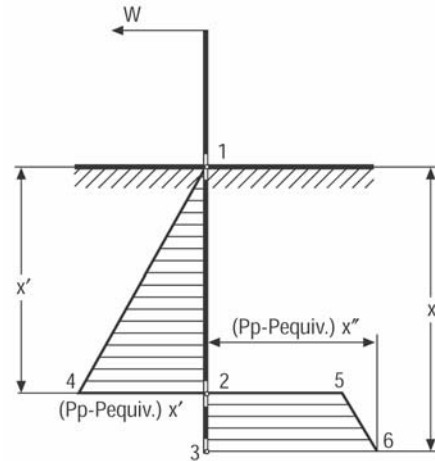


Fig. 89

For simplicity, study the condition shown in Fig. 88 adjoining, in which the piling wall is driven into earth and is loaded at the top with a single force W . The wall tends to turn about some point 2 as indicated by the dotted line. This displacement is resisted by the passive earth pressure which acts toward the right between points 1 and 2 and to the left between points 2 and 3.

The ideal load diagram, indicating the distribution of the resisting pressure, is shown in Fig. 89 adjoining. On the side from which the wall would be displaced, the active earth pressure $p_{equiv.}$ (see note below) is present and acts in the direction of motion, while on the other side the passive earth pressure p_p is, in effect, in the direction opposite to the motion.

The net effective pressure increment is, therefore, $(p_p - p_{equiv.})$

At point 2 the pressure would change instantly from $(p_p - p_{equiv.})x'$ acting toward the left to the same pressure acting toward the right.

NOTE— $p_{equiv.}$ corresponds to p_e , $p_{e\ in\ w}$ or $p_{comb.}$ in pounds per square foot, as developed on pages 2 to 6.

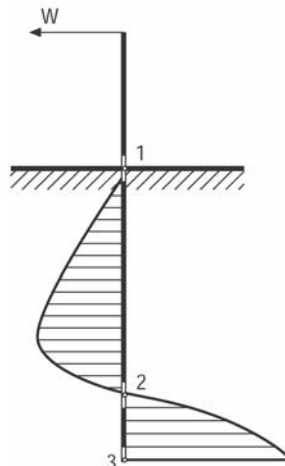


Fig. 90

The soil, however, has some elasticity and a sharp reversal of pressures would not occur. Instead, the actual load distribution would have a more rounded form, as in Fig. 90 adjoining. The actual form of the curve is unknown. It depends on the elastic properties of the soils which vary for each case and on which there is practically no test data.

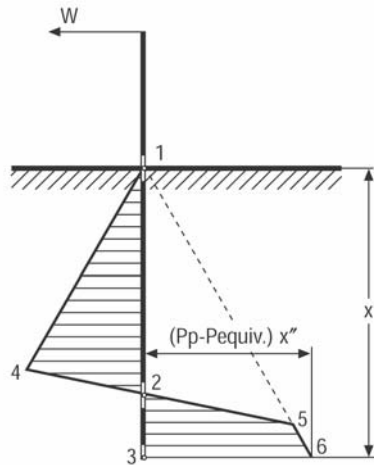


Fig. 91

For practical calculations, therefore, it is necessary to replace the curved lines of Fig. 90 by straight lines as shown in Fig. 91 adjoining, in such proportions as to give an equivalent pressure distribution. This is accomplished by substituting for the horizontal line 4-5 of Fig. 89 an inclined straight line 4-5 in Fig. 91. The inclination of this line, however, is unknown, varying with the properties of the soil.

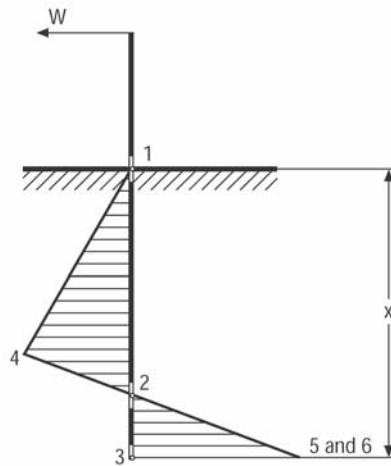


Fig. 92

This inclination may vary between the limits as shown in Fig. 89 and Fig. 92, in the latter of which point 5 coincides with point 6. The inclination need not be known exactly, because it makes little difference in the final result since, as the slope of line 4-5 is increased, the area illustrating the loading is decreased but the distance between the centers of gravity of the areas is increased and the depth to which the sheet piling must be driven, in order to resist the moment caused by the load W , varies only slightly.

The distribution of pressure in Fig. 92, because it is the most unfavorable, is chosen as the standard for all further calculations. It has the additional advantage that all equations of equilibrium can be brought into comparatively simple form, which is not true of the other loading diagrams.

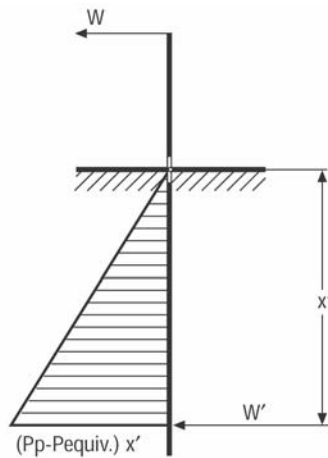


Fig. 93

For practical use, this can be further simplified by assuming the distribution of the pressures as illustrated in Fig. 93 adjoining. This consists of a single triangle plus a concentrated load W' at the lower end acting at a level where the earth resistance on the left vanishes.

If the steel sheet piling is driven to a sufficient depth, it acts as a beam fixed at the lower end, and the moment. caused by the restraining action of the earth will be termed the "constraining moment."

Load distributions, moments and deflection curves of sheet piling walls driven to various depths are shown in Fig. 94 below.

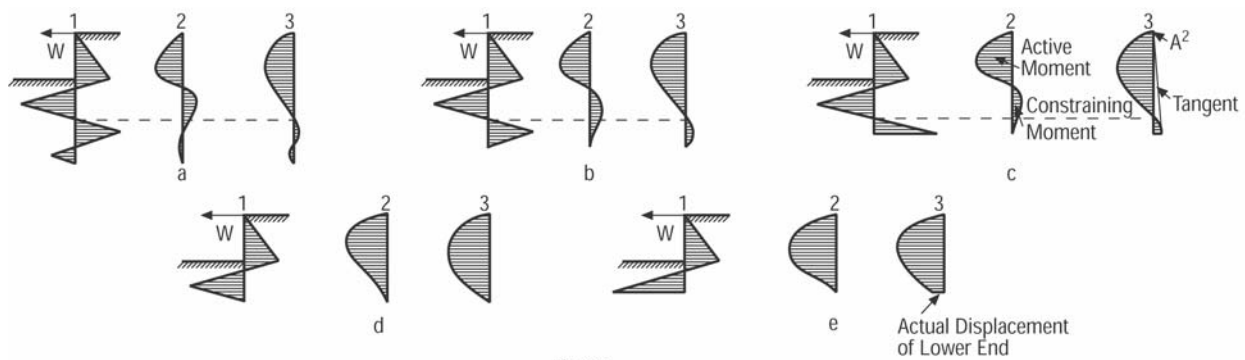


Fig. 94

If the piling wall is driven to an unlimited depth, Fig. 94-a, the deflection curve has a form similar to the well-known “damped vibration” waves encountered in dynamics and radio. The deflections alternate toward either side, decreasing in magnitude with increasing depth. In Fig. 94-b the wall is driven only deep enough so that there is no deflection of the lower end and there are but two points of contraflexure. In Fig. 94-c the penetration is reduced still further and the lower end is displaced somewhat, making but one point of contraflexure, and the tangent to the deflection curve at the lower end passes through point A_2 . Evidently, as the penetration is decreased, the constraining moment decreases while the active moment, due to the lateral loads, increases. If the penetration is still further decreased, a point is reached at which the constraining moment vanishes and the active moment attains its maximum value. Fig. 94-d and -e illustrate this case and they differ only in the lower boundary of the passive earth resistance area, depending on whether the deflection at the lower end is zero or has a small positive value. In both cases the wall acts as a beam on two supports.

Somewhere between case c and d the constraining action vanishes. Comparison of the different figures shows that the constraining moment is always less than the active moment, no matter how deep the piling is driven, and that the active moment is less for a, b and c than for d and e but the penetration must be greater.

There are two alternatives in the choice of depth of penetration in the design of sheet piling walls:

1. Choose the smallest possible depth, in which case the bending moment is a maximum and the necessary section modulus larger.
2. Increase the depth sufficiently so that the constraining action of the earth is effective. This reduces the moment and allows less section modulus but requires longer lengths of sheet piling.

The material utilization factor is, $\text{Length} \times \sqrt{\text{Maximum Moment}}$

This factor is nearly the same in both of the above alternatives, with or without constraining action, so that apparently neither method has the advantage over the other.

However, in case 2, which utilizes the constraining action, the anchor tension is less and the cost of the wales, tie rods and anchorage decreases. The greatest advantage of case 2 lies in its greater actual safety, for if the sheet piling is driven only to the minimum depth of case 1, the factor of safety against pushing out at the toe is only unity while the factor of safety of the steel work is usually three or more (considering the steel in the sheet piling to have a minimum ultimate strength of 70,000 pounds per square inch). If too favorable passive earth pressures have been assumed, the wall will move outward at the toe.

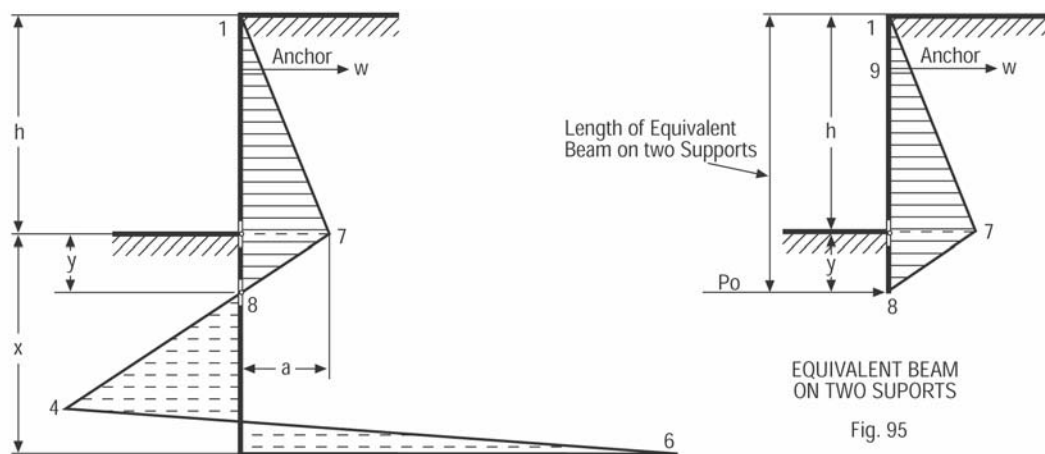
If too favorable passive earth pressures have been assumed and the sheet piling is driven to the greater depth than necessary in case 2, the maximum effect is that the constraining moment vanishes and the bending moment is increased, but not over 40 per cent. The factor of safety of the steel is reduced to two but the stress is still below the elastic limit and the wall is absolutely safe against failure due to movement at the toe.

Determination of Bending Moment

The calculation of the bending moment for a beam on two supports is very simple, whereas that of a constrained beam on elastic supports presents great difficulties. Even if the complicated theories developed in treatises on advanced mechanics were applied, the results would be incorrect on account of the imperfectly elastic nature of the soil into which the sheet piling is driven.

Dr. Blum has, therefore, studied the resulting moments and deflections of sheet piling walls driven to various depths and over a wide range of earth pressures including submerged earth, various angles of repose, friction of the earth on steel walls and different locations of the anchor, in order to determine whether an equivalent beam on two supports could be substituted for a constrained beam. He based the investigation on a pressure distribution as illustrated in Fig. 94-c, and used chiefly the graphical method of finding the deflection curve. This method is adaptable to a great variety of complicated loading conditions.

He found that the constrained beam can be replaced for calculation purposes by a beam on two supports and that the depth of the “equivalent beam” can, by very simple methods, be determined quite accurately, and certainly within the limits of accuracy of the data on the soil properties for the various conditions. This is the same procedure as outlined on page 12.



The method of finding the necessary depth of penetration is illustrated in Fig. 95 above. The active earth pressure is laid out in accordance with the Rankine-Coulomb method (pages 1 to 6), or the Krey method (pages 40 to 50 following). This determines point 7. Point 8 is the intersection with the sheet piling wall of a line drawn from point 7, having a slope for which the increment is $p_p - p_{equiv}$. Analytically, as illustrated in Fig. 95 above,

$$y = \frac{a}{p_p - p_{equiv}} \text{ (see Note 1)}$$

Dr. Blum recommends that the effect of wall friction be disregarded (theory of H. Krey, following) and that both active and passive earth pressures be figured by the Rankine-Coulomb methods. **He also advises that the passive earth pressure thus figured be multiplied by 2 to allow for the additional resistance due to the undisturbed and naturally compact soil into which the sheet piling is usually driven.** (See page 10.)

The length of the equivalent beam, then, is the distance between points 9 and 8, Fig. 95 (or between tie rod and line of support, Fig. 18, page 12). The equivalent reaction at the lower end, P_0 , can be found by equating to zero the sum of all the moments about the point of application of the anchor force, point 9.

The depth to which the sheet piling should be driven to obtain full advantage of the constraining action of the earth on the lower end is figured from the equation

$$x = K_2 \left[y + \sqrt{\frac{6P_0}{2p_p - p_{equiv.}}} \right] \text{ (see Note 2)}$$

x =depth of penetration in feet.

Y =distance, in feet, below bottom of the equivalent beam or line of support.

P_0 =equivalent reaction in pounds per foot of width.

p_p =equivalent liquid passive earth pressure in pounds per square foot, see pages 9 and 10.

$p_{equiv.}$ =equivalent liquid active earth pressure in pounds per square foot corresponding to p_e , $p_{e \text{ in } w}$ or $p_{comb.}$, see pages 2 to 6.

K_2 =a factor ranging between 1.1 and 1.2. Normally it is 1.1 but if the earth behind the wall is loose fill, it may rise to 1.2.

NOTE 1—This formula was developed in paragraph A, page 12. The equivalent liquid passive pressure p_p , in pounds per square foot, is determined by the Coulomb formula at bottom of page 9.

NOTE 2—The derivation of this equation is involved and is obtained from the conditions of equilibrium as follows: (a) The loads on the two sides must balance. (b) The moments about the toe must balance.

Determination of Bending Moment by Graphical Method—Example

Let the conditions be as shown in Fig. 96, following page.

$$h = 22 \text{ feet.}$$

$$h' = 12 \text{ feet.}$$

$$h'' = 10 \text{ feet.}$$

$$w_e \text{ (dry)} = 100 \text{ pounds per cubic foot. Voids in earth 30 per cent.}$$

$$w_w = 62.5 \text{ pounds per cubic foot.}$$

$$w_s = 700 \text{ pounds per square foot.}$$

$$\phi = 30 \text{ degrees, angle of repose of dry earth.}$$

$$\phi' = 25 \text{ degrees, angle of repose of submerged earth.}$$

Then

$$\tan^2 (45^\circ - \frac{1}{2}\phi) = .333$$

$$\tan^2 (45^\circ - \frac{1}{2}\phi') = .406$$

$$p_s = 700 \times .333 = 233 \text{ pounds per square foot.}$$

$$p'_s = 700 \times .406 = 284 \text{ pounds per square foot.}$$

$$p_e = 100 \times .333 = 33.3 \text{ pounds per square foot.}$$

$$p_e h'' = 33.3 \times 10 = 333 \text{ pounds per square foot.}$$

$$p'_e = 100 \times .406 = 40.6 \text{ pounds per square foot.}$$

$$p'_e h'' = 40.6 \times 10 = 406 \text{ pounds per square foot.}$$

$$w_{e \text{ in } w, \text{ in}} = 100 - \left(\frac{100 - 30}{100} \times 62.5 \right) = 56.3 \text{ pounds per cubic foot.}$$

$$p_{e \text{ in } w} = 56.3 \times .406 = 22.9, \text{ say } 23 \text{ pounds per square foot.}$$

$$p_{e \text{ in } w} h' = 23 \times 12 = 276 \text{ pounds per square foot.}$$

These values are laid out graphically on the loading diagram, Fig. 96, all figures applying to one foot of width of wall.

To find $p_p - p_{e \text{ in } w}$ for the submerged earth at the bottom of the sheet piling wall,

$$p_p = 56.3 \tan^2 \left(45^\circ + \frac{25^\circ}{2} \right) = 138.7, \text{ say } 139 \text{ pounds per square foot.}$$

To take into account the increase in the passive pressure of the earth due to its undisturbed, compact condition (not fill), Dr. Blum recommends multiplying this by 2, or

$p_p = 139 \times 2 = 278$ pounds per square foot and $(2p_p - p_{e \text{ in } w}) = 278 - 23 = 255$ pounds per square foot, equivalent liquid pressure, which is the increment of the slope of line 7 – 8. Point 8 is found to be

$$\frac{966}{255} = 3.79 \text{ feet below the bottom.}$$

The method of laying out the loading diagram shown at the left of Fig. 96 is outlined in Fig. 12, page 6. The diagram is constructed by using the wall, earth, water, and tie rod elevations in feet, the lateral earth pressures in pounds per square foot and the total lateral loads in pounds per foot of width of wall to any convenient scales, as noted. The total height of wall (length of beam) is divided into any number of convenient sections which need not all be of equal width. In Fig. 96 the length was divided into 13 equal sections of 2 foot width, except the bottom section. For each area the earth pressure is considered as a total concentrated load acting through the center of gravity, the magnitude of the load being determined by scaling off the mean length of the strip.

A vector diagram is then drawn as indicated. On a horizontal base line, commencing at the right, the successive loads for the sections from the bottom to the top of the equivalent beam are laid off end to end. A pole O is then selected at a distance from the base line, in inches, equal to

Selected scale of moments (inch-pounds per inch)

Scale of lateral loads on base (pounds per inch) \times scale of lengths (inches per inch).

(Moments and loads are given in pounds per foot of width of wall.)

The moment scale is so selected as to give a convenient size of diagram. The lateral location of O is not important but may be above the approximate middle of the base line. From point O lines are drawn to the ends of all the load lines on the base line. The moment diagram is then drawn as follows:

Starting at the bottom, line O' – 1' is drawn parallel to line O – 1 of the vector diagram, intersecting the first load line at 1'; through the point 1' line 1' – 2' is drawn parallel to O – 2 intersecting the section load line at 2', etc. The process is continued through all the sections. At the top, the line drawn from point 13' parallel to O – 14 extends upwards to the right as 13' – 14' and also downward to the left as 13' – A'. The point A' is its intersection with the line of the anchor force. Through A' a straight line is drawn to the starting point O'.

This straight line is the base line of the moment diagram and may be vertical or may lean to the right or left depending on the lateral location of O in the vector diagram, but its slant does not affect the results. Another straight line A – O parallel to A' – O' is drawn through O in the vector diagram, and its intersection with the base line at A gives the magnitude of the anchor tension and the equivalent reaction at the base. These, determined by scaling, are found to be 8,450 and 7,311 pounds per foot of width, respectively.

By scaling the maximum horizontal distance from line A' – O' to the curve in the moment diagram, the maximum bending moment M is found to be 490,000 inch-pounds per foot of width at a distance of 15 feet from the top of the wall.

The necessary section modulus S is the Moment M divided by the working stress, or

$$S = \frac{490,000}{24,000} = 20.4 \text{ in}^3$$

The moment in the cantilever portion above the tie rod is negative and is maximum at the tie rod. This moment is determined by scaling the horizontal distance from A' to the moment curve. In Fig. 96 this is found to be approximately 10,000 inch-pounds.

The depth to which the sheet piling is to be driven is found to be

$$x = 1.1 \left[3.79 + \sqrt{\frac{6 \times 7311}{278 - 23}} \right] = 18.6 \text{ feet}$$

For simple loading, as in this example, the calculation by the analytical method would not require much more time than by the graphical method, but in the cases of more complicated loading the latter method is simpler.

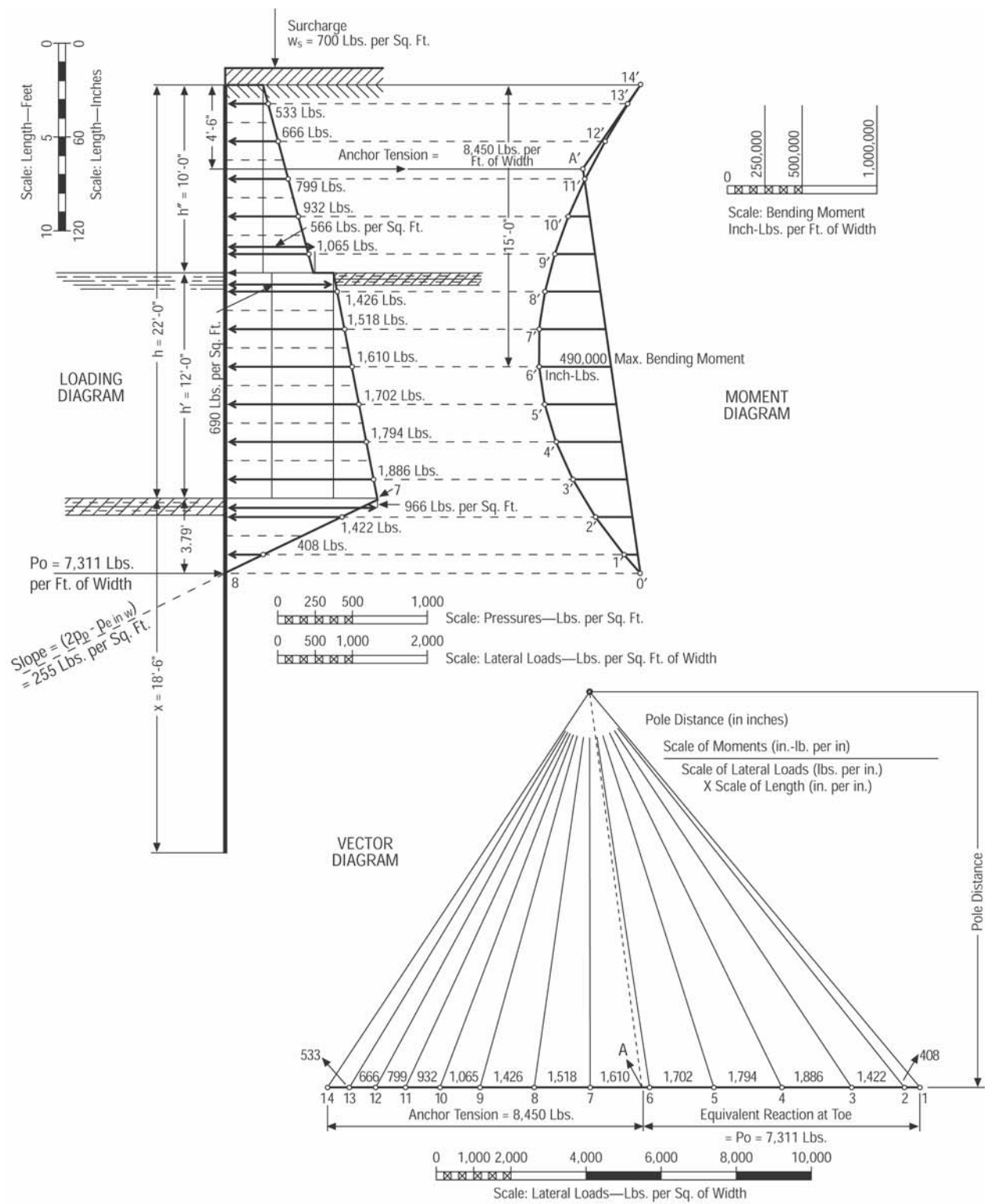


Fig. 96

Determination of Bending Moment by Analytical Method-Example

The same problem for which the graphical method was used on the preceding pages is solved below by the analytical methods outlined on pages 7 to 12. Fig. 97 below illustrates the problem.

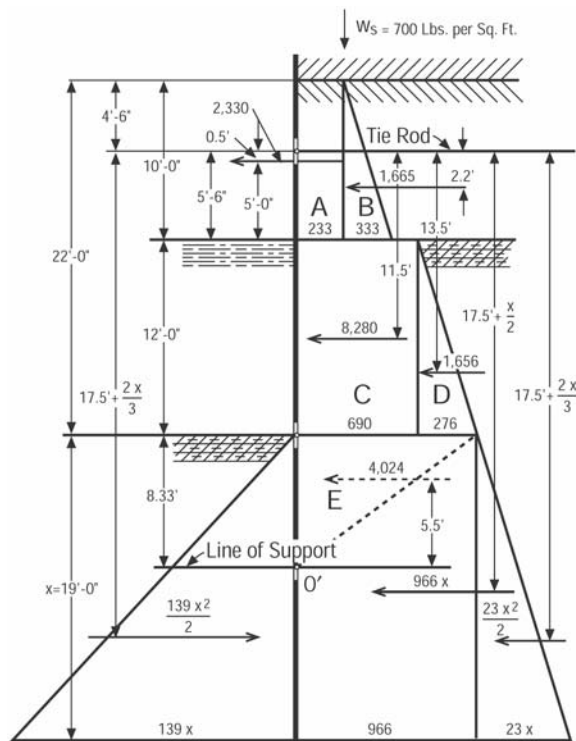


Fig. 97

1. Depth of Penetration.

(See page 11). Try $x = 19$ feet penetration.

$$\frac{139(19^2)}{2} \left(17.5 + \frac{2(19)}{3} \right) \geq 2330(.5) + 1665(2.2) + 8280(11.5) + 1656(13.5) + 966(19) \left(17.5 + \frac{19}{2} \right) +$$

$$\frac{23(19^2)}{2} \left(17.5 + \frac{2(19)}{3} \right)$$

$$756,950 > 743,212$$

The above equation shows that the passive pressure resisting movement at the toe slightly overbalances the active earth pressures and the trial penetration of 19 feet agrees as closely as necessary with the penetration of 18.6 feet derived by H. Blum's method on page 30.

2. Line of Support.

(See page 12). The distance y, in feet, of the line of support below the bottom is

$$y = \frac{690 + 276}{139 - 23} = \frac{966}{116} = 8.33$$

This compares with 3.79 feet obtained by the Blum method, the difference arising because Blum doubles the passive pressure p, as derived by the Coulomb formula.

3. Bending Moment

(See page 12, sec. 3).

$$\text{Area } A = 233 \times 5.5 = 1,282$$

$$B = 150 \times 5.5 + \frac{183 \times 5.5}{2} = 1,328$$

$$C = 690 \times 12 = 8,280$$

$$D = \frac{276 \times 12}{2} = 1,656$$

$$E = \frac{966 \times 8.33}{2} = 4,024$$

Total area = 16,570 which
represents the total lateral load per foot width of wall.

$$\text{Bending moment } M = \frac{16,570 \times 25.83 \times 12}{8} = 642,000 \text{ inch-pounds per foot width of wall.}$$

$$S \text{ required} = \frac{642,000}{24,000} = 26.7 \text{ in.}^3 \text{ per foot width of wall.}$$

This compares with a section modulus of 20.4 in.³ as developed by the Blum graphical method in the preceding problem.

4. Tie Rod and Wale Load.

(See page 8, sec. 2). The load in pounds per foot width of wall is obtained by taking moments about the line of support O', Fig. 97. Taking moments about the line of support,

$$\begin{aligned}\text{Load} \times 25.83 &= (2,330 \times 25.33) + (1,665 \times 23.63) + (8,280 \times 14.33) + (1,656 \times 12.33) + (4,024 \times 5.55) \\ \text{Load} &= 10,050 \text{ pounds per foot width of wall.}\end{aligned}$$

This compares with 8,180 pounds as developed by the Blum method, and the heavier load by the above method is to be expected because of the lower line of support and the larger area in triangle E, Fig. 97, and the heavier load represented.

Determination of Bending Moment by Graphical Method—Example

Consider the problem as illustrated in Fig. 99 with the following assumptions:

$$\begin{aligned}
 w_e &= 112 \text{ pounds per cubic foot.} \\
 \phi &= 35^\circ, \text{ therefore } \tan^2 (45^\circ - \frac{1}{2}\phi) = .272. \\
 p_e &= 112 \times .272 = 30.5, \text{ say } 31 \text{ pounds per square foot.} \\
 w_{e \text{ in } w} &= 69 \text{ pounds per cubic foot.} \\
 \phi' &= 30^\circ, \text{ therefore } \tan^2 (45^\circ - \frac{1}{2}\phi') = .334. \\
 p_{e \text{ in } w} &= 69 \times .334 = 23 \text{ pounds per square foot.} \\
 p_{comb.} &= 23 + 62.5 = 85.5 \text{ pounds per square foot.} \\
 w_s &= 200 \text{ pounds per square foot.} \\
 p_s &= 200 \times .272 = 54.4, \text{ say } 54 \text{ pounds per square foot.} \\
 p_s' &= 200 \times .334 = 66.8, \text{ say } 67 \text{ pounds per square foot.} \\
 p_p &= 69 \tan^2 (45^\circ + \frac{35^\circ}{2}) = 69 \times 3.69 = 254.6, \text{ say } 255 \text{ pounds per square foot.} \\
 (2p_p - p_{e \text{ in } w}) &= 510 - 23 = 487 \text{ pounds per square foot.} \\
 w_e \times 8 \times .334 \text{ (for } \phi') &= 299 \text{ pounds per square foot.}
 \end{aligned}$$

The loading diagram at the left in Fig. 99 is constructed by the methods previously described, line 4 – 5 having a slope equal to $(2p_p - p_{e \text{ in } w})$ pounds per foot. On the right side of the wall, the earth resistance represented by distance 3 – 5 is equal to

$$\begin{aligned}
 &[\text{weight of column of earth on the right side one foot square and height } (h + x) \times \tan^2 (45^\circ - \frac{1}{2}\phi)] \\
 &- [\text{weight of column of earth on the left side one foot square and height } x \times \tan^2 (45^\circ - \frac{1}{2}\phi)] \\
 &+ [\text{water pressure on right side}] - [\text{water pressure on left side}].
 \end{aligned}$$

Weight of column of earth one foot square on right side,

$$\begin{aligned}
 112 \times 8 &= 896 \\
 69 \times 15 &= 1035 \\
 69 \times 11.8 &= 814 \\
 2745 \times 3.69 &= 10,130 \text{ pounds}
 \end{aligned}$$

Weight of column of earth one foot square on left side,

$$\begin{aligned}
 69 \times 11.8 &= 814 \text{ pounds.} & 814 \times .272 &= 222 \text{ pounds.} \\
 10,130 - 222 + (62.5 \times 26.8) - (62.5 \times 24.8) &= 10,033 \text{ pounds.}
 \end{aligned}$$

You will note that the passive pressure on the right side is not multiplied by 2 as is that on the left side because the displacement of these points of the wall near the bottom may be too small to call into action the full maximum passive earth resistance.

The location of point 7 is unknown but it should, to be exact, coincide in level with the point of zero displacement (point 7' of the deflection diagram). However, the latter is not known at the start. For a trial point 7 is chosen anywhere near the bottom of the wall.

Moment Diagram.

The moment diagram in Fig. 99 is drawn by exactly the same procedure as described for Fig. 96 on page 31 with the following precautions:

1. At the top of the moment diagram, the curve reverses after the line to point 11' is drawn. The straight line 11' – A' must be drawn parallel to the line O – 11 (which is at the extreme left of the vector diagram for moments) to the intersection with the line of the anchor at A. The closing line of the moment diagram is then drawn from A' to C'.
2. When this closing line is transferred to the vector diagram for moments as line O – 12, in order that condition 2 on page 33 may be fulfilled, the point 12 thereby determined must coincide with the end of the anchor load line and also, as will be evident from the vector diagram, with the end of the load line of the lowest load acting to the left or W-18. If it does not coincide, the slope of the line 6 – 7 of the pressure diagram must be changed, slightly reducing load W-17 and increasing W-18, or vice versa. A very slight change of the slope of line 6 – 7 in the pressure diagram modifies these two loads considerably and, with a little judgment, the desired coincidence is easily obtained. This results in a slight shift in the closing line of the moment diagram and changes to some extent the relation of the moments above the bottom to those below.

The difference between the method described on page 29 and this one is that, in the former, the problem is the study of a simple beam on two supports in which all loads are predetermined. Hence the closing line can have but one position. In the latter, loads W_{17} and W_{18} are not predetermined but must be adjusted to suit conditions 1 and 2, page 33. To fulfill condition 1, the right end of the anchor load must come vertically above the right end of the load W_{18} ; and to fulfill condition 2, the closing line of the vector diagram must parallel the closing line of the moment diagram.

Deflection Diagram.

The graphical method of constructing the deflection curve illustrated in Fig. 99 is based on the fact that the area enclosed within the moment diagram can be designated as an "elastic weight" with which the beam is loaded. The deflection curve then bears the same relation to the moment curve as the latter does to the pressure diagram. These "elastic weights" F_1, F_2 , etc., pass through the centers of gravity of the strips into which the area between the moment curve and its base line is divided. These strips need not be of equal width, and in Fig. 99 their boundaries are the lines of action of the loads W_1, W_2 , etc. Each "elastic weight" F_1, F_2 , etc., is equal to the average moment of its corresponding strip multiplied by the width of the strip in inches. The units of these "elastic weights" are therefore pounds \times inches².

Thus for the strip immediately below the maximum moment, the average moment for the strip is 388,300 inch-pounds, the width of the strip is 24 inches and the "elastic weight" $F_9 = 9,320,000$ pounds \times inches².

In the vector diagram for deflections in Fig. 99, the lowermost five forces, which act toward the right, are laid off in that direction beginning at the vertical dotted line with F_{16} first, F_{15} to the right, etc. The pole distance x is determined from the relation:

$$x \text{ (in inches)} = \frac{\text{Scale of deflections (pounds} \times \text{inches}^3 \text{ per inch)}}{\text{Scale of lengths (inches per inch)} \times \text{Scale of "elastic weights" (pounds} \times \text{inches}^2 \text{ per inch)}}.$$

This determines the location of pole O'' , from which the slanting lines are drawn to the division points of the base line of the vector diagram. In the deflection diagram, commencing at the bottom, a line $16' - 15'$ is drawn parallel to line $O'' - 16$ of the vector diagram. From point $15'$ line $14' - 15'$ is drawn parallel to $O'' - 15$, etc., until point $11'$ is reached. The lines must begin and end on the lines of action of the forces F_{16}, F_{15} , etc.

For clearness, the base line of the vector diagram is shown at a higher level for forces F_1 to F_{11} . These forces act toward the left and are laid off in that direction from point 12 in the order F_{11}, F_{10}, F_9 , etc. Pole O''' is of course vertically above pole O'' . The slanting lines are drawn to correspond, and the same procedure as in the moment diagram is used to complete the deflection curve. If the assumed depth is correct, the vertical dotted line, drawn through a point $7'$ tangent to the curve at the same level of point 7, will also pass through the point where the deflection curve is intersected by the line of action of the anchor force at A_2 , showing no displacement at this point and thus fulfilling condition c on page 27. In the example, Fig. 99, the deflection curve shown is the result of several trials with slightly different depths x .

Thus, for an 11' – 0" depth the curve was too far to the right, intersecting the line of the anchor at A_2'' ; for a 12' – 0" depth it was too far to the left, point A_2' ; for 11.8' depth, point 9' almost coincides with A_2 . A very slight increase of depth would be required for exact coincidence, hence the final result will be:

Depth of driving, 11' – 9" to 11' – 10".

Least maximum bending moment, 385,000 inch-pounds per foot width of wall.

It will be noticed that the scale of deflections is not inches per inch but pounds \times inches³ per inch. The reason for this is that the curve represents the deflections applicable to any beam of uniform section, and for any particular beam the actual deflection in inches is equal to the latter so-called "general deflections" divided by EI (modulus of elasticity \times moment of inertia). For the present purpose, however, the cross section of the beam is immaterial as it is possible to judge whether requirements c and d have been met from the "general" deflection curve just as well as though the actual deflection curve had been drawn.

The actual length of the equivalent beam on two supports is $(h + y)$, in which y is the distance from the bottom to point 10 of the moment curve where the moment is zero. For the simplified method described in Fig. 94, which was devised to avoid the calculation of the deflection curve, it was assumed that distance y' is the same as the distance y from the lower earth level (point 2 to point 5) where the pressure line cuts the plane of the wall. Usually these two points do not exactly coincide. In the example, $y = 1.72$ feet while $y' = 1.50$ feet. For practical purposes the differences of 2 ½ inches is negligible.

Rankine-Coulomb and Krey Methods.

In the example the effect of friction of the earth on the wall has not been taken into account as in the Krey method following, but the Rankine-Coulomb method for the earth loads was used. Further investigation by Dr. Blum showed that, except for unusual anchor locations, the passive earth pressures or earth resistances are not affected by wall friction, while for active earth pressures the wall friction may, if desired, be taken into account by the use of Dr. Krey's tables. (Krey's work was published in 1926 and Blum's in 1931.)

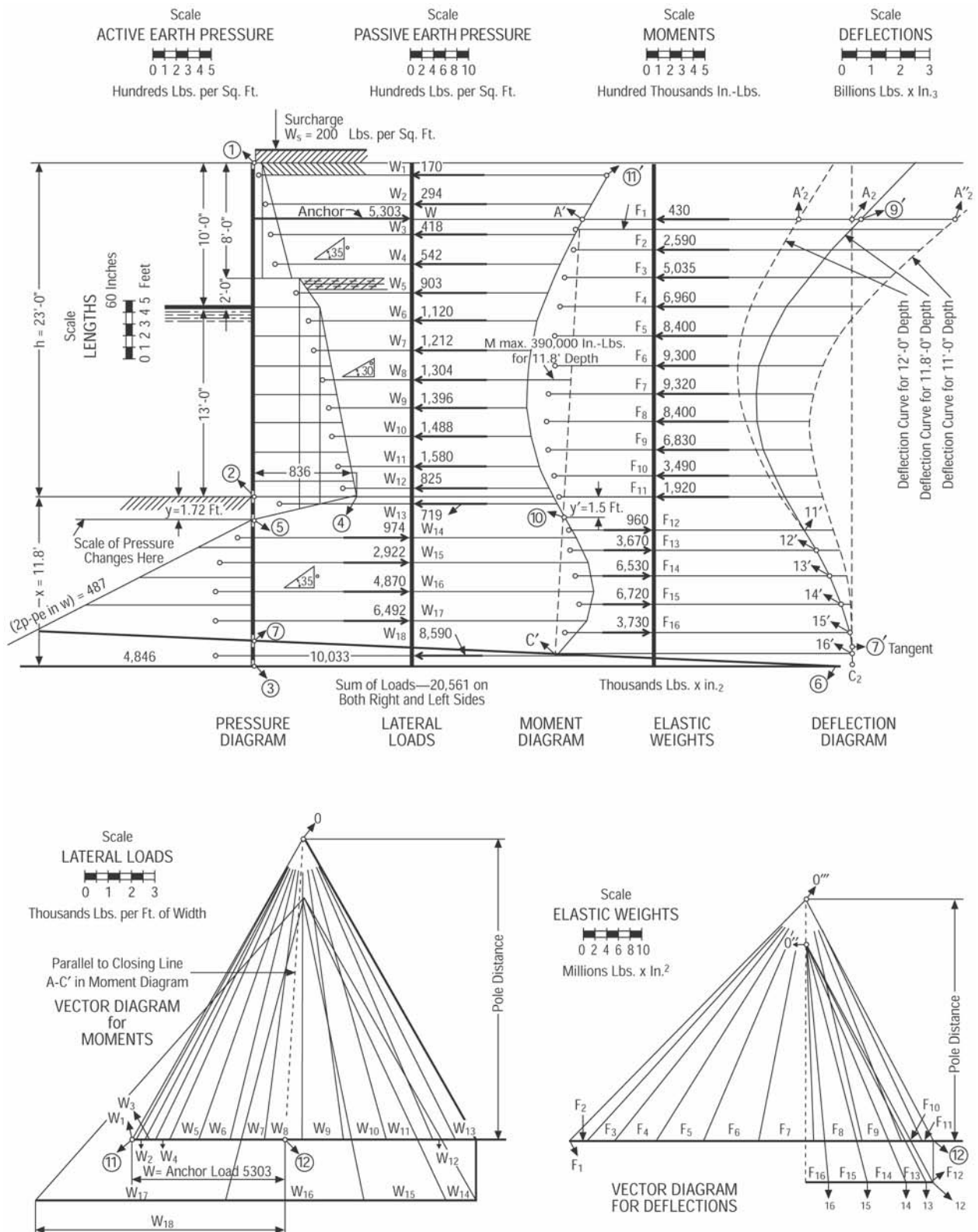


Fig. 99

In the following pages is given a condensed and rearranged translation of "Erddruck, Erdwiderstand und Tragfaehigkeit des Baugrundes Gesichtspunkte fuer die Berechnung" by Dr. Ing. H. Krey, published in Germany in 1926. In this book is set forth a system of design generally accepted as modern practice in Europe. Dr. Krey maintains that, since the basic assumptions of earth conditions are theoretical and may vary, to introduce subsequent calculations based on higher mathematics, or to carry out the figures into decimals, is useless because of the original inaccuracy.

The lateral loads on a steel sheet piling wall, as determined by the Krey principles, are somewhat less than those derived by the Rankine-Coulomb method as set forth on pages 2 to 6. In the former, the relieving effect of the friction of the earth upon the wall is taken into account and it is necessary, therefore, to determine this frictional value as well as the weight and angle of repose of the earth. Dr. Krey emphatically recommends that extremely careful tests be made to determine these characteristics of the earth to be dealt with.

The use of the Krey method will result in a saving of material and the following translation is offered to those of the Engineering profession designing steel sheet piling structures who are willing to perform the additional preliminary work necessary in order to avail themselves of this economy.

Lateral Pressures on Walls

Retaining walls are subjected to lateral pressures from the following load conditions:

1. Water.
 2. Earth.
 3. Earth submerged in water.
- or a combination at different elevations of:
4. Water and earth submerged in water.
 5. Earth and earth submerged in water.

Conditions 2 and 5 may be supporting a surcharge load which produces additional lateral pressure.

In the construction of such walls, steel sheet piling may act as a cantilever and, if the height of the wall is great, it may be braced or tied back to anchors, so as to bring it into the condition of a beam supported at both ends. In any event, the strength of the steel piling between the supports should be sufficient to sustain the lateral pressures, and its penetration into firm soil must be such as to prevent movement at the toe.

1. Water Pressures.

The direction of water pressure on an immersed plane is always normal to the plane. The fundamental laws of liquid pressures apply as follows:

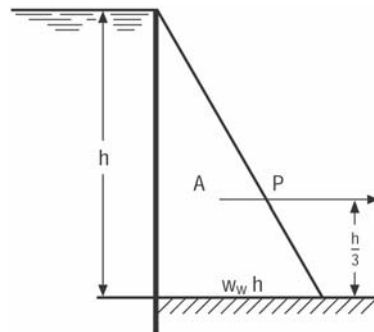


Fig. 1

$w_w h$ = pressure in pounds per square foot at depth h .

h = depth of water in feet.

w_w = weight of a cubic foot of water in pounds and also represents the hydrostatic pressure increment in pounds per square foot per foot of depth.

$w_w = 62.5$ for fresh water and 64.0 for salt water.

The total load on the wall, P , from the water level to the first support at h feet, is,

$$P = \frac{1}{2} W_w h^2, \text{ in pounds per foot of width.}$$

The total load is distributed or applied as illustrated in Triangle A, Fig. 1 adjoining.

The total load, P , can be considered as concentrated at a distance of $\frac{1}{3}h$ from the bottom. The product $P \times \frac{1}{3}h$ is useful in the calculation of the overturning moment in a cellular cofferdam.

The following table will prove useful in giving the values of $w_w h$ and P for various depths of fresh water.

Table 5. Hydrostatic Pressures
Unit Bottom and Total Pressures on Surface One Foot in Width

Depth, Feet	Pressure, Pounds		Depth, Feet	Pressure, Pounds	
	Bottom, $w_w h$	Total, P		Bottom, $w_w h$	Total, P
1	62.5	31	31	1937.5	30031
2	125	125	32	2000	32000
3	187.5	281	33	2062.5	34031
4	250	500	34	2125	36125
5	312.5	781	35	2187.5	38281
6	375	1125	36	2250	40500
7	437.5	1531	37	2312.5	42781
8	500	2000	38	2375	45125
9	562.5	2531	39	2437.5	47531
10	625	3125	40	2500	50000
11	687.5	3781	41	2562.5	52531
12	750	4500	42	2625	55125
13	812.5	5281	43	2687.5	57781
14	875	6125	44	2750	60500
15	937.5	7031	45	2812.5	63281
16	1000	8000	46	2875	66125
17	1062.5	9031	47	2937.5	69031
18	1125	10125	48	3000	72000
19	1187.5	11281	49	3062.5	75031
20	1250	12500	50	3125	78125
21	1312.5	13781	51	3187.5	81281
22	1375	15125	52	3250	84500
23	1437.5	16531	53	3312.5	87781
24	1500	18000	54	3375	91125
25	1562.5	19531	55	3437.5	94531
26	1625	21125	56	3500	98000
27	1687.5	22781	57	3562.5	101531
28	1750	24500	58	3625	105125
29	1812.5	26281	59	3687.5	108781
30	1875	28125	60	3750	112500

2. Earth Pressures.

Krey's method of calculating earth pressures differs chiefly in the following respects from the RankineCoulomb method, given on pages 2 to 6:

1. Friction of the earth on the wall is taken into account. The line of action of the earth pressure is, therefore, inclined to the horizontal, usually having a downward component, depending on the probable relative motion of the earth and wall at incipient failure.
2. The total earth pressure, considered as a concentrated load, acts, not at the height from the base, as in triangle A, Fig. 1, page 39, but slightly higher than this.

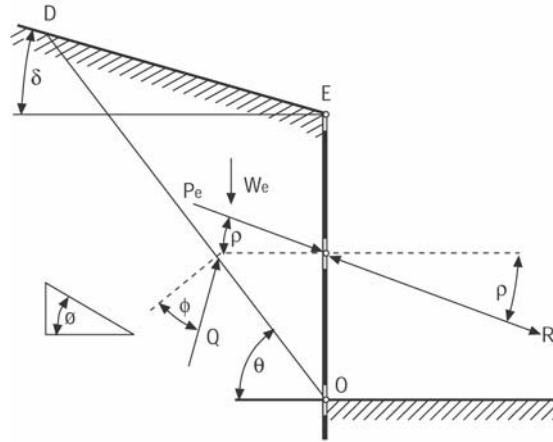


Fig. 100

It is considered that, if failure of the wall were to occur, the prism of earth ODE, as illustrated in Fig. 100 adjoining, would slide to the right and downward along plane DO called the "slide line." While doing so, the earth prism would exert on the wall the force P_e at an angle ρ to the normal to the wall surface, (or to the horizontal if the wall is vertical), equal to the friction angle of the earth on the wall. In order to resist failure, this force would have to be balanced by the equal and opposite resistance of the wall, namely the force R . For the earth wedge ODE to be in equilibrium, its weight W_e must be balanced by two other forces, the resistance R of the wall and the force Q exerted on it by the earth underneath.

For equilibrium, therefore, if the sliding surface is to be considered a plane,

$$P_e = -R = W_e \frac{\sin(-\phi)}{\cos(\theta - \phi - \rho)}.$$

Force Q acts at an angle ϕ to the normal to the surface on which sliding occurs, and ϕ is the friction angle of earth on earth, or the natural angle of repose. The angle θ of the "slide line" is unknown and is not the same as the angle of repose ϕ . The weight W_e is also unknown, depending on the angle θ . The initially unknown angle θ which the "slide line" makes with the horizontal is determined as that angle for which the force $P_e = -R$ is a maximum.

There are several methods of finding the maximum P_e for any combination of angles ϕ , ρ , and δ , all of which, however, are complicated. To avoid making these involved calculations for each case, the maximum values of earth pressure coefficients have been determined for many different combinations of angles, and are tabulated as dimensionless numbers in Tables 30, 31, and 32 following, denoted by the symbol K .

As explained more fully on page 43, the variation of the unit earth pressure with depth is not represented exactly by a straight line, but by a slightly curved line, as at the right of the dotted line in Fig. 101. This means, in strict accuracy, that the increment of the pressure is not constant as in the Rankine-Coulomb method (page 2, paragraph 2 (a)) but is somewhat greater near the top than near the bottom. For practical purposes, however, Krey assumes that the difference is negligible and that the total pressure is proportional to the square of the depth. On that basis, the increment can also be assumed as constant over the whole depth. Furthermore, while the earth pressure has a downward component, he assumes that it can, for practical purposes, be taken as acting horizontally (see page 44).

The earth pressure may then be transferred to an equivalent horizontal liquid pressure as follows: If P_e is the increment, in pounds per square foot, corresponding to W_w for water, then

$$p_e = K w_e$$

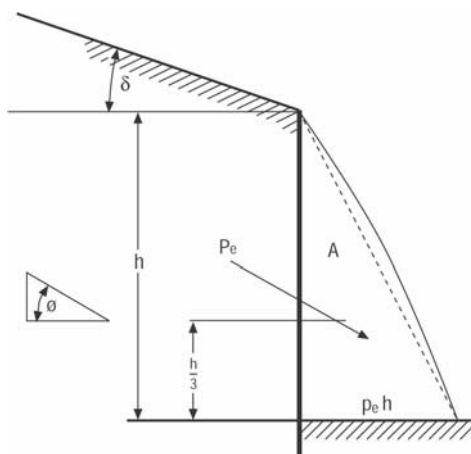


Fig. 101

The fundamental laws of liquid pressures then apply, and the pressure in pounds per square foot at depth h , in feet, is

$$p_e h.$$

The total load on the wall P_e , in pounds per foot of width, is

$$P_e = K w_e \frac{h^2}{2} = p_e \frac{h^2}{2}.$$

The total load is distributed or applied as shown by the modified triangle A, Fig. 101 adjoining.

Table 30. Earth Pressure Coefficients - K

Wall Vertical

$$\delta = +0$$

Earth Surface Horizontal

ρ Positive (Pressure has Downward Component)

Values of K for Active Earth Pressure

[illegible]

Wall Vertical
 $\delta = +10$
 Earth Surface Inclined at an Angle to Horizontal
 ρ Positive (Pressure has Downward Component)

[illegible]

Table 32. Earth Pressure Coefficients - K

Wall Vertical

$\delta = +30^\circ$

Earth Surface Inclined at an Angle to Horizontal

ρ Positive (Pressure has Downward Component)

Values of K for Active Earth Pressure

Angle, ρ +	Angle of Repose, ϕ							Angle, ρ +
	35°	37° 30'	40°	42° 30'	45°	50°	60°	
0°	0.434	0.372	0.318	0.274	0.236	0.171	0.086	0°
5°	0.426	0.365	0.312	0.268	0.231	0.168	0.085	5°
10°	0.421	0.360	0.307	0.264	0.228	0.166	0.084	10°
15°	0.420	0.358	0.306	0.262	0.226	0.165	0.084	15°
17° 30'	0.421	0.358	0.305	0.262	0.226	0.165	0.084	17° 30'
20°	0.423	0.360	0.306	0.263	0.227	0.165	0.084	20°
22° 30'	0.426	0.362	0.307	0.264	0.228	0.166	0.084	22° 30'
25°	0.430	0.364	0.309	0.265	0.230	0.167	0.084	25°
27° 30'	0.435	0.368	0.311	0.267	0.231	0.168	0.085	27° 30'
30°	0.440	0.372	0.315	0.270	0.233	0.170	0.086	30°
32° 30'	0.447	0.378	0.320	0.273	0.237	0.172	0.088	32° 30'
35°	0.455	0.384	0.324	0.278	0.240	0.175	0.090	35°
37° 30'	0.392	0.330	0.282	0.245	0.178	0.092	37° 30'
40°	0.336	0.289	0.250	0.181	0.094	40°
42° 30'	0.295	0.255	0.185	0.096	42° 30'
45°	0.261	0.190	0.100	45°
50°	0.202	0.106	50°
60°	0.125	60°

The values to be used for the unit weight of the earth W_e , the angle of repose ϕ , and the friction angle ρ of the earth on the wall should be determined in every case by an actual test of the particular kind of earth encountered. The angle of repose can be found by heaping up a pile of the material and measuring the greatest angle of the sides of the pile. If the earth will be entirely or partly submerged, the angle should be measured in water also, since this usually decreases the friction.

The friction angle ρ of the earth on the wall depends to some extent on the roughness of the latter as well as on the nature of the soil. It can never exceed the earth-on-earth friction angle ϕ , but may vary between $+\phi$ and zero. This angle can be determined by test, but may be estimated closely enough from the value of ϕ , when this has been determined. H. Blum states that the friction angle ρ of earth on steel is $\frac{1}{3}$ to $\frac{1}{2} \phi$.

More important than the actual value of this friction angle is the **direction** of the earth pressure, i. e., whether the vertical component acts upward or downward. This can be determined by carefully studying how the wall would move if it started to fail, and in which direction the earth particles would slide on it at various levels. For the majority of cases, the component acts downward, and the K factors given in Tables 30, 31 and 32 are to be used. An exception would be where the earth under the wall is more yielding than the filling behind the wall, in which case the earth pressure might have a steep slope upward.

Careful thought should be given to the possible effect of vibrations. In most cases these are absent, but in the vicinity of railroad tracks, cranes, or when blasting is done nearby, not only may the angle of repose ϕ of the earth itself and the friction angle ρ of the earth on the wall be decreased, but the tamping action of repeated vibrations may so change the pressure conditions that the pressure of the fill behind the wall increases until it is equal to the full weight of the column, or $K = \text{unity}$. For cases where it is known that vibrations are likely to occur, Krey recommends that ϕ be decreased somewhat from the values determined by test, and that ρ be assumed to increase uniformly from zero at the earth surface to its full static value at depth h , or, for simplicity, may be taken as zero over the upper half of the wall and at its full static value over the lower half of the wall.

The effect of submergence in water and of action of vibrations is illustrated by Table 33 following, giving the results of experiments by Krey. *The values are illustrative only and are not to be used in the calculations except as a rough guide. In every case the angle should be determined by tests.*

Table 33. Values of Friction Angle or Angle of Repose ϕ .

Material	Without Vibration		Vibrated
	Dry	Submerged in Water	Dry
	Angle ϕ	Angle ϕ	Angle ϕ
Sieved Sand (Grains 0.036 to 0.48 inch).....	32° 30'	32° 30'	30°
River Sand.....	32° 30'
Pure, Finely Ground Clay.....	45°	15°

Distribution of Earth Pressures.

According to Krey's theory, the distribution of the earth pressures along the height of the wall is not represented exactly by a triangle.

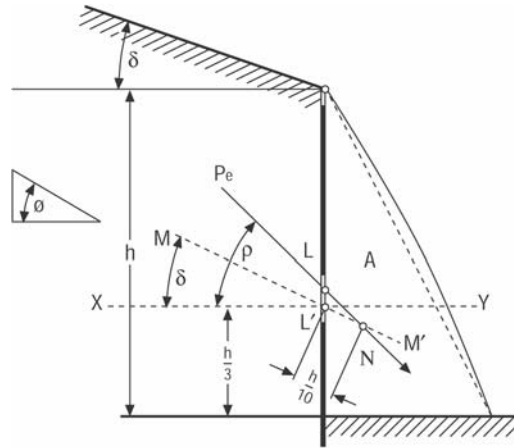


Fig. 102

Instead of the hypotenuse of the triangle being straight, as illustrated by the dotted line, Fig. 102, adjoining, it is the slightly convex, solid line. Consequently, the point of application of the total load P_e , considered as a concentrated force, is not at a distance of $\frac{1}{3}h$ above the bottom, but slightly higher up.

He gives the following empirical method for locating this point of application. See Fig. 102.

At a distance $\frac{1}{3}h$ from the base of the triangle A draw the horizontal line XY, intersecting the contact surface of the earth with sheet piling wall at L'. Through this point draw line MM' parallel to the upper earth surface, hence at angle δ to the horizontal. On MM' lay off distance L' equal to $\frac{1}{10}$ the height of the wall. Through point N draw the line of action of the total load or force, P_e , at an angle ρ to the horizontal.

The intersection with the contact surface of the earth and wall at L is the point of application of the total load, P_e .

When there is a surcharge load, the distribution of the combined lateral loads due to the surcharge and earth is illustrated by a trapezoid rather than a triangle, and the line XY is drawn through the center of gravity of the trapezoid (see Fig. 106, page 46). Similarly, when only a section of height h of the wall is being considered, the distribution of the lateral load is trapezoidal, and the line XY is drawn through the center of gravity of the trapezoid, while the distance L'N, Fig. 102, page 43 is made equal to $\frac{1}{10}h$.

Krey recommends that, once the direction and point of application of the total load P_e have been determined, this force be considered as acting horizontally, stating that, in view of the unavoidable uncertainty of the various factors, such a refinement as resolving the force to obtain its horizontal component is hardly justifiable.

(a) **Level Bank.** The earth pressure is transferred to an equivalent horizontal liquid pressure, p_e , in pounds per square foot, as follows, and p_e is the increment in pounds per square foot corresponding to p_w , for water. Then

$$p_e = K W_e.$$

where w_e , = weight per cubic foot of earth in pounds.

and K = coefficient from Table 30 for the angle of repose ϕ of the earth and the friction angle ρ of the earth on the sheet piling wall, as determined by tests of the soil.

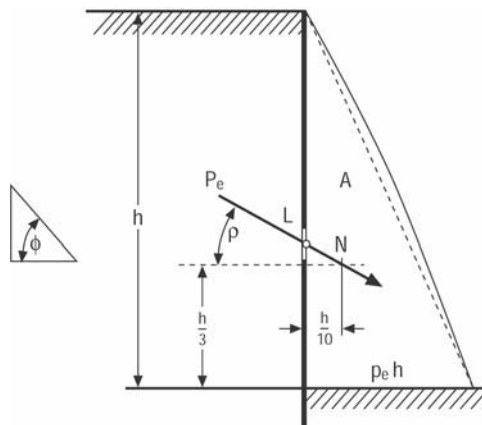


Fig. 103

The fundamental laws of liquid pressures then apply, which gives, as illustrated in Fig. 103 adjoining, and

$$p_e h \quad \text{or} \quad K w_e h$$

is the pressure in pounds per square foot at any depth h , in feet.

The total load on the wall, P_e , in pounds per foot of width, is

$$P_e = K w_e \frac{h^2}{2} \quad \text{or} \quad P_e = \frac{1}{2} P_e h^2.$$

The total load is applied or distributed approximately as illustrated in the modified triangle A in Fig. 103 above, and the point of application of the equivalent concentrated load is at point L.

Example 1. No Vibration.

$$h = 30 \text{ feet.}$$

$$\phi = 32^\circ 30'.$$

$$\rho = \text{Approximately } \frac{2}{3} \times 32^\circ 30' = 22^\circ.$$

$$w_e = 90 \text{ pounds per cubic foot.}$$

$$K = 0.271, \text{ from Table 30.}$$

$$P_e = 0.271 \times 90 \times \frac{30^2}{2} = 10,976 \text{ pounds per foot of width.}$$

(This is about 10 per cent lower than if figured by the Rankine-Coulomb method.)

Distance of point of action L above the bottom is

$$\frac{1}{3} h + \frac{1}{10} h \tan \rho = \frac{30}{3} + \frac{30}{10} \times 0.4040 = 11.21 \text{ feet.}$$

Example 2. With Vibration.

Let the conditions be the same as in Example 1 but subjected to continuous vibration. See Fig. 104. adjoining.

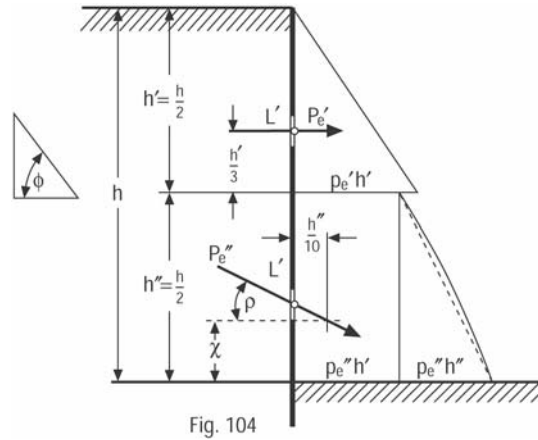


Fig. 104

Let $\phi = 25^\circ$.

$\rho = 0$ on upper half of wall.

$\rho' = \frac{2}{3} \times 25^\circ = 16^\circ$ on lower half of wall.

$h' = 15$ feet.

$h'' = 15$ feet.

For upper half of wall,

$K = 0.406$, from Table 30.

$$P_e' = 0.406 \times 90 \times \frac{15^2}{2} = 4,110 \text{ pounds per foot of width.}$$

This is the total pressure in pounds per foot of width acting at a distance from the bottom of $15 + \frac{15}{3} = 20$ feet.

For lower half of wall,

From Table 30, by interpolation, $K = 0.363$.

Pressure at top of bottom section, $p_e'' h' = 0.363 \times 90 \times 15 = 490$ pounds per square foot.

Pressure at bottom of bottom section,

$$p_e'' h' + p_e'' h'' = 490 + 0.363 \times 90 \times 15 = 490 + 490 = 980 \text{ pounds per square foot.}$$

$$\text{Total pressure on lower half of wall, } P_e'' = 490 \times 15 + 490 \times \frac{15}{2} = 11,025 \text{ pounds per foot of width.}$$

Total pressure on entire wall = $11,025 + 4,110 = 15,135$ pounds per foot of width.

This is almost 50 per cent greater than for Example 1.

Distance x , center of gravity of trapezoid from base, Fig. 104, above,

$$x = \frac{h''}{2} \times \left(\frac{p_e'' h' + \frac{1}{3} p_e'' h''}{p_e'' h' + \frac{1}{2} p_e'' h''} \right) = \frac{15}{2} \times \left(\frac{490 + \frac{1}{3} \times 490}{490 + \frac{1}{2} \times 490} \right) = 6.67 \text{ feet.}$$

Distance of point of application of the total load P_e'' from the bottom,

$$x + \frac{h''}{10} \tan \rho = 6.67 + \frac{15}{10} \tan 16^\circ = 7.10 \text{ feet.}$$

(b) **Inclined Bank.** The same method applies in this case but the values of K are taken from Tables 31 and 32, for inclination angles, δ , of $+10^\circ$ and $+30^\circ$ respectively. For other angles the values of K are obtained by interpolation between the values in Tables 30 and 31, or 31 and 32.

The problem is illustrated in Fig. 105, adjoining.

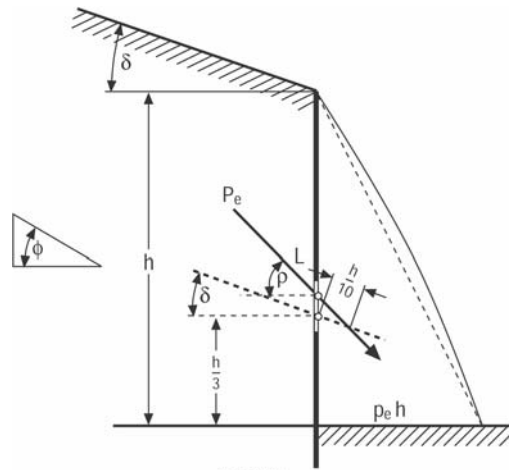


Fig. 105

Example 3.

$h = 25$ feet.

$\phi = 32^\circ 30'$.

$\delta = 20^\circ$.

$\rho = 32^\circ 30'$.

$w_e = 95$ pounds per cubic foot.

From Table 31, for $\delta = +10^\circ$, $K = 0.32$ for these angles and from Table 32, for $\delta = +30^\circ$, $K =$ (about) 0.50.

Interpolating between these values for $\delta = +20^\circ$, $K = 0.41$ approximately.

$$P_e K w_e \frac{h^2}{2} = 0.41 \times 95 \times \frac{25^2}{2} = 12,180 \text{ pounds per foot of width.}$$

Distance of point of application of total load, P_e , above bottom,

$$\frac{h}{3} - \frac{h}{10} \sin \delta + \frac{h}{10} \cos \delta \tan \rho =$$

$$25 \times \left(\frac{1}{3} - \frac{1}{10} \sin 20^\circ + \frac{1}{10} \cos 20^\circ \tan 32^\circ 30' \right) = 8.97 \text{ feet.}$$

(C) Surcharge Load. For the case of a surcharge load, Krey's method does not differ in principle from the method previously explained on page 2. In detail it differs to the extent that the surcharge load is considered equivalent to an additional load for height equal to

$$\frac{w_s}{w_e} = h', \text{ in feet,}$$

in which w_s is the surcharge load in pounds per square foot, and w_e is the weight of the earth in pounds per cubic foot.

The lateral pressure p_s in pounds per square foot, due to the surcharge alone, is

$$p_s = h' K w_e, \quad \text{or} \quad p_s = K w_s,$$

where $K =$ coefficient from Table 30.

The lateral pressure p_s is not subject to the laws of liquid pressures but is a uniform load throughout the height h , in feet, of the piling wall; and the total load, in pounds per foot of width, due to the surcharge is

$$P_s = p_s h.$$

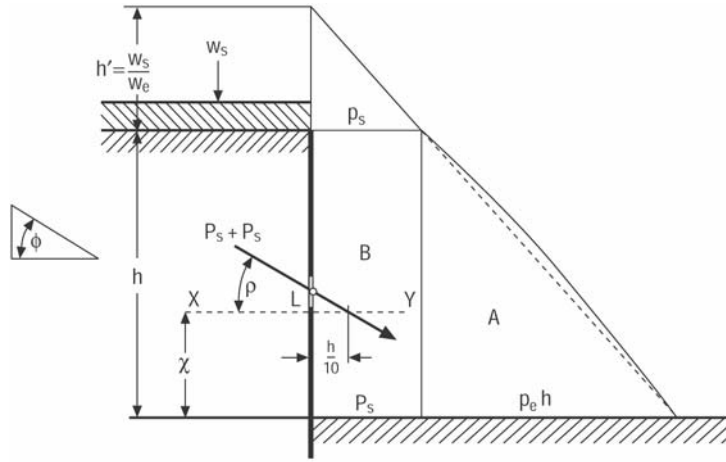


Fig. 106

Figure 106, adjoining, shows a piling wall which retains earth pressure and is loaded with a surcharge of railroad cars or piled materials which increases the lateral pressure. The total load, due to the surcharge alone, is applied or distributed as illustrated in rectangle B.

The lateral loads, due to the earth loads alone are calculated as described in (a) preceding, and since this pressure is liquid, the total load for these earth loads is illustrated in modified triangle A.

The pressure, in pounds per square foot, at any point h feet from the top of the wall, due to surcharge and earth is, therefore,

$$p_s + p_e h = K (w_s + w_e h),$$

where K = coefficient from Table 30.

The total lateral load on the wall, in pounds per foot of width, is

$$P_s + P_e = K w_s h + K w_e \frac{h^2}{2}.$$

The line of action of this combined load or force passes through point L at a distance in feet from the bottom equal to

$$\frac{h}{2} \times \left(\frac{p_s + \frac{1}{3} p_e h}{p_s + \frac{1}{2} p_e h} \right) + \frac{h}{10} \tan \rho.$$

Example 4.

$w_s = 750$ pounds per square foot.

$w_e = 90$ pounds per cubic foot.

$\phi = 30^\circ$.

$\rho = 20^\circ$.

$h = 30$ feet.

$K = .300$, from Table 30.

$$\frac{w_s}{w_e} = \frac{750}{90} = 8 \frac{1}{3} \text{ feet, height of an equivalent column of earth.}$$

$$p_s = 8 \frac{1}{3} \times .300 \times 90 \text{ or } .300 \times 750 = 225 \text{ pounds per square foot.}$$

Total load, in pounds per foot of width, is

$$\text{Due to earth } .300 \times 90 \times \frac{30^2}{2} = 12,150 \text{ pounds.}$$

$$\text{Due to surcharge } 225 \times 30 = 6,750 \text{ pounds.}$$

$$\text{Total Load } 18,900 \text{ pounds.}$$

$$\text{Height of L from bottom} = \frac{30}{2} \times \left(\frac{750 + \frac{1}{3} \times 30 \times 90}{750 + \frac{1}{2} \times 30 \times 90} \right) + \frac{30}{10} \tan 20^\circ = 12.87 \text{ feet.}$$

3. Earth Submerged in Water.

Krey's method of allowing for the effect of submergence of earth in water is exactly the same in principle as already explained in assumption 2 on page 4, that is, the full hydrostatic pressure is assumed to act horizontally and the weight of earth is reduced by the amount of the buoyancy produced by the water. Thus if v is the percentage of voids in the material, w_e the weight of the dry earth, and w_w , the weight of water, then the submerged weight of the earth in pounds per cubic foot is

$$w_{e \text{ in } w} = w_e - \frac{(100 - v)}{100} w_w.$$

(a) Submerged Earth on One Side Only.

Let $w_{e \text{ in } w}$ = weight, in pounds, of a cubic foot of earth submerged in water.

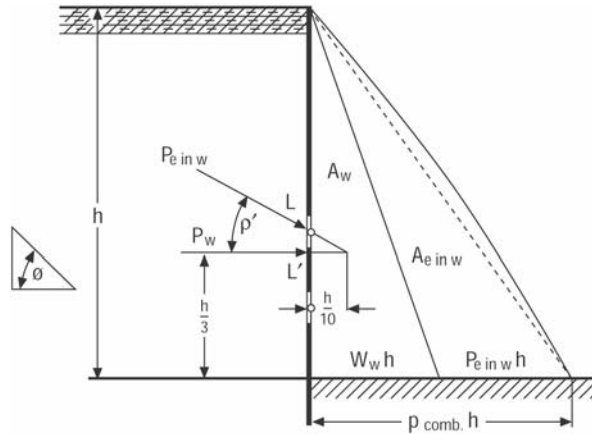


Fig. 107

The problem is illustrated in Fig. 107, adjoining.

The pressure due to water only (see paragraph 1, page B-18), in pounds per square foot at depth h , in feet, is

$$\text{pressure} = w_w h$$

where $w_w = 62.5$ pounds for fresh water and 64.0 pounds for salt water.

The pressure due to submerged earth only, at depth h feet, is

$$p_{e \text{ in } w} = K' w_{e \text{ in } w} h.$$

where K' is coefficient from Table 30 for the angles θ' and ρ' appropriate for earth submerged in water.

The total pressure, p_{comb} , due to water and submerged earth, in pounds per square foot at depth h feet are not added directly, since they do not act in the same direction. Approximately, however,

$$P_{comb.} = w_w h + K' w_{e \text{ in } w} h$$

The total load due to water only, in pounds per foot of width, is

$$P_w = \frac{1}{2} w_w h^2$$

This acts horizontally at a point $\frac{1}{3}h$ above the lower earth level and is applied or disturbed as illustrated in triangle A_w in Fig. 107, page 47.

The total load due to submerged earth only, in pounds per foot of width, is

$$P_{e \text{ in } w} = K' w_{e \text{ in } w} \frac{h^2}{2}$$

This load, applied or distributed as illustrated in modified triangle $A_{e \text{ in } w}$, acts at angle ρ' to the horizontal through point L at a distance of

$$\frac{h}{3} + \frac{h}{10} \tan \rho' \text{ feet above the bottom}$$

and the total combined load due to earth and water is

$$P_{comb.} = P_w + P_{e \text{ in } w}$$

Example 5.

w_w	= 62.5 pounds per cubic foot.
w_e (dry)	= 90.0 pounds per cubic foot.
v (voids in earth)	= 40 percent.
ϕ'	= 25° .
ρ'	= $17^\circ 30'$.
h	= 25 feet.
K	= 0.361, from Table 30.

$$w_{e \text{ in } w} = 90 - \frac{100 - 40}{100} \times 62.5 = 52.5 \text{ pounds per cubic foot.}$$

The load due to water is

$$P_w = 62.5 \times \frac{25^2}{2} = 19,530 \text{ pounds per foot of width}$$

acting at a distance of $\frac{25}{3} = 8.33$ feet from the bottom.

The load due to submerged earth only, is

$$P_{e \text{ in } w} = 0.361 \times 52.5 \times \frac{25^2}{2} = 5,923 \text{ pounds per foot of width.}$$

This load acts at an angle of $17^\circ 30'$ to the horizontal at a distance of

$$8.33 + \frac{25}{10} \tan 17^\circ 30' = 9.12 \text{ feet from the bottom.}$$

Approximately, the two separate loads can be considered as replaced by a single load

$$P_{comb.} = P_w + P_{e \text{ in } w} = 19,350 + 5,923 = 25,453 \text{ pounds per foot of width.}$$

(b) Submerged Earth on One Side, Water on Other Side.

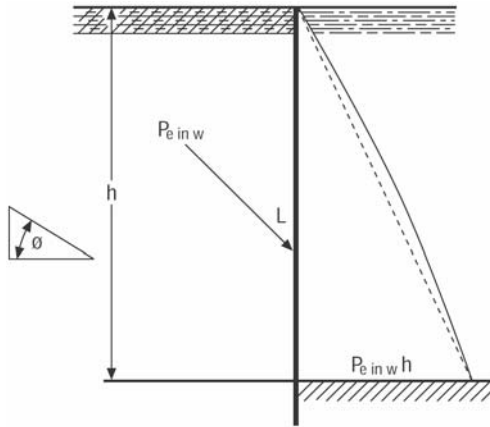


Fig. 108

Fig. 108, adjoining, illustrates the problem.

The water pressures on the two sides balance; hence, the total pressure is that of the submerged earth only.

The pressure per square foot at any depth h feet is, therefore,

$$p_{e \text{ in } w} = K' w_{e \text{ in } w} h.$$

The total lateral pressure, in pounds per foot of width, is

$$P_{e\ in\ w} = K' w_{e\ in\ w} \frac{h^2}{2}.$$

K' is the coefficient from Table 30 for the appropriate values of ϕ' and ρ' .

The location of the point of application L is outlined on page 43.

4. Combinations of Water and Earth Submerged in Water.

The problem of calculating the loads for the various combinations resolves itself into reducing all of the individual conditions, which cause the lateral load on the wall, to the individual lateral loads. For all except a surcharge load, this means to their equivalent liquid pressures. The individual loads are combined, after each one is studied as before described, and the total load laterally on the wall, for the most usual conditions, are best illustrated by the following sketches with the load diagrams.

The symbols shown in the sketches have been developed in the three preceding paragraphs but for convenient reference they are summarized as follows:

All heights (h and h') are in feet; all pressures (P , etc.) are in pounds per square foot; all angles in degrees.

Equivalent Liquid Lateral Pressures

w_w = Water.

p_e = Earth only.

$p_{e\ in\ w}$ = Earth submerged in water.

$p_{comb.}$ = Combined water and earth submerged in water.

Uniformly Distributed Lateral Pressure

p_s = Due to surcharge load w_s , in pounds per square foot.

Angles

ϕ = Angle of repose of earth.

ϕ' = Angle of repose of earth submerged in water.

ρ = Friction angle of earth on steel sheet piling wall.

ρ' = Friction angle of earth submerged in water on steel sheet piling wall.

Lateral Pressure Coefficients

K = Coefficient as read from Table 30 for angles ϕ and ρ .

K' = Coefficient for earth submerged in water as read from Table 30 for angles ϕ' and ρ' .

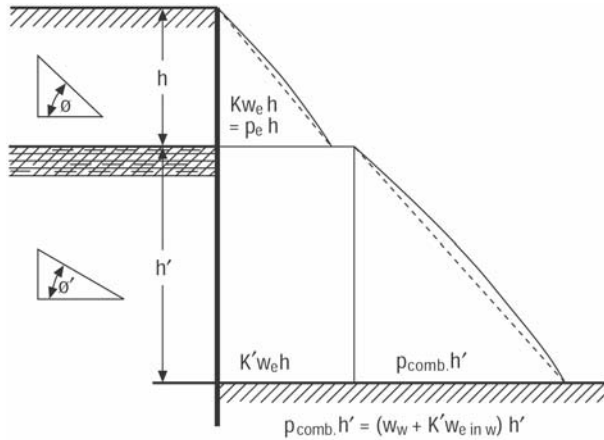


Fig. 109

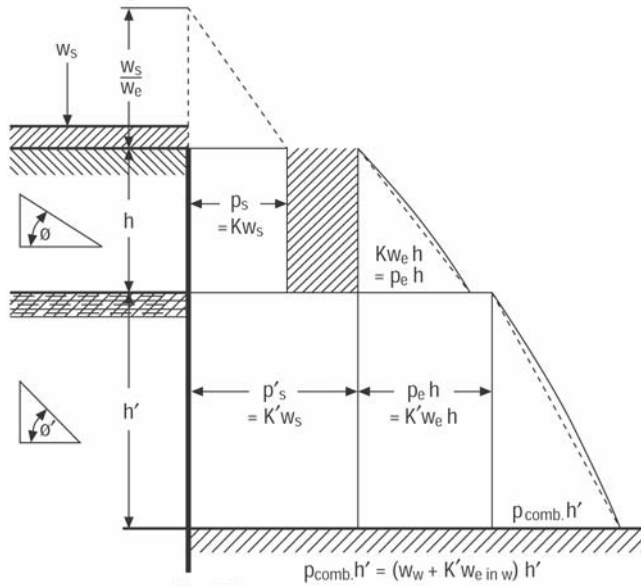


Fig. 110

Dry Earth over Submerged Earth on One Side.
Side.

Dry Earth over Submerged Earth, Water on Opposite
Side.

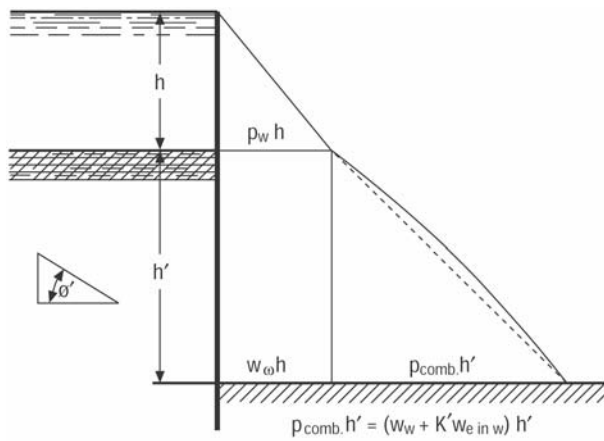


Fig. 111

Surcharged on Dry Earth over Submerged Earth on One Side

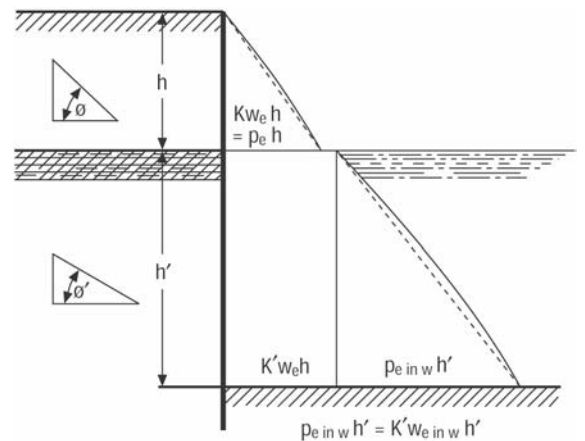


Fig. 112

Surcharge on Dry Earth over Submerged Earth, Water on Opposite Side.

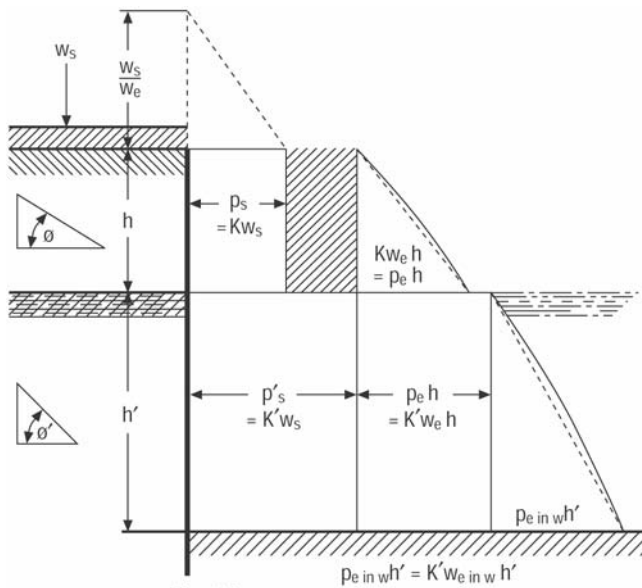


Fig. 113

Water on Top of Submerged Earth on One Side.

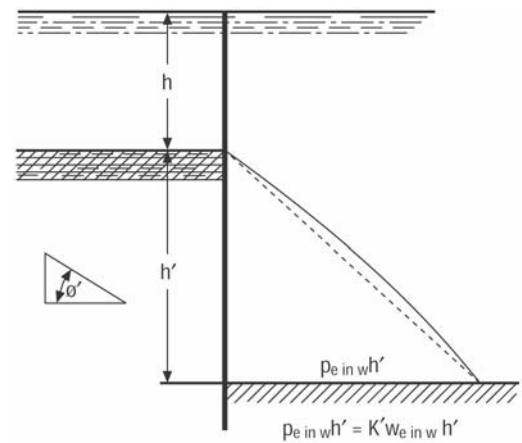


Fig. 114

Water on Top of Submerged Earth, Water on Opposite Side.

After the lateral loads on the sheet piling wall have been determined by the above methods the subsequent procedure leading to the final design of the sheet piling structure is outlined on pages 7 to 14 and 25 to 37.